*Journal of Agricultural and Applied Economics,* 42,4(November 2010):711–718


# The Effect of Increased Energy Prices on Agriculture: A Differential Supply Approach

**Charles B. Moss, Grigorios Livanis, and Andrew Schmitz**

The increase in energy prices between 2004 and 2007 has several potential consequences for aggregate agriculture in the U.S. We estimate the derived input demand elasticities for energy as well as capital, labor, and materials using the differential supply formulation. Given that the derived input demand for energy is inelastic, it is more price-responsive than the other inputs. The results also indicate that the U.S. aggregate agricultural supply function is responsive to energy prices.

*Key Words:* differential input demand, concavity constrained, energy

**JEL Classifications:** C30, Q11, Q42

This study examines the effect of increased energy prices on agriculture by estimating agriculture's elasticity of demand for energy. During 2004, crude oil prices in the U.S. increased almost 14%, from $27.63/barrel on January 2, 2004, to $32.07/barrel on December 31, 2004. Since that time, crude oil prices have continued to increase (Figure 1) reaching a maximum of $69.52/barrel on August 11, 2006. As of July 6, 2007, the crude oil price stood at $67.65/barrel. Figure 2 shows the effect of the increased oil prices on gasoline prices in the U.S. Similar to the increase in oil prices, gasoline prices increased by 17% in 2004. Given that fuel is an important input for the agricultural sector, these price increases would appear to bode ill for agriculture in the U.S. However, some speculate that agriculture could benefit from the fact that ethanol from either corn or cellulose could increase the demand for agricultural output in the U.S. For example, Senator Tom Harkin from Iowa has recently introduced legislation entitled the "Farm-to-Fuel Investment Act" which would "...provide transition assistance for farmers to grow dedicated energy crops (crops like switchgrass grown solely for the purpose of producing energy)" (Harkin, 2007). The net impact of ethanol on agriculture in the U.S. is dependent on its derived demand for energy. Specifically, the derived demand for energy in agriculture may be fairly elastic or inelastic. To answer these questions, we estimate the elasticity of the energy input demand for agriculture using the differential approach. Unfortunately, little empirical estimates exist on the derived demand input elasticities for U.S. agriculture of which the demand for energy is a key component (Schmitz and Stevens, 2000). This makes it difficult for policy analysts who deal with such topics as the future of biofuels.

Charles B. Moss is professor, and Andrew Schmitz is Ben Hill Griffin, Jr. eminent scholar and professor, Food and Resource Economics Department, University of Florida, Gainesville, FL. Grigorios Livanis is assistant professor, International Business and Strategy, College of Business Administration, Northeastern University, Boston, MA.

## The Differential Supply System

Like the familiar Rotterdam (Theil, 1981) formulation of the consumer demand model, the

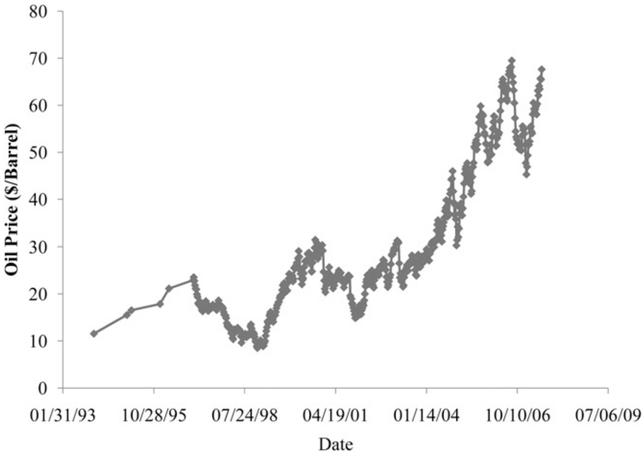


**Figure 1.** Oil Prices

differential supply model starts from the choice of cost-minimizing inputs subject to a given level of production ($\ln(z) = h(q)$). The Lagrange multiplier is

$$(1) \qquad L(q,p) = \sum_{i=1}^{n} p_i q_i + \rho(\ln(z) - h(q))$$

where $L(q,p)$ is the constrained cost of production, $p_i$ is the price of the input $i$, $q_i$ is the level of output $i$, $\rho$ is the marginal cost of the constraint $\rho = \partial L(q,p)/\partial \ln(z)$, $\ln(z)$ is the natural logarithm of the level of output $z$, and $h(q)$ is the logarithmic production function. Applying the differential approach to the optimizing conditions for producers, the univariate production function depicted in Equation (1) can be used to derive a differential formulation of the input decision

$$(2) \quad f_i d\ln(q_i) = \theta_i d\ln(z) - \psi \sum_{i=1}^{n} \theta_{ij} d\ln(^{p_i}/_{P})$$

where $f_i$ is the share of cost expended on factor $i$, $d\ln(q_i)$ denotes the logarithmic change in the quantity of input $i$ demand, $\theta_i$ is the share of the overall cost expended on factor $i$ as the logarithm of output increases ($d\ln(z)$), $\psi$ is the flexibility of marginal cost with respect to the overall level of output, $\theta_{ij}$ are parameters that capture the relative change in demand in

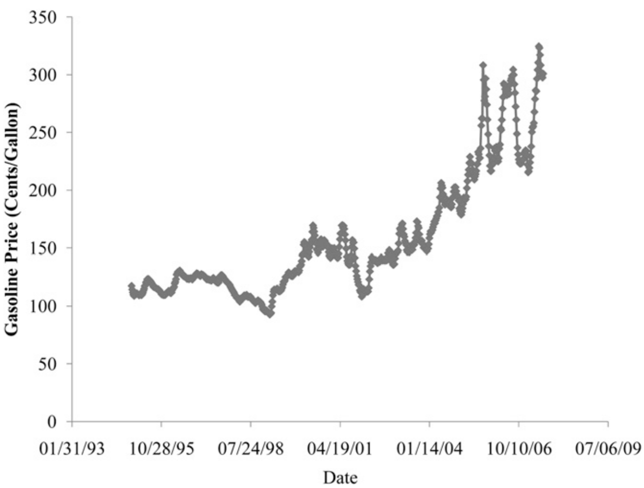


**Figure 2.** Gasoline Price (All Grades)

response to a change in each input price, and $P$ is a Frisch price index for inputs (an overview of the derivation is provided in the Appendix). The system of derived demand curves presented in Equation (2) is identical to the demand relationships from the Rotterdam formulation substituting the level of output in the derived demand relationship for income in the consumer demand model. In addition, an empirical model for the derived demand curve can be generated from this differential model using the same approach. Substituting discrete changes $(d\ln(x_{it}) \Rightarrow D\ln(x_{it}) = \ln(x_{it}) - \ln(x_{i,t-1}))$ and the average input share $(f_{it} \Rightarrow \bar{f}_{it} = 1/2(f_{it} + f_{i,t-1}))$ into Equation (2) yields an empirical counterpart of

$$\text{(3)} \quad \bar{f}_{it} D\ln(q_{it}) = \theta_i D\ln(z_t) + \sum_{i=1}^{n} \pi_{ij} D\ln(p_{it}) + \varepsilon_{it}$$

where $\pi_{ij}$ is symmetric ($\pi_{ij} = \pi_{ji}$), negative semidefinite, and homogeneous ($\sum_{j=}^{n} \pi_{ij} = 0$ for all $j$), and $\varepsilon_{it}$ is the error term for equation $i$ in period $t$.

## Estimation Issues

Like in the empirical implementation of the Rotterdam demand system, the empirical estimates of the system of derived input demand equations specified in Equation (3) often deviate from some of the theoretical restrictions (i.e., homogeneity, symmetry, and concavity). One is faced with two alternatives: 1) testing the statistical significance of these failures; or 2) simply imposing the theoretical restrictions. This analysis follows the latter approach. Specifically Laitinen (1978) concludes that the standard test for homogeneity of demand systems overstates the level of significance leading to excessive type II error. Moss and Theil (2003) expand on this increase in type II error. Similarly, Meisner (1979) finds that symmetry restrictions are rejected too often using standard tests, especially in small samples. In addition to the well-established problems with symmetry and homogeneity, the estimation and testing of demand systems raises potential difficulties with the concavity restrictions. Thus, this study imposes concavity using the approach suggested by Terrell (1996).

Following Terrell, we first estimate the system of factor demands using maximum likelihood imposing homogeneity and symmetry. Given these estimates, we then test for concavity by computing the maximum eigenvalue for the $\pi_{ij}$ matrix in Equation (3). If the maximum eigenvalue is greater than zero, the system is not concave. Based on this test, we then bootstrap the estimator 10,000 times retaining the estimates whose $\pi_{ij}$ matrix is concave. The concave estimator is then the average $\pi_{ij}$ matrix. In addition, the sample of estimated vectors can be used to construct robust estimates of the parameters and elasticities along with their respective variances.

The Terrell approach is very different from either estimating the Cholesky decomposition of the second moment matrix (Featherstone and Moss, 1994) or constraining the eigenvalues of $\pi_{ij}$ matrix to be less than zero (Shumway, Alexander, and Talpaz, 1990). Under both of these approaches, at least one of the eigenvalues is constrained to zero at the point of estimation: thus, the need to impose an additional linearity into the $\pi_{ij}$ matrix. Thus, instead of $\text{rank}(\pi_{ij}) = n - 1$ as implied by $\sum_{j=1}^{n} \pi_{ij} = 0$ the estimated rank of the system of input, derived demand equations results in $\text{rank}(\pi_{ij}) = n - 2$ using either the Cholesky decomposition approach or by constraining the eigenvalues to be less than zero.

## Data

To estimate the derived input demand elasticities, we use the KLEM (K, Capital; L, Labor; E, Energy; and M, Materials) (Jorgenson, 2010; Jorgenson and Stiroh, 2000). These data report the quantity of agricultural output along with the price received by farmers and price paid by consumers along with the expenditures on each input and a price for each input for 1960 through 2006. Following the differential formulation, we use the quantity of output as $z$ and divide the expenditure on each input by the price for each input to yield the quantity of each input used ($q_{it}$).

## Estimated Demand System

We estimate the derived input demand system depicted in Equation (3) imposing symmetry and homogeneity (by normalizing on the materials input) conditions (Table 1). The maximum eigenvalue of these unrestricted estimates is 0.5753, implying that the unrestricted estimates fail the concavity restriction. Given this result, we then bootstrap the estimation 10,000 times. From these results, 55 of the samples obey the concavity restriction. Averaging across these 55 estimates, we obtain the concavity imposed estimates (Table 1). The maximum eigenvalue for the concavity imposed estimates is –0.1454, implying that the estimated $\pi_{ij}$ matrix is negative definite as opposed to negative semidefinite. Imposing concavity improves the fit of the demand system (i.e., in the restricted formulation, five parameters are statistically significant at the 0.05 level of significance and all of the diagonal elements are statistically significant). However, the statistical significance is overstated in that we only consider variations over solutions that obey the concavity conditions.

**Table 1.** Estimated Derived Demand for Parameters for Aggregate U.S. Agriculture, 1958–2005 (*100)

| Variable | Without Concavity | Concavity Imposed |
|---|---|---|
| $\theta_1$(Capital) | 1.284 | 3.243*** |
| | (1.142)[a] | (0.903) |
| $\theta_2$(Labor) | –6.316* | –3.943 |
| | (4.735) | (4.040) |
| $\theta_3$(Energy) | 1.601* | 2.303*** |
| | (1.034) | (0.905) |
| $\pi_{11}$ | 0.289 | –0.256** |
| | (0.320) | (0.127) |
| $\pi_{12}$ | 1.023* | 0.806* |
| | (0.682) | (0.283) |
| $\pi_{13}$ | 0.427** | 0.190 |
| | (0.247) | (0.172) |
| $\pi_{22}$ | –6.806** | –7.992*** |
| | (3.113) | (2.646) |
| $\pi_{23}$ | –0.191 | –0.507 |
| | (0.637) | (0.487) |
| $\pi_{33}$ | –0.540* | –0.863** |
| | (0.408) | (0.401) |

[a] Values in parentheses denote standard errors of estimates.
* Denotes statistical significance at the 0.10 level of confidence.
** Denotes statistical significance at the 0.05 level of confidence.
*** Denotes statistical significance at the 0.01 level of confidence.

The estimated elasticities (Table 2) indicate that the demands for all inputs are inelastic with respect to price. The derived demand for energy is less price inelastic than the derived demand for labor but more elastic than the derived demand for both capital and materials. Furthermore, although the elasticity is statistically significant at the 0.05 confidence level for labor and materials, the elasticity of demand for capital and energy are only statistically significant at the 0.10 confidence level.

Examining the cross-price elasticities, we see that increased energy prices lead to a reduction in the demand for labor but an increase in the demand for both capital and materials. These interactions could be interpreted in a number of ways. First, we may anticipate that increased energy prices would reduce the demand for capital items. Specifically, a large portion of agriculture's capital investment is in tractors, combines, and other mobile equipment. Hence, we would hypothesize that increased energy costs would reduce the demand for these energy-consuming items. However, since World War II, agriculture has seen a continual trend toward larger equipment. This trend coincides with a reduction in the number of farm operators, which is evident in the positive but statistically insignificant cross-price elasticities between labor and capital in our results. A secondary effect may be that this larger equipment is relatively more fuel-efficient than older, smaller capital items.

A similar explanation may be possible for the relationship between materials and energy. Looking back on row-crop agriculture, numerous row operations were often required to control weeds in cotton. However, at the same time larger equipment arrived (e.g., two-row to four-row and six-row planters), pesticides were introduced that reduced the necessity of some of these row operations. This replacement ultimately culminated with the introduction of low-till and no-till technologies for many crops in which a vast majority of energy-based operations have been replaced by material applications.

## Implications and Conclusions

The empirical results suggest that imposing concavity on the differential cost system

**Table 2.** Compensated Input Elasticities

| Change in Demand for | Output Level | Elasticity with Respect to Capital Prices | Labor Prices | Energy Prices | Materials Prices |
|---|---|---|---|---|---|
| Capital | 0.1589** | –0.0126* | 0.0394* | 0.0093 | –0.0362 |
| | (0.0542)[a] | (0.0067) | (0.0228) | (0.0087) | (0.0220) |
| Labor | –0.1968 | 0.0402 | –0.3989** | –0.0253 | 0.3840** |
| | (0.2108) | (0.0241) | (0.1650) | (0.0255) | (0.1571) |
| Energy | 0.9078* | 0.0748 | –0.1999 | –0.3403* | 0.4654* |
| | (0.4501) | (0.0732) | (0.2061) | (0.1896) | (0.2687) |
| Materials | 1.7971*** | –0.0135* | 0.1405*** | 0.0216* | –0.1486*** |
| | (0.1759) | (0.0078) | (0.0473) | (0.0069) | (0.0471) |

[a] Numbers in parentheses denote standard deviations.
* Denotes statistical significance at the 0.10 level of confidence.
** Denotes statistical significance at the 0.05 level of confidence.
*** Denotes statistical significance at the 0.01 level of confidence.

significantly improves the estimated system of demand equations for aggregate U.S. agriculture. After imposing concavity, agriculture's energy demand, although inelastic, appears to be more sensitive to price changes than any other input. The estimated input demand elasticity for energy is –0.3403 compared with an own price elasticity of –0.3989 for labor, –0.1486 for materials, and –0.0126 for capital inputs. Furthermore, the largest cross-price effect between input prices appears to be between energy and labor followed by a substitution of labor for materials. Thus, we conclude that increases in energy prices will affect the supply of agricultural products more significantly than other inputs. Also, that increase in energy prices will have a significant impact on agriculture's labor demand. However, as we expand the specification in an attempt to estimate the effect of energy prices on the supply of agricultural outputs, we are plagued by additional concavity concerns. Specifically, although the empirical results in that the estimated parameter on energy prices is negative but insignificant at any conventional confidence level, the estimates suffer anomalies of the output price and other input prices. Furthermore, these discrepancies cannot be solved using the procedure outlined by Terrell.

Given that the empirical results of our analysis are somewhat mixed, several alternatives may provide additional insight. One possibility involves generalizations of the differential supply system. First, the data set KLEM provides an aggregate agricultural output, which may average out the effect of energy prices on crop vs. livestock operations. Laitinen and Theil (1978) provide a multiproduct version of the differential model of the firm. However, data work would be required to produce livestock and crop output indices comparable to the Jorgenson KLEM data. An alternative extension would be the incorporation of quasifixed inputs particularly for capital and farmland. A multiproduct model of the differential model including quasifixed variables is presented in Livanis and Moss (2006). Finally, Livanis (2004) presents a more flexible formulation of the effect of changes in output level on relative input shares.

*[Received July 2007; Accepted June 2010.]*

## References

Featherstone, A.M., and C.B. Moss. "Measuring Economies of Scale and Scope in Agricultural Banking." *American Journal of Agricultural Economics* 76,3(1994):655–61.

Harkin, T. "Chairmen Harkin and Conrad, Senator Klobuchar Introduce the Farm-to-Fuel Investment Act." Internet site: http://harkin.senate.gov/news.cfm?id=274367 (Accessed July 13, 2007).

Jorgenson, D.W. "Professor Dale Jorgenson's Data Sets." Internet site: www.economics.harvard.edu/faculty/jorgenson/data_sets_jorgenson (Accessed June 24, 2010).

Jorgenson, D.W., and K.J. Stiroh. "U.S. Economic Growth at the Industry Level." *The American Economic Review* 90,2(2000):161–67.

Laitinen, K. "Why Is Demand Homogeneity So Often Rejected?" *Economics Letters* 1,3(1978): 187–91.

Laitinen, K., and H. Theil. "Supply and Demand of the Multiproduct Firm." *European Economic Review* 11,2(1978):107–54.

Livanis, G.T. "The Differential Production Model with Quasi-Fixed Inputs: A Panel Data Approach to U.S. Banking." Unpublished PhD dissertation, University of Florida, 2004.

Livanis, G.T. and C.B. Moss. "Quasi-fixity and Multiproduct Firms." Economics Letters 93(2 ): 228–34.

Meisner, J.F. "The Sad Fate of Asymptotic Slutsky Symmetry Tests for Large Systems." *Economics Letters* 2,3(1979):231–33.

Moss, C.B., and H. Theil. "Homogeneity Testing Revisited." *Journal of Agricultural and Applied Economics* 35(2003):3–4.

Schmitz, A. and T. Stevens. "Rising Energy Costs and Profitability of U.S. Agriculture." Mimeographed paper, Food and Resource Economics Department, University of Florida, March 20, 2000.

Shumway, C.R., W.P. Alexander, and H. Talpaz. "Texas Field Crops: Estimation with Curvature." *Western Journal of Agricultural Economics* 15(1990):45–54.

Terrell, D. "Incorporating Monotonicity and Concavity Conditions in Flexible Functional Forms." *Journal of Applied Econometrics* 11, 2(1996):179–94.

Theil, H. *Theory and Measurement of Consumer Demand*. Vol 1. Amsterdam: North-Holland Publishing Co., 1981.


## Appendix: Derivation of the Differential Supply System

Taking the first-order conditions of Equation (1) with respect to $\ln(q_j)$ yields

$$\text{(A.1)} \qquad \frac{\partial L(q,p)}{\partial \ln(q_j)} = p_j \frac{\partial q_j}{\partial \ln(q_j)} - \rho \frac{\partial h(q)}{\partial \ln(q_j)} = 0.$$

Substituting for the logarithmic differentiation in the first term on the right-hand side of Equation (A.1) yields

$$\text{(A.2)} \qquad \frac{\partial L(q,p)}{\partial \ln(q_j)} = p_j q_j - \rho \frac{\partial h(q)}{\partial \ln(q_j)} = 0.$$

Substitution of $f_j = p_j q_j / C$ where $C = \sum_{i=1}^{n} p_i q_i$ gives

$$\text{(A.3)} \qquad \frac{\partial L(q,p)}{\partial q_j} = f_j - \frac{\rho}{C} \frac{\partial h(q)}{\partial \ln(q_j)} = 0$$

Further substituting

$$\text{(A.4)} \qquad \begin{aligned} \rho \equiv \frac{\partial C}{\partial \ln(z)} \Rightarrow \frac{1}{C}\frac{\partial C}{\partial \ln(z)} &= \frac{\partial C/C}{\partial \ln(z)} \\ &= \frac{\partial \ln(C)}{\partial \ln(z)} \equiv \gamma \end{aligned}.$$

Substituting this result back into Equation (A.3) yields

$$\text{(A.5)} \qquad \frac{\partial L(q,p)}{\partial \ln(q_j)} = f_j - \gamma \frac{\partial h(q)}{\partial \ln(q_j)} = 0$$

and solving Equation (A.5) yields Theil's expression $\partial h(q)/\partial \ln(q_j) = f_j/\gamma$. Next differentiating Equation (A.5) at the point of optimization gives

$$\text{(A.6)} \qquad \begin{aligned} \frac{\partial L(q,p)}{\partial \ln(q_j)\partial \ln(q_i)} = \frac{\partial f_j}{\partial \ln(q_i)} \\ - \gamma \frac{\partial h(q)}{\partial \ln(q_j)\partial \ln(q_j)} \end{aligned}$$

To simplify the derivation, we introduce two matrix derivatives into Equation (A.6). Focusing on the first term on the right-hand side of Equation (A.6)

$$\text{(A.7)} \qquad \frac{\partial f_j}{\partial \ln(q_i)} = \frac{\partial \left(\frac{q_j p_j}{C}\right)}{\partial \ln(q_i)} = \begin{cases} \frac{q_j p_j}{C} & i = j \\ 0 & i \neq j \end{cases}.$$

Thus, we construct a matrix $F$ whose diagonal elements are $q_i p_i / C$ following the results of Equation (A.7). Next, we define $H$ as the matrix of second logarithmic derivatives

$$\text{(A.8)} \qquad H = \left[\frac{\partial^2 h(q)}{\partial \ln(q_j)\partial \ln(q_i)}\right]_{i,j=1,\ldots n}$$

Thus, Equation (A.6) can be rewritten as

$$\text{(A.9)} \qquad \frac{\partial L(q,p)}{\partial \ln(q_j)\partial \ln(q_i)} = F - \gamma H$$

Differentiating Equation (A.2) with respect to the natural logarithm of the level of outputs ($\ln(z)$) yields

$$\text{(A.10)} \qquad \begin{aligned} \frac{\partial^2 L(q,p)}{\partial \ln(q_j)\partial \ln(z)} &= p_j q_j \frac{\partial \ln(q_j)}{\partial \ln(z)} \\ &- \rho \frac{\partial h(q)}{\partial \ln(q_j)} \frac{\partial \ln(\rho)}{\ln(z)} \\ &- \rho \sum_{i=1}^{n} \frac{\partial^2 h(q)}{\partial \ln(q_j)\partial \ln(q_i)} \frac{\partial \ln(q_i)}{\partial \ln(z)} \\ &- \rho \frac{\partial h(q)}{\partial \ln(q_j)\partial \ln(z)} = 0 \end{aligned}.$$

Imposing the first-order condition from Equation (A.2) ($q_j p_j - \rho \partial h(q)/\partial \ln(q_j) = 0 \Rightarrow \partial h(q)/\partial \ln(q_j) = q_j p_j/\rho$) into the second term on the right-hand side of Equation (A.10) yields

$$\text{(A.11)} \qquad \begin{aligned} & q_j p_j \frac{\partial \ln(q_j)}{\partial \ln(z)} - q_j p_j \frac{\partial \ln(\rho)}{\partial \ln(z)} \\ &- \rho \sum_{i=1}^{n} \frac{\partial^2 h(q)}{\partial \ln(q_j)\partial \ln(q_i)} \frac{\partial \ln(q_i)}{\partial \ln(z)} \\ &- \rho \frac{\partial^2 h(q)}{\partial \ln(q_j)\partial \ln(z)} = 0 \end{aligned}$$

Multiplying Equation (A.11) by $1/C$, substituting $\gamma = \rho/C$, and the definition of input shares and collecting like terms yields

$$\text{(A.12)} \qquad (F - \gamma H)\frac{\partial \ln(q)}{\partial \ln(z)} - F\iota \frac{\partial \ln(\rho)}{\partial \ln(z)} = \gamma H^*$$

where $\iota$ is a vector of ones and $H^* = [\partial^2 h(q)/\partial \ln(q_j)\partial \ln(z)]_{j=1,\ldots n}$.

Next, we differentiate Equation (A.2) with respect to the natural logarithm of input prices

$$\text{(A.13)} \qquad \begin{aligned} \frac{\partial^2 L(q,p)}{\partial \ln(q_j)\partial \ln(p_i)} &= \delta_{ji} q_j p_j + q_j p_j \frac{\partial \ln(q_j)}{\partial \ln(p_i)} \\ &- q_j p_j \frac{\partial \ln(\rho)}{\partial \ln(p_i)} - \rho \sum_{k=1}^{n} \frac{\partial^2 h(q)}{\partial \ln(q_j)\partial \ln(q_k)} \\ &\times \frac{\partial \ln(q_k)}{\partial \ln(p_j)} = 0 \end{aligned}$$

where $\delta_{ji}$ is the Kronecker delta, which is 1 if $i = j$ and 0 otherwise. With this substitution

$$\text{(A.14)} \qquad (F - \gamma H)\frac{\partial \ln(q)}{\partial \ln(p')} - F\iota \frac{\partial \ln(\rho)}{\partial \ln(p')} = -F$$

It is necessary to solve for changes in endogenous variables (the quantity of the vector of inputs used and marginal cost of production) with respect to changes in exogenous variables (the vector of input prices and level of output). We first totally differentiate the output constraint first with respect to the natural logarithm of the level of output

$$\text{(A.15)} \qquad \begin{aligned} & \sum_{j=1}^{n} \frac{\partial h(q)}{\partial \ln(q_j)} \frac{\partial \ln(q_j)}{\partial \ln(z)} - \frac{\partial h(q)}{\partial \ln(z)} \\ &= 0 \Rightarrow \iota' F \frac{\partial \ln(q)}{\partial \ln(z)} = \gamma \end{aligned}$$

and then with respect to the natural logarithm with respect to input prices

$$\text{(A.16)} \qquad \sum_{j=1}^{n} \frac{\partial h(q)}{\partial \ln(q_i)} \frac{\partial \ln(q_i)}{\partial \ln(p_j)} = 0 \Rightarrow \iota' F \frac{\partial \ln(q)}{\partial \ln(p')}$$

Combining Equations (A.12), (A.14), (A.15), and (A.16) into a matrix equation yields

$$\text{(A.17)} \qquad \begin{bmatrix} F - \gamma H & \iota \\ \iota' & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \ln(q)}{\partial \ln(z)} & \frac{\partial \ln(q)}{\partial \ln(p')} \\ -\frac{\partial \ln(\rho)}{\partial \ln(z)} & \frac{\partial \ln(\rho)}{\partial \ln(p')} \end{bmatrix} = \begin{bmatrix} \gamma H^* & -F \\ \gamma & 0 \end{bmatrix}.$$

We derive our differential demand model for the supply function by solving the system of equations implicit in Equation (A.17). We first make refinements in Equation (A.17) to yield the supply equivalent to Barten's fundamental matrix equations. By taking the component of Equation (A.17) that corresponds to Equation (A.14) and multiplying both sides of this equation by $F^{-1}$ one obtains

$$\text{(A.18)} \qquad \begin{aligned} & F^{-1}\left[(F - \gamma H)\frac{\partial \ln(q)}{\partial \ln(p')} - F\iota \frac{\partial \ln(\rho)}{\partial \ln(p')}\right] = F^{-1}[-F] \; F^{-1}(F - \gamma H)\frac{\partial \ln(q)}{\partial \ln(p')} - \iota \frac{\partial \ln(\rho)}{\partial \ln(p')} = -I \\ & F^{-1}(F - \gamma H)F^{-1}F \frac{\partial \ln(q)}{\partial \ln(p')} - \iota \frac{\partial \ln(\rho)}{\partial \ln(p')} = -I \end{aligned}$$

By a similar transformation to Equation (A.12), the matrix transformation from Equation (A.17) becomes

$$\begin{bmatrix} F^{-1}(F-\gamma H)F^{-1} & \iota \\ \iota' & 0 \end{bmatrix} \times \begin{bmatrix} F\frac{\partial \ln(q)}{\partial \ln(z)} & F\frac{\partial \ln(q)}{\partial \ln(p')} \\ -\frac{\partial \ln(\rho)}{\partial \ln(z)} & \frac{\partial \ln(\rho)}{\partial \ln(p')} \end{bmatrix} = \begin{bmatrix} \gamma F^{-1}H^{*} & -I \\ \gamma & 0 \end{bmatrix} \tag{A.19}$$

Solving Barten's fundamental equation in Equation (A.19) yields

$$\begin{bmatrix} F\frac{\partial \ln(q)}{\partial \ln(z)} & F\frac{\partial \ln(q)}{\partial \ln(p')} \\ -\frac{\partial \ln(\rho)}{\partial \ln(z)} & \frac{\partial \ln(\rho)}{\partial \ln(p')} \end{bmatrix} = \begin{bmatrix} F^{-1}(F-\gamma H)F^{-1} & \iota \\ \iota' & 0 \end{bmatrix}^{-1} \times \begin{bmatrix} \gamma F^{-1}H^{*} & -I \\ \gamma & 0 \end{bmatrix} \tag{A.20}$$

where

$$\begin{bmatrix} F^{-1}(F-\gamma H)F^{-1} & \iota \\ \iota' & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \psi(\Theta-\theta\theta') & \theta \\ \theta' & -1/\psi \end{bmatrix}. \tag{A.21}$$

This last result implies that

$$\begin{aligned} \Theta &= \frac{1}{\psi}F(F-\gamma H)^{-1}F \\ \psi &= \iota' F(F-\gamma H)^{-1}F\iota \\ \theta &= \Theta\iota \end{aligned} \tag{A.22}$$

Thus, taking the results of Equation (A.21)and (A.22), we have

$$F\frac{\partial \ln(q)}{\partial \ln(p')} = -\psi(\Theta-\theta\theta'). \tag{A.23}$$

To complete the input demand system, we start by totally differentiating the input level for input $j$

$$\begin{aligned} d\ln(q_j) &= \frac{\partial \ln(q_j)}{\partial \ln(z)}d\ln(z) \\ &+ \frac{\partial \ln(q)}{\partial \ln(p')}d\ln(p') \end{aligned} \tag{A.24}$$

Substituting the solution from Equations (A.20) and (A.22) into Equation (A.24) yields

$$f_i d\ln(q_i) = \theta_i d\ln(z) - \psi\sum_{i=1}^{n}\theta_{ij}d\ln(p_i/P). \tag{A.25}$$

which can be estimated using the standard Rotterdam empirical assumptions.