AJAE Appendix:

Grading, Minimum Quality Standards, and the Labeling of Genetically Modified Products

Harvey Lapan and **GianCarlo Moschini**

September 2006

The material contained herein is supplementary to the article named in the title and published in the ***American Journal of Agricultural Economics*** (***AJAE***).

Harvey Lapan is University Professor and GianCarlo Moschini is Professor and Pioneer Chair in Science and Technology Policy, both with the Department of Economics, Iowa State University, Ames, IA.

Proof of Proposition 5 – Comparative Statics Results

With a uniform distribution of types $H(\beta)=\beta$, and thus in the uncovered market when both GM and non-GM products are produced, market demands are, respectively, $D_g=\hat{\beta}$ and $D_n=\tilde{\beta}-\hat{\beta}$, so that total demand is $D_T\equiv D_n+D_g=\tilde{\beta}$. Upon recalling the arbitrage relations of competitive equilibrium, that is,

$$p_n^0=p_g^0+\delta$$

$$p_g^1=p_g^0+\eta$$

$$\left(p_n^1-p_g^1\right)F(R)=\delta+\sigma$$

we have

$$\tilde{\beta}(R)\equiv\frac{u-p_g^0-\eta-\dfrac{\delta+\sigma}{F(R)}}{a\overline{s}(R)}$$

$$\hat{\beta}(R)=\frac{\delta+\sigma}{aF(R)\left[1-\overline{s}(R)\right]}\,.$$

In what follows we simplify notation and omit the functional dependence on R by writing $F(R)=F$, $f(R)=f$ and $\overline{s}(R)=\overline{s}$. Also, we define $k\equiv\delta+\sigma$, $A\equiv u-p_g^0-\eta$, and $P\equiv p_g^0$, so that

$$\hat{\beta}=\frac{k}{Fa(1-\overline{s})}$$

$$\tilde{\beta}=\frac{AF-k}{Fa\overline{s}}\,.$$

Aggregate consumer surplus here is $CS=\frac{1}{2}a\left[\tilde{\beta}^2\overline{s}+\hat{\beta}^2(1-\overline{s})\right]$. Substituting and simplifying obtains

$$CS=\frac{1}{2a}\left[\frac{\left(A-\dfrac{k}{F}\right)^2}{\overline{s}}+\frac{\left(\dfrac{k}{F}\right)^2}{(1-\overline{s})}\right].$$

Hence, the welfare function is

$$W = \Pi(P) + \frac{1}{2a}\left[\frac{\left(A - \frac{k}{F}\right)^2}{\bar{s}} + \frac{\left(\frac{k}{F}\right)^2}{(1-\bar{s})}\right]$$

where $\Pi(P)$ is producer surplus. The optimality conditions for welfare maximization (yielding the optimal standard purity R^* and the competitive farm-level equilibrium price P^*) are

(1) $$W_P = \Pi'(P) - \frac{1}{a}\frac{\left(A - \frac{k}{F}\right)}{\bar{s}} = 0$$

(2) $$W_R = \frac{1}{2a}\left[\frac{2\left(A - \frac{k}{F}\right)\frac{k}{F^2}f}{\bar{s}} - \frac{\left(A - \frac{k}{F}\right)^2}{\bar{s}^2}\frac{f}{F}(R-\bar{s}) - \frac{2\left(\frac{k}{F}\right)^2\frac{f}{F}}{(1-\bar{s})} + \frac{\left(\frac{k}{F}\right)^2}{(1-\bar{s})^2}\frac{f}{F}(R-\bar{s})\right] = 0.$$

Upon substitution and simplification we obtain

(3) $$W_R = \left(\frac{af}{2F}\right)\left(\tilde{\beta} - \hat{\beta}\right)\left[\hat{\beta}\left((1-\bar{s}) + (1-R)\right) - \tilde{\beta}\left(R-\bar{s}\right)\right] = 0 \rightarrow \left[\hat{\beta}\left((1-\bar{s}) + (1-R)\right) - \tilde{\beta}\left(R-\bar{s}\right)\right] = 0.$$

Consider now the comparative statics effect of the parameter $k \equiv \sigma + \delta$. Differentiating the optimality conditions in (1) and (2) and expressing the results in matrix form yields

$$\begin{bmatrix} W_{RR} & W_{RP} \\ W_{PR} & W_{PP} \end{bmatrix}\begin{bmatrix} R_k \\ P_k \end{bmatrix} = \begin{bmatrix} -W_{Rk} \\ -W_{Pk} \end{bmatrix}.$$

Solving by Cramer's rule obtains

$$R_k = \frac{1}{\Delta}\begin{vmatrix} -W_{Rk} & W_{RP} \\ -W_{Pk} & W_{PP} \end{vmatrix} = \frac{-W_{Rk}W_{PP} + W_{Pk}W_{RP}}{\Delta}$$

$$P_k = \frac{1}{\Delta}\begin{vmatrix} W_{RR} & -W_{Rk} \\ W_{PR} & -W_{Pk} \end{vmatrix} = \frac{-W_{RR}W_{Pk} + W_{PR}W_{Rk}}{\Delta}$$

where $\Delta = \begin{vmatrix} W_{RR} & W_{RP} \\ W_{PR} & W_{PP} \end{vmatrix} < 0$, $W_{RR} < 0$ and $W_{PP} > 0$ by the second-order conditions of the welfare optimization problem (saddle point).

We now compute the partial effects that enter these comparative statics expressions. Differentiating the optimality conditions in (1) and (2) yields

$$W_{PR} = -\frac{1}{a}\frac{k}{F^2}\frac{f}{\bar{s}} + \frac{1}{a}\frac{\left(A - \frac{k}{F}\right)}{\bar{s}^2}\frac{f}{F}(R - \bar{s})$$

which can be simplified to

$$W_{PR} = \frac{f}{F\bar{s}}\left[-\hat{\beta}(1-\bar{s}) + \tilde{\beta}(R-\bar{s})\right].$$

Evaluating this partial effect at the optimality conditions, such that (3) holds, we obtain

$$W_{PR} = \frac{f}{F\bar{s}}\hat{\beta}(1-R) \quad > 0 .$$

Next, differentiating (1) we find

$$W_{Pk} = \frac{1}{aF\bar{s}} > 0 .$$

And differentiating (2) we obtain

$$W_{Rk} = \frac{1}{a}\left[-\frac{kf}{F^3\bar{s}} + \left(A - \frac{k}{F}\right)\frac{1}{\left(F\bar{s}\right)^2}\left(f\bar{s} + f(R-\bar{s})\right) - \frac{k}{F^3}\frac{f}{(1-\bar{s})} + \frac{k}{F^2}\frac{1}{(1-\bar{s})^2}\frac{f}{F}(R-\bar{s})\right]$$

which simplifies to

$$W_{Rk} = \frac{f}{F^2}\frac{1}{\bar{s}(1-\bar{s})}\left[\tilde{\beta}R(1-\bar{s}) - \hat{\beta}\left((1-\bar{s})^2 + \bar{s}(1-R)\right)\right].$$

Evaluating this partial effect at the optimality conditions, such that (3) holds, we obtain

$$W_{kR} = \frac{f}{F^2}\frac{1}{\bar{s}(1-\bar{s})}\hat{\beta}\left[\frac{(1-R)\left[R(1-\bar{s}) - \bar{s}(R-\bar{s})\right] + \bar{s}(1-\bar{s})^2}{(R-\bar{s})}\right].$$

Thus, a sufficient condition for $W_{kR} > 0$ is $R(1-\bar{s}) \geq \bar{s}(R-\bar{s})$, which does hold because $R \geq \bar{s}$ and $R \leq 1$. Hence, we conclude that $W_{kR} > 0$.

The foregoing partial effects allow us to sign the comparative statics effect on farm price:

$$P_k = \frac{W_{RR}W_{Pk} - W_{PR}W_{Rk}}{-\Delta} < 0 .$$

But $sign(R_k) = sign(W_{Rk}W_{PP} - W_{Pk}W_{RP})$. Note that

$$W_{PP} = \Pi''(p_g^0) + \frac{1}{a\bar{s}}.$$

Because $\Pi''(p_g^0) = S'(p_g^0) > 0$ (the profit function is convex) and $W_{Rk} > 0$, to conclude that $R_k > 0$ it suffices to show that $Z \equiv W_{Rk}\frac{1}{a\bar{s}} - W_{Pk}W_{RP} \geq 0$. From earlier derivations,

$$Z \equiv \frac{f}{F^2}\frac{1}{\bar{s}(1-\bar{s})}\hat{\beta}\left[\frac{(1-R)\left[R(1-\bar{s}) - \bar{s}(R-\bar{s})\right] + \bar{s}(1-\bar{s})^2}{(R-\bar{s})}\right]\frac{1}{a\bar{s}} - \frac{1}{aF\bar{s}}\frac{f}{F\bar{s}}\hat{\beta}(1-R)$$

which can be simplified to yield

$$Z = \frac{1}{a}\frac{f}{F^2}\frac{1}{\bar{s}^2(1-\bar{s})}\hat{\beta}\left[\frac{\bar{s}(1-R)^2 + \bar{s}(1-\bar{s})^2}{(R-\bar{s})}\right] > 0$$

and so we can conclude that $R_k > 0$. Recalling that $k \equiv \delta + \sigma$, we have therefore established parts (i) and (ii) of Proposition 4.

The comparative statics analysis for the parameter k is readily adapted to the comparative statics of the "GM aversion" parameter a. Specifically,

$$R_a = \frac{W_{Ra}W_{PP} - W_{Pa}W_{RP}}{-\Delta}$$

$$P_a = \frac{W_{RR}W_{Pa} - W_{PR}W_{Ra}}{-\Delta}.$$

The partial effects of interest here are

$$W_{Pa} = \frac{1}{a^2F\bar{s}}(AF - k) = \frac{1}{a}\tilde{\beta} > 0$$

$$W_{Ra} = -\frac{1}{a}W_R = 0$$

and so we find

$$P_a = \frac{W_{RR}W_{Pa}}{-\Delta} < 0$$

$$R_a = \frac{-W_{Pa}W_{RP}}{-\Delta} < 0$$

which establishes part (iii) of Proposition 4.

Finally, concerning the parameters u and η, we note that they enter the problem only through the term $A \equiv u - p_g^0 - \eta$. For the comparative statics of this term we have

$$R_A = \frac{W_{RA}W_{PP} - W_{PA}W_{RP}}{-\Delta}$$

$$P_A = \frac{W_{RR}W_{PA} - W_{PR}W_{RA}}{-\Delta}.$$

The partial effects of interest here are

$$W_{PA} = -\frac{1}{a\bar{s}} < 0$$

and $W_{RA} < 0$ because $W_{RA} = -W_{PR}$ and we showed earlier that $W_{PR} > 0$. Thus we can immediately conclude that $P_A > 0$. The sign of R_A is the sign of $Z \equiv \left(W_{RA}W_{PP} - W_{PA}W_{RP}\right)$. By using $W_{RA} = -W_{PR}$ we find $Z \equiv W_{RA}\left(W_{PP} + W_{PA}\right)$, and by noting that $W_{PP} = \Pi''(p_g^0) - W_{PA}$ we get $Z \equiv W_{RA}\Pi''(p_g^0) < 0$, and so we conclude that $R_A < 0$. Recalling again that $A \equiv u - p_g^0 - \eta$, this concludes the comparative statics of parameters u and η (part (iv) of Proposition 5). ■