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Input Demand and Substitution in the Australian Sheep Industry

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The demand for production inputs by the average property in the Australian sheep industry and substitution between these inputs was examined in this paper by estimating the set of input share demand equations derived from a transcendental logarithmic cost function. The following five input categories were examined: labour, land, livestock, capital, and materials and services. While the demand for labour was inelastic with respect to its own price, the demand for capital was elastic. All cross price demand elasticities estimated were less than one. In contrast with earlier Australian studies, the elasticity of substitution between labour and capital was found to be greater than unity. Technical change has been relatively labour and land saving and relatively capital, livestock, and materials and services using.

Introduction

The primary aim of this paper is to investigate the substitutability of various inputs and the price elasticity of input demand for the average property in the Australian sheep industry.[1] Knowledge of these economic relationships is essential in adequately assessing the production effects of changes in policy and other parameters that influence input prices. Information on the magnitude of these elasticities is essential in assessing the likely effects of such changes as the removal of some indirect input tax/subsidy such as the investment allowance or a relatively rapid increase in the price of a particular input, such as labour.

In addition to estimating the Allen-Uzawa elasticities of substitution[2] between each pair of inputs and the own price elasticity of demand for each input, the bias in factor augmenting technical change[3] was estimated. As a guide to the likely future direction of technological progress this information has implications for the possible future relative demand for production inputs.

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[1] The Australian sheep industry is defined here to consist of all properties with 200 or more sheep. Hence it encompasses the majority of Australia's grazing and broadacre cropping properties.

[2] The Allen-Uzawa elasticity of substitution between inputs i and j (σ_{ij}, $i \neq j$) is the proportional rate of change in the input ratio divided by the proportionate rate of change of the rate of technical substitution between these two inputs ($-dx_i/dx_j$), *i.e.*,

$$\sigma_{ij} = \frac{d \log (x_i/x_j)}{d \log (f_j/f_i)}.$$

where $f(x)$ is the production function and f_i is the partial derivative of f with respect to x_i. This elasticity may be derived from the parameters of the dual cost function ($c(w;y)$) as follows:

$$\sigma_{ij} = \frac{c(w;y) \;.\; c_{ij}(w;y)}{c_i(w;y) \;.\; c_j(w;y)}$$

[3] Technical change is defined to be input i saving or biased in favour of input i if the marginal product of input i is raised relative to that of other inputs as a whole.

Elasticity estimates were obtained in this study by estimating the set of input share equations derived from a transcendental logarithmic cost function. These share equations were estimated for the following five input types; labour (L), land (N), livestock (V), capital (C), and materials and services (M). The time period covered is from 1952–3 to 1976–7.

After briefly reviewing the findings of other researchers on this subject, the methodology used in this study is discussed in the third section. Data employed are the subject of section four. Parameter estimates, along with the associated elasticities, are presented and analysed in section five. Conclusions are given in the sixth section, and the data used in the study are presented in the appendix.

Previous Australian findings

Estimates of elasticities of substitution for Australian agriculture have been reported by Powell (1969), Duncan (1972), Bates (1974) and Vincent (1977). Powell fitted a variation of the restrictive CES production specification to Australian production data collected mainly by Juhasz and Hillsdon (1964) for the period 1948 to 1961. Duncan fitted a CES production function to data for the New South Wales arid zone for the period 1892 to 1968 and various subperiods. Similarly, Bates fitted a CES labour side condition to Powell's (1974) data for Australian agriculture for the period 1920 to 1970. These three studies all concentrated on capital and labour inputs. In all cases these two factors were found to be weakly substitutable, *i.e.*, the estimated elasticity of substitution was positive but less than one.

Ryan and Duncan (1974) formulated estimates of the elasticity of net income with respect to changes in the per unit price of labour using BAE and CBCS data for Australian agriculture for the period 1949 to 1968. Although no elasticity of substitution estimates were derived in this study, estimates of the price elasticity of input demand are presented. The estimated own price elasticity of demand for hired labour was -0.58 while the cross-elasticity of demand for hired labour with respect to the price of capital was 0.75.

Vincent, on the other hand, derived elasticities of substitution for hired labour, capital and land from derived demand functions using Powell's (1974) data for Australian agriculture for the period 1921 to 1970. However, he used a value-added output specification. Parks (1971) points out that the use of a value-added framework will be valid only in the polar cases where intermediate inputs enter the production function in fixed proportions and where intermediate inputs are perfect substitutes with each other and with the factors of production. Where these conditions are not fulfilled, the resulting estimates of the elasticities of substitution may be biased. As was the case in the above-mentioned studies, Vincent's elasticity estimates were all close to zero, indicating that factor inputs exhibited a low responsiveness to relative price changes.

Methodology

As with most econometric production studies in which more than two inputs are used, the choice of a functional form which is sufficiently flexible is a major consideration. Many production function studies have adopted functional forms which have held the elasticities of substitution equal to either some constant or to each other. While there are now some functional forms of the production function which are sufficiently flexible to allow elasticities of substitution to vary, there are several advantages in using a dual cost function approach to derive estimates of production parameters. Cost functions, $C(w; y)$ where w is the vector of input prices (w_i) and y the quantity of output, are linear homogeneous in prices, regardless of the homogeneity properties of the production function, and the estimation equations have prices as independent variables instead of factor quantities, which more closely approximates the situation faced by individual firms.

A fundamental duality exists between production and cost functions. It has been shown (Diewert, 1974) that, if producers minimize costs in competitive markets, the cost function contains sufficient information under relatively weak regularity conditions to completely describe the production technology. Thus a functional form may be specified for the cost function instead of explicitly specifying a particular production function, and the factor demand functions derived from the cost function by partially differentiating the cost function with respect to input prices (Shephard, 1953). Given a suitable functional form for the underlying cost function, this leads to the estimating equations being linear and estimatable by ordinary multivariate regression techniques.

The flexible cost function used in this study was the transcendental logarithmic cost function developed by Christensen, Jorgenson and Lau (1973) and applied, for example, by Binswanger (1974) and Denny and May (1978).[4] This functional form provides a second order approximation in prices to an arbitrary cost function[5] and, hence, to any arbitrary elasticity of substitution matrix (Diewert, 1974). However, in the econometric model estimated here, this transcendental logarithmic cost function was not viewed as an approximation to an unknown function. Rather it was assumed to be the true data generating function. This permitted additive disturbance terms (e_i) to be specified for the derived input share equations and interpreted as deviations of the endogenous left-hand variables about their cost minimizing values.

To satisfy the necessary conditions for duality with a corresponding production function, the translog cost function was assumed to be—(i) a positive real valued function for all positive input prices (w), and output quantity (y); (ii) a non-decreasing function of w and y; (iii) positive linear homogeneous in w; and (iv) concave in input prices w. Furthermore, non-neutral efficiency differences arising from biased technological change were allowed for by the inclusion of a time variable as a proxy for an index of technology. This has been achieved by generalizing the conventional translog cost function to incorporate time as an explanatory variable. Assuming separability with respect to inputs and output this translog cost function may be represented as follows:

$$(1)\quad \ln C(w; y,T) = b_o + b_y \ln y + \sum_{i=1}^{n} b_{io} \ln w_i + \tfrac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} \ln w_i \ln w_j + \sum_{i=1}^{n} b_{it} \ln w_i \ln T + b_t \ln T + b_{ty} \ln T \ln y + \tfrac{1}{2} b_{yy} (\ln y)^2 + \tfrac{1}{2} b_{tt} (\ln T)^2$$

[4] It is acknowledged that this cost function approach does not overcome econometric difficulties associated with changes in the composition of output if, as is the case with the majority of Australia's "sheep properties", there is more than one output. A consideration of the potential implications of aggregating output in this type of analysis is beyond the scope of this study.

[5] Diewert's (1971) generalized Leontief cost function also provides a second order approximation and could alternatively be used.

where C is total cost, w_i is the price of the i'th input and T is time.[6] Linear homogeneity of the cost function and symmetry of the input price parameters impose the following restrictions on the parameters:

$$(2) \quad \sum_{i=1}^{n} b_{io} = 1; \quad b_{ij} = b_{ji}; \quad \sum_{i=1}^{n} b_{ij} = 0 \text{ for all } j; \quad \sum_{i=1}^{n} b_{it} = 0.$$

The input demand functions derived by differentiating the translog cost function with respect to input prices (Shephard's lemma) are not linear in the unknown parameters. However, the input share equations derived from the above translog cost function are linear in unknown parameters. That is, assuming cost minimizing behaviour by producers who are input price takers, the following linear equations represent the cost shares of the n inputs:

$$(3) \quad S_i = \partial \ln C / \partial \ln w_i = b_{io} + \sum_{j=1}^{n} b_{ij} \ln w_j + b_{it} \ln T, \; i = 1, \ldots, n$$

where S_i is the share of input i in total cost. The b_{it} coefficients measure the rates of bias of technical change. A positive value of the coefficient indicates that technical change has been relatively input i—using while a negative value indicates that it has been relatively input i—saving. If all b_{it} coefficients are zero, Hicks neutral technical change is indicated.

The Allen-Uzawa elasticities of substitution between inputs i and j (σ_{ij}) can be derived from the first and second partial derivatives of the cost function with respect to the i'th and j'th input prices as follows:

$$(4) \quad \sigma_{ij} = C \, . \, C_{ij} / C_i \, . \, C_j$$

where subscripts on C refer to partial derivatives. Diewert (1974, p. 114) aptly describes this elasticity of substitution σ_{ij} as "a normalization of the response of input i to a change in the price of input j, $\partial x_i / \partial w_j$ where the normalization is chosen so that $\sigma_{ij} = \sigma_{ji}$ and so that σ_{ij} is invariant to changes in the scale of measurement of the inputs". In the case of the translog cost function equation (4) reduces to

$$(5) \quad \sigma_{ij} = \frac{b_{ij}}{S_i S_j} + 1, \text{ for all } i, j; \; i \neq j$$

$$(6) \quad \sigma_{ii} = S_i^{-2} (b_{ii} + S_i^2 - S_i) \text{ for all } i$$

The σ_{ii} terms are related to the own-price elasticities of demand (η_{ii}) as follows:

$$(7) \quad \eta_{ii} = S_i \sigma_{ii} = \frac{b_{ii}}{S_i} + S_i - 1 \text{ for all } i$$

Similarly, the cross-price partial elasticities of input demand can be derived as:

$$(8) \quad \eta_{ij} = S_j \sigma_{ij} = \frac{b_{ij}}{S_i} + S_j \text{ for all } i, j; \; i \neq j$$

[6] We have adopted a more general approach than Binswanger which, while resulting in the same estimating equations, avoids a number of the difficulties he encountered. By explicitly assuming separability between inputs and output our approach overcomes the need for a rather lengthy and frail argument to replace the output variable, y, with time, t. Note also that the assumption of separability with respect to inputs and output implies that scale changes have a neutral effect on the composition of inputs and hence permits the cross product terms $\sum_{i=1}^{n} b_{iy} \ln w_i \ln y$ to be omitted. The reality of this assumption may be taken up in future work.

All the parameters required to estimate the elasticities can be derived from the system of input share equations.

Due to the absence of adjustment constraints within this production model these are all implicitly long run elasticities. There is assumed to be no delay in the implementation of producer decisions regarding the optimal level and composition of inputs. The extent to which these are not full adjustment, long run elasticities will depend upon the degree to which this implicit assumption is an abstraction from reality.

The error terms in the input share equations were assumed to be contemporaneously correlated. The fact that the input shares must sum to unity, along with the contemporaneous residual correlation assumption, leads to the singularity of the residual covariance matrix. Following Denny and May (1978), this problem was overcome by dropping one of the equations from the estimating system and employing a generalized least squares estimator with linear cross-equation constraints. The last input share equation (that of materials and services) was, therefore, dropped from the estimating system and the remaining equations estimated simultaneously with the restrictions on parameters expressed in equation (2) imposed (Byron, 1970).

In view of the linear homogeneity and symmetry restrictions ($b_{ij} = b_{ji}$) expressed in equation (2) the four remaining equations can be simplified by expressing the explanatory variables in terms of price ratios. This leads to the following estimating system:

$$(9) \quad S_L = b_{LO} + b_{LL} \ln \frac{w_L}{w_M} + b_{LN} \ln \frac{w_N}{w_M} + b_{LV} \ln \frac{w_V}{w_M} + b_{LC} \ln \frac{w_C}{w_M} + b_{Lt} \ln T + e_1$$

$$(10) \quad S_N = b_{NO} + b_{LN} \ln \frac{w_L}{w_M} + b_{NN} \ln \frac{w_N}{w_M} + b_{NV} \ln \frac{w_V}{w_M} + b_{NC} \ln \frac{w_C}{w_M} + b_{Nt} \ln T + e_2$$

$$(11) \quad S_V = b_{VO} + b_{LV} \ln \frac{w_L}{w_M} + b_{NV} \ln \frac{w_N}{w_M} + b_{VV} \ln \frac{w_V}{w_M} + b_{VC} \ln \frac{w_C}{w_M} + b_{Vt} \ln T + e_3$$

$$(12) \quad S_C = b_{CO} + b_{LC} \ln \frac{w_L}{w_M} + b_{NC} \ln \frac{w_N}{w_M} + b_{VC} \ln \frac{w_V}{w_M} + b_{CC} \ln \frac{w_C}{w_M} + b_{Ct} \ln T + e_4$$

The associated parameter estimates for the omitted equation may be recovered from the homogeneity constraints.

Data

The data used in this study were an extension of the Tornqvist indexes of input and output quantities developed by Lawrence and McKay (1980). This is the index number formula that is superlative for the translog functional form used here to estimate the production technology (Diewert, 1976). For this study, Tornqvist price indexes and the corresponding implicit quantities were computed for the five input categories used by Lawrence and McKay (labour, land, livestock, capital, and materials and services). Conceptually, these price indexes should be those relevant to the producer's decision problem. For example, allowance should be made in the price of capital for both the investment allowance and accelerated depreciation rates. However, data limitations rendered such refinement impossible. The indexes developed for this study were derived from the BAE Australian Sheep Industry Survey average per property data for the period 1952–3 to 1976–7. The data employed in this study are presented in the Appendix, together with a discussion of the method employed in computing them. Perfect price expectations were implicitly assumed throughout this study. Alternatives such as myopic price expectations, together with a one year planning horizon, could be adopted but there seems little *a priori* reason to prefer such alternatives and an analysis of the formation of expectations is not the focus of this paper.

Results

Constrained generalized least squares estimates of the parameters of the input share equations are presented in Table 1. The parameters of the equation for materials and services were obtained using the homogeneity constraint.[7] As all the estimated shares were positive (Table 2) the estimated cost function is positive and a non-decreasing function of input prices as required for duality (at least for the range of relative prices observed over this 25-year period). Unfortunately, the estimated cost function was not generally concave in input prices as the matrix of partial elasticities of substitution was not negative semi-definite. However, all own price elasticities of input demand were negative.

Given the potential for misinterpretation of goodness-of-fit measures in systems of equations, no goodness-of-fit statistics are reported in Table 1. However, an indication of the goodness-of-fit is provided by the correlation squared between actual and estimated cost shares. The correlation squared values were: labour 0.81, land 0.88, livestock 0.71, and capital 0.77.[8] No conclusive evidence for or against the presence of first order autocorrelation was found in the estimated equations as the OLS Durbin-Watson d statistics were: labour 1.2, land 1.6, livestock 1.6, and capital 1.7.

Most of the b_{ij} parameter estimates were significantly different from zero at the 95 per cent confidence level. This implies that a Cobb-Douglas specification would be inappropriate for the Australian sheep industry since, if all b_{ij} and b_{it} coefficients are equal to zero, the translog specification collapses to that of the Cobb-Douglas. In fact, the null hypothesis of a Cobb-Douglas specification was strongly rejected with the estimated chi-square test statistic being 1,255 which is greater than the critical value at 1 per cent level of significance. The b_{ij} coefficients were used to derive elasticities of substitution and demand as outlined in the preceding section.

The significantly negative coefficients on the time variable in the labour and land equations indicate that technical change has been labour and land saving. Consequently, given this pattern of technical change the cost shares of labour and land would have declined even if all input prices had remained constant. Similarly, the positive time coefficients in the capital, livestock, and materials and services equations indicate that technical change has been relatively using in these inputs.

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[7] As indicated by theory the parameter estimates were found to be independent of which share equation was dropped.

[8] The R^2 statistics from ordinary least squares (OLS) estimation give an indication of the goodness-of-fit of each equation in isolation. The OLS R^2 values were: labour 0.83, land 0.93, livestock 0.83 and capital 0.81.

Table 1: Restricted Estimates of the Coefficients of the Input Share Demand Functions

Dependent variables[a] i	Explanatory variables[b]						
	b_{iL}	b_{iN}	b_{iV}	b_{iC}	b_{iM}[c]	b_{it}	b_{io}
L	0.0224 (0.60)	−0.0447 (−3.74)	−0.0542 (−5.21)	0.0050 (0.15)	0.0715 (2.53)	−0.0246 (−4.26)	0.3018 (26.36)
N	−0.0447 (−3.74)	0.1134 (10.28)	−0.0507 (−7.37)	0.0479 (4.85)	−0.0659 (−4.37)[d]	−0.0250 (−6.43)	0.2060 (26.67)
V	−0.0542 (−5.21)	−0.0507 (−7.37)	0.1019 (8.87)	−0.0249 (−3.49)	0.0279 (2.17)	0.0298 (6.17)	0.0964 (8.78)
C	0.0050 (0.15)	0.0479 (4.85)	−0.0249 (−3.49)	−0.0652 (−1.70)	0.0372 (1.90)	0.0076 (1.70)	0.1466 (17.65)
M[c]	0.0715 (2.53)	−0.0659 (−4.37)[d]	0.0279 (2.17)	0.0372 (1.90)	−0.0707 (−1.78)	0.0122 (1.92)	0.2492 (20.22)

[a] The letters L, N, V, C and M refer to labour, land, livestock, capital, and materials and services, respectively.

[b] Parametric restrictions imposed $\sum_{i=1}^{n} b_{io} = 1$; $\sum_{j=1}^{n} b_{ij} = 0$, $b_{ij} = b_{ji}$ $\sum_{i=1}^{n} b_{it} = 0$ for all i, j. t-statistics in parentheses.

[c] Implied estimates computed using the homogeneity constraint. t-statistics for these parameters were obtained by estimating the system of equations generated by omitting the land share equation rather than that for materials and services.

[d] t-statistic was obtained by estimating the set of share equations generated by dropping the capital equation.

Table 2: Input Cost Shares in the Australian Sheep Industry 1952–53 *to* 1976–77[a] *(percentages)*

Year	S_L		S_N		S_V		S_C		S_M	
	Actual	Estimated	Actual	Estimated	Actual	Estimated	Actual	Estimated	Actual	Estimated
1952–53	26.9	30.0	14.9	16.4	16.3	13.6	13.7	14.3	28.1	25.7
1953–54	27.6	26.6	14.5	13.0	16.1	18.6	13.8	14.1	27.9	27.8
1954–55	27.7	27.1	14.5	13.7	16.4	16.7	14.7	14.9	26.7	27.5
1955–56	26.2	26.2	14.2	13.3	18.7	17.9	15.0	14.8	25.9	27.8
1956–57	25.3	24.9	14.4	13.4	19.7	19.0	14.9	14.8	25.8	27.9
1957–58	25.8	24.9	15.2	15.6	16.2	16.7	16.1	16.1	26.7	26.6
1958–59	27.2	25.3	15.7	16.0	13.7	15.6	17.4	16.2	26.0	26.8
1959–60	27.4	25.1	15.0	15.4	14.6	15.8	16.7	16.2	26.3	27.4
1960–61	24.9	23.9	15.6	15.9	16.8	16.8	16.4	16.2	26.3	27.3
1961–62	25.4	23.8	16.0	16.4	15.4	16.1	16.8	16.4	26.4	27.4
1962–63	25.1	23.9	15.4	16.2	15.4	16.0	16.6	16.5	27.5	27.4
1963–64	22.6	23.1	14.5	15.0	17.2	17.7	16.5	16.0	29.1	28.2
1964–65	21.0	21.7	15.4	16.5	18.5	17.6	16.4	16.5	28.8	27.7
1965–66	20.3	21.7	15.3	16.1	19.5	17.9	16.6	16.6	28.3	27.7
1966–67	21.3	21.7	15.3	15.3	15.7	18.5	16.7	16.5	31.0	28.0
1967–68	19.4	20.9	19.6	19.2	13.9	15.8	18.8	18.3	28.3	25.8
1968–69	19.3	20.6	18.8	19.4	16.5	15.8	17.7	18.2	27.7	26.1
1969–70	18.9	20.2	19.6	20.3	16.9	15.2	18.5	18.1	26.1	26.1
1970–71	18.8	20.7	20.6	21.5	16.0	13.6	19.2	18.5	25.4	25.7
1971–72	20.0	21.7	20.2	21.4	16.1	12.4	17.6	18.5	26.0	26.0
1972–73	18.6	20.3	19.7	19.7	16.8	14.9	16.7	17.9	28.1	27.2
1973–74	18.9	17.4	19.6	18.9	18.4	18.3	16.5	17.0	26.6	28.4
1974–75	25.2	24.0	24.1	21.3	7.0	8.8	16.4	18.0	27.2	27.8
1975–76	24.4	23.1	21.5	20.3	9.3	11.0	17.1	17.3	27.6	28.3
1976–77	23.8	23.4	20.2	20.5	9.2	9.9	19.1	17.5	27.8	28.8

[a] The letters L, N, V, C and M refer to labour, land, livestock, capital, and materials and services, respectively.

The actual and estimated shares of the five inputs in total cost are presented in Table 2. The cost shares of labour and livestock have been falling throughout the period. However, the most noticeable decline in livestock's share came in 1974–5 with the substantial reduction in livestock prices which occurred at that time. The shares going to both land and capital have increased over the period while the share of materials and services has remained relatively constant.

The estimated Allen-Uzawa elasticities of substitution are presented in Table 3 for five of the twenty-five years, spaced at six-yearly intervals, and for mean input prices. A negative value of the partial elasticity of substitution indicates that the pair of inputs are complements, while a positive value indicates they are substitutes. Five pairs of inputs—labour and capital, labour and materials, capital and land, livestock and materials, and capital and materials—were found to be strongly substitutable.

Table 3: Estimated Allen-Uzawa Elasticities of Substitution[a] [b]

E.S.	1952–3	1958–9	1964–5	1970–1	1976–7	Mean Prices
σ_{LN}	0.090	−0.104	−0.249	−0.004	0.068	−0.103
σ_{LV}	−0.330	−0.370	−0.416	−0.931	−1.343	−0.588
σ_{LC}	1.116	1.122	1.140	1.131	1.122	1.132
σ_{LM}	1.926	2.054	2.190	2.345	2.063	2.159
σ_{NV}	−1.284	−1.026	−0.744	−0.736	−1.500	−0.853
σ_{NC}	3.047	2.846	2.765	2.205	2.341	2.577
σ_{NM}	−0.567	−0.535	−0.445	−0.191	−0.117	−0.333
σ_{VC}	−0.281	0.020	0.144	0.009	−0.443	0.028
σ_{VM}	1.800	1.665	1.572	1.799	1.981	1.670
σ_{CM}	2.007	1.852	1.813	1.779	1.739	1.801

[a] The letters L, N, V, C and M refer to labour, land, livestock, capital, and materials and services, respectively.

[b] While acknowledging that it would be desirable to have confidence limits for our elasticity estimates we do not know of any means of calculating them. We are aware that Binswanger (1974) computed what he called "standard errors" for his elasticity estimates as follows:

$$SE(\sigma_{ij}) = SE(b_{ij})/S_iS_j \quad \text{etc.}$$

However, this is incorrect as S_i and S_j are functions of the estimated parameters and also have standard errors. The standard error of σ_{ij} is a function not simply of the standard error of b_{ij} but standard errors of S_i and S_j and the relationship between b_{ij}, S_i and S_j as well.

Previous Australian studies have concentrated on the elasticity of substitution between labour and capital and have found the value of this elasticity to be low. The estimate obtained in this study for the corresponding partial elasticity, however, was relatively high (1.1 at mean prices), indicating that capital and labour can substitute for one another with relative ease. This is close to Binswanger's (1974) estimate of the elasticity of substitution between labour and machinery in U.S. agriculture of 0.9. In contrast, Vincent's (1977) estimate for the elasticity of substitution between labour and capital was 0.1.

High positive values (*i.e.*, values greater than one) of the partial elasticity of substitution were also obtained for land and capital, capital and materials, labour and materials, and livestock and materials. This reflects the alternative intensities with which these input pairs can be used. The elasticity of substitution between livestock and capital, although positive, at mean prices was close to zero for all observed sets of prices. Similarly, while labour and land were weakly complementary at mean prices, the sign of this elasticity changed over the period as the vector of input prices were changed.

The three input pairs; labour and livestock, land and livestock, and land and materials and services were found to be complements for all observed combinations of input prices. This suggests that livestock input was connected with both land and labour in such a way that an increase in the price of any one of these inputs would have led to a decrease in the quantity demanded of all three. When the five inputs specified in this study were considered together these three inputs were found to be complementary. Hence, an increase in wages relative to the price of other inputs would have led not only to a decrease in the quantity of labour employed but to a decrease in the quantity of livestock and land demanded and presumably a decline in the relative importance of the livestock enterprise.

The own and cross partial price elasticities of input demand are reported in Table 4. The own-price elasticity of demand measures the change in quantity demanded of an input in response to a change in its own price. For normal goods, the own-price elasticity of demand will be negative. All estimated own-demand elasticities were negative. The estimated own-price elasticity of demand for capital at mean prices was -1.2 while those for labour, land and livestock were -0.7, -0.2 and -0.2, respectively. The results correspond reasonably closely to both Binswanger's (1974) estimates of the elasticity of demand for machinery, labour and land in U.S. agriculture of -1.1, -0.9 and -0.3, respectively, and Ryan and Duncan's (1974) estimates for Australian agriculture of the elasticity of demand for total capital and labour of -0.9 and -0.5, respectively. These results suggest that although the quantity of labour employed would fall in response to a wage increase this fall would not be sufficient to avoid an increase in the average property's wage bill. The own-price elasticity of the demand for materials and services was close to unity.

The cross-price elasticity of input demand measures the change in the quantity demanded of an input in response to a change in the relative price of another input. The estimates of the cross-price elasticities of input demand obtained in this study were generally small in magnitude and in all cases the value was less than one.

Conclusion

The estimation of a system of generalized least squares input share equations derived from a translog cost function has provided estimates of the elasticities of substitution between inputs and of own price elasticities of input demand for the Australian sheep industry. Allowance for biased technological change has also provided information on the nature of technical change in the sheep industry and on the appropriateness of the assumption of Hicks-neutral technical change.

Table 4: Price Elasticities of Input Demand[a]

E.D.	1952–3	1958–9	1964–5	1970–1	1976–7	Mean Prices
LL	−0.625	−0.658	−0.680	−0.685	−0.670	−0.675
LN	0.015	−0.017	−0.041	−0.001	0.014	−0.018
LV	−0.045	−0.058	−0.073	−0.126	−0.133	−0.089
LC	0.160	0.182	0.188	0.209	0.196	0.191
LM	0.495	0.551	0.606	0.603	0.593	0.592
NL	0.027	−0.026	−0.054	−0.001	0.016	−0.023
NN	−0.143	−0.130	−0.146	−0.257	−0.241	−0.189
NV	−0.174	−0.160	−0.131	−0.100	−0.148	−0.129
NC	0.437	0.462	0.456	0.408	0.409	0.435
NM	−0.146	−0.144	−0.123	−0.049	−0.034	−0.091
VL	−0.099	−0.094	−0.090	−0.193	−0.314	−0.132
VN	−0.210	−0.164	−0.123	−0.158	−0.308	−0.154
VV	−0.113	−0.192	−0.246	−0.113	−0.129	−0.177
VC	−0.040	0.003	0.024	0.002	−0.077	0.005
VM	0.463	0.447	0.435	0.463	0.570	0.458
CL	0.335	0.284	0.247	0.234	0.263	0.255
CN	0.499	0.456	0.456	0.474	0.480	0.465
CV	−0.038	0.003	0.025	0.001	−0.044	0.004
CC	−1.312	−1.239	−1.230	−1.167	−1.199	−1.217
CM	0.516	0.497	0.502	0.458	0.500	0.494
ML	0.579	0.520	0.475	0.485	0.483	0.486
MN	−0.093	−0.086	−0.073	−0.041	−0.024	−0.060
MV	0.244	0.260	0.277	0.244	0.196	0.254
MC	0.288	0.301	0.299	0.329	0.304	0.304
MM	−1.018	−0.995	−0.978	−1.017	−0.958	−0.983

[a] The letters L, N, V, C and M refer to labour, land, livestock, capital, and materials and services respectively.

While the demand for labour was inelastic with respect to its own price the demand for capital was elastic. All cross-price demand elasticities estimated were less than one. In contrast with earlier Australian studies, the Allen-Uzawa elasticity of substitution between labour and capital was found to be close to unity. Hence, the capital to labour ratio has been reasonably sensitive to changes in the price of labour relative to that of capital.

Technical change has been relatively labour and land saving and relatively capital, livestock, and materials and services using.

Clearly, policy and other changes which affect relative prices influence the desired level and composition of inputs in the Australian sheep industry.[9] More specifically, as the elasticity of substitution between capital and labour has been close to one, the investment allowance policy for example has lead to an increase in the capital to labour ratio by causing the price of capital relative to that of labour to be lower than it would have been otherwise. Similarly, if the past pattern of technical change continues, in the absence of relative input price changes, there will be a decline in the demand for labour and land relative to that for capital, livestock, and materials and services.

[9] The robustness of these policy implications inevitably depends upon the accuracy with which this model reflects the production relationships within the Australian sheep industry.

Appendix

The data employed in this analysis were Tornqvist price indexes for the five input categories, labour, land, livestock, capital, and materials and services, together with the corresponding implicit quantity indexes. These average per property indexes are presented in Table A.1, and the shares in total costs of the five input categories appear in Table 2.

The Tornqvist (1936) index is quite flexible as it is based on a homogeneous translog production function which provides a second-order approximation to an arbitrary production function at any given point (Christensen, Jorgensen and Lau, 1973). It can precisely reflect an arbitrary set of substitution possibilities at any given feasible point. The formula for the Tornqvist price index in log change form for an input group consisting of n items is as follows:

$$\log (P_t/P_{t-1}) = \sum_{i=1}^{n} v_{it} \log (p_{it}/p_{it-1})$$

where

$$v_{it} = (p_{it}\, q_{it} / \sum_{j=1}^{n} p_{jt}\, q_{jt} + p_{it-1}\, q_{it-1} / \sum_{j=1}^{n} p_{jt-1}\, q_{jt-1})/2$$

p_{it} is the price of item i at time t;

q_{it} is the quantity of item i at time t; and

P_t is the price of the input group at time t.

Clearly, a price, quantity and dollar value were required for each item in each of the five input groups to calculate the five Tornqvist price indexes. The value of inputs was also used in calculating the market share of each input category.

Labour inputs included all labour used in operating the property. That is, the operator's own labour, his family's labour and all farm operation contracts were included in addition to hired labour. Similarly, all materials and services used by the property, including maintenance of plant and improvements, were contained in the materials and services category.

The price of the three durable input categories (land, livestock and capital) attempts to represent the price of the service flow from those inputs as opposed to the stock price of the durable inputs. Durable inputs are not completely consumed in the year of purchase but provide a flow of services over several years. The value of this service flow consists of two components: depreciation and opportunity cost. A lack of data precluded the incorporation of capital gains. The value of both purchases and any reductions in inventories was added in the case of livestock. The quantity of service flow from capital and land was assumed to be proportional to the quantity of the stock of the input held. Livestock input quantity was taken to be proportional to the sum of opening numbers, purchases and reductions in on-farm inventories during the year. A more detailed discussion of the treatment of durable inputs and of the components of each input category is presented in Lawrence and McKay (1980).

Appendix Table A.1: Tornqvist Price Indexes and Implicit Quantities[a] *(average per property)*

Year	Labour price index	Land price index	Livestock price index	Capital price index	Materials and services price index	Implicit labour quantity	Implicit land quantity	Implicit livestock quantity	Implicit capital quantity	Implicit materials and services quantity
1952–53	82.192	69.757	105.830	72.728	88.819	39.044	25.466	18.333	22.505	37.745
1953–54	85.556	70.034	145.145	72.502	87.522	39.761	25.530	13.678	23.494	39.315
1954–55	85.411	70.483	108.254	75.147	87.371	40.228	25.573	18.848	24.351	37.891
1955–56	86.157	73.981	116.746	81.514	91.262	40.520	25.592	21.303	24.431	37.691
1956–57	90.075	84.426	133.487	90.180	96.989	42.296	25.630	22.257	24.871	40.086
1957–58	93.237	100.724	113.217	94.230	100.383	45.888	24.936	23.650	28.391	44.095
1958–59	95.337	100.535	99.270	98.409	100.012	45.549	25.024	22.084	28.244	41.600
1959–60	100.000	100.000	100.000	100.000	100.000	46.040	25.212	24.618	28.004	44.212
1960–61	101.143	112.359	114.219	108.030	103.622	44.965	25.432	26.930	27.785	46.319
1961–62	104.754	117.375	108.684	112.276	103.991	45.711	25.631	26.760	28.179	47.825
1962–63	104.758	116.165	104.205	112.662	105.307	46.405	25.687	28.580	28.454	50.450
1963–64	104.897	112.043	117.835	112.887	103.419	42.985	25.870	29.181	29.073	56.196
1964–65	110.915	137.828	132.021	120.602	106.541	42.318	24.954	31.298	30.293	60.302
1965–66	116.267	143.344	137.760	125.682	112.899	40.700	25.001	32.994	30.910	58.609
1966–67	120.752	142.287	144.200	129.208	117.868	42.123	25.662	26.061	30.870	62.705
1967–68	124.540	197.554	130.809	133.685	121.384	40.977	26.064	27.970	37.027	61.196
1968–69	128.334	204.345	133.835	140.004	122.359	42.789	26.127	35.043	36.013	64.296
1969–70	132.099	219.707	133.290	149.746	121.545	42.066	26.118	37.282	36.164	62.893
1970–71	137.461	236.882	119.226	159.591	125.167	39.946	25.364	39.029	35.020	59.058
1971–72	147.041	232.127	107.612	169.021	130.235	41.772	26.688	46.028	32.030	61.375
1972–73	160.613	240.611	145.273	177.944	136.637	40.747	28.868	40.722	33.058	72.361
1973–74	195.180	312.072	255.601	216.183	154.003	44.906	29.117	33.420	35.327	79.997
1974–75	275.438	326.232	123.027	307.055	205.306	39.035	31.483	24.084	22.818	56.504
1975–76	299.457	362.341	166.224	361.103	235.691	40.779	29.582	28.070	23.701	58.537
1976–77	350.289	395.917	167.994	396.797	251.539	41.978	31.542	33.885	29.751	68.339

[a] Implicit quantity indexes were obtained by dividing the current dollar value of each input by its corresponding Tornqvist price index. Hence the dollar value of each input can be obtained by multiplying the Tornqvist price index by the relevant implicit quantity.

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