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Corner solutions in the allocation of environmental water: an application of inframarginal economics

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Inframarginal economics is a combination of marginal and total cost-benefit analysis (across corner solutions). It has been applied extensively in analysing trade issues, however, there have been few environmental applications. While there is debate over the contribution of inframarginal economics to the analysis of aggregate economic phenomena, inframarginal economics is central to understanding agent-level decisions.

This paper applies inframarginal methods to investigate the efficient allocation of water among ecosystems. The Australian Government is acquiring billions of dollars of water for environmental uses through a number of programs. Allocating this water efficiently will require information on preferences and environmental production functions, as well as the development of analytical frameworks capable of examining corner solutions.

Within a general inframarginal framework, this paper investigates the conditions under which corner solutions are likely to be efficient. In particular, corner solutions may arise when environmental production functions are convex but are also possible under 'well behaved' functions.

Introduction

The ecological health of many of Australia's river systems has declined over time. The regulation of rivers and diversions for agriculture and other uses have changed flow regimes, affecting floodplain and instream habitats. According to Arthington and Pusey (2003), around 90 per cent of floodplain wetlands in the Murray-Darling Basin no longer exist. In New South Wales, around 50 per cent of coastal wetlands have been lost, while around 75 per cent of wetlands on the Swan Coastal Plain of Western Australia have also been lost. These changes in habitat have contributed to a decline in native fish and bird populations in some areas (Davies et al. 2008; Gehrke et al. 2003; Harris and Gehrke 1997; Kingsford 2000; Kingsford and Thomas 1995). In the Murray-Darling Basin, approximately half of native fish species are

considered threatened, while common carp was the dominant species in Sustainable River Audit surveys, accounting for 58 per cent of total fish biomass (Davies et al. 2008).

Additional water and other resources will probably have to be allocated to environmental uses if the declining health of many of Australia's river systems is to be slowed or reversed. This could happen a number of ways. In Australia, most environmental water is allocated by governments. In the Murray-Darling Basin, a central government agency – the Murray-Darling Basin Authority – is responsible for determining how much water to allocate to thousands of ecosystems across the Basin. Some or all of this water will be sourced through a $3.1 billion plan to buy water rights from irrigators and $5.8 billion of investment in water infrastructure projects. Economists have also examined the potential for decentralised public systems where public water could be sourced and managed by local water trusts or public land and river managers (PC 2010, Young 2010). Finally, there could be some role for the private sector (individuals, charities and businesses) in restoring degraded river systems, as is common in the western United States.

There are two primary motivations for developing an analytical framework to investigate efficient environmental water allocation decisions. First, understanding the nature of the allocation problem makes it easier to assess which system, or mix of systems, is most likely to resolve the allocation problem. Second, irrespective of the system adopted, understanding the allocation problem should lead to more efficient environmental allocation decisions and better outcomes for the community.

What does the literature say about the allocation of environmental water? (The allocation of other resources will not be considered.) According to the Productivity Commission's Water Buyback study, 'the efficient allocation of water resources occurs when the marginal net benefits of water are equated across all uses, including consumptive and environmental uses' (PC 2010, p.61). This is similar to the definitions set out in introductory environmental economics textbooks, and is most easily demonstrated with a simple diagram. Figure 1 examines the allocation of a given bundle of water between two ecosystems. The volume of water available is given by the distance between the left vertical axis and the right vertical axis. The marginal net benefit of watering ecosystem A is given by MBa, which should be interpreted left-to-right (from the left vertical axis). The marginal net benefit of watering ecosystem B is given by MBb, which should be interpreted right-to-left (from the right vertical axis).

In the case drawn in PC (2010), both marginal net benefit curves are downward sloping and intersect between the vertical axes and above the horizontal axis. Under these conditions, and in the absence of other complications, the marginal approach gives the efficient allocation of water, and some water is allocated to both ecosystems. But what happens, for example, if the marginal net benefit curves are upward sloping or they do not intersect between the vertical axes? In this case, the marginal solution either does not exist or represents the least beneficial allocation of environmental water, and efficiency requires that some ecosystems receive no water.

Figure 1: a valid marginal solution in the allocation of environmental water
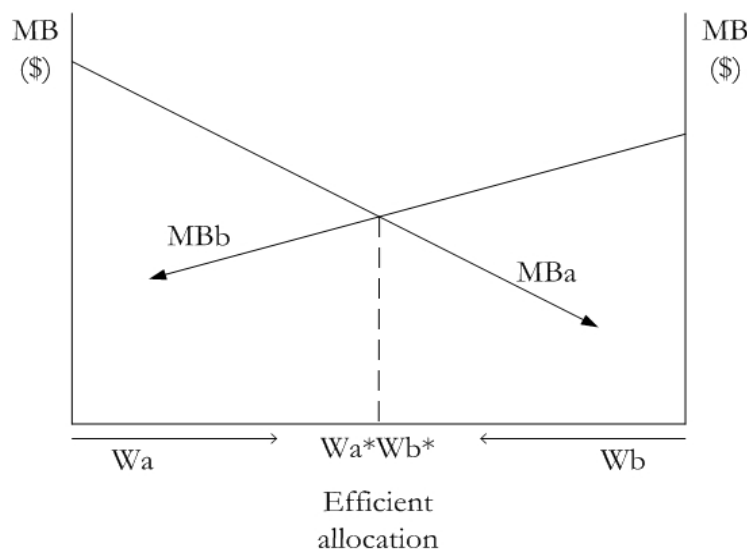

The inframarginal approach proposed below provides a solution to these problems but does introduce additional complexities, and hence, it is reasonable to consider whether the marginal approach is likely to represent a good approximation of the underlying problem. There are a number of ways of considering this. One approach is to examine the environmental production functions (which show the relationship between inputs, such as water, and environmental outputs). The environmental benefit functions estimated by Horne et al. (2009) (for river flow on the Goulburn River in Victoria) have upward sloping segments and other complexities that could render marginal analysis insufficient (figure 2). Indeed, these empirically-derived curves look very different to the 'textbook style' curves in figure 1.

An alternative way to assess the applicability of the marginal approach is to consider whether the implication — that it would be efficient to water all 30 000 environmental assets in the Murray-Darling Basin — is reasonable. It is unlikely that any economist would find such a conclusion credible. Indeed, some economists have recognised the necessity to prioritise

environmental water allocations by not applying water to some wetlands and riparian forests (sometimes known as 'triage' decisions). As such, it is likely to be important to use an analytical framework that allows for some ecosystems to go without water.

Figure 2: an example of an empirically-derived marginal benefit curve
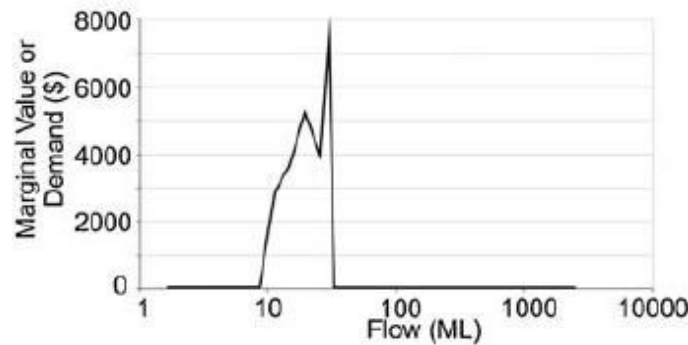
Source: Horne et al. 2009.

Goddard et al. (2009) introduce such a framework in a dynamic programming model with stochastic rainfall. Like the model discussed above, the authors examine the allocation of water between two ecosystems. In addition, water can be held in storage, introducing a marginal user cost. The ecosystems have fixed water requirements, and need to be watered at least every five years or suffer irreversible damage. Goddard et al. (2009) calculate the optimal management rule based on hypothetical data and find that it is sometimes efficient to water only one ecosystem, retaining more water in storage, even when there is sufficient water to keep both ecosystems healthy in the short run. This strategy increases the probability of the watered ecosystem surviving, but reduces the probability of the other ecosystem surviving. The simplified environmental production functions used in Goddard et al. (2009) do not allow for a marginal solution (although it can generate interior solutions where both ecosystems receive water) or corner solutions caused by convexities in the production functions. Thus, while Goddard et al. (2009) provides a valuable example illustrating one possible cause of corner solutions, the task of developing a more general framework remains.

This paper attempts this task, illustrating the framework using quadratic production functions. In doing so, it draws heavily on inframarginal economics, especially Cheng et al. (2000). Inframarginal economics applies non-classical mathematical programming techniques to solve problems that might have corner solutions. It has been applied most extensively to examine specialisation in the context of international trade. Cheng et al. (2000) develops a simple Ricardian trade model which examines countries' decisions regarding whether to produce two goods in an environment characterised by 'increasing returns' (notionally caused by learning costs) and exogenous comparative advantage. There are a number of possible

'market structures' from autarky at one extreme to complete specialisation at the other. In some respects this is similar to the problem at hand — with autarky corresponding to the case where both ecosystems are watered and specialisation corresponding to the case where only one ecosystem is watered. As a result, this paper adopts a broadly similar analytical approach to Cheng et al. (2000).

While the non-linear programming approach used below is entirely conventional, some of the applications of inframarginal economics have been criticised by Dixon (2006). Dixon's main argument was that most problems can be adequately addressed with marginal analysis when analysed with sufficient aggregation. While this may, or may not, be the case for aggregate trade models (see Tombazos (2006) for counter arguments), the model developed in this paper examines decision making at a highly disaggregated level where corner solutions are certainly plausible. In this sense, the water allocation problem discussed below is more like Rosen (1978), which examines the division of labour within a firm.

Model and results

This section introduces a deterministic, single period environmental water allocation model. The model seeks to maximise returns, Π, by allocating a given bundle of water, $\overline{W}$, between two ecosystems, A and B.

Maximise:
$$\Pi = P_A f(W_A, W_B) + P_B g(W_B, W_A) \tag{1}$$

Subject to:
$$W_A + W_B \leq \overline{W} \tag{2}$$
$$W_A, W_B \geq 0 \tag{3}$$

where P_A and P_B are the values of a unit of output from ecosystems A and B (assumed to be constant), while $f(W_A, W_B)$ and $g(W_B, W_A)$ are twice differentiable environmental production functions that relate the volumes of water applied to ecosystems A and B, W_A and W_B, to environmental output. To illustrate technical interdependence between the ecosystems, the volume of water applied to one ecosystem is allowed to influence the output of the other ecosystem. This kind of interdependence could be a result of return flows or similar phenomena.

Configurations

In this model there are four possible configurations: (i) water both ecosystems, (ii) only water ecosystem A, (iii) only water ecosystem B, and (iv) water neither ecosystems. The water constraint is assumed to be binding. This rules out configuration (iv) and simplifies the analytics for the other configurations.

First order conditions

This problem gives rise to the following Khun-Tucker conditions:

$$P_A f_{WA} + P_B g_{WA} - \lambda \leq 0 \,;\; W_A \geq 0 \,;\; W_A(P_A f_{WA} + P_B g_{WA} - \lambda) = 0 \,;$$

$$P_B g_{WB} + P_A f_{WB} - \lambda \leq 0 \,;\; W_B \geq 0 \,;\; W_B(P_B g_{WB} + P_A f_{WB} - \lambda) = 0 \,;$$

$$\overline{W} - W_A - W_B = 0 \tag{4}$$

where f_{WA}, g_{WA}, f_{WB} and g_{WB} are the partial derivatives of $f(W_A, W_B)$ and $g(W_B, W_A)$ with respect to W_A and W_B, and λ is the Lagrangian multiplier.

To examine the parameter subspace and solution values associated with different configurations a specific function form for the environmental production functions is assumed. For illustrative purposes, a quadratic relationship is assumed between the water inputs and environmental outputs:

$$f(W_A, W_B) = a_0 + a_1 W_A + .5a_2 W_A^2 + a_3 W_A W_B + a_4 W_B + .5a_5 W_B^2 \,;$$

$$g(W_B, W_A) = b_0 + b_1 W_B + .5b_2 W_B^2 + b_3 W_B W_A + b_4 W_A + .5b_5 W_A^2 \tag{5}$$

where $a_0, \ldots, a_5$ and $b_0, \ldots, b_5$ are parameters.

The first step is to evaluate:

- the conditions under which there is an interior stationary point,

- the values of W_A and W_B associated with that point (if it exists), and

- the conditions under which the corners are local maxima or minima.

Both W_A and W_B would exceed zero under any interior stationary point. Thus, the complementary slackness conditions in (4) imply that the marginal conditions would hold with strict equality and the problem reduces to the standard marginal problem:

$$P_A f_{WA} + P_B g_{WA} - \lambda = 0$$
$$P_B g_{WB} + P_A f_{WB} - \lambda = 0$$
$$\overline{W} - W_A - W_B = 0 \tag{6}$$

Substituting in the partial derivatives of (5) and solving for the values of W_A and W_B associated with the interior stationary point (if it exists):

$$X_A = \frac{P_A\left[a_1 - a_4 + \overline{W}(a_3 - a_5)\right] - P_B\left[b_1 - b_4 + \overline{W}(b_2 - b_3)\right]}{D}$$

$$X_B = \frac{P_B\left[b_1 - b_4 + \overline{W}(b_3 - b_5)\right] - P_A\left[a_1 - a_4 + \overline{W}(a_2 - a_3)\right]}{D} \tag{7}$$

where

$$D = P_A\left[2a_3 - a_2 - a_5\right] + P_B\left[2b_3 - b_2 - b_5\right] \tag{8}$$

An interior stationary point exists when $X_A > 0$ and $X_B > 0$. D can be either positive or negative. If $D > 0$, the following conditions apply for an interior stationary point:

$$P_A\left[a_1 - a_4 + \overline{W}(a_3 - a_5)\right] > P_B\left[b_1 - b_4 + \overline{W}(b_2 - b_3)\right]$$
$$P_B\left[b_1 - b_4 + \overline{W}(b_3 - b_5)\right] > P_A\left[a_1 - a_4 + \overline{W}(a_2 - a_3)\right] \tag{9}$$

On the other hand, the inequalities are reversed if $D < 0$ and the following conditions apply:

$$P_A\left[a_1 - a_4 + \overline{W}(a_3 - a_5)\right] < P_B\left[b_1 - b_4 + \overline{W}(b_2 - b_3)\right]$$
$$P_B\left[b_1 - b_4 + \overline{W}(b_3 - b_5)\right] < P_A\left[a_1 - a_4 + \overline{W}(a_2 - a_3)\right] \tag{10}$$

If (9) or (10) are satisfied, the stationary point will occur at $W_A = X_A$ and $W_B = X_B$.

To identify the conditions under which the corners are local maxima or minima, the problem must be solved four times, as both a maximisation and minimisation problem at each corner. We will briefly examine the results for corner A (the conditions for corner B are symmetrical). The maximisation problem for the corner where only ecosystem A is watered is similar to (6) except that the second equation is characterised by a weak inequality.

$$P_A f_{WA} + P_B g_{WA} - \lambda = 0$$

$$P_B g_{WB} + P_A f_{WB} - \lambda \leq 0$$

$$\overline{W} - W_A - W_B = 0 \tag{11}$$

Watering only ecosystem A will be a local maximum when:

$$P_B \left[b_1 - b_4 + \overline{W}(b_3 - b_5) \right] \leq P_A \left[a_1 - a_4 + \overline{W}(a_2 - a_3) \right] \tag{12}$$

To solve the corresponding minimisation problem some of the inequalities in (4) must be reversed. Watering only ecosystem A will be a local minimum when:

$$P_B \left[b_1 - b_4 + \overline{W}(b_3 - b_5) \right] \geq P_A \left[a_1 - a_4 + \overline{W}(a_2 - a_3) \right] \tag{13}$$

Second order conditions

The second step is to evaluate:

- whether there is the potential for more than one stationary point, and
- the conditions under which the allocation problem is concave or convex.

One way of doing this is to examine the second order total differential of (1).

$$d^2 \Pi = h_{WAWA} dW_A{}^2 + 2 h_{WAWB} dW_A dW_B + h_{WBWB} dW_B{}^2 \tag{14}$$

where h_{WAWA}, h_{WAWB}, h_{WBWB} are the second partial derivatives with respect to the variables W_A and W_B. If, as above, the water constraint is assumed to be binding, we can take the total differential of (2) and impose the following restriction on the relationship between dW_A and dW_B:

$$dW_A = -dW_B \tag{15}$$

Substituting (15) into (14):

$$d^2\Pi = \left(h_{W_A W_A} - 2h_{W_A W_B} + h_{W_B W_B}\right)dW_A^{\ 2} \tag{16}$$

Adopting the specific functional forms assumed in (1) and (5):

$$d^2\Pi = -\left[P_A\left(2a_3 - a_2 - a_5\right) + P_B\left(2b_3 - b_2 - b_5\right)\right]dW_A^{\ 2} \tag{17}$$

The terms inside the square brackets are equal to the denominator in (7) (D). The sign of $d^2\Pi$ does not depend on the values of W_A and W_B. This means that it is not possible for there to be more than one stationary point. (17) also allows us to determine whether the problem is convex or concave. In particular, the problem will be concave when $D > 0$ and convex when $D < 0$.

Solutions

The third step is to identify all possible 'problem types', and then determine the parameter subspace and solution values associated with different configurations. There are four possible 'problem types':

- Interior stationary point and $D > 0$. The problem is concave, and hence the stationary point corresponds to a local maximum. Since there is only one stationary point, this must also be a global maximum. This is the only 'problem type' that results in an interior solution. It is illustrated in Figure 3 which shows Π for different combinations of W_A and W_B that satisfy the water constraint.

- Interior stationary point and $D < 0$. The problem is convex, and hence the stationary point corresponds to a local minimum. Since there is only one stationary point, the global maximum associated with (10) must be a corner — more specifically, the corner with the higher value. In figure 4, Π is higher at point c than point d and so the optimal strategy is to only water ecosystem A. Returning to the algebraic example, in the context of an interior stationary point being a minimum, it will be optimal to only water ecosystem water A when:

$$P_A\left(a_1 + 0.5a_2\overline{W}\right) + P_B\left(b_4 + 0.5b_5\overline{W}\right) > P_A\left(a_4 + 0.5a_5\overline{W}\right) + P_B\left(b_1 + 0.5b_2\overline{W}\right) \text{(18)}$$

and only water ecosystem B when:

$$P_A\left(a_1 + 0.5a_2\overline{W}\right) + P_B\left(b_4 + 0.5b_5\overline{W}\right) < P_A\left(a_4 + 0.5a_5\overline{W}\right) + P_B\left(b_1 + 0.5b_2\overline{W}\right) \text{(19)}.$$

- Underlined One of the corners is a local maximum and $D > 0$. The problem is concave, and the existence of a corner as a local maximum rules out the possibility of an interior stationary point (as it has been established that there are either zero or one stationary points). Hence, any feasible movement away from the corner which is the local maximum will reduce Π and the that corner will also be a global maximum. This possibility is shown in figure 5 for the case where it is optimal to only water ecosystem A.

- Underlined One of the corners is a local minimum and $D < 0$. The problem is convex, and the existence of a corner as a local minimum rules out the possibility of an interior stationary point (as in the case above). Hence, any feasible movement away from the corner which is the local minimum will increase Π and the opposite corner will be a global maximum. This possibility is shown in figure 6 for the case where it is optimal to only water ecosystem B.

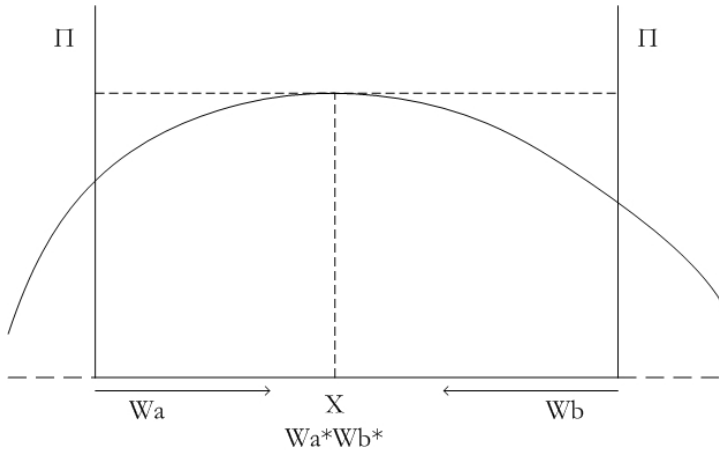Figure 3: illustrative interior solution

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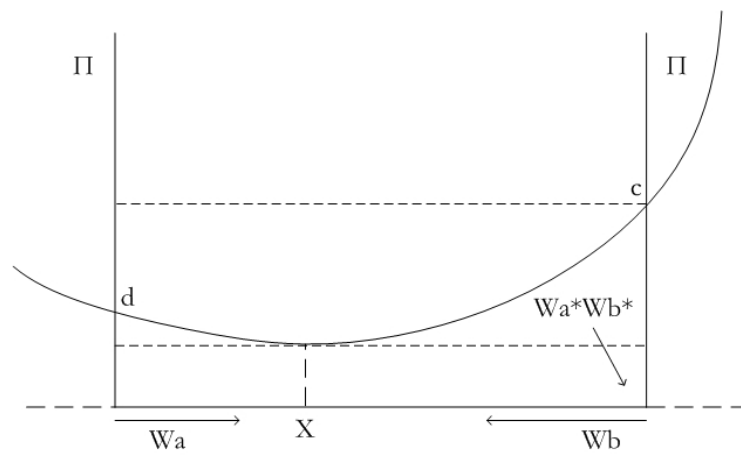Figure 4: illustrative corner solution with interior stationary point

Figure 5: illustrative corner solution without interior stationary point, $D > 0$
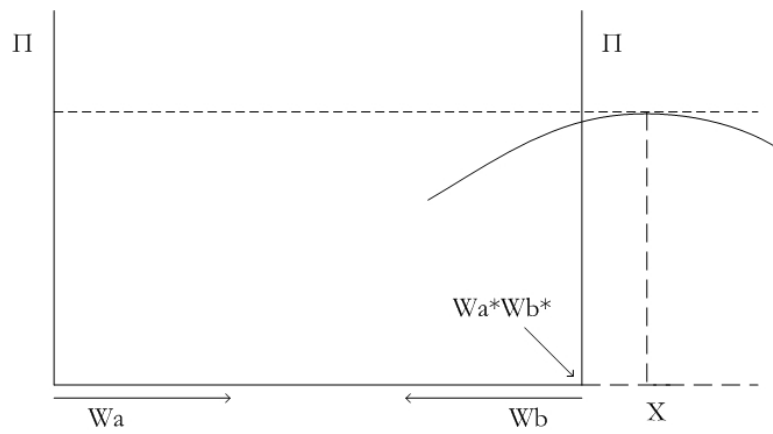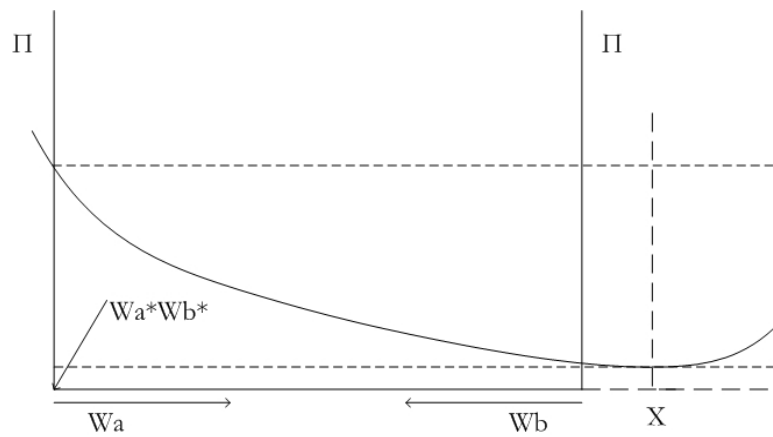
Figure 6: illustrative corner solution without interior stationary point, $D < 0$

It is now possible to examine the parameter subspace and solution values associated with different configurations (table 1).

Table 1: parameter subspace and solution values associated with different configurations, assuming that $\lambda > 0$

Configuration	Parameter subspace		Solution
Both A and B (i)	$D > 0$	$P_A\left[a_1 - a_4 + \overline{W}(a_3 - a_5)\right] > P_B\left[b_1 - b_4 + \overline{W}(b_2 - b_3)\right]$ $P_B\left[b_1 - b_4 + \overline{W}(b_3 - b_5)\right] > P_A\left[a_1 - a_4 + \overline{W}(a_2 - a_3)\right]$	$W_A{}^* = \dfrac{P_A\left[a_1 - a_4 + \overline{W}(a_3 - a_5)\right] - P_B\left[b_1 - b_4 + \overline{W}(b_2 - b_3)\right]}{D}$ $W_B{}^* = \dfrac{P_B\left[b_1 - b_4 + \overline{W}(b_3 - b_5)\right] - P_A\left[a_1 - a_4 + \overline{W}(a_2 - a_3)\right]}{D}$
A only (ii)	$D > 0$	$P_B\left[b_1 - b_4 + \overline{W}(b_3 - b_5)\right] \leq P_A\left[a_1 - a_4 + \overline{W}(a_2 - a_3)\right]$	$W_A{}^* = \overline{W}\,;\; W_B{}^* = 0$
	$D < 0$	$P_A\left[a_1 - a_4 + \overline{W}(a_3 - a_5)\right] < P_B\left[b_1 - b_4 + \overline{W}(b_2 - b_3)\right]$ $P_B\left[b_1 - b_4 + \overline{W}(b_3 - b_5)\right] < P_A\left[a_1 - a_4 + \overline{W}(a_2 - a_3)\right]$ $P_A\left(a_1 + 0.5a_2\overline{W}\right) + P_B\left(b_4 + 0.5b_5\overline{W}\right) > P_A\left(a_4 + 0.5a_5\overline{W}\right) + P_B\left(b_1 + 0.5b_2\overline{W}\right)$ $P_A\left[a_1 - a_4 + \overline{W}(a_3 - a_5)\right] \geq P_B\left[b_1 - b_4 + \overline{W}(b_2 - b_3)\right]$	
B only (iii)	$D > 0$	$P_A\left[a_1 - a_4 + \overline{W}(a_3 - a_5)\right] \leq P_B\left[b_1 - b_4 + \overline{W}(b_2 - b_3)\right]$	$W_B{}^* = \overline{W}\,;\; W_A{}^* = 0$
	$D < 0$	$P_A\left[a_1 - a_4 + \overline{W}(a_3 - a_5)\right] < P_B\left[b_1 - b_4 + \overline{W}(b_2 - b_3)\right]$ $P_B\left[b_1 - b_4 + \overline{W}(b_3 - b_5)\right] < P_A\left[a_1 - a_4 + \overline{W}(a_2 - a_3)\right]$ $P_A\left(a_1 + 0.5a_2\overline{W}\right) + P_B\left(b_4 + 0.5b_5\overline{W}\right) < P_A\left(a_4 + 0.5a_5\overline{W}\right) + P_B\left(b_1 + 0.5b_2\overline{W}\right)$ $P_B\left[b_1 - b_4 + \overline{W}(b_3 - b_5)\right] \geq P_A\left[a_1 - a_4 + \overline{W}(a_2 - a_3)\right]$	

$D = P_A\left[2a_3 - a_2 - a_5\right] + P_B\left[2b_3 - b_2 - b_5\right]$

Comparative statics

We can investigate the relationship between the parameters and optimal allocations by taking the partial derivatives of W_A* with respect to the parameters. The following equations demonstrate that an increase in a_1, a_2, b_4 and b_5 results in a higher W_A* (within the interior solution).

$$\frac{\partial W_A*}{\partial a_1} = \frac{P_A}{D} > 0 \qquad (20)$$

(assuming $P_A > 0$ and $D > 0$, the latter being a condition for an interior solution)

$$\frac{\partial W_A*}{\partial b_4} = \frac{P_B}{D} > 0 \qquad (21)$$

(assuming $P_B > 0$)

$$\frac{\partial W_A*}{\partial a_2} = \frac{P_A N}{D^2} > 0 \qquad (22)$$

(assuming $N > 0$, a condition for an interior solution)

$$\frac{\partial W_A*}{\partial b_5} = \frac{P_B N}{D^2} > 0 \qquad (23)$$

where

$$N = P_A\big[a_1 - a_4 + \overline{W}(a_3 - a_5)\big] - P_B\big[b_1 - b_4 + \overline{W}(b_2 - b_3)\big] \qquad (24)$$

This is because an increase in a_1, a_2, b_4 or b_5 increases the marginal benefits of applying water to ecosystem A at all levels. The relationship between a_2 and W_A* can be illustrated graphically for a given hypothetical set of parameters (figure 7; table 2). At values of $a_2 < -0.6$ the interior solution holds and water is allocated to both ecosystems. As expected,

an increase in a_2 increases $W_A\text{*}$. The model shifts to a corner solution at $a_2 = -0.6$ beyond which only ecosystem A is watered.
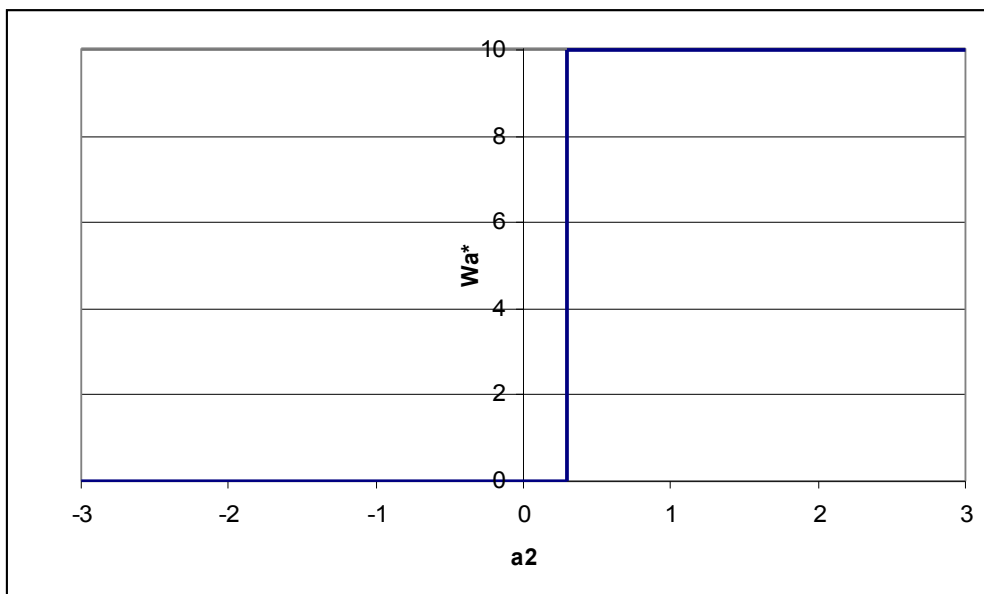
Figure 7: relationship between a_2 and $W_A\text{*}$ [a]
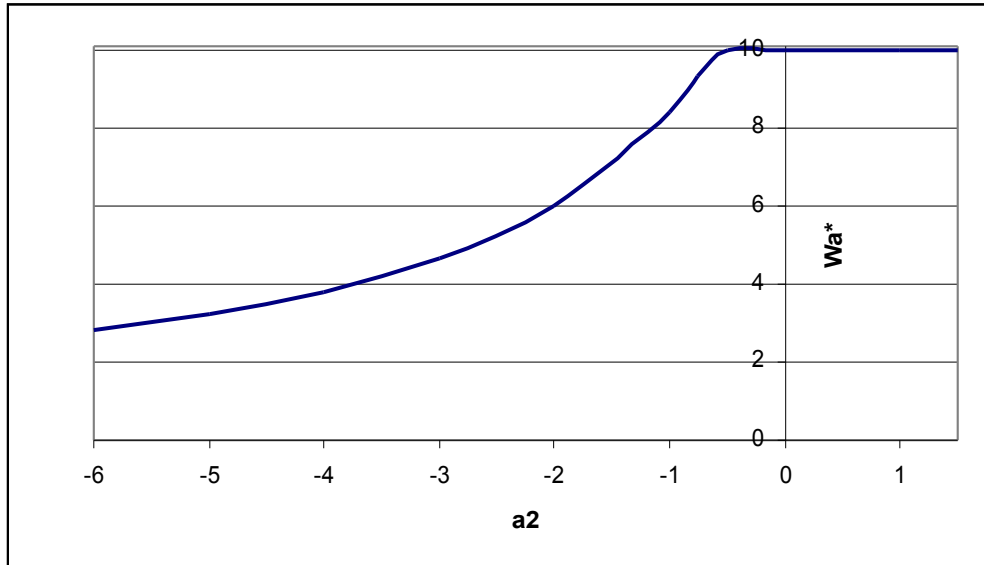
[a] assumed parameter values given in table 2, $b_2 = -1$

Setting $b_2 = 2$ changes the problem substantially (figure 8). In this case, the problem is convex and jumps discontinuously between the two corner solutions at $a_2 = 0.3$.

Figure 8: relationship between a_2 and $W_A\text{*}$ [a]

[a] assumed parameter values given in table 2, $b_2 = 2$

Table 2: hypothetical parameter values used in figures 7 to 12

Parameter	Value(s)
P_A	1
a_1	20
a_2	-2
a_3	0.2
a_4	2
a_5	-0.1
P_B	1
b_1	10
b_2	-1 or 2
b_3	0
b_4	0
b_5	0
$\overline{\overline{W}}$	10

An increase in b_1, b_2, a_4 and a_5 increases the marginal benefits of applying water to ecosystem B. This increases the opportunity cost of watering ecosystem A, therefore reducing $W_A{}^*$ (within the interior solutions). The partial derivatives with respect to these parameters are as follows:

$$\frac{\partial W_A{}^*}{\partial a_4} = \frac{- P_A}{D} < 0 \tag{25}$$

$$\frac{\partial W_A{}^*}{\partial b_1} = \frac{- P_B}{D} < 0 \tag{26}$$

$$\frac{\partial W_A{}^*}{\partial a_5} = \frac{P_A\left(N - \overline{W}D\right)}{D^2} < 0 \tag{27}$$

(assuming $\dfrac{N}{D} < \overline{W}$, a condition for an interior solution)

$$\frac{\partial W_A{}^*}{\partial b_2} = \frac{P_B\left(N - \overline{W}D\right)}{D^2} < 0 \tag{28}$$

We now turn to the interaction terms, a_3 and b_3. Within the interior solution, an increase in a_3 and b_3 increases W_A* when more water is applied to ecosystem B than ecosystem A, and decreases W_A* when more water is applied to ecosystem A than ecosystem B. This means that increases in a_3 and b_3 result in a more balanced allocation of water among the ecosystems. This is shown below.

$$\frac{\partial W_A*}{\partial a_3} = \frac{P_A\left(\overline{\overline{W}}D - 2N\right)}{D^2} ; \ \frac{\partial W_A*}{\partial a_3} > 0 \ \text{if} \ \frac{N}{D} < \frac{\overline{\overline{W}}}{2} ; \ \frac{\partial W_A*}{\partial a_3} < 0 \ \text{if} \ \frac{N}{D} > \frac{\overline{\overline{W}}}{2} \qquad (29)$$

$$\frac{\partial W_A*}{\partial b_3} = \frac{P_B\left(\overline{\overline{W}}D - 2N\right)}{D^2} ; \ \frac{\partial W_A*}{\partial b_3} > 0 \ \text{if} \ \frac{N}{D} < \frac{\overline{\overline{W}}}{2} ; \ \frac{\partial W_A*}{\partial b_3} < 0 \ \text{if} \ \frac{N}{D} > \frac{\overline{\overline{W}}}{2} \qquad (30)$$

Figure 9 shows that a decrease in a_3 has the effect of increasing watering of the more highly watered ecosystem (within the interior solutions), which is ecosystem A in this example. It can also result in corner solutions. For example, at $-2 < a_3 < -1.2$ it is optimal to only water ecosystem A. By contrast, at the limit as $a_3 \to \infty$, $W_A* \to 5$ (a perfectly balanced allocation of water).

Figure 9: relationship between a_3 and W_A* [a]

[a] assumed parameter values given in table 2, $b_2 = -1$

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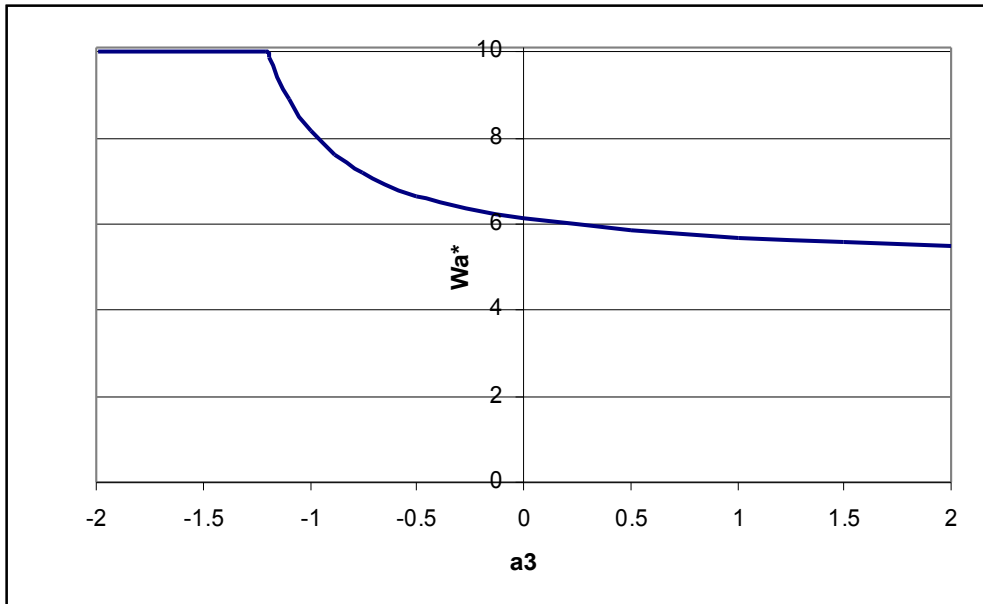
The effect of P_A and P_B on W_A* is ambiguous.

$$\frac{\partial W_A*}{\partial P_A} = \frac{\left[a_1 - a_4 + \overline{w}(a_3 - a_5)\right]D - [2a_3 - a_2 - a_5]N}{D^2};$$

$$\frac{\partial W_A*}{\partial P_A} > 0 \text{ if the numerator of } \frac{\partial W_A*}{\partial P_A} \text{ is positive;}$$

$$\frac{\partial W_A*}{\partial P_A} < 0 \text{ if the numerator of } \frac{\partial W_A*}{\partial P_A} \text{ is negative} \tag{31}$$

$$\frac{\partial W_A*}{\partial P_B} = -\left\{ \frac{\left[b_1 - b_4 + \overline{w}(b_2 - b_3)\right]D + [2b_3 - b_2 - b_5]N}{D^2} \right\}$$

$$\frac{\partial W_A*}{\partial P_B} < 0 \text{ if the numerator of } \frac{\partial W_A*}{\partial P_B} \text{ is positive;}$$

$$\frac{\partial W_A*}{\partial P_A} > 0 \text{ if the numerator of } \frac{\partial W_A*}{\partial P_B} \text{ is negative} \tag{32}$$

Without the interaction terms, the numerator of $\frac{\partial W_A*}{\partial P_A}$ will be positive when $D > 0$ and

$\lambda > 0$, both of which are conditions for the interior solution described in table one. An

increase in P_A will increase W_A* under these conditions. However, with the interaction

terms it is possible that an increase in P_A could reduce W_A*. This is because an increase in

P_A could theoretically increase the marginal benefits of watering ecosystem B more than it

increases the marginal benefits of watering ecosystem A. $\frac{\partial W_A*}{\partial P_A}$ happens to be positive in

figure 10.

Figure 10: relationship between P_A and $W_A *$ [a]
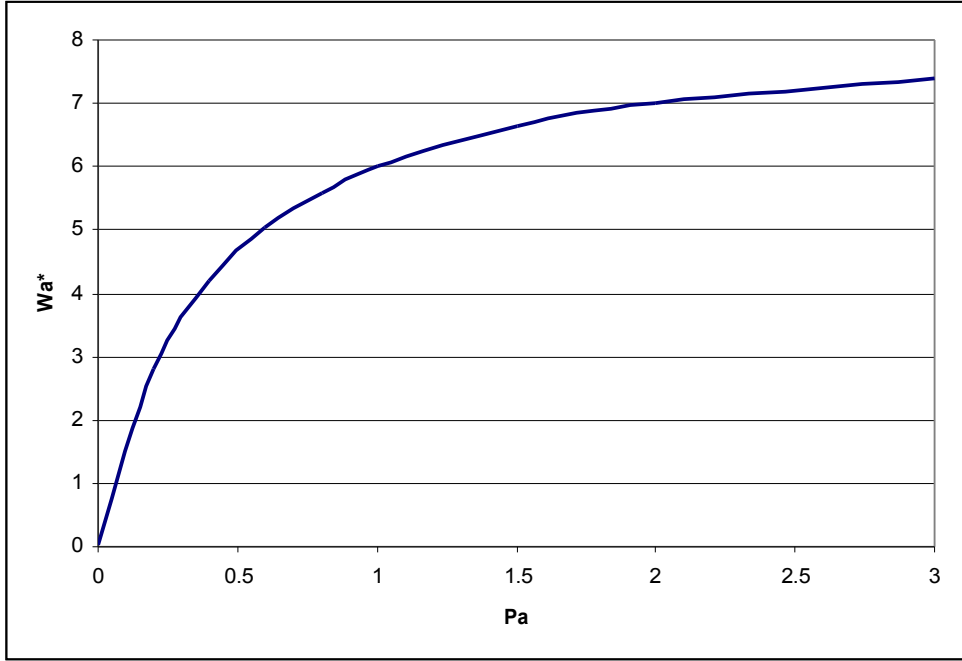
[a] assumed parameter values given in table 2, $b_2 = -1$

An increase in the volume of water available could have a positive or negative effect on $W_A *$.

$$\frac{\partial W_A *}{\partial \overline{W}} = \frac{P_A(a_3 - a_5) - P_B(b_2 - b_3)}{D};$$

$$\frac{\partial W_A *}{\partial \overline{W}} > 0 \text{ if the numerator of } \frac{\partial W_A *}{\partial \overline{W}} \text{ is positive;}$$

$$\frac{\partial W_A *}{\partial \overline{W}} < 0 \text{ if the numerator of } \frac{\partial W_A *}{\partial \overline{W}} \text{ is negative} \qquad (33)$$

This depends on $\frac{\partial W_A *}{\partial \overline{W}}$, the numerator of which is the negative of the slope of the marginal benefit curve associated with watering ecosystem B. Figures 11 and 12 illustrate these possibilities, with parameter values of $b_2 = -1$ and $b_2 = 2$. In figure 11, the corner solution A prevails for $0 < \overline{W} < 3.6$, while the interior solution described in table one prevails for $3.6 < \overline{W} < 24.2$. Since the numerator in (33) is positive, there is also a positive relationship between $\overline{W}$ and $W_A *$ within the interior solution.

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Figure 10 also illustrates a corner solution where it is efficient to only water ecosystem A for lower values of $\overline{\overline{W}}$, followed by an interior solution. However, the relationship between $\overline{\overline{W}}$ and W_A* is negative within the interior solution as the benefits at the margin of watering ecosystem B increase as more water becomes available (because of its convex production function). At values of $\overline{\overline{W}}$ in excess of 4.7 the other corner solution prevails and it is efficient to only water ecosystem B.

Figure 11: relationship between $\overline{\overline{W}}$ and W_A* [a]
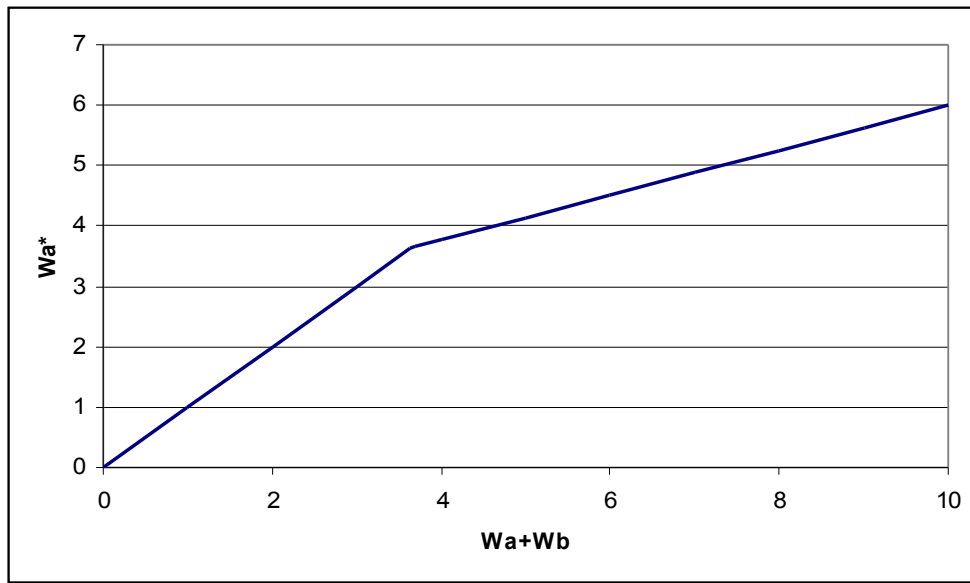
a assumed parameter values given in table 2, $b_2 = -1$

Figure 12: relationship between $\overline{\overline{W}}$ and W_A* [a]

a assumed parameter values given in table 2, $b_2 = 2$

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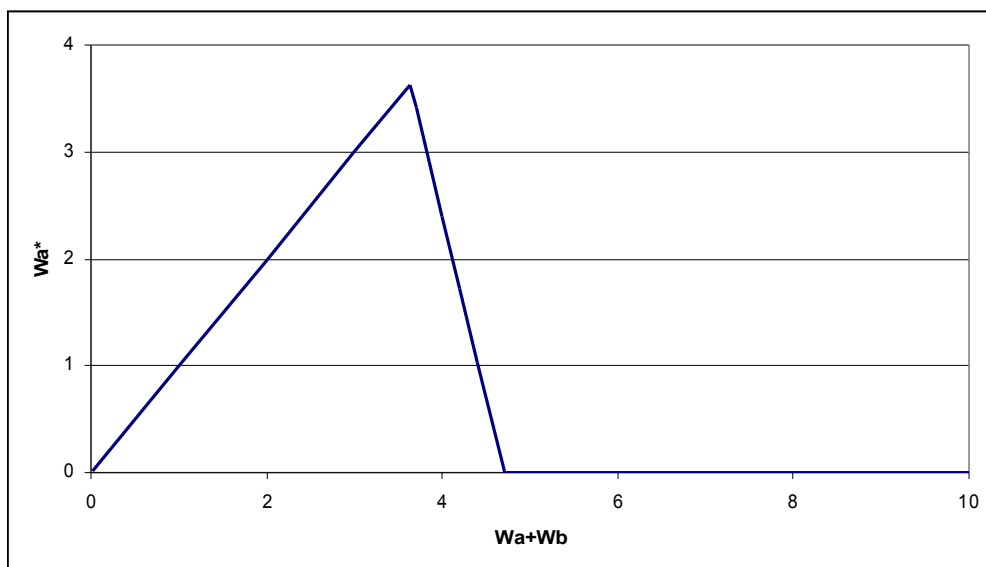
Discussion

The analysis above demonstrates that corner solutions are a genuine theoretical possibility in a two ecosystem model. As a result, the responses of optimal water allocation to changes in parameters may not be smooth or continuous, even with quadratic environmental production functions. What can be reasonably concluded about the possibilities of corner solutions? First, corner solutions are possible, even when both environmental production functions are concave (as in figure 7). This is because the marginal net benefit of applying the first unit of water to one ecosystem could be less than the marginal net benefit associated with the last unit of water applied to the other ecosystem, even when the second ecosystem receives all available water. Hence, with concave environmental production functions, the interior solution only applies over a limited parameter subspace. In this setting, a corner solution is more likely the lower the interaction term (figure 9) and the smaller the overall volume of water available (figure 11).

Second, corner solutions are assured when both environmental production functions are convex. (If one environmental production function is concave and the other is convex, the allocation problem could be either concave or convex depending on the magnitudes involved.) Where the problem is convex, the responses to changes in parameters can look very different. For example, there could be discontinuous jumps between corner solutions (figure 8).

The model also presents some potentially counter-intuitive insights. For example, an increase in the price of output from an ecosystem could decrease the efficient production from that ecosystem, while an increase in the overall volume of water available could reduce the optimal quantity of water applied to an ecosystem (figure 12). Revealing the possibility of these counter-intuitive outcomes and deriving the conditions under which they will occur is an advantage of approaching the water allocation problem with a degree of formalism.

This work has parallels with Kuosmanen and Laukkanen (2009), which employs a dynamic model to examine efficient abatement strategies where there are a number of interacting pollutants. Applying their model to eutrophication and climate change they conclude that 'the optimal policy is often a corner solution, in which abatement is focused on a single pollutant' and that 'corner solutions may arise even in well-behaved problems with concave production functions' (p. 1). Although their application is different, their approach and results have similarities. This suggests that the general framework outlined above is likely to apply a range

of environmental (and other economic) problems, and that the allocation of environmental water is one of many potential applications.

Potential areas for future research include adding dynamics and uncertainty surrounding inflows and environmental responses; the ability to carryover water in storage; more than two ecosystems; and more realistic environmental production functions (with thresholds, and so on). Incorporating empirically-derived on ecological responses and preferences would also be worthwhile.

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