Comparative Advantage: From an Individual to the Economy

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Selected Paper prepared for presentation at the Agricultural & Applied Economics Association 2010 AAEA, CAES, & WAEA Joint Annual Meeting, Denver, Colorado, July 25-27, 2010

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ABSTRACT

This paper identifies an internal inconsistency in the Heckscher-Ohlin (H-O) models of international exchange. The inconsistency stems from assuming homogeneity of inputs within a population. This assumption annihilates individual comparative advantage, benefits from exchange and, consequently, existence of autarky prices. In order to remove this inconsistency, I build a two-good multi-individual model by using the microeconomic concept of individual comparative advantage stemming from differences in endowments of qualitatively heterogeneous inputs. The model shows how differences in the distribution of individual production possibilities result in individual specialization, exchange and differences in autarky prices between hypothetically isolated economies. Next, the effect of preference heterogeneity, learning by doing and supply restrictions is examined. In addition to bringing internal logical consistency into the theory of cross-border exchange by demonstrating how price differences between hypothetically isolated economies can be derived from the general neoclassical and Austrian subjectivist principles, this paper addresses the criticisms raised by the labour value theorists. The model can be refined to include comparative advantage in the production of capital goods and differences in the distribution of ownership over natural resources and capital goods. This theoretical approach to inter-local exchange has important policy implications. While the H-O framework lends itself well to conflicting interventionist policies of production allocation based on different interpretations of ambiguous aggregate data, the alternative microeconomic approach acknowledges the importance of the institutional setting in which individual comparative advantage, unknown to an external observer, is discovered, enhanced, and expressed.

Introduction

The concept of comparative advantage has had a central role in modern economics for more than two hundred years. However, there are important differences in how the concept has been interpreted and analyzed within different economic schools of thought. For some theorists, following the marginalist tradition of Menger ([1870], 2007), comparative advantage can only apply to acting individuals. This is consistent with the principles of methodological individualism that the theorists such as von Mises ([1949], 1996), Rothbard ([1962], 2009), von Hayek (1945), Buchanan ([1969], 1978), Kirzner (1978), Lucas (1996), and Boettke (2001) adhere to. Within this approach, macroeconomic phenomena are strictly derived from the actions of individuals.

Unlike the followers of the Mengerian approach, some theorists that follow the Walrasian tradition of mathematical modeling use macroeconomic models that often take for granted the fact that economic activity is performed by motivated individuals. Often times, the analysis starts with aggregate variables with no clear relation to individual action. Within this approach, the concept of comparative advantage is presented both at the individual and aggregate levels but without explicit cross-reference (Ohlin, [1924], 1991; Stolper and Samuelson, 1941; Samuelson 1953; Vanek, 1968; McAfee, 2006; Salemi, 2008; Onuma, 2008).

While the approach taken by authors following the Mengerian tradition is less mathematically formal, it does offer an indication that an overly mathematical approach might lead to logical inconsistencies. For example, aggregation of physical entities by definition requires qualitative homogeneity of these entities. But, we know that heterogeneity (or differences) among individuals is the basis for specialization and exchange – the key manifestation of individual comparative advantage. However, the main switch in the neoclassical

economic analysis of comparative advantage in the last century was from the Ricardian to the Heckscher-Ohlin macroeconomic view, with little or no reference to the relationship between the individual and the economy. Only recently have there been some (mainly cosmetic) attempts of introducing heterogeneity into the macroeconomic models of cross-boundary movements of goods.

This paper argues that:

1. The implicit assumption of the homogeneity of inputs, imposed by the mathematical formalism of the standard H-O model, annihilates the logical possibility of autarky prices;
2. This inconsistency has been used by the proponents of the labour theory of value to successfully bring into question the neoclassical concept of capital, and, at least in their view, the subjective theory of value[1];
3. The inconsistency stems from ignoring the principles of methodological individualism, and thus the remedy is to incorporate these principles back into the theory of comparative advantage;
4. Bringing internal consistency into the theory of comparative advantage will contribute to better understanding of the role of individuals and institutions in shaping comparative advantage as a dynamic process of information discovery. More generally, this approach contributes to the defense of the subjective theory of value.

The following sections elaborate on the above presented points.

[1] This seems to have lead to the complete abandonment of the notion of capital in the recent versions of the H-O model. Instead, skilled labour is used as a surrogate for capital.

Ohlin vs. Ricardo: Change of Paradigm within Macroeconomics

Ohlin (1924, [1991]) argued that the Ricardian model of international exchange suffers from a serious inadequacy – inability to explain partial specialization of countries. Ricardo, similar to Ohlin, implicitly assumed homogeneous inputs[2]. However, Ricardo used a fixed proportions aggregate technology which, Ohlin argued, was due to Ricardo's adherence to the labour theory of value[3]. Since capital and labour in the Ricardian model are used in fixed proportions in production of all goods and at any level of output, the resulting economy Production Possibilities Frontier (PPF)[4] is linear. This means that the marginal productivity of labour is constant at any aggregate level of production. Opening of trade between two nations would, according to this model, lead to complete specialization of nations. In reality, countries produce and import some products at the same time. Thus, the strict application of the Ricardian model could not explain partial specialization at the economy level nor the effect of supply and demand for final products on market prices.

Ohlin (1924, [1991]), following the initial idea of Heckscher ([1919], 1991), suggested that this inconsistency can be overcome by assuming variable proportions technology. He believed that this is what was needed to break away from Ricardo's labour theory of value and

[2] It needs to be said that Ohlin initially acknowledges the fact that intra-national markets are a consequence of heterogeneity of inputs owned by different individuals but then argues that assuming homogeneity is a useful abstraction. However, it is not obvious for what purpose this abstraction is useful other than for creating an apparent conceptual difference between exchanges based on which side of the border the participants in the exchange happen to be. This apparent difference was later cemented by Samuelson's mathematical formulation of the H-O model.

Ohlin builds his argument on the claim that factors of production are less mobile across than within country borders and that international trade in goods is a way of compensating for the inability to exchange inputs between countries. However, in the world of heterogeneous inputs, as is the world we live in, even if the exchange of *all* inputs was possible, and *all* inputs were perfectly mobile, exchange of inputs is not a substitute for the exchange of outputs. Exchange of heterogeneous inputs only results in a different distribution of property rights over these heterogeneous inputs. There is no reason why some individuals in each country would not have a comparative advantage in one of the outputs compared to some individuals in the other country. Thus, movement of goods across the border is possible in both directions even in the case of two identical countries.

[3] Assuming variable proportions technology would imply that same product could be produced using different quantities of labour, which would, within the context of the labour theory of value, imply different value (and price) for every unit produced. This result is inconsistent with the labour theory of value.

[4] The concept of Production Possibilities Frontier was introduced by Haberler (1930).

bring the theory in line with the subjective theory of value[5]. Stolper and Samuelson (1941) and Samuelson (1953) demonstrated mathematically how marginal input productivity in Ohlin's model declines with increases in output due to differences in relative factor-ratios of different outputs. This implied that the two trading countries would reach a point where marginal input productivities and, consequently, prices in the two countries are equalized.

However, while Ohlin's textual exposition of his theory left some ambiguous space for factor heterogeneity within a country, the Stolper-Samuelson mathematical formalization in the form of aggregate production function and factor ratios removed input heterogeneity as a theoretically inconvenient feature. While this may be seen as an innocent simplification to allow for analytical tractability, the implications of this approach are far-reaching. The next section elaborates on this.

Neoclassicals vs. Sraffians vs. Austrians

Boettke (1997) made a general statement about the legacy of Paul Samuelson that could be applied to his formalization of the Heckscher-Ohlin model of comparative advantage as well:

> Samuelson's synthesis created a rather strange mix of general equilibrium microeconomics with Keynesian macroeconomics. As Robert Lucas repeatedly pointed out in the early 1970s, graduate students were taught one thing during their Monday/Wednesday microeconomic theory courses, and another thing on Tuesdays and Thursdays in their macroeconomic theory courses (p. 36).

The Stolper-Samuelson formalization of the H-O theory ultimately treats the whole economy as a single optimizing entity. The qualitative homogeneity of inputs within this entity is necessary

[5] However, as noted by von Mises ([1949], 1996), the model of comparative advantage itself does not imply any theory of value. But, applying the model to a collective rather than to an individual can create a conflict with the principles of the subjective theory of value.

for arriving at mathematically tractable solutions. However, the implicit factor homogeneity makes it problematic to explain why different individuals within the economy choose to specialize in the first place.

While the Stolper-Samuelson approach was correct in identifying decreasing marginal productivity at the aggregate level (i.e. as the supply of a product increases, marginal productivity per unit of labour-time decreases), it ignored a more logical explanation for diminishing marginal productivity – input heterogeneity. Most inputs other than labour *time* are, in reality, different physical objects with different purposes in different contexts. In addition, subjective human characteristics (i.e. human capital) are not even observable, let alone measurable in meaningful units. Thus, it is logical that the capital inputs (physical and human) of lower appropriateness for production of a given output would only become profitable at higher output prices.

Finally, as recently pointed out by Subasat (2003), the unclear meaning of the aggregate quantity of heterogeneous capital creates circular reasoning within the H-O model. Similarly, Cohen and Harcourt (2003) argue persuasively that heterogeneity of capital, leading to instability of capital demand, raises serious questions about the neoclassical meaning of "capital-scarcity." Samuelson (1966), when confronted with these criticisms by Piero Sraffa and Joan Robinson in the 1950's and 60's, finally resorted to the Austrian concept of capital as time.

When it comes to comparative advantage, Austrian theorists, on the other hand, avoid the aggregate approach altogether and use essentially the same approach as most of today's microeconomics textbooks in the two-good, two-individual case (von Mises, [1949], 1996; Rothbard, [1962], 2009). However, given the general preference for textual logic within this approach, the Austrians did not expand this two-individual model to a many-individual case

using the mathematical language preferred by the followers of the Walrasian tradition to show the link between individual comparative advantage and the market exchange ratios. Similarly, neither did the followers of the Walrasian approach follow the logic of their two-individual case into an n-individual economy. Instead, they made an ontological leap of logic from the individual differences in the *quality* of inputs to the aggregate difference in the *quantity* of inputs formulated in the Heckscher-Ohlin (H-O) paradigm.

The Stolper-Samuleson formalization of the Hecksher-Ohlin model has undergone numerous modification and extensions since 1941 by Samuelson (1953), Rybczynski (1955), Vanek (1968) and many others. However, the unit of analysis – whole economy or country – has remained unchanged. Firms and markets are assumed but, ultimately, the logical disconnect between the existence of different industries, market prices and the input homogeneity implied by the quantitative factor-ratio framework is not resolved.

Subasat (2003) claims that the H-O model had a ideological mission – to discourage policies directed at capital accumulation in the "labour-abundant" countries in the first half of the 20^{th} century, mainly former colonies. While the truthfulness of this claim may never be determined, there are other important implications of the factor-ratio theory. Putting the source of comparative advantage in the framework of endowments of factors indistinguishable among individuals creates an impression of determinism and that cross-boundary exchange is less of an entrepreneurial choice compared to within-boundary exchange. While removing the institutional obstacles for exchange is a political issue, there is no apparent reason why motivations of individuals to engage in exchange with other individuals across borders would stem from economic principles different from the principles governing any individual exchange.

The thesis of this paper is that these internal inconsistencies can be overcome by a more complete integration of the principles of methodological individualism and subjective theory of value – the alleged cornerstones of neoclassical economics – into the analysis of comparative advantage and cross-boundary price differences. According to methodological individualism and subjectivism, choice is individual and based on subjective costs and benefits (Menger, [1870], 2007, Buchanan, [1969], 1978). Thus, choice to specialize and trade can potentially be explained as an expression of individual costs and benefits stemming from differences in individual production possibilities and preferences. Consequently, any aggregate economic phenomena involving movement of goods across national boundaries logically follows from choices of individuals.

The purpose of this paper is to take the initial formal step in integrating the principles of methodological individualism and subjectivism into the theory of comparative advantage. In the remainder of the paper, I first present the standard Heckscher-Ohlin model in more detail, and then summarize the major extensions and critiques. Next, the specifics of the inconsistency between the model's assumptions and results are identified. This is followed by the main body of the paper that illustrates how the concept of individual comparative advantage and differences in autarky prices between hypothetically isolated markets can be integrated.

The Heckscher-Ohlin Model, Extensions and Critiques

The model known today as the Heckscher-Ohlin model of international trade was mathematically formalized by Stolper and Samuelson (1941) using Ohlin's (1924) textual exposition of the Factor Endowment Theory. The model has retained this form to date. I will use the Stolper-Samuelson graphical representation to illustrate the basic components.

The model assumes that there are two goods, F and C,[6] two homogeneous inputs, capital and labour, and two countries, Home and Abroad. There is a limited amount of each input in each country. Identical Constant Returns to Scale (CRS) technology in both countries is used in production of F and C. However, the technologies differ between F and C. F is more labour-intensive and C is more capital-intensive at given input prices. This means that, in order to produce one unit of C, more capital per unit of labour is used at given factor prices. The two goods are produced by two perfectly competitive industries (i.e. no transaction costs or externalities, complete information, firms are price-takers). Inputs are perfectly mobile across industries but immobile across country boundaries.

Stolper and Samuelson then take the next step in which they analyze the whole economy as a single entity. The logic is as follows: Since inputs are homogeneous, they can be aggregated across firms and industries. Thus, the production within a country can be characterized by a single production function. The total amount of labour and capital used in the production of F and C at any given input price ratio is defined by the contract curve.

The upper panel in Figure 3.1 shows two production Edgeworth boxes for the two countries. The box $O_C^H K_0^H O_F^H L_0^H$ represents Home, while the box $O_C^A K_0^A O_F^A L_0^A$ represents Abroad. The labour-to-capital ratios in the two countries are represented by the slope of the diagonals: $O_C^H O_F^H$ for Home, and $O_C^A O_F^A$ for Abroad. From this, it can be seen that Home has a lower labour-to-capital ratio compared to Abroad. At the same time, Home has a higher capital-to-labour ratio, since this is just a reciprocal value of the labour-to-capital ratio.

The locus of optimal production points in each country is defined by the contract curves. Each country's contact curve lies below the diagonal connecting the two origins. This follows

[6] Samuelson's nomenclature of goods (good A and good B) is modified to reflect the traditional examples used by Ricardo and Mill (Food and Clothing).

from the fact that C is relatively capital-intensive and F is more labour-intensive. Thus, this is only true for the area below the box diagonal.

In the output space, the contract curve translates into the economy Production Possibilities Frontier (PPF), represented in the lower panel. Starting at O_C^H, Home produces only C, The capital-to-labour ratio is the highest at the origin and declines along the contact curve. Since *C* is relatively capital-intensive, this means that the marginal productivity in *C* declines as *C* increases. Same is true for *F*. Since *F* is relatively labour-intensive, marginal productivity for *F* declines as the output of *F* increases. As a result, the economy PPF is concave.

Since both countries use the same technology, the isoquant C represents the same quantity of C in both countries. The isoquants F^H and F^A represent quantities of F at Home and Abroad respectively for a given quantity of C. The Abroad contract curve is steeper compared to Home at any quantity of C. This means that the capital-to-labour ratio used in production of any quantity of C at Home is higher compared to Abroad. Since C is the capital-intensive product, marginal input productivity in C at Home is higher at any given level of C. This means that for a given change in F, Home will have a greater change in C, at any level of C. This implies that the Home PPF is steeper than Abroad, when C is on the vertical axis.

In other words, at any given level of C, Home has to "give up" less F in order to increase its output of C by one unit. This is interpreted as a lower opportunity cost of C at Home compared to Abroad. It is said that the capital-abundant Home has a comparative advantage in the capital-intensive good, C, while the labour-abundant Abroad has a comparative advantage in the labour-intensive good, F.

While Ohlin ([1924], 1991) assumes identical preferences for all individuals in the two countries and concludes that a difference in factor endowments between two countries is a

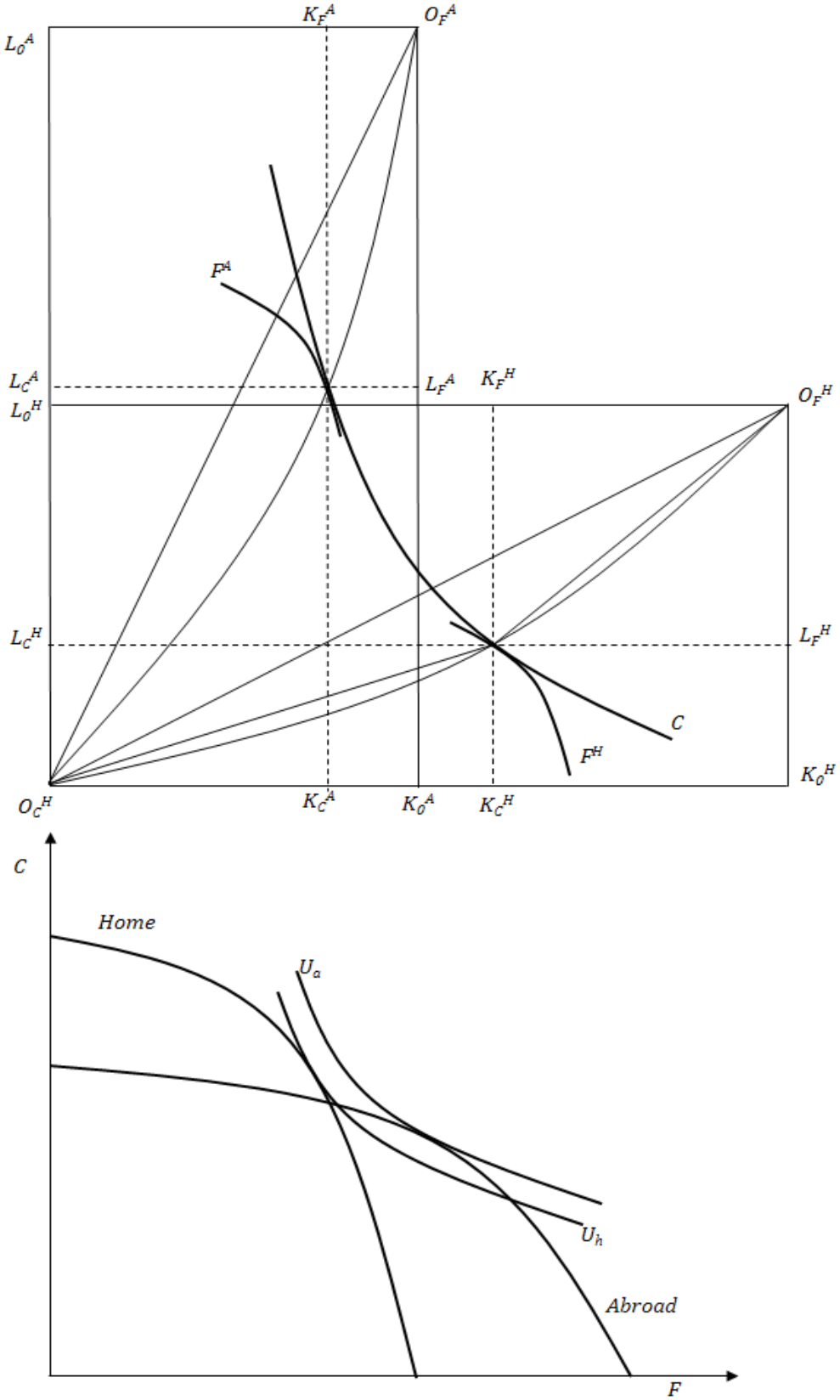


Figure 3.1. A graphical representation of the Heckscher-Ohlin model as described by Stolper and Samuelson (1941)

necessary condition for trade, Vanek (1968) finds that if preferences are identical and homothetic, trade is a linear function of factor endowments. In equilibrium, the slope of the economy PPF equals the slope of the indifference curve for the representative individual (i.e U^H at Home and U^A Abroad) and defines the price ratio for the two products and the input price ratio. It follows that the country with a higher capital-to-labour ratio (Home) has a lower pre-trade price of C compared to the labour abundant country (Abroad). Equivalently, the price of F will be lower Abroad compared to Home.

Model Extensions

There have been several major theoretical refinements and extensions of the standard H-O model. Stolper and Samuelson (1941) find that in the H-O model defined above, the factor price ratio is equal to the output price ratio (i.e. Stolper-Samuelson Theorem). Next, Samuelson (1953) showed that in trade equilibrium, real wages in two countries are equalized (i.e. Factor Price Equalization Theorem). Rybczynski (1955) demonstrated that an increase in the supply of a factor will cause an increase in the supply of the output that uses that factor relatively intensively and a decrease in the supply of the other output. In addition, there have been several major extensions of the model.

Jones (1957) extended the model from two to three commodities and found that the general results would stay unchanged in the sense that "[o]rdering the commodities with respect to the capital-labor ratios employed in production is to rank them in order of comparative advantage. Demand conditions merely determine the dividing line between exports and imports" (p. 85). Bhagwati (1972) extended the model further to a multi-commodity case. Vanek (1968), on the other hand, extended the number of factors from 2 to N. Horiba (1974), introduced a

multi-country case, while Mussa (1978) examined dynamic adjustments in a two-sector case when transfer of capital from one industry to the other requires use of resources. Chen (1992) developed a long run dynamic H-O model with endogenous savings and labour supply and found that the initial factor endowments lead to continued trade in the long run.

The validity of different variations of the H-O factor endowment model has been questioned mainly on the grounds of its predictive power. With respect to the empirical literature, the evidence is mixed. Leontief (1953) found that, contrary to the H-O prediction, United States exported labour-intensive products. According to the factor-ratio hypothesis, the opposite should be the case, since the United States was considered relatively capital-abundant. However, Leamer (1980) claimed that Leontief used an inappropriate measure for factor-intensity.

Bowen et al. (1987) tested the H-O hypothesis in a multi-factor multi-output model using tests developed by Leamer (1980) but found little supportive evidence. Grubel and Lloyd (1975) documented a significant amount of two-way trade, even in highly disaggregated data, suggesting that factor endowments cannot explain this trade. Leamer (1995) reexamines empirical evidence and finds a correlation between factor supplies and net exports of machinery and chemicals, after accounting for home bias and/or technological differences.

Some of the empirical difficulties with the H-O theory have motivated the development of the New Trade Theory that focuses on increasing returns to scale and preference heterogeneity as the major sources of international exchange (Krugman, 1979, 1985, 1996). Unlike this theoretical approach, which did not raise any explicit criticism of the underlying structure of the H-O paradigm (i.e. quantitative differences in aggregate factor endowments may still be a factor in international trade, but often not a major one), some authors have expressed a strong criticism

of the structure of the H-O theory, especially in relation to the nature of capital. The next section provides more details on this.

Model Critiques

The most common critiques of the H-O model have been related to its unrealistic assumptions (Wood, 1995). However, these critiques mostly focus on the relation of the model's assumptions with the real world but rarely focus on the interrelationships between the model's assumptions and outcomes, and ultimately on the meaning of the derived results.

A more substantial critique that questions the meaning of the model's results was raised by Subasat (2003), who points out the unresolved "Cambridge Capital Theory Controversy", a debate that took place in the 1950's and 1960's between some economists of the Cambridge School in the UK (i.e. Joan Robinson and Piero Sraffa) and some MIT economists (i.e. Paul Samuelson and Robert Solow) over the nature of capital. The UK side was arguing against Samuelson's and Solow's assumption of homogeneous capital or the attempts to derive any consistent theoretical results through capital aggregation (Robinson 1953; Blaug 1975; Harcourt 1972, 1976).

According to Cohen and Harcourt (2003), this controversy was never fully resolved due to Sraffa's death, and this makes the neoclassical paradigm vulnerable to future criticisms. The controversy spurred intense research in the theory of aggregation (Sollow, 1963; Fisher,1971, 1983, 1987, 1993; Harris, 1973). Felipe and Fisher (2003), conclude that besides having an empirical appeal stemming from the underlying income accounting identity, "the most important conclusion is that the conditions under which a well-behaved aggregate production function can be derived from micro production functions are so stringent that it is difficult to believe that

actual economies satisfy them. Therefore, aggregate production functions do not have a sound theoretical foundation" (p. 208). This suggests that the UK side's point was eventually appreciated.

Capital heterogeneity, at least partly, Subasat (2003) argues, explains the failure to find robust empirical evidence in favour of the factor endowment hypothesis. The empirical literature relies on the aggregate market value of capital as a measure of capital abundance. However, the value of capital depends on the prices of capital goods, which is part of the result, not a parameter in the H-O model. Thus, capital abundance, in this framework, rests on circular reasoning.

In addition, Subasat points out that physical capital is not an endowment but a product of human labour. Consequently, the economic meaning of capital intensity is unclear. Capital abundance is then a consequence of past individual labour allocation decisions and entrepreneurial activity. The actual motivation for production of capital is to increase individual labour time productivity. Thus, high labour wages in industrialized countries, Subasat argues, are a result of higher labour productivity resulting from the use of technologically advanced capital inputs (i.e. different *kind* of capital), not relative labour scarcity, as it is implied by the H-O theory.

However, Subasat (2003) uses the above argument to reintroduce the labour theory of value as the correct theory by interpreting the fact that capital is a product of human labour as proof that the value of capital is determined by the amount of labour needed to produce it. However, as it will later be shown, the reasoning goes in the opposite direction. The producers determine to which activity to allocate their labour time depending on the consumer valuations of the final product and on the individual labour time productivity. Capital goods are produced in

order to be used in the production of consumption goods. The value of capital goods is determined by individual subjective valuations of the usefulness of the final consumption goods (Rorhbard, [1962], 2009). In other words, as for any other goods, if capital goods cannot be used as a means of satisfying human ends, they are worthless. Thus, the value of a capital good to an individual producer will depend on the quantity and consumer valuations of a final good that can be produced using this capital good. Capital goods that yield more output (demanded by consumers) per unit of time would be valued more regardless of the amount of labour used to produce them.

Input Homogeneity and Impossibility of Markets

There is another troubling implication of assuming input homogeneity in the H-O model that has not been pointed out up to date, and this will be the focus of the remainder of this paper – if inputs are homogeneous, there is no particular reason for the existence of separate industries. It seems that ignoring the fact that allocation of labour is determined by the choices of individuals has contributed to assuming away input heterogeneity.

To clarify, in the standard H-O model it is said that all firms within an industry use identical constant returns to scale technology[7]. Consequently, labour productivity does not change with the scale of production. Thus, there is no reason why a firm would not consist of only one individual. In addition, inputs are homogeneous. This means that each individual's use of a given amount of labour and capital produces the same amount of output. Each individual could produce both goods without any loss of productivity.

[7] Some literature (i.e. Krugman's New Trade Theory) deals with economies of scale as a source of international exchange. However, the input homogeneity assumption still implies that there is no particular reason for the existence of two separate groups of specialized individuals. Pooling all inputs in one industry that produces both products makes equal sense as having two separate industries under increasing returns to scale.

At the same time, each individual also consumes both goods and preferences are identical across individuals. Since there are no differences in relative productivity across individuals, there would be no gains from exchange for any two individuals in the economy, and thus no market prices[8]. This shows that, given the identical CRS technology and homogeneous inputs, there is no particular reason for any exchange among the individuals in a society. *Individual* comparative advantage does not exist within the standard H-O framework. The prices derived in the model are *shadow* prices equal to the marginal rates of technical substitution at the aggregate isoquants and Production Possibilities Frontier. These mathematical functions do not embody individual decisions in a satisfactory way.

Consequently, in the world described by Hecscher-Ohlin type models – a world of identical technology; homogeneous inputs, constant returns to scale, and zero transaction costs, no individual would find it beneficial to specialize and trade. Rather, in Samuelson's mathematical formulation of the H-O model, an aggregate production function is assumed and distribution of labour and capital among industries is imposed by the mathematical properties of the model. The implications of this level abstraction are quite serious, because, in this case, the meaning of the derived market prices is dubious. Why are there exchange ratios (prices) in a society in which there are no benefits from exchange?

In order for an actual market price to arise, there needs to be individual motivation for exchange. It needs to be recognized that *individual* comparative advantage is expressed through choices of individuals to specialize and exchange goods and services, where choices are based on the individuals' knowledge of their own production possibilities and preferences. Without individual comparative advantage there would be no market or market prices to be compared

[8] It should be noted that, unlike the Block et al. (2007) critique of von Mises and Rothbard, this paper looks at homogeneity in the pure mathematical sense implied by the H-O model. Inputs available to any individual are identical in all possible respects.

across regions or countries. Thus, a logical starting point in explaining differences in prices between two isolated economies is to examine the sources of individual comparative advantage. Rothbard ([1962], 200) provides a concise description of the sources of individual specialization in a simple Crusoe example:

> It is clear that conditions for exchange, and therefore increased productivity for the participants, will occur where each party has a superiority in productivity in regard to one of the goods exchanged—a superiority that may be due either to better nature-given factors or to the ability of the producer. If individuals abandon attempts to satisfy their wants in isolation, and if each devotes his working time to that specialty in which he excels, it is clear that total productivity for each of the products is increased. If Crusoe can produce more berries per unit of time, and Jackson can kill more game, it is clear that productivity in both lines is increased if Crusoe devotes himself wholly to the production of berries and Jackson to hunting game, after which they can exchange some of the berries for some of the game. In addition to this, full-time specialization in a line of production is likely to improve each person's productivity in that line and intensify the relative superiority of each (p.97).

Thus, according to Rothbard, input heterogeneity ("better nature-given factors or the ability of the producer") is the source of individual comparative advantage, specialization and formation of prices. In addition, specialization has a further effect on comparative advantage through learning by doing. It is then crucial not to abandon the heterogeneity of inputs when explaining differences in prices between two spatially isolated markets. Furthermore, when it comes to a more complex, real world economy, Rothbard points out:

> It is evident, as will be explained further in later sections on indirect exchange, that the contractual society of the market is a genuinely co-operative society. Each person *specializes in the task for which he is best fitted*, and each serves his fellow men in order to serve himself in exchange [Emphasis added] (p. 99).

This argument, while intuitive, is absent from the aggregate models of interregional price differences. Nevertheless, there is a tendency towards disaggregation in the recent literature. The recent attempts to introduce heterogeneity into models of international exchange at the firm level include Melitz (2003) and Bernard et al. (2006). While Melitz (2003) employs

only one homogeneous input (labour) which is used by firms that differ in productivity, Bernard et al. (2006) replaced the standard capital-labour production function by the unskilled-skilled labour production function[9]. However, in either of the two models the individual allocation of labour is not based on individual comparative advantage. The existence of different lines of production and market prices does not depend on input or firm heterogeneity. If the firm heterogeneity were to be assumed away, the Melitz model would be reduced to the standard Ricardian model and the Bernard model would be reduced to the standard H-O model. These models would still produce market prices, even in the absence of any input or firm heterogeneity. This is a logical contradiction that requires attention at the microeconomic level.

One of the objectives of this paper is to continue the emerging disaggregation tendency but in a different manner – by translating the individual input heterogeneity argument made by Rothbard ([1962], 2009) into the general equilibrium language where production and consumption decisions are made by individuals and the existence of separate industries is derived from individual comparative advantage rather than assumed without relation to individual pursuit of self interest. As Rosen (1997) argues, there may be some "gains from trade" between the Walrasian neoclassical language of mathematical logic and the verbal logic of the Austrian neoclassicals. This paper is an attempt to achieve some of these gains, in hope of not repelling either side by wedding the two methods of analysis.

[9] The Cambridge Capital Theory Controversy seems to have contributed to the abandonment of capital altogether in the H-O modeling. However, any recognition of human capital as an input brings back the same problem, even more insurmountable because of the intangibility of human capital. In addition, the dichotomy between unskilled and skilled labour is logically artificial. It is not that different industries require more or less skill but they require different *kinds* of skill. A highly skilled carpenter is unlikely to be a highly skilled economics professor, but the same applies to the economics professor as well. Thus, "factor-ratio" can only have a metaphorical but not quantitative meaning (i.e. an individual will be a relatively more productive as a carpenter if he has a higher ratio of carpentry skills compared to university professor skills).

An Alternative: Input-Heterogeneity instead of Input-Intensity

Compared to differences in input-intensity and variable proportions technology, differences in input *kind* is a much more meaningful explanation for diminishing labour productivity and especially for individual specialization. This is the key link to methodological individualism. It is individual humans who posses human and physical capital of varying qualities, who produce, specialize and exchange in order to satisfy their ends. Thus, providing the link between individuals' pursuit of their comparative advantage and the aggregate outcomes is essential.

To provide this link, I take the conventional microeconomic model of an optimizing agent and follow through Hayek's (1945) idea that market prices reflect and coordinate choices of self-interested individuals. However, an important difference between Hayek's approach and this model is that this model assumes that individuals, as well as the external analyst, know each other's production possibilities and preferences, while Hayek stresses the importance of the fact that knowledge is individual, subjective and thus not directly available to other individuals. This assumption brings with it important implications, which require special attention. However, this is beyond the scope of this paper, where the purpose is to demonstrate how the fact that individuals benefit from specialization and exchange relates to the aggregate outcome – difference in exchange ratios between hypothetically isolated populations.

Since labour *time* is the only input that is expressed in homogeneous units for all individuals, individual production functions in this model have only labour *time* as the explanatory variable, while the differences in productivity per unit of time are explained using qualitative, verbal logic rather than the quantitative logic of arithmetic. This approach removes the criticisms put forth by the proponents of the labour theory of value (i.e. the Cambridge

Capital Theory Controversy) and demonstrates that cross-boundary exchange can consistently be explained within the neoclassical subjectivist framework.

In this model, goods are not characterized by input-intensity. It is not the difference between goods that cause diminishing marginal productivity of labour *time* at the aggregate level – it is differences between individual producers-consumers. Individual producers can be more or less productive in different lines of production, depending on the kind of inputs owned (human and physical capital). As the market exchange ratio between two goods changes, marginal producers switch from the production of one good to the production of the other good. As the price of one good increases, relatively less productive individuals find it beneficial to start producing that good. On the other hand, relatively more productive individuals remain in the production of the other good[10]. As a result, marginal productivity of labour time at the aggregate level diminishes as the total output of a good increases. This approach solves two problems at the same time: it explains (1) complete individual specialization, left unexplained by the H-O model, and (2) partial national specialization.

As to reasons why no such attempt has been done before, I can only speculate. One reason could be the preference for mathematical language within the Walrasian current of neoclassical economics. Assuming input heterogeneity carries with it problems of expressing aggregate production in a form of a mathematical function. As Fisher (2003) and many others clearly pointed out, under input heterogeneity, there is no such thing as an aggregate production function. There can only be individual production functions. This is why, in this paper, the attempt is made to mathematically aggregate the outcomes of individual actions, not the inputs used.

[10] This all takes time, destruction (deterioration) and creation of new capital in different lines of production, depending on the changes in the relative demand for different products and entrepreneurial abilities of the producers.

The next section introduces a model with three individuals who have identical preferences but own inputs that vary in quality. The reason for using three individuals as opposed to two is that the three-individual example is still simple enough to be represented graphically but also provides the first step between the two-individual and the n-individual case. It is also assumed that specialization does not affect individual technological knowledge (i.e. there is no learning by doing). The model will gradually be expanded to n-individuals with heterogeneous preferences. In addition, the possibility of learning by doing will be assessed.

A Three-Individual Economy

In this initial model, each of the three individuals owns some quantity of heterogeneous natural resources, human capital of a certain kind and some length of time that can be allocated to labour. The labour time is used to produce consumption goods that can be used to meet the ends of a given individual. Preferences over the consumption goods are homothetic and identical across individuals. Each of the individuals can employ his or her labour time and human capital in transforming the natural resources into capital goods that can then be used, in conjunction with labour, human capital and the remaining natural resources for production of the final consumption goods.

There are two goods (food (F) and clothing (C)), that can be used to meet the ends of the three individuals. Each of the individuals is able to produce both goods but in different proportions. This may be due to the differences in the quantity and/or quality of the natural resources and/or quality of human capital (i.e. physical and mental abilities, technological knowledge). As a result, all inputs other than labour *time* are heterogeneous, and thus the production schedule can be defined only between labour time and output. Thus, the

heterogeneity of the natural resources, technological knowledge, and physical and mental characteristics of the three individuals results in differences in the productivity of their labour time. The production functions for the three individuals are specified in the following manner[11]:

Individual 1 : $F_1 = aL_{F_1}$ and $C_1 = bL_{C_1}$ (3.1)

Individual 2: $F_2 = cL_{F_2}$ and $C_2 = dL_{C_2}$ (3.2)

Individual 3: $F_3 = eL_{F_3}$ and $C_3 = fL_{C_3}$ (3.3)

where

$F_1,\ F_2, F_3$	are the quantities of food produced by individuals 1, 2, and 3 respectively;
$C_1,\ C_2, C_3$	are the quantities of clothing produced by individuals 1, 2, and 3 respectively;
$L_{F_1}, L_{F_2}, L_{F_3}$	are the lengths of labour time spent in production of food by individuals 1, 2, and 3 respectively;
$L_{C_1}, L_{C_2}, L_{C_3}$	are the lengths of labour time spent in production of clothing by individuals 1, 2, and 3 respectively;
a, c, and e,	are the amounts of food produced per unit of time by individuals 1, 2, and 3 respectively;
b, d, and f	are the amounts of clothing produced per unit of time by individuals 1, 2, and 3 respectively.

Assuming each individual has the same length of labour time, L_0, at his or her disposal to complete the production cycle, the Production Possibilities Frontier (PPF) for each individual is defined as:

Individual 1 : $C_1 = bL_0 - \frac{b}{a}F_1$ (3.4)

Individual 2: $C_2 = dL_0 - \frac{d}{c}F_2$ (3.5)

[11] Continuity of the production function with respect to the continuous variable – time – is an abstraction that does not seriously violate the logic of production. Aside from introducing (unrealistic) infinitesimal divisibility of outputs, the basic logic, more time – more output, is preserved. Thus, this abstraction is useful for analytical purposes. While increasing returns to labour time (due to learning through specialization) are likely, for simplicity, constant returns will be assumed.

Individual 3: $C_3 = dL_0 - \frac{f}{e}F_3$ (3.6)

The PPF expresses the quantity of one good as a function of the other. The terms $b, d, d,$ represent the C intercepts of the three PPFs, while the ratios $\frac{b}{a}$, $\frac{d}{c}$, and $\frac{f}{e}$ represent the slopes of the three individual PPFs. These functions depict the maximum combinations of food and clothing that each individual can produce with the inputs he or she owns. In other words, given some quantity of C on the PPF, there is no higher quantity of F that can be achieved other than the quantity associated with the given quantity C on the PPF. Thus, the combination of F and C that each individual can consume in the case when he or she is satisfying his or her ends using his or her own production (in self-sufficiency) is limited by his or her PPF. The next section assesses the individual production and consumption choices in self-sufficiency.

Self-sufficiency

First, I will assume that the three individuals meet their needs in isolation. Each individual's objective is to maximize his or her utility by consuming C and F. The optimization problem for each individual is to maximize utility subject to his or her production possibilities constraint.

Individual 1: $maxU(F_1, C_1)$ s.t. $C_1 = bL_0 - \frac{b}{a}F_1$ (3.7)

Individual 2: $maxU(F_2, C_2)$ s.t. $C_2 = dL_0 - \frac{d}{c}F_2$ (3.8)

Individual 3: $maxU(F_3, C_3)$ s.t. $C_3 = fL_0 - \frac{f}{e}F_3$ (3.9)

The First Order Conditions imply that the optimum is achieved at the production-consumption bundle of F and C where each individual's indifference curve is tangent to his or her Production Possibilities Frontier:

Individual 1: $\frac{U_{C_1}}{U_{F_1}} = \frac{b}{a}$ (3.10)

Individual 2: $\frac{U_{C_2}}{U_{F_2}} = \frac{d}{c}$ (3.11)

Individual 3: $\frac{U_{C_3}}{U_{F_3}} = \frac{f}{e}$ (3.12)

This result depicts the optimal bundle of food and clothing produced by each of the 3 individuals in isolation. This optimal solution determines the allocation of labour time by each individual in each of the two production activities. Since the current allocation is the best each of the 3 individuals can do in isolation, any potential for improvement must involve interpersonal exchange. The next section describes a situation where exchange is preferred over self-sufficiency by all 3 individuals.

Interpersonal exchange

I will now assume complete information (i.e. each individual knows all the relevant characteristics of the other two individuals). This implies no transaction costs. The three individuals can trade without search, negotiating, or concluding costs. Food produced by any of the three individuals is a perfect substitute for food produced by the other two. The same applies for clothing. Goods can be exchanged at a constant per unit price in terms of the quantity of the other good[12].

The individuals face the following decision: to increase production of one good by some amount and exchange some amount for a set per unit amount of the other good. Each individual needs to choose which good to produce for exchange. The market exchange between the three

[12] This assumption is used as a precursor of the law of one price within a market consisting of many individuals. It does not imply that a single exchange ratio must arise. Rather, the question asked here is – given a single exchange ratio for all individuals, can they be better off compared to self-sufficiency.

individuals is in the interest of all three individuals if the opportunity cost of specialization and exchange is less than the opportunity cost of self-sufficiency for all of them. What determines the opportunity costs is the utility that each individual attaches to the different consumption possibilities that are attainable under the different scenarios.

Essentially, the individual decision to specialize and trade depends on whether one can obtain more of some good through market transactions than by reallocating his own resources. If the exchange ratio is such that this is possible, then it would be beneficial to specialize and trade. In the mathematical language, this means that the exchange ratio is higher (lower) than the slope of the individual PPF, where the good being sold is on the horizontal (vertical) axis.

This idea is illustrated in Figure 3.2. The panel on the left illustrates a situation when the exchange ratio for F is less than the slope of the individual PPF $\left(P_F < \frac{C(L_0)}{F(L_0)}\right)$. In self-sufficiency, this individual would produce and consume at the point where U^0 is tangent to his or her PPF. In this situation, the individual is better off if he or she specializes in C and exchanges some of it for F because he or she can reach a higher level of utility (U^1) compared to self-sufficiency (U^0).

In essence, this is because this individual is able to obtain more F per unit of C through exchange than by reallocating his own resources from the production of C to the production of F. At the point of specialization, his or her production of C is $C^*(L_0)$ while he or she demands C_d. The difference between one's production (supply) and demand can be defined as the individual excess supply. At the same time, the individual demands F_d but does not produce any F. The difference between the actual demand and supply of F is the individual excess demand for F.

The panel on the right illustrates a situation when the exchange ratio is higher than the slope of the individual PPF. Again, this individual would, in self sufficiency, produce and consume at the point where his or her indifference curve, U^0, is tangent to his or her PPF.

However, this individual could reach a higher level of utility, U^1, if he or she specialized in F. Then, he or she would produce $F^*(L_0)$, consume F_d and sell the difference. In return, he or she would buy C_d. A similar argument applies as before. This individual is able to obtain more C per unit of F than he or she could by reallocating his or her resources from the production of F to the production of C.

While the above argument shows that there may be situations when a hypothetical individual would prefer specialization and exchange over self sufficiency, it still needs to be demonstrated that each individual in a group of three would prefer exchange to self sufficiency, given the production possibilities and preferences of each of them. Figure 3.3 demonstrates this graphically for a situation where the individual utility function is $U(F_i, C_i) = {F_i}^{0.5}{C_i}^{0.5}$, and the individual production possibilities are defined as in Table 3. **1**. The only criterion used for selecting these specific production possibilities was that the three PPFs differ in slope. This represents differences in relative productivities among the three individuals arising from the differences in the quality and/or quantity of inputs owned by each individual.

Table 3. 1. Distribution of production possibilities in a hypothetical 3-individual economy

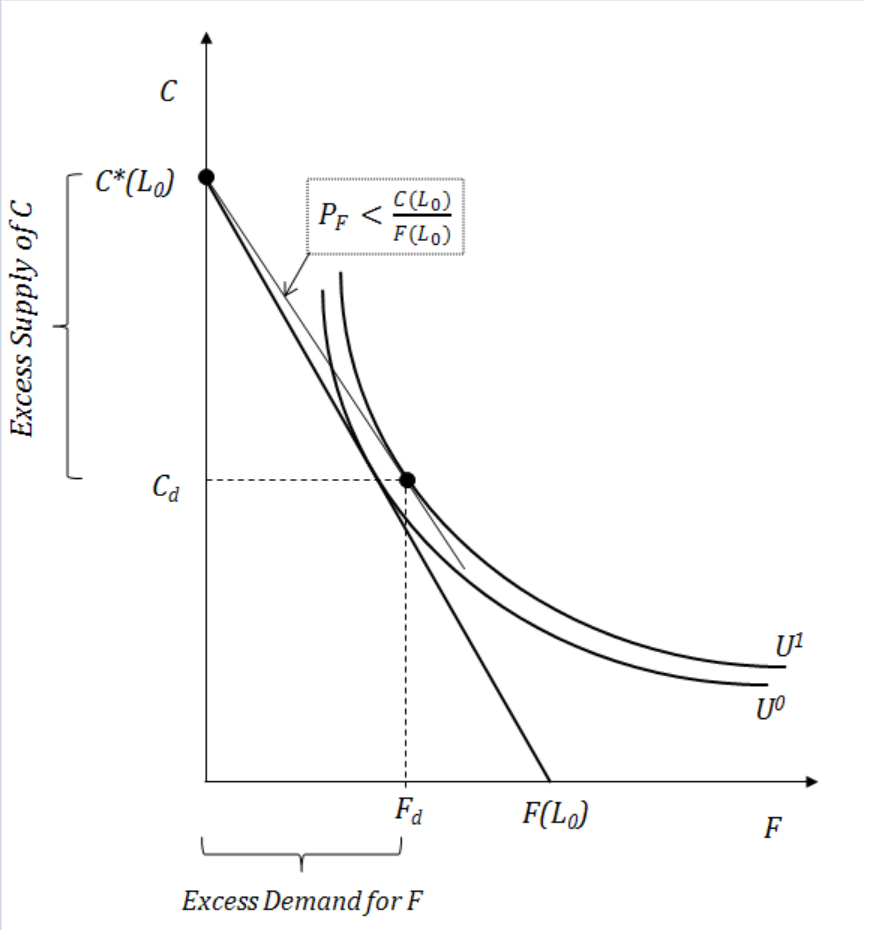
	Maximum Supply of Clothing ($C(L_0)$)	Maximum Supply of Food ($F(L_0)$)
Individual 1	7	2.5
Individual 2	10	5
Individual 3	5	10

The solution to this particular problem can be found analytically. The market-clearing exchange ratio in Figure 3.3 satisfies the following conditions:

1. The same exchange ratio applies to all three individuals. This is the exchange ratio expressed in quantity of one good per unit of quantity of the other good[13]:

[13] All prices in this model are expressed as quantity ratios.

A situation when the exchange ratio is lower than the slope of the individual PPF



A situation when the exchange ratio is higher than the slope of the individual PPF

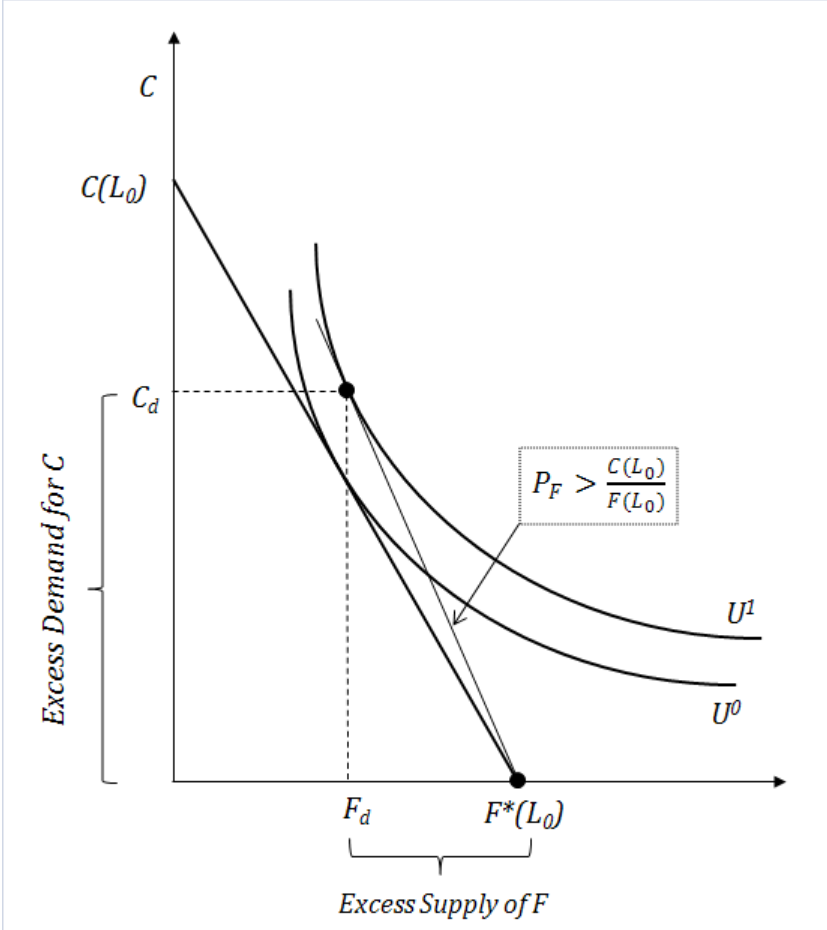


Figure 3.2. Individual decision to specialize and trade depending on the exchange ratio

$$\frac{bL_0 - C_1}{F_1} = \frac{C_2}{cL_0 - F_2} = \frac{fL_0 - C_3}{F_3} \tag{3.13}$$

2. The bundles (F_1, C_1), (F_2, C_2), and (F_3, C_3) are on the price line tangent to each individual's indifference curve[14]:

$$\frac{U_F(F_1, C_1)}{U_C(F_1, C_1)} = \frac{U_F(F_2, C_2)}{U_C(F_2, C_2)} = \frac{U_F(F_3, C_3)}{U_C(F_3, C_3)} \tag{3.14}$$

3. The market is cleared at the price where the sum of excess demands for F by Individual 1 and Individual 3 is equal to the excess supply by Individual 2:

$$cL_0 - F_2 = F_1 + F_3 \tag{3.15}$$

At the same time, the excess demand for C by Individual 2 is matched by the combined excess supply from Individuals 1 and Individual 3:

$$C_2 = (bL_0 - C_1) + (fL_0 - C_3) \tag{3.16}$$

In this particular example, the resulting market-clearing exchange ratio is 1.7 units of clothing per one unit of food. Individual 1 produces 10 units of clothing, of which he or she exchanges 5 units in return for 2.94 units of food. Individual 2 produces 10 units of food, of which he or she exchanges 5 units in return for 8.5 units of clothing. Five of these 8.5 units come from Individual 1 and the remaining 3.5 come from Individual 3. Individual 3 produces 7 units of clothing, of which 3.5 are exchanged for 2.06 units of food. These 2.06 units, together with the 2.94 units bought by Individual 1, make up the 5 units of food provided by Individual 2.

Figure 3.3 illustrates this graphically. It shows that when Individual 1 and Individual 3

[14] It should be noted that Rothbard rejected the concept of indifference (i.e. indifference implies absence of choice and this is contrary to the postulates of human action). However, indifference could be interpreted differently. Indifference here can simply mean that different bundles of clothing and food satisfy one's needs equally well. This means that these bundles can be viewed as different units of the same thing – a bundle of food and clothing. Different units of the same thing are not a subject of choice because they are identical in the mind of an individual. However, if prices are attached to food and clothing, then, every bundle will have a different price because it contains a different combination of food and clothing. Now, these bundles are different (take different places on one's value scale) and choice takes place.

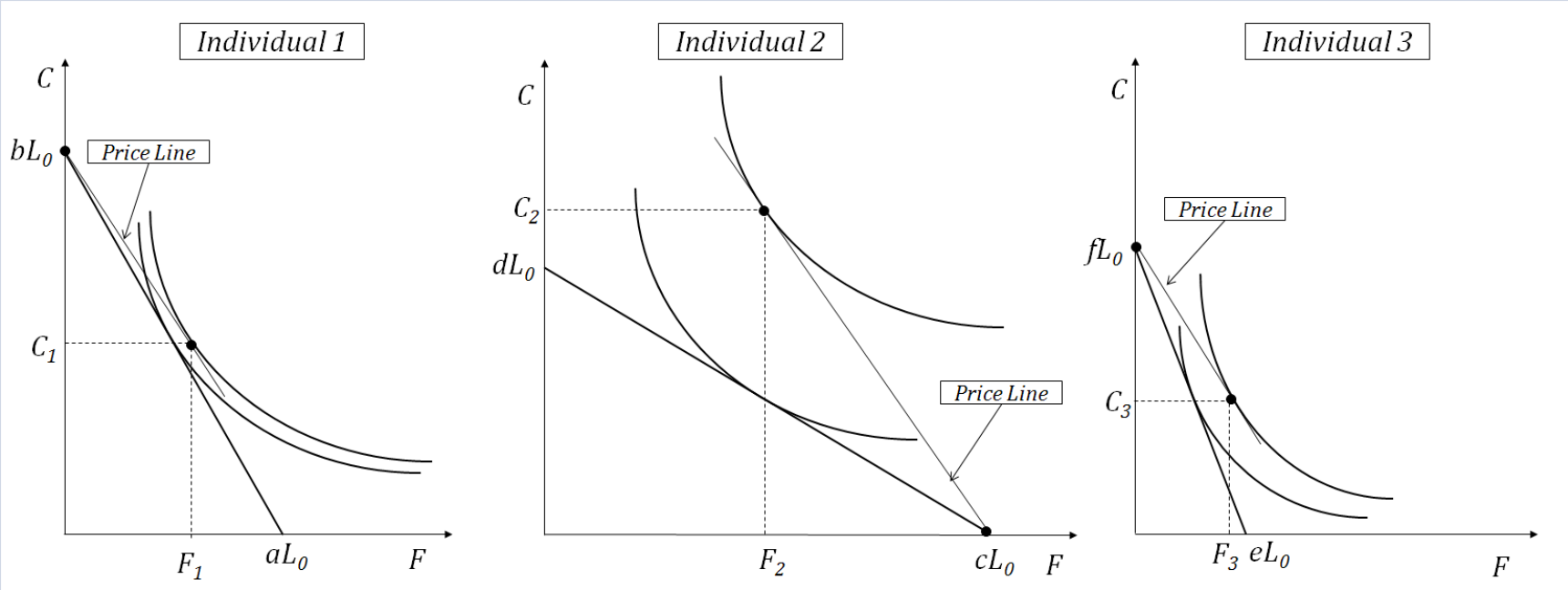


Figure 3.3. Market equilibrium in a three-individual economy

specialize in clothing and Individual 2 specializes in food, each individual could receive enough of the other good in exchange to be better off compared to self-sufficiency. This is because Individual 2 can provide more food than Individual1 and Individual 3 could produce for their own consumption in self-sufficiency. At the same time, Individual 1 and Individual 3 can provide more clothing than Individual 2 could produce for his or her own consumption in self-sufficiency. The exchange ratio is higher than the slope of the PPF for individual 2, while it is less for Individual 1 and Individual 3. Consequently, Individual 2 specializes in F and individual 1 and Individual 3 specialize in C. Individual 2 has excess supply of F and excess demand for C, while individuals 1 and 3 have excess supply of C and excess demand for F.

This situation is obviously a very simplified representation of reality for more than one reason. One of the most immediately noticeable reasons is that the real economy consists of many individuals. The next section generalizes the situation in Figure 3.3 to a large number of individuals using a general Cobb-Douglass utility function. In this section, I define market-clearing conditions by deriving aggregate excess demand and excess supply from individual specialization and exchange decisions.

A Closed Economy with n Individuals

A discrete distribution of individual production possibilities

Now, I will extend the model by assuming that there are n individuals in the economy. Each individual has the same amount of labour time L_0 available for production activities during the production cycle:

$C_i(L_0)$ is individual i's maximum production of C (using all available labour time)

$F_i(L_0)$ is individual i's maximum production of F (using all available labour time)

A Cobb-Douglas utility function identical for all individuals is assumed[15]:

$$U(F_i, C_i) = {F_{D_i}}^{\alpha} {C_{D_i}}^{1-\alpha} \tag{3.17}$$

where

F_{D_i} and C_{D_i} are the quantities of food and clothing demanded (consumed) by individual *i*.

As explained earlier in Figure 3.2, individual *i* specializes in C if the market exchange ratio, P_F, is less than $\frac{C_i(L_0)}{F_i(L_0)}$, and he or she specializes in F if P_F is greater or equal to $\frac{C_i(L_0)}{F_i(L_0)}$[16].The stocks of the physical product that can be exchanged on the market are: ${F_i}^*(L_0)$ for an individual specializing in F, and ${C_i}^*(L_0)$ for an individual specializing in C. This is the income of the specialized individual because this physical product of one's production activity is exchanged for consumption goods. As a result, each individual maximizes the utility of consumption, subject to the income constraint. From this optimizing behaviour, it follows then that the individual demand for F is:

$$F_{D_i} = \frac{\alpha {C_i}^*(L_0)}{P_F} \quad \text{if } P_F < \frac{C_i(L_0)}{F_i(L_0)} \tag{3.18}$$

$$F_{D_i} = \alpha F_i^*(L_0) \quad \text{if } P_F \geq \frac{C_i(L_0)}{F_i(L_0)} \tag{3.19}$$

where

$${C_i}^*(L_0) = C_i(L_0) \text{ and } {F_i}^*(L_0) = 0 \text{ if } P_F < \frac{C_i(L_0)}{F_i(L_0)} \tag{3.20}$$

$${F_i}^*(L_0) = F_i(L_0) \text{ and } {C_i}^*(L_0) = 0 \text{ if } P_F \geq \frac{C_i(L_0)}{F_i(L_0)} \tag{3.21}$$

[15] This assumption will later be relaxed to include heterogeneous preferences.

[16] It should be admitted that equality is included for the sake of modeling convenience. In fact, an individual whose PPF has the slope equal to the market exchange ratio would be indifferent between self sufficiency and specializing in any of the two commodities.

This means that if $P_F < \frac{C_i(L_0)}{F_i(L_0)}$, individual *i* specializes in C. The total income available for exchange is then $C_i(L_0)$. From this, the demand function, F_{D_i}, is derived. Similarly, if $P_F \geq \frac{C_i(L_0)}{F_i(L_0)}$, individual *i* specializes in F. The total income available for exchange is $F_i(L_0)$. However, this individual will not buy F but use his or her own. Consequently the demand for F is $\alpha F_i^*(L_0)$. These derivations are shown in Appendix 1 and Appendix 2.

The aggregate demand for F is the sum of all individual demands:

$$Q_{D_F} = \sum_{i=1}^{n} F_{D_i} \tag{3.22}$$

Individual supply of F is:

$$F_{S_i} = F_i{}^*(L_0) \quad \text{if} \quad P_F \geq \frac{C_i(L_0)}{F_i(L_0)} \tag{3.23}$$

$$F_{S_i} = 0 \qquad \text{if} \quad P_F < \frac{C_i(L_0)}{F_i(L_0)} \tag{3.24}$$

The aggregate supply is the sum of all individual supplies:

$$Q_{S_F} = \sum_{i=1}^{n} F_{S_i} \tag{3.25}$$

In market-clearing, the aggregate demand equals the aggregate supply:

$$\sum_{i=1}^{n} \frac{\alpha C_i{}^*(L_0)}{P_F} + \sum_{i=1}^{n} \alpha F_i{}^*(L_0) = \sum_{i=1}^{n} F_i{}^*(L_0) \tag{3.26}$$

or

$$\frac{\alpha}{P_F} \sum_{i=1}^{n} C_i{}^*(L_0) = \sum_{i=1}^{n} [F_i{}^*(L_0) - \alpha(F_i{}^*(L_0))] \tag{3.27}$$

The expression on the left side of equation (3.27) represents the sum of individual excess demands for F across individuals specializing in C, while expression on the right represents the sum of excess supplies of F across individuals specializing in F. Equivalently:

$$\frac{\alpha}{P_F}\sum_{i=1}^{n} {C_i}^*(L_0) = (1-\alpha)\sum_{i=1}^{n} {F_i}^*(L_0) \tag{3.28}$$

After isolating P_F in equation (3.28), it results in equation (3.29):

$$P_F = \frac{\alpha}{1-\alpha}\frac{\sum_{i=1}^{n} {C_i}^*(L_0)}{\sum_{i=1}^{n} {F_i}^*(L_0)} \tag{3.29}$$

Equation (3.29) restates the condition that needs to be satisfied in order for P_F to be the market-clearing price of F. At this price, the sum of individual excess demands equals the sum of individual excess supplies.

It should be noted that while the market-clearing price determines which individuals would find it beneficial to specialize in either of the two goods, this model does not provide the price discovery mechanism[17]. It merely demonstrates that if there is an exchange ratio at which all individuals would be at least as well off as in self sufficiency, this exchange ratio needs to equate the total quantity of food that is offered for sales by the individuals that specialize in food and the total quantity that is demanded by the individuals that specialize in clothing. Moreover, given some distribution of individual production possibilities, all individuals for whom the exchange ratio differs from the slope of their PPF would be better off than in self-sufficiency.

Similar to the demand for F, the individual demand for C is:

[17] Given the complete information assumption, the price discovery lacks meaning in this setting. If all individuals have a complete knowledge of all other individuals' production possibilities and preferences, there is no need for price discovery. The mutually beneficial exchanges are arrived at instantly without search, negotiating, or concluding costs. The unrealism of this assumption needs special attention but it is beyond the scope of this paper.

$$C_{D_i} = \frac{(1-\alpha)F_i^*(L_0)}{P_C} \qquad \text{if } P_C < \frac{F_i(L_0)}{C_i(L_0)} \tag{3.30}$$

$$C_{D_i} = (1-\alpha)C_i^*(L_0) \quad \text{if } P_C \geq \frac{F_i(L_0)}{C_i(L_0)} \tag{3.31}$$

where

$$C_i^*(L_0) = C_i(L_0) \quad \text{and} \quad C_i^*(L_0) = 0 \qquad \text{if } P_C < \frac{F_i(L_0)}{C_i(L_0)} \tag{3.32}$$

$$C_i^*(L_0) = 0 \qquad \text{and} \quad C_i^*(L_0) = C_i(L_0) \quad \text{if } P_C \geq \frac{F_i(L_0)}{C_i(L_0)} \tag{3.33}$$

The derivations of the individual demands for C are shown in Appendix 1 and Appendix 2. The aggregate demand for C is the sum of the individual demands:

$$Q_{D_C} = \sum_{i=1}^{n} C_{D_i} \tag{3.34}$$

while the individual supply of C is:

$$C_{S_i} = C_i^*(L_0) \quad \text{if} \quad P_C \geq \frac{F_i(L_0)}{C_i(L_0)} \tag{3.35}$$

$$C_{S_i} = 0 \qquad \text{if} \quad P_C < \frac{F_i(L_0)}{C_i(L_0)} \tag{3.36}$$

and the aggregate supply of C is:

$$Q_{S_C} = \sum_{i=1}^{n} C_{S_i} \tag{3.37}$$

In market-clearing, the aggregate demand equals the aggregate supply:

$$\sum_{i=1}^{n} \frac{(1-\alpha)F_i^*(L_0)}{P_C} + \sum_{i=1}^{n}(1-\alpha)C_i^*(L_0) = \sum_{i=1}^{n} C_i^*(L_0) \tag{3.38}$$

$$\frac{(1-\alpha)}{P_C}\sum_{i=1}^{n} F_i^*(L_0) = \alpha \sum_{i=1}^{n} C_i^*(L_0) \tag{3.39}$$

The expression on the left side of equation (3.39) represents the sum of excess demands for C by the individuals specializing in F, while the expression on the right represents the sum of

individual excess supplies of C by the individuals specializing in C. Isolating P_C in equation (3.39) results in equation (3.40).

$$P_C = \frac{1-\alpha}{\alpha}\frac{\sum_{i=1}^{n} F_i^*(L_0)}{\sum_{i=1}^{n} C_i^*(L_0)} = \frac{1}{P_F} \tag{3.40}$$

Equation (3.40) defines the condition that needs to be satisfied in order for P_C to be the market-clearing price of C. By definition, this is the reciprocal value of the price of food, P_F.

The next step is to show that there is, indeed, such an exchange ratio where most individuals would be better off and no one would be worse off compared to being self-sufficient in both food and clothing. I will show this by examining the properties of the aggregate excess supply and demand functions in relation to the exchange ratio. In addition some examples of hypothetical markets consisting of multiple individuals will be presented.

Aggregate Excess Demand and Supply and Market Price

Aggregate excess demand and excess supply are step functions. The aggregate excess demand for food has two multiplicative elements: $\frac{\alpha}{P_F}$ and $\sum_{i=1}^{n} C_i^*(L_0)$. The term $\frac{\alpha}{P_F}$ decreases continuously with price, while the term $\sum_{i=1}^{n} C_i^*(L_0)$ decreases incrementally, as each individual *i* switches from producing C to producing F, as P_F changes, at the point where $P_F \geq \frac{C_i(L_0)}{F_i(L_0)}$. As a result, the aggregate excess demand is a product of a continuous and a step function. Thus, it is a step function itself.

The term $\sum_{i=1}^{n} F_i^*(L_0)$, in the aggregate excess supply function for food, increases incrementally with price at the point where $P_F \geq \frac{C_i(L_0)}{F_i(L_0)}$. Thus, at the point when individual *i* switches from producing C to producing F, aggregate excess demand for food decreases by a

finite increment equal to the excess demand of the i^{th} individual. At the same time, excess supply of food increases by a finite increment equal to the excess supply of the i^{th} individual. Thus, the aggregate excess demand for food decreases with P_F while the aggregate excess supply increases with P_F.

The intercept of the aggregate excess demand for food is equal to the slope of the steepest individual PPF (when food is on the horizontal axis) and the intercept of the aggregate excess supply is equal to the slope of the flattest individual PPF. Consequently the market-clearing price corresponds to some non-zero quantity of food exchanged on the market. An analogous argument applies to the aggregate excess demand and supply of clothing as well.

The market-clearing exchange ratio determines the dividing line between producers of food and producers of clothing. Everyone whose PPF slope is less than P_F[18], specializes in F, while everyone whose PPF slope is greater than (or equal to) P_F specializes in C.

If the price of food is too high (i.e. higher than the market-clearing price), there will be too many individuals specializing in F (and too few specializing in C) and there will be excess supply of food and excess demand for clothing. On the other hand, if P_F is too low (i.e. lower than the market-clearing price), there will be too few individuals specializing in F (and too many specializing in$C$). This would result in excess demand for F and excess supply of C. The next section provides examples to illustrate the above points.

Examples

This section shows 3 examples of aggregate excess demand and supply for economies consisting of 3, 12, and 15 individuals with different PPFs and identical preferences. The preferences in each example are described by a Cobb-Douglass utility function:

[18] F is on the horizontal axis.

$$U(F_i, C_i) = F_i^{0.5} C_i^{0.5} \tag{3.41}$$

where

F_i and C_i are quantites of food and clothing consumed by individual *i*.

Three Individuals

The production possibilities for each of the three individuals are defined as in Table 3. **1**. Figure 3.2 shows the aggregate excess demand and supply curves for food derived for this particular example using equations (3.22) and (3.25). Aggregate excess demand is zero for any exchange ratio that is higher than the slope of the PPF of the individual comparatively most productive in clothing – Individual 3. This means that at high prices (above 2.8 units of clothing per unit of food), all individuals would want to specialize in food (if they could). On the other hand, no one would want to specialize in clothing, and thus no one demands food in return for clothing – aggregate excess demand is 0. Obviously, this is an unsustainable situation.

Once the price of food falls below 2.8 C/F, Individual 3 would want to specialize in clothing. This individual is the first to start producing clothing because he or she is comparatively most productive in this line of production. Only for him or her, it is "profitable" to produce clothing at such high food prices (and low clothing prices). Thus, the aggregate excess supply of food declines by Individual 3's previous contribution to this supply. At the same time, Individual 3 starts to demand food in exchange for clothing. Consequently, aggregate excess demand for food has increased by Individual 3's contribution.

As the price of food falls between 2.8 C/F and 2 C/F, Individual 3 demands more food. Once the price falls below 2 C/F, which is the slope of Individual 1's PPF, Individual 1 moves away from producing food to producing clothing. This is because at this point the food price

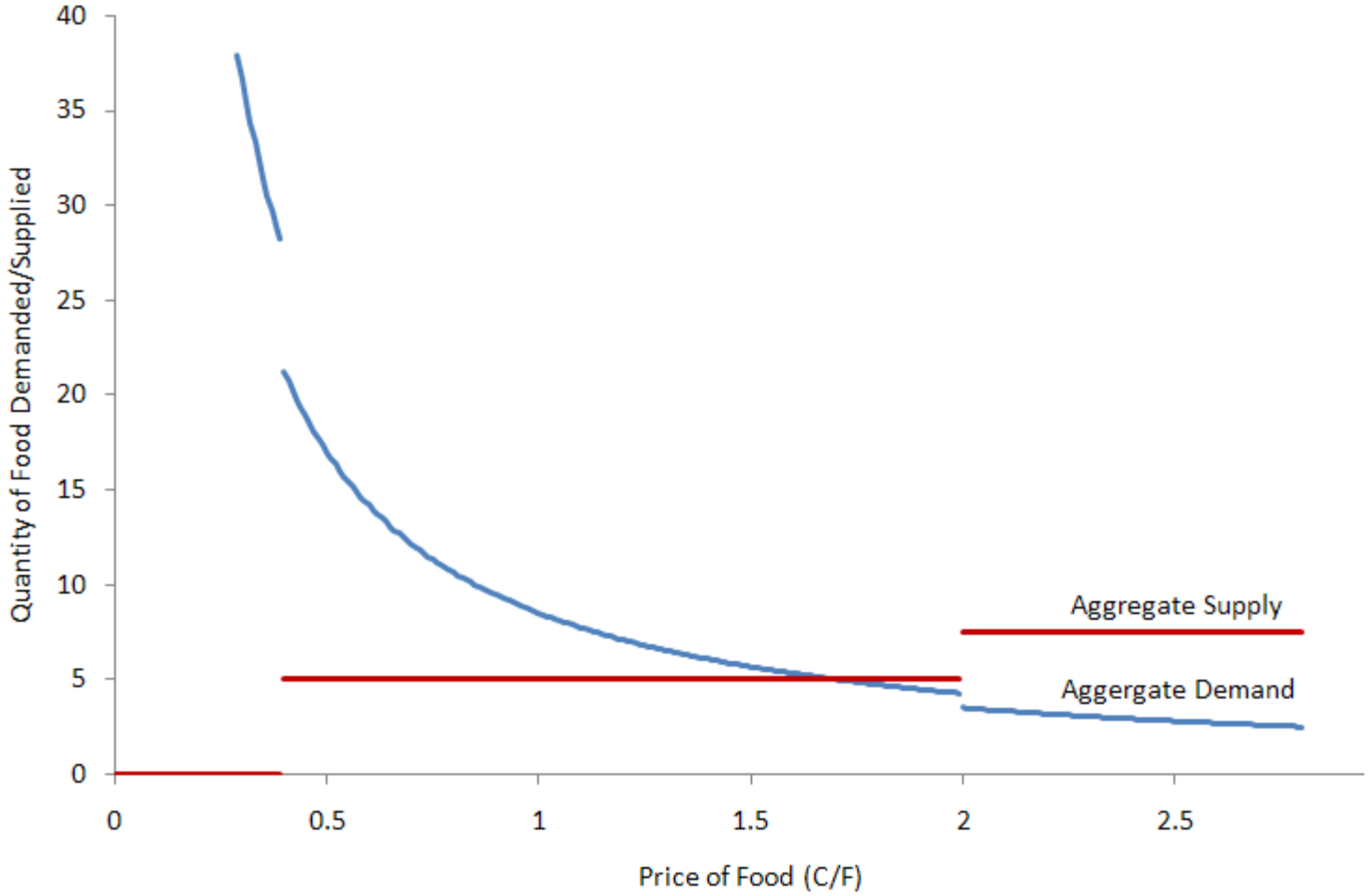


Figure 3.4. Aggregate excess supply and demand for food in a 3-individual economy

became too low to be profitable for Individual 2. He or she was able to stay longer in the production of food compared to Individual 3 because he or she was relatively more productive in food. As a consequence, now both Individual 1 and Individual 3 demand food in exchange for clothing. Moreover, the excess supply of food has been reduced by Individual 1's contribution.

The aggregate excess demand increases as the price of food goes down. It is evident from the graph that the aggregate excess demand and excess supply are equated at a price that is between 1.5 and 2 C/F. As shown earlier, the exact market-clearing price is 1.7 C/F. As the price is further reduced below 0.4 C/F, no individual would want to produce food, and the aggregate excess supply falls to zero. Simultaneously, the aggregate excess demand increases as food becomes cheaper.

12 Individuals

The distribution of individual production possibilities in this example is shown in Table 3.2, and the derived aggregate excess supply and demand functions are presented in Figure 3.5. These aggregate functions are derived using the same approach as in the previous example. It can be seen that the market-clearing price is somewhat above 1 C/F (1.06). Table 3.3 summarizes the individual production and consumption in self-sufficiency; individual excess supplies and demands at the market-clearing price; and (ordinal) utility before and after the exchange. It is evident that each of the 12 individuals prefers specialization and exchange over self-sufficiency.

15 Individuals

Table 3.4 presents hypothetical production possibilities of 15 individuals. The resulting aggregate excess supply and demand are shown in Figure 3.6. Unlike the previous two examples,

the market-clearing price is not directly defined by the intersection of the aggregate excess supply and demand. This is due to the discontinuity and the step nature of the two functions. However, Table 3.5. demonstrates that this is not a critical issue. The difference between the aggregate excess supply and demand at the closest defined exchange ratio is small relative to the number of individuals in the market. If this excess quantity is distributed across all the individuals, the marginal condition (utility function tangency) would not be satisfied, but the gains from specialization and exchange for each individual are still large enough that specialization is preferred to self-sufficiency.

As the number of individuals increases, the difference between the aggregate excess demand and supply becomes even smaller when distributed across all market participants. Asymptotically, this difference tends to 0. The next section further examines the asymptotical properties of the aggregate excess demand and supply functions.

Table 3.2. Distribution of production possibilities in a 12-individual economy

	Maximum Supply of Clothing ($C(L_0)$)	Maximum Supply of Food ($F(L_0)$)
Individual 1	20	8
Individual 2	19	9
Individual 3	18	10
Individual 4	17	11
Individual 5	16	12
Individual 6	15	13
Individual 7	14	14
Individual 8	13	15
Individual 9	12	16
Individual 10	11	17
Individual 11	10	18
Individual 12	9	19

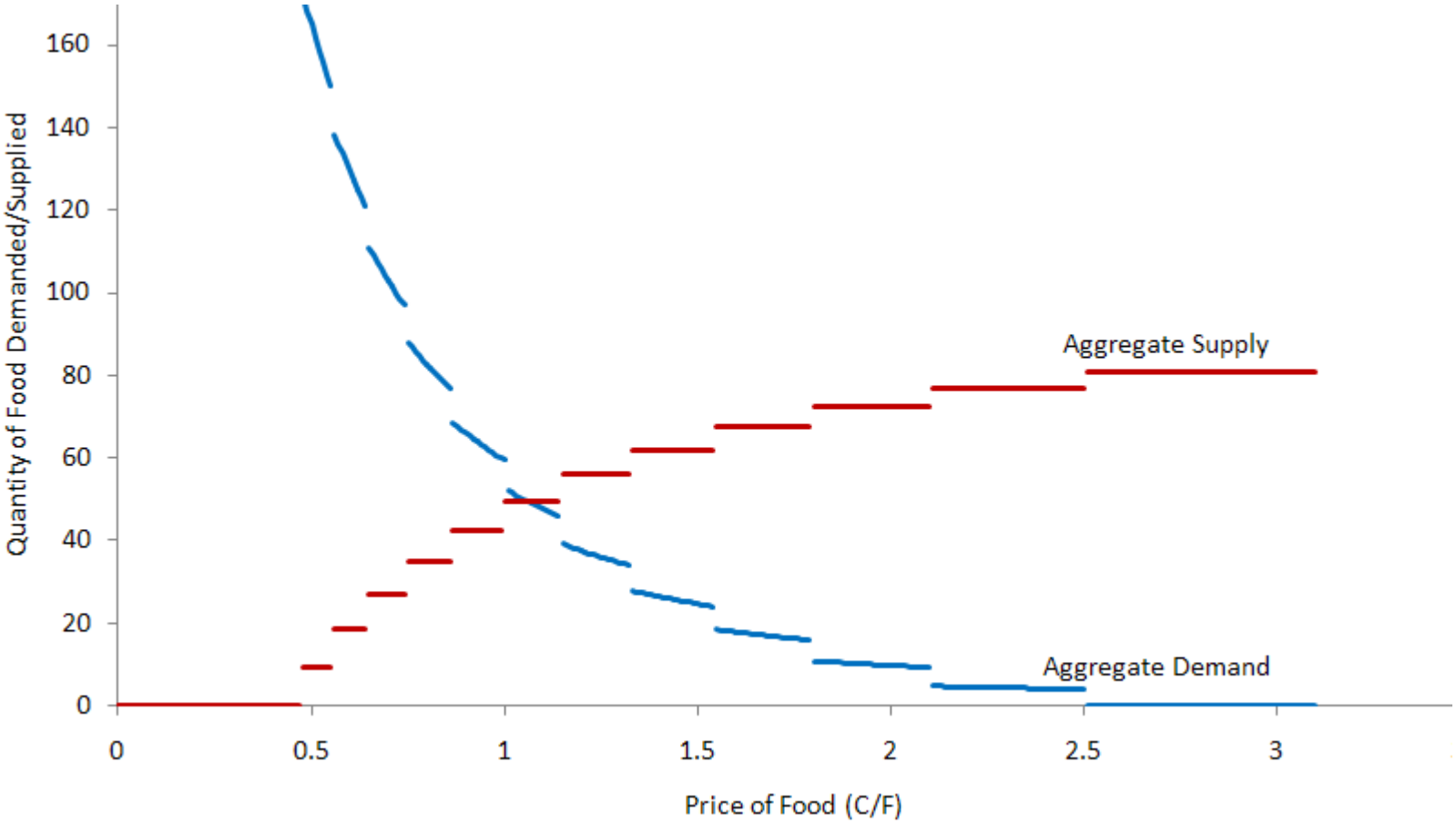


Figure 3.5. Aggregate excess supply and demand for food in a 12-individual economy

Table 3.3. Individual excess demand and supply in a 12-individual economy

	Slope of PPF (C/F)	Demand for and supply of F In self Sufficiency	Excess Demand @ 1.06 C/F	Excess Supply @ 1.06 C/F	Utility Before exchange	Utility After Exchange
Individual 1	2.50	4	9.4	0	6.32	9.71
Individual 2	2.11	4.5	9.0	0	6.54	9.22
Individual 3	1.80	5	8.5	0	6.71	8.74
Individual 4	1.55	5.5	8.0	0	6.84	8.25
Individual 5	1.33	6	7.5	0	6.93	7.77
Individual 6	1.15	6.5	7.1	0	6.98	7.28
Individual 7	1.00	7	0	7.0	7.00	7.21
Individual 8	0.87	7.5	0	7.5	6.98	7.72
Individual 9	0.75	8	0	8.0	6.93	8.24
Individual 10	0.65	8.5	0	8.5	6.84	8.75
Individual 11	0.56	9	0	9.0	6.71	9.27
Individual 12	0.47	9.5	0	9.5	6.54	9.78
Total			49.5	49.5		

Table 3.4. Distribution of production possibilities in a 15-individual economy

	Maximum Supply of Clothing ($C(L_0)$)	Maximum Supply of Food ($F(L_0)$)
Individual 1	23	5
Individual 2	22	6
Individual 3	21	7
Individual 4	20	8
Individual 5	19	9
Individual 6	18	10
Individual 7	17	11
Individual 8	16	12
Individual 9	15	13
Individual 10	14	14
Individual 11	13	15
Individual 12	12	16
Individual 13	11	17
Individual 14	10	18
Individual 15	9	19

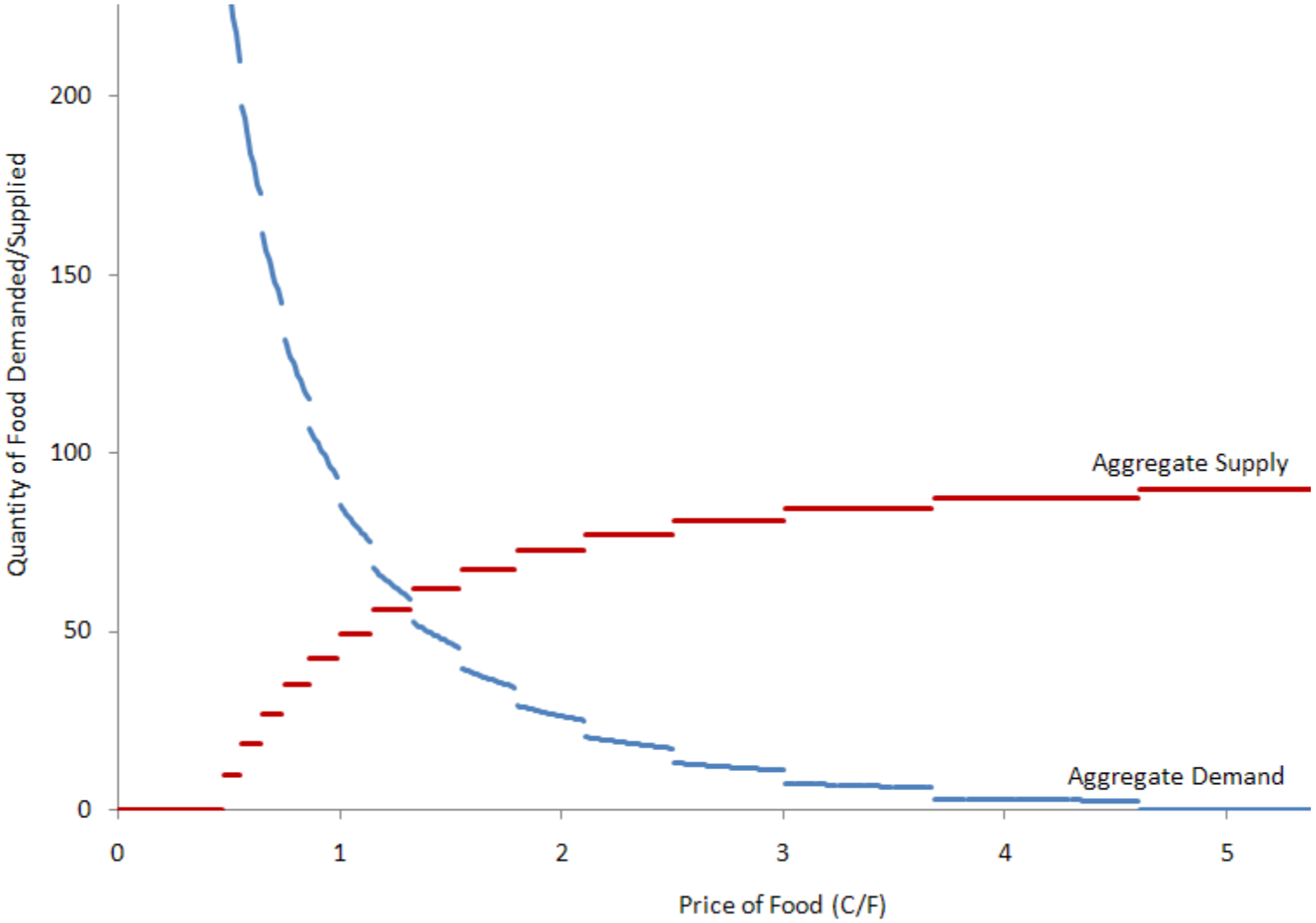


Figure 3.6. Aggregate excess supply and demand for food in a 15-individual economy

Table 3.5. Individual excess demand and supply in a 15-individual economy

	Slope of PPF (C/F)	Demand for and supply of F In self Sufficiency	Excess Demand @ 1.33 C/F[1]	Excess Supply @ 1.33 C/F	Excess demand adjustment[2]	Adjusted Excess Demand	Utility Before exchange	Utility After Exchange
Individual 1	4.60	2.5	8.63	0	-0.41	8.21	5.36	9.72
Individual 2	3.67	3	8.25	0	-0.39	7.86	5.74	9.30
Individual 3	3.00	3.5	7.88	0	-0.38	7.50	6.06	8.87
Individual 4	2.50	4	7.50	0	-0.36	7.14	6.32	8.45
Individual 5	2.11	4.5	7.13	0	-0.34	6.79	6.54	8.03
Individual 6	1.80	5	6.75	0	-0.32	6.43	6.71	7.61
Individual 7	1.55	5.5	6.38	0	-0.30	6.07	6.84	7.18
Individual 8	1.33	6	6.00	0	0.00	6.00	6.93	6.93
Individual 9	1.15	6.5	0	6.50	0	0	6.98	0.00
Individual 10	1.00	7	0	7.00	0	0	7.00	0.00
Individual 11	0.87	7.5	0	7.50	0	0	6.98	0.00
Individual 12	0.75	8	0	8.00	0	0	6.93	0.00
Individual 13	0.65	8.5	0	8.50	0	0	6.84	0.00
Individual 14	0.56	9	0	9.00	0	0	6.71	0.00
Individual 15	0.47	9.5	0	9.50	0	0	6.54	0.00
		90	58.50	56.00	-2.50	56.00		

Notes:

1. The exchange ratio of 1.33 C/F is where the difference between the aggregate excess supply and demand is the smallest.
2. The excess demand adjustments are calculated by allocating a share of the total difference between the aggregate excess demand and supply (2.5 F) at 1.33 C/F to each of the individuals. The individual adjustment shares are equal to the share of each individual excess demand in the aggregate excess demand at 1.33 C/F. The only criterion used here was to use some consistent method of determining the excess supply and demand adjustments. There are other consumption patterns where all individuals would prefer exchange over self-sufficiency.

A Near-continuous Distribution of Individual Production Possibilities

The next step in the model development involves an abstraction. Namely, when the aggregate supply and demand functions in Figure 3.4, Figure 3.5, and Figure 3.6 are compared, it becomes evident that as the number of individuals in the economy increases, the function steps become smaller relative to the total supply and demand. Thus, these functions are asymptotically continuous (i.e. when the supply/demand of each individual is infinitely small compared to the total supply/demand). Consequently, the price interval for which no one switches from being a producer of C to being a producer of F tends to 0. This means that aggregate excess demand and supply tend to continuous functions as the number of individuals increases.

Thus, treating the aggregate excess supply and demand functions as continuous is a reasonable simplification for an economy with many participants. In these cases, the step functions for the market-clearing condition can be approximated by the integral:

$$\frac{\alpha}{P_F}\int_{C^*{}_1(L_0)}^{C^*{}_n(L_0)} C^*(L_0)dC^* = (1-\alpha)\int_{F^*{}_1(L_0)}^{F^*{}_n(L_0)} F^*(L_0)\,dF^* \tag{3.42}$$

At $P_F = 0$, the expression on the left side of equation (3.42) is infinite, while the expression on the right is equal to 0. The left side is monotonically decreasing in P_F and reaches 0 at P_F equal to the slope of the steepest individual PPF[19]. The expression on the right is increasing in price. It is 0 at $P_F = 0$, becomes non-zero at the point where (P_F) is equal to the slope of the flattest individual PPF, and reaches some finite value at P_F equal to the slope of the steepest individual PPF. It follows that there is a positive P_F, for which the equality between the aggregate excess supply and demand holds. Analogous logic applies to P_C, where $P_C = \frac{1}{P_F}$.

[19] Assuming food is on the horizontal axis.

Figure 3.7 shows the market-clearing in the market for food. The intercept of the aggregate excess supply (S_F) is equal to the slope of the flattest individual PPF $\left(\frac{C_i(L_0)}{F_i(L_0)}\right)_{min}$. This is because, as shown in Figure 3.4, Figure 3.5, and Figure 3.6, at the exchange ratio lower than the slope of the flattest PPF, all individuals would want to specialize in C.

The aggregate excess supply is monotonically increasing in P_F. Assuming there is only comparative advantage across individuals; that is, no individual has absolute advantage over another individual, aggregate excess supply is increasing in slope. This is because the first to specialize in the production of food (i.e. at the lowest price) are the individuals with the flattest PPFs. These are the individuals relatively most productive in F. As the price increases, those with steeper PPFs and lower productivity start producing F. Thus, equal increases in price add progressively less supply to the aggregate supply of F.

The intercept of the aggregate excess demand for F is at $\left(\frac{C_i(L_0)}{F_i(L_0)}\right)_{max}$. This is the slope of the steepest individual PPF. As the price of F decreases, more individuals specialize in C and demand F in exchange. Similar to aggregate excess supply, aggregate excess demand increases in slope with the increases in price (Appendix 3).

At the market-clearing price, P_F, the quantity of food exchanged on the market is F_e and the total quantity produced in the economy is $\frac{F_e}{(1-\alpha)}$ (see equation (3.28)). Some proportion of the population specializes in F while the rest specializes in C. Producers of F sell excess F in exchange for some C from the producers of C. Similarly, producers of C sell excess C in exchange for F. The total amount of C exchanged on the market at price $P_c = \frac{1}{P_F}$ is C_e and the total quantity of C produced is $\frac{C_e}{\alpha}$ (see equation (3.39).

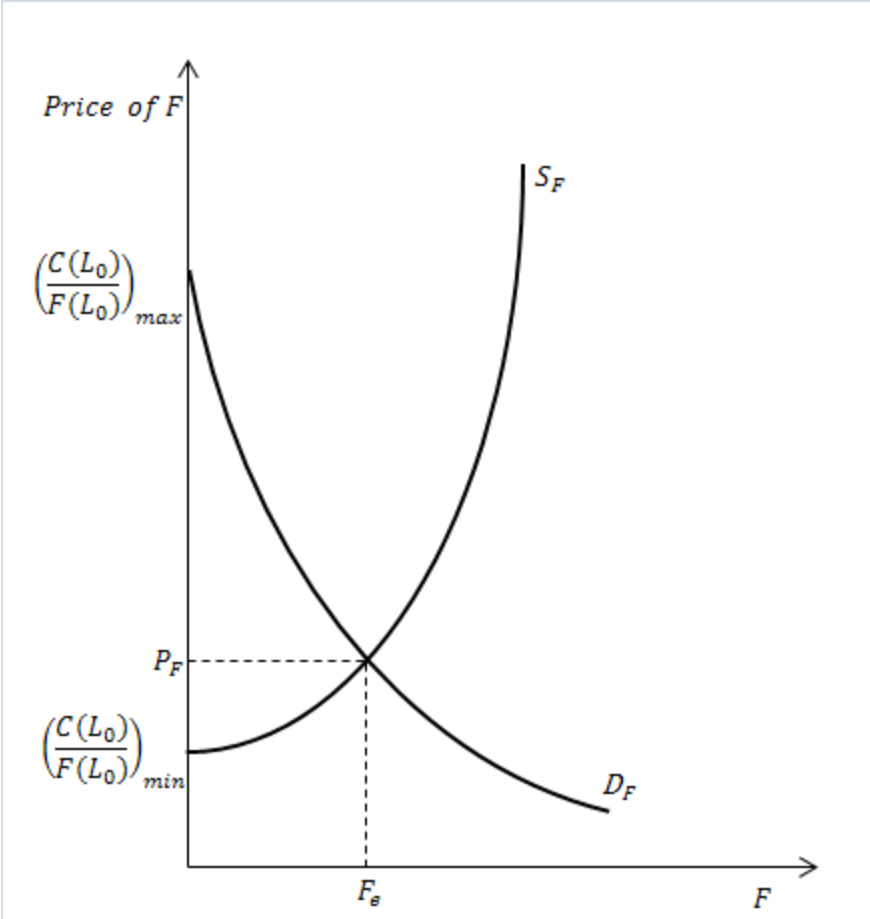


Figure 3.7. Aggregate excess supply and demand for food and equilibrium price

So far, this model explains individual specialization in an exchange economy and defines the market-clearing exchange ratio. The next section will examine how prices can differ between two economies depending on the differences in the distribution of individual production possibilities. The difference in the distribution of individual production possibilities may stem, as in the case of individual exchange, from better (or worse) nature-given factors in different areas or skill of some producers in either line of production (either in transforming the natural resources into capital goods or in combining capital goods and labour time into one of the finished products). Quantification of the relationship between the qualitative factor endowment and quantitative productivity is, of course, impossible.

However, given there are differences in the distribution of the individual production possibilities between two hypothetically isolated economies, some statements about exchange ratios in the two economies can be derived. The necessary condition for differences in the distribution of individual production possibilities is that the two hypothetical areas are not identical in all respects. In the real world, this condition is regularly fulfilled. However, this is not a sufficient condition. Even two different distributions of individual production possibilities can produce identical autarky prices. However, this would be an accident rather than regularity. Even in such an unlikely event, cross-boundary trade is not excluded, since on both sides of the border there may be some individuals with a comparative advantage in either of the two products. The next section looks at the more likely event, when the autarky exchange ratios differ between the two economies.

Differences in Autarky Prices

The previous analysis has demonstrated how individual production possibilities and preferences determine the market exchange ratios in a 2-good multi-individual economy. Thus, if there are two isolated economies consisting of individuals where the distributions of production possibilities or preferences differ between the two countries, the market-clearing prices in the two economies may differ as well. This section presents a stylized example to demonstrate this point.

First, equation (3.42) can be rewritten as:

$$\frac{\alpha}{P_F}\int_{C_{min}}^{C_{max}} C^*(L_0)dC^* = (1-\alpha)\int_{F_{min}}^{F_{max}} F^*(L_0)\,dF^* \tag{3.43}$$

After integrating, equation (3.43) becomes:

$$\frac{\alpha}{P_F}(C_{max}^2 - C_{min}^2) = (1-\alpha)(F_{max}^2 - F_{min}^2) \tag{3.44}$$

Isolating P_F in equation (3.44), leads to:

$$P_F = \frac{\alpha}{(1-\alpha)}\frac{(C_{max}^2 - C_{min}^2)}{(F_{max}^2 - F_{min}^2)} \tag{3.45}$$

Equivalently, using equation (3.40):

$$P_C = \frac{1}{P_F} = \frac{(1-\alpha)}{\alpha}\frac{(F_{max}^2 - F_{min}^2)}{(C_{max}^2 - C_{min}^2)} \tag{3.46}$$

Equations (3.45) and (3.46) can be used to compare prices in two isolated economies that differ in the distribution of individual production possibilities. Assuming continuous, uniform distributions in both countries:

Home: $F^h \sim U[{F^h}_{min}, {F^h}_{max}]$ and $C^h \sim U[{C^h}_{min}, {C^h}_{max}]$ (3.47)

Abroad: $F^a \sim U[{F^a}_{min}, {F^a}_{max}]$ and $C^a \sim U[{C^a}_{min}, {F^a}_{max}]$ (3.48)

where

F^h is the distribution of the quantity of F that each resident of Home can produce using all available labour time (L_0)

F^a is the distribution of the quantity of F that each resident of Abroad can produce using all available labour time (L_0)

C^h is the distribution of the quantity of C that each individual at Home can produce using all available labour time (L_0)

C^a is the distribution of the quantity of C that each individual Abroad can produce using all available labour time (L_0)

For tractability, I will assume that the correlation between individual productivity in the two goods for both countries is – 1. The purpose of this assumption is to enable specification of the intercept of the aggregate excess demand and supply, given the specification of the two distributions of individual production possibilities. The assumption implies that there is only comparative advantage across individuals (i.e. no absolute advantage). The individual most productive in F (per unit of time) is also the least productive in C. Thus, this is the individual that would specialize in F at the lowest price of F compared to all other individuals. From this, we know that the slope of this individual's PPF represents the intercept of the aggregate supply.

On the other hand, the individual that is the least productive in F, is at the same time the most productive in C. Thus, this is the individual with the steepest PPF (given F is on the horizontal axis). This individual would specialize in F at the highest price of F, compared to all other individuals in the country. This means that this individual would otherwise specialize in C and demand F in exchange. Thus, the slope of this individual's PPF determines the intercept of the aggregate excess demand for food.

I will examine two cases to demonstrate how the market-clearing prices in the two economies can be determined analytically and graphically: (i) when the distribution of individual

production possibilities is such that the productivity of labour time of the most productive individual in F Abroad is higher than the productivity of the most productive individual at Home[20], while the productivities of the most productive individual in C and the least productive individual in F are identical between the two countries and (ii) when the productivity of labour time of the most productive individual in C is lower Abroad than at Home, while the productivities of the most productive individual in F and the least productive individual in C are identical between the two countries[21]:

(i) $$F^h{}_{min} = F^a{}_{min} \;;\; C^h{}_{min} = C^a{}_{min} \;;\; C^h{}_{max} = C^a{}_{max} \;;\; F^h{}_{max} < F^a{}_{max} \quad (3.49)$$

Substituting the above conditions into equations (3.45) and (3.46) results in:

$$P_F^a < P_F^h \text{ and } P_C^a > P_C^h \quad (3.50)$$

The price of F is lower Abroad compared to Home, and the price of C is lower at Home compared to Abroad.

(ii) $$F^h{}_{min} = F^a{}_{min} \;;\; C^h{}_{min} = C^a{}_{min} \;; C^h{}_{max} > C^a{}_{max} \;;\; F^h{}_{max} = F^a{}_{max} \quad (3.51)$$

Substituting the above conditions into equations (3.45) and (3.46) results in:

$$P_F^a < P_F^h \text{ and } P_C^a > P_C^h \quad (3.52)$$

The price of F is lower Abroad compared to Home, and the price of C is lower at Home compared to Abroad.

The market-clearing prices can also be derived graphically, using the aggregate excess supply and demand curves. The next section shows this procedure.

[20] Since uniform distributions are assumed in both countries, the two distributions will be different not only in extremes but in all parameters. However, specifying extremes is useful in defining the aggregate supply and demand and the price difference

[21] Note that this also implies differences in average labour productivity between countries.

Supply-Demand Graphical Representation

The slope of the PPF for the most productive individual in F is

$$\left(\frac{C^i}{F^i}\right)_{min} = \frac{C^i{}_{min}}{F^i{}_{max}} \tag{3.53}$$

The slope of the PPF for the least productive individual in F

$$\left(\frac{C^i}{F^i}\right)_{max} = \frac{C^i{}_{max}}{F^i{}_{min}} \tag{3.54}$$

where i denotes country. Then:

$\left(\frac{C^h}{F^h}\right)_{min}$ and $\left(\frac{C^a}{F^a}\right)_{min}$ are the intercepts for the aggregate excess supply of F at home and abroad respectively

$\left(\frac{C^h}{F^h}\right)_{max}$ and $\left(\frac{C^a}{F^a}\right)_{max}$ are intercepts for the aggregate excess demand for F at home and abroad respectively

It follows that in (*i*):

$$\left(\frac{C^h}{F^h}\right)_{min} > \left(\frac{C^a}{F^a}\right)_{min} ; \quad \left(\frac{C^h}{F^h}\right)_{max} = \left(\frac{C^a}{F^a}\right)_{max} \tag{3.55}$$

and in (*ii*):

$$\left(\frac{C^h}{F^h}\right)_{min} = \left(\frac{C^a}{F^a}\right)_{min} ; \quad \left(\frac{C^h}{F^h}\right)_{max} > \left(\frac{C^a}{F^a}\right)_{max} \tag{3.56}$$

In (*i*), there is a difference in aggregate excess supply between the two countries while in (*ii*) the difference is in aggregate excess demand. The two situations are represented in Figure 3.8.

The use of the term price difference as opposed to national comparative advantage is intentional here. Since comparative advantage is individual and is expressed through voluntary market transactions, the market prices reflect this individual comparative advantage. However, it would be wrong to say that a lower autarky price is the same as a "national" comparative advantage because only individuals have the ability to recognize and express comparative advantage. Even in the importing country, there are individuals who specialize (and thus

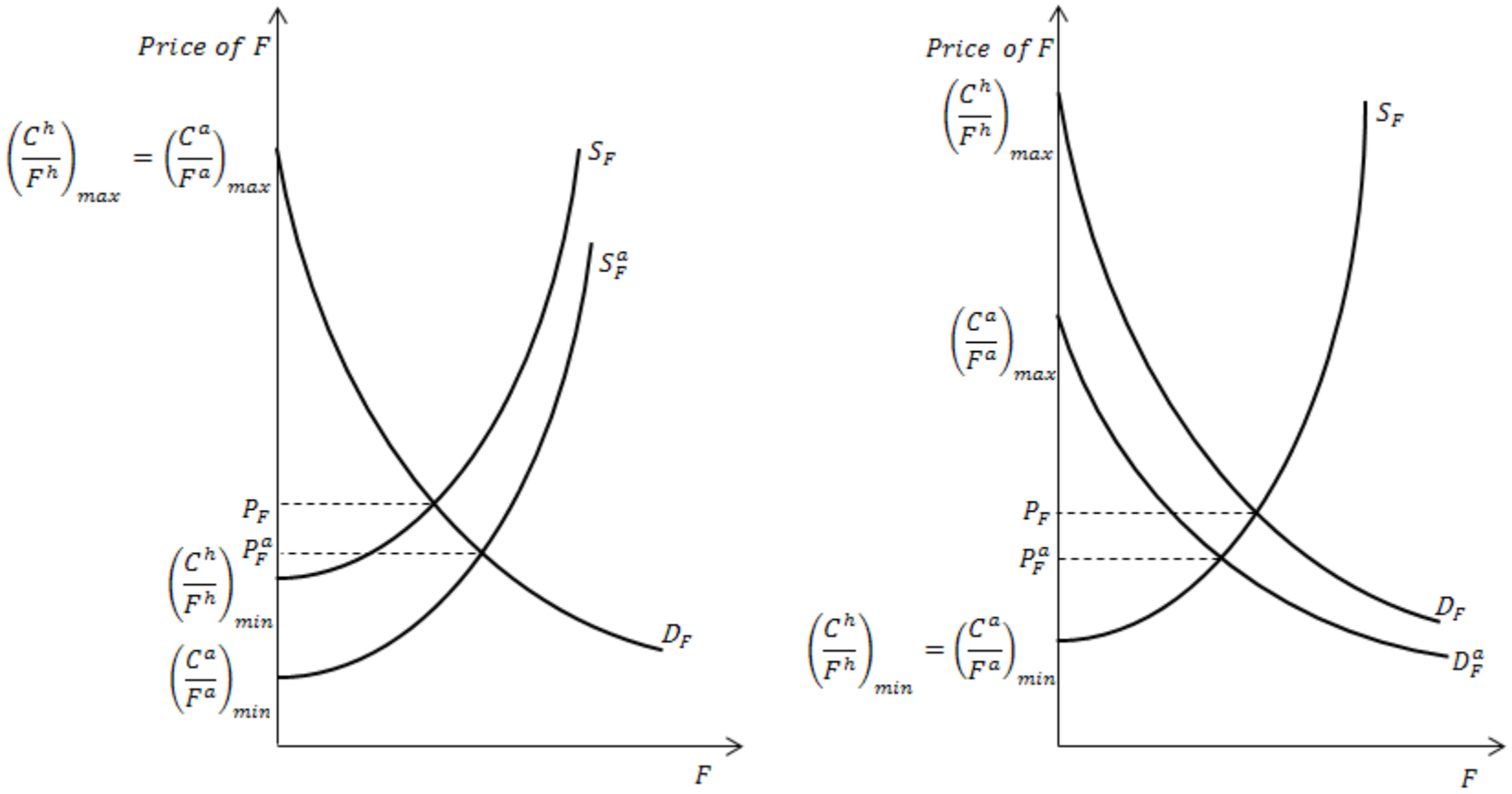


Figure 3.8. Differences in autarky prices across economies depending on the distribution of individual production possibilities

have a comparative advantage) in producing the imported product[22].

Non-identical Preferences between Populations

Up to this point, the model consisted of individuals with identical preferences both within and across countries. In order to bring the model closer to the underlying subjective theory of value[23], I will first relax the assumption of identical preferences across countries and maintain identical preferences within countries as well as identical distribution of individual production possibilities. Let the preferences of all the individuals at Home be described by the utility function shown in equation (3.57):

$$U(F_i, C_i) = {F_i}^{\alpha} {C_i}^{1-\alpha} \tag{3.57}$$

and let the preferences Abroad be represented by the utility function shown in equation (3.58)

$$U(F_i, C_i) = {F_i}^{\beta} {C_i}^{1-\beta} \tag{3.58}$$

and let $\alpha > \beta$.

This means that the preferences for food at Home are relatively stronger than Abroad. Then, from equation (3.29), it follows that the price of F at Home is

$${P_F}^{H} = \frac{\alpha}{(1-\alpha)} \frac{(C_{max}^2 - C_{min}^2)}{(F_{max}^2 - F_{min}^2)} \tag{3.59}$$

and the price of food Abroad is:

[22] However, even in this model, the underlying assumption of complete information makes it possible for individuals to know not only their own comparative advantage but also other people's comparative advantage. While obviously unrealistic, the purpose of this assumption was to illustrate how and why individuals benefit by pursuing their own comparative advantage through market transactions, and how this would translate into market prices under these simplified conditions.

[23] Note that there is still a gap between the Austrian subjectivism and this model because it is assumed here that an external modeler is able to specify individual preferences. However, Hayek stresses the fact that individual information is not directly available to other individuals in the economy. Nevertheless, it is a useful exercise to examine what *might* happen rather that what *does* happen in the real economy.

$$P_F{}^A = \frac{\beta}{(1-\beta)} \frac{(C_{max}^2 - C_{min}^2)}{(F_{max}^2 - F_{min}^2)} \tag{3.60}$$

Since individual production possibilities are identical by assumption, any price difference would result from differences in preferences. It follows that $P_F{}^H > P_F{}^A$ (and $P_C{}^H < P_C{}^A$). The Home price of food, $P_F{}^H$, is higher than Abroad, $P_F{}^A$. The opposite applies for the price of C, $P_C{}^H$.

This argument can be extended to different combinations of production possibilities and preferences. It can be shown that if preferences and individual production possibilities are related in the same direction, the price gap between two markets will be smaller. For example if the Home population is relatively more productive in food (i.e. productivity distribution with higher maximum and minimum) but also has stronger preferences for food relative to clothing compared to Abroad, the price-lowering effect of higher productivity will be offset by a higher demand. On the other hand, if productivity and preferences are inversely related (i.e. a population relatively more productive in food (clothing) demands less food (clothing)) the price gap between the two economies will be wider.

Non-identical Preferences within Populations

So far, I assumed that all individuals within an economy are characterized by identical preferences. However, preferences may differ from individual to individual. Some may prefer having more clothing while others may want to have more food.

In the case where each individual has specific preferences, equation (3.29) is transformed into equation (3.61):

$$P_F = \frac{\sum_{i=1}^{n} \alpha_i C_i{}^*(L_0)}{\sum_{i=1}^{n} (1-\alpha_i) F_i{}^*(L_0)} \tag{3.61}$$

where α_i is the individual-specific demand parameter.

Multiplying the numerator and denominator of equation (3.61) by $\frac{1}{n}$ results in equation (3.62):

$$P_F = \frac{\frac{1}{n}\sum_{i=1}^{n} \alpha_i {C_i}^*(L_0)}{\frac{1}{n}\sum_{i=1}^{n}(1-\alpha_i){F_i}^*(L_0)} \tag{3.62}$$

$$= \frac{mean(\alpha_i) \cdot mean\left({C_i}^*(L_0)\right) + cov(\alpha_i, {C_i}^*(L_0))}{mean(1-\alpha_i) \cdot mean\left({F_i}^*(L_0)\right) + cov(1-\alpha_i, {F_i}^*(L_0))}$$

In the previous case, when α was constant across individuals, equation (3.29) reduces to equation (3.63):

$$P_F = \frac{\alpha}{1-\alpha} \cdot \frac{mean\left({C_i}^*(L_0)\right)}{mean\left({F_i}^*(L_0)\right)} \tag{3.63}$$

Comparing equations (3.62) and (3.63), it can be seen that the difference in the price under preference heterogeneity compared to the price under uniform preferences depends on the covariance between individual preferences and the pattern of specialization. Given that α is the mean of the distribution, in the absence of covariance (i.e. when individual productivity and preferences are uncorrelated), the resulting market-clearing price is identical to the price found when preferences are identical across all individuals. This is because the difference between individual demand under preference heterogeneity and the situation when preferences are uniform is a random variable. As long as the distribution of the parameter α is symmetric around the mean, the distribution of differences will be symmetric around zero. Thus the positive and negative differences would cancel out.

${C_i}^*(L_0)$ and ${F_i}^*(L_0)$ are negatively correlated by assumption. Consequently, when the covariance between α and ${C_i}^*(L_0)$ is negative, the covariance between $1-\alpha$ and ${F_i}^*(L_0)$ is also negative. This means that individuals that have a comparative advantage in food also have a

stronger preference for food compared to the situation when α is fixed at the mean. At the same time, individuals that have a comparative advantage in clothing, prefer more clothing compared to the situation when all preferences are identical. As a result, the effect of the preference heterogeneity on market price depends on the relative magnitudes of the two covariance terms.

Table 3.6 shows that, if the distribution of the individual specialization and the parameter α are relatively symmetric (production possibilities are not perfectly symmetric while α is) there is almost no change in the market price. Table 3.6 represents the earlier 12-individual example, shown in Table 3.2 and Figure 3.5, where α is 0.5 for all individuals. In this table, α is distributed uniformly between 0.08 and 0.92, with the mean of 0.5. The resulting price (1.064) units of C per unit of F) is almost identical to the price obtained under identical preferences (0.61).

This result can be interpreted in the following manner: Those individuals that specialize in food are also the ones whose demand for food is higher (compared to the situation when α is fixed at the mean). Thus, their excess supply of food is lower compared to the situation when α is fixed at the mean. At the same time, individuals that specialize in clothing will be the ones whose demand for food is lower (compared to the situation when α is fixed at the mean), and the reduction in excess supply and excess demand cancel each other out. This is evident from Table 3.6 as well. Both excess supply and excess demand were reduced by about a half. This simply means that those that specialize in food (clothing), also consume more food (clothing), which leaves less food (clothing) for exchange.

On the other hand, if the correlation between productivity and preferences is negative, individuals that specialize in food, would demand less food (and more clothing) compared to the situation when preferences are uniform. This would bring the excess supply of food up. At the

same time, individuals that specialize in clothing, would demand more food (and less clothing), which would bring the excess demand up. Again, the net effect on the price depends on the magnitude of the two covariance terms relative to the respective means. In the cases when the effect of the two covariance terms are close in relative magnitude (the denominator and the numerator in equation (3.62) change by the same proportion relative to equation (3.63)), the two effects cancel each other out, and the effect on price is small.

However, the distribution of preferences and individual specialization may not be symmetric. In this case the effect of heterogeneity of preferences on the market price is more complex and depends on the actual distributional parameters of individual productivity and preferences.

Table 3.7 shows a hypothetical asymmetric distribution where α is distributed non-uniformly around the mean, 0.5. The resulting market-clearing price (1.103 C/F) in this situation is higher than in the case of a uniform, symmetric distribution However, due to the relatively small number of individuals in the economy, the market-clearing price has not changed sufficiently to alter the pattern of specialization. In a larger model, where even small prices would cause a shift of marginal producers from one industry to the other, this effect is possible.

Further computations would be needed to tract the effect of different distributional parameters. In any case, this section has demonstrated that heterogeneity of preferences, when preferences are correlated with productivity, has an effect on the quantities of goods exchanged on the market and can have an effect on prices. In the cases when preferences affect the market-clearing prices, this is accompanied by a change in the pattern of specialization (because market prices affect the individual trade-offs between specializing in either of the two goods). While equation (3.62) provides the general condition that the market-clearing price needs to satisfy,

Table 3.6. Specialization pattern and market-clearing for a hypothetical uniform and symmetric distribution of individual preferences

α	$1 - \alpha$	$C^*(L_0)$	$F^*(L_0)$	$\alpha C^*(L_0)$	$(1 - \alpha)F^*(L_0)$
0.08	0.92	20	0	1.54	0.00
0.15	0.85	19	0	2.92	0.00
0.23	0.77	18	0	4.15	0.00
0.31	0.69	17	0	5.23	0.00
0.38	0.62	16	0	6.15	0.00
0.46	0.54	15	0	6.92	0.00
0.54	0.46	0	14	0.00	6.46
0.62	0.38	0	15	0.00	5.77
0.69	0.31	0	16	0.00	4.92
0.77	0.23	0	17	0.00	3.92
0.85	0.15	0	18	0.00	2.77
0.92	0.08	0	19	0.00	1.46
means					
0.5	0.5	8.75	8.25	2.24	2.11
$Cov(a,C^*(L_0))$	$Cov(1 - a,C^*(L_0))$	P_F (α fixed)		P_F (α varies)	
-2.13	-2.02	1.061		1.064	
		Excess Demand of F	Excess Supply of F	Excess Demand of F	Excess Supply of F
		49.5	49.5	26.92	26.92

Table 3.7. Specialization pattern and market-clearing for a hypothetical asymmetric non-uniform distribution of individual preferences

α	$1 - \alpha$	$C^*(L_0)$	$F^*(L_0)$	$\alpha C^*(L_0)$	$(1 - \alpha)F^*(L_0)$
0.20	0.80	20	0	4.00	0.00
0.21	0.79	19	0	3.99	0.00
0.22	0.78	18	0	3.96	0.00
0.23	0.77	17	0	3.91	0.00
0.29	0.71	16	0	4.64	0.00
0.45	0.55	15	0	6.75	0.00
0.50	0.50	0	14	0.00	7.00
0.55	0.45	0	15	0.00	6.75
0.70	0.30	0	16	0.00	4.80
0.80	0.20	0	17	0.00	3.40
0.90	0.10	0	18	0.00	1.80
0.95	0.05	0	19	0.00	0.95
mean					
0.5000	0.5000	8.75	8.25	2.27	2.06
$Cov(\alpha, C^*(L_0))$	$Cov(1 - \alpha, C^*(L_0))$	P_F (α fixed)		P_F (α varies)	
-2.10	-2.07	1.061		1.103	
		Excess Demand	Excess Supply	Excess Demand	Excess Supply
		49.5	49.5	27.25	27.25

going into more detailed computations for specific distributional parameters would be beyond the purpose and scope of this paper.

Specialization and learning by doing

Now, I will turn to formalizing the second part of Rothbard's argument – gain in total productivity resulting from specialization does not need to be limited to a better allocation of resources through pursuing individual comparative advantage. Individuals may become more productive after devoting all their attention to producing only one good. Many authors recognize these gains in productivity due to acquisition of skill, specific techniques and tools (i.e. human and physical capital). This acquisition of specific capital resources through specialization can be incorporated into the model developed in the preceding sections.

Consider an individual with production possibilities similar to the one presented in Figure 3.9. Depending on the market price, an individual would specialize either in food or clothing. The panel on the left shows the situation when the market price of food is lower than the slope of the individual's PPF. The individual specializes in the production of clothing because he can obtain greater utility of consumption through exchange compared to being self-sufficient. However, after specialization, he or she devotes all his or her time only to the production of clothing. It is likely that, due to this focus on only one commodity, he or she would obtain better skills and/or design better tools, specific for the production of clothing. This all contributes to a higher productivity of his or her labour time. Now, instead of being able to produce C^* units of clothing in L_0 units of time, he or she may be able to produce ${C_{K_C}}^*(L_0)$, where the subscript K_C represents the specific inputs obtained as a result of specialization in the production of clothing. These inputs include both human and physical capital and are different in kind from the inputs obtained by any other individual.

Under the given market price, this individual now enjoys utility U^2, higher than before he or she increased his or her productivity (i.e. U^1). Thus, individual motivation for an increase in productivity is evident. Similarly, an individual who originally specialized in the production of food can now devote all his or her time to the production of food (shown in the right panel). This is may enable him or her to obtain specific knowledge and design tools to become more productive in the production of food and increase production from $F^*(L_0)$, to ${F_{K_F}}^*(L_0)$. This is followed by an increase in utility from U^1 to U^2.

An increase in productivity of many individuals would lead to a change in the market supply and demand for the two products. A simultaneous increase in productivity of both groups of individuals, those specialized in food and those specialized in clothing, may or may not change the market price. This depends on the relative increases in productivity of the two groups. Suppose that the increase in productivity in both groups is proportional so that the market price remains unchanged. What is certain then is that the utility of all individuals increased due to higher consumption possibilities.

This model can also show why specialized individuals may be reluctant to switch from the production of one commodity to the other when the price of their product declines. If an individual deems the price change to be a short-run decline, he or she might stay in the current line of production since his or her ability to develop the inputs specific to the other line of production are limited in the short run. This means that the level of utility that could be obtained by switching is likely to be lower in the short run than the utility in the current line of production, even at a lower price. However, if an individual deems the price decline to be permanent, this also implies that the price of the other commodity has increased permanently, and, depending on

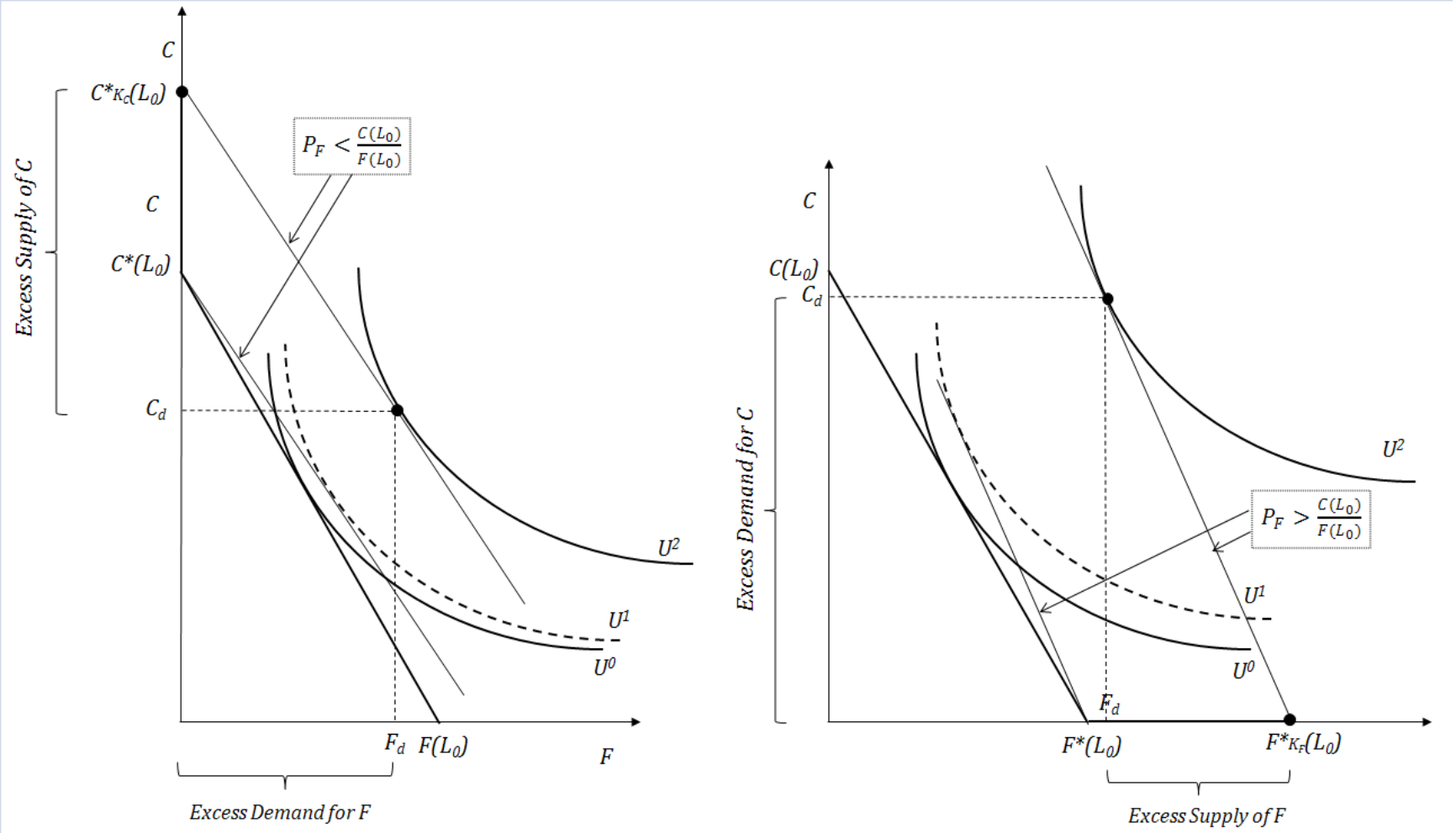


Figure 3.9. Learning through specialization

the individual's judgment of his or her own capability of developing the inputs specific to the other line of production, one may choose to switch.

This section has shown how individual motivation for learning by doing and short vs. long-run production decisions can be explained using the micro-foundations approach consistent with the logic of a utility-maximizing agent. The next section looks at why some specialized individuals might support policies that are directed at restricting the total supply of the commodity that these individuals produce.

Supply Restrictions

The inadequacy of H-O type models becomes particularly apparent in cases when supply is restricted by an administrative decision. As shown in Figure 3.10, the version of the H-O model with identical and homothetic preferences within and across populations (Vanek, 1968) implies that the optimal production-consumption ratio of goods is at the point where the indifference curve of the representative consumer (i.e. all consumers have identical and homothetic preferences) is tangent to the economy PPF (Point A). Then, restricting the supply of any of the two goods away from this point would cause a movement along the economy PPF (from A to B), up to the restricted quantity of one of the goods. This consumption bundle is Pareto inferior to the competitive outcome because it lays on a lower indifference curve for all consumers.

But, all individuals in the economy are consumers. The indifference curve U_p represents the preferences of *all* individuals in the economy. Thus, these are the preferences of the clothing producers as well as the food producers. The higher food price resulting from a reduced supply

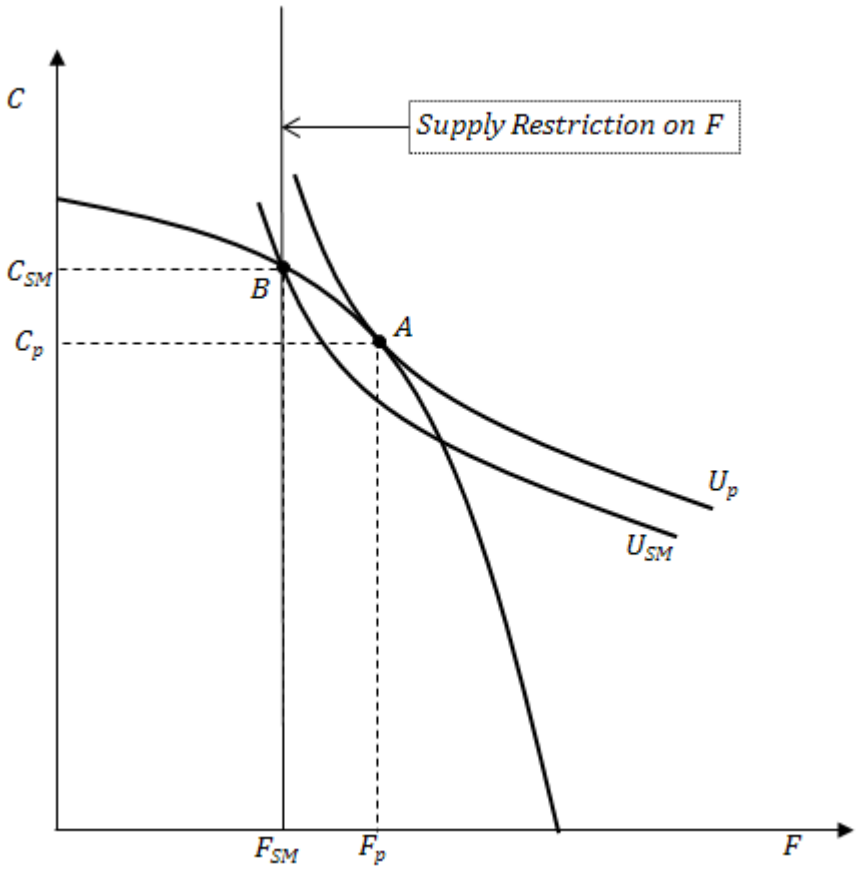


Figure 3.10. The Heckscher-Ohlin-Vanek model and supply management

would be preferred by the food producers only if it enables them to acquire more preferable bundles of food and clothing. However, this aggregate model implies that all individuals would be able to consume only less preferred bundles of food and clothing as a consequence of this policy. Thus, it turns out that no one would support supply restrictions because the consumption bundle under the unrestricted supply is preferred by all. This is sharply at odds with the current partial equilibrium analysis of supply management that indicates that there would be at least some individuals (i.e. some producers) that are better off due to higher prices.

The model developed in this paper can explain why some individuals (i.e. producers) would prefer a reduction in supply even though they are also consumers at the same time. This is illustrated in Figure 3.11 and Figure 3.12. Figure 3.11 shows how a supply restriction of food raises the market-clearing price of food (and, equivalently, lowers the market-clearing price of clothing). A restriction on the market supply can be implemented if, for example, every producer is required to reduce his or her production by a certain percentage.

The combination of the individual production reduction and an increase in the market price are illustrated in Figure 3.12. The panel on the left represents a producer of clothing. Since the price of clothing has dropped relative to the price of food, the clothing producer can now buy less food for the same amount of clothing that he or she produced. This is indicated by the steeper price line. Consequently, the clothing producer's demand for food will decline from F_d to $F_{d_{SM}}$. At the same time, the food producer has reduced his production from $F_{K_F}(L_0)$ to his or her production quota, F_{SM}. He or she is now facing the new, steeper price line for his or her product. This means that he or she can potentially still buy the same or larger amount of clothing as before using less food. This new consumption bundle is indicated by the point on the new indifference curve U^3.

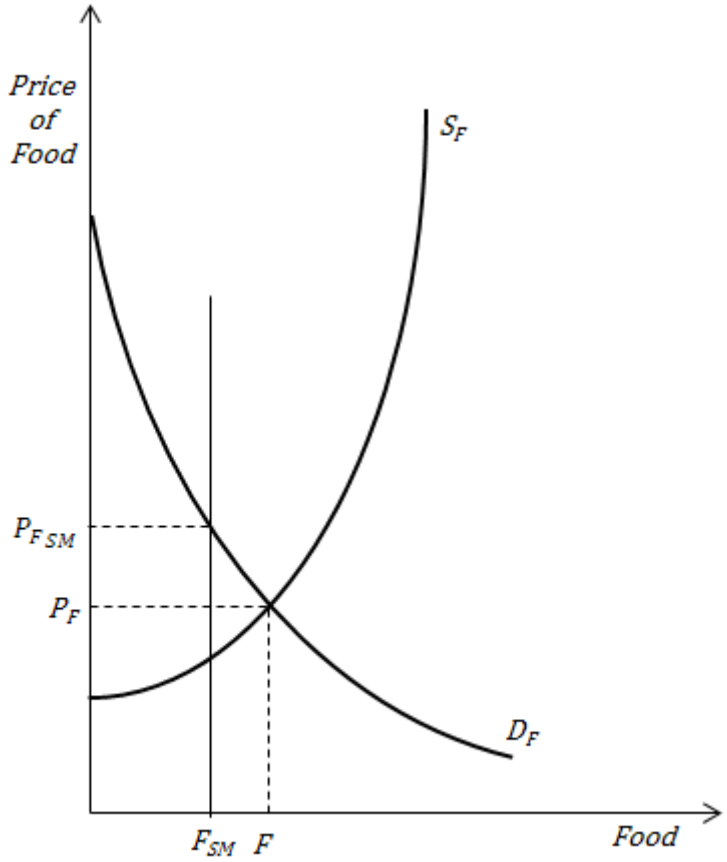


Figure 3.11. Supply restriction and market price

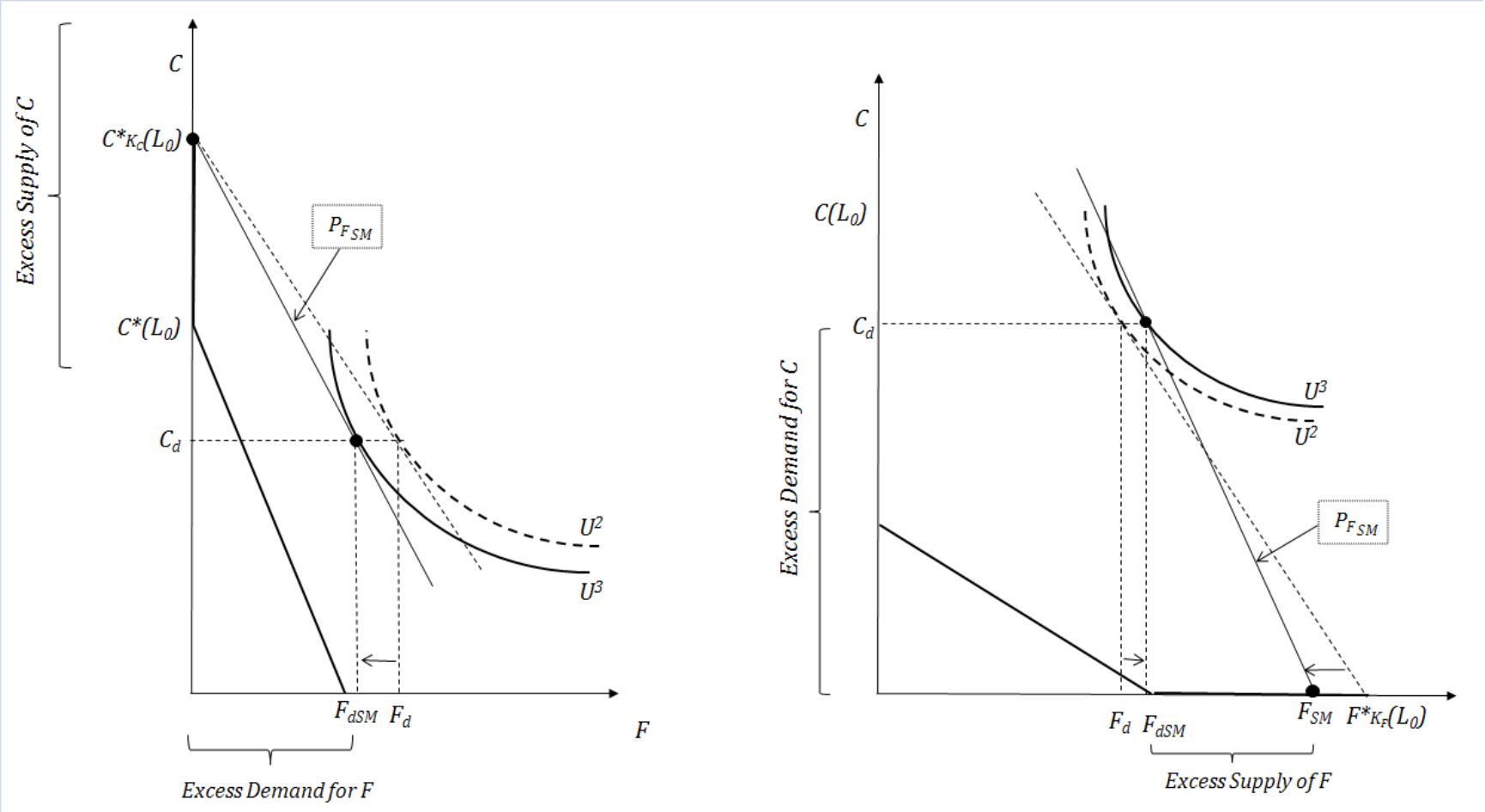


Figure 3.12. Individual consumption bundles and preferences under supply management

Discussion and Conclusions

The model developed in this paper employs the assumptions used in the Stolper-Samuelson stylized version of the Heckscher-Ohlin model: (1) constant returns to scale, (2) no transportation costs, (3) no transaction costs, (4) perfect factor mobility across industries and (5) no factor mobility between countries. However, the input-intensity assumption is replaced with the input-heterogeneity assumption. This is what bridges the gap between individual decisions and aggregate outcomes.

Factors of production are owned by the inhabitants of a given country. Factors of production are mobile across industries because the owners of factors are free to engage in the production of either good. Since this exchange model is defined for two consumption goods, it is implied that individual comparative advantage exists only for those two consumption goods. In other words, no individual has a comparative advantage in production of any capital good. This is why all individuals own capital goods and none are exchanged on the market. Consequently, factors of production are immobile across countries as well.

Existence of a market for capital goods would imply that some individuals are better off by producing only capital goods and exchanging them for consumption goods. Simultaneously, some individuals would be better off purchasing capital goods instead of producing them. Additionally, there may be individuals that do not own any natural resources or capital goods. These individuals would not be able to produce any capital goods or final products without using natural resources or capital goods owned by other individuals. Thus, individuals that own only their labour time could obtain consumption goods only by renting their labour services to the individuals that own natural resources or capital goods. Introducing comparative advantage in capital goods and differences in the ownership structure may provide a more complex and more

realistic model. However, this would add little to the primary purpose of this paper – to bridge the conceptual gap between the individual comparative advantage and aggregate outcomes in a two-good model of autarky price differences.

With respect to the role of the overall capital structure, some general statements can be made. Since the purpose of producing any capital good is to obtain more production per unit of time in the future, a general improvement in the level of capital would lead to an increase in labour time productivity[24]. There is no reason to a priori expect that one industry would exhibit higher improvements in labour productivity due to the improvements in the capital level (i.e. replacement of a horse with a tractor vs. replacement of a needle with a sewing machine). However, if such circumstances occur, then the productivity of individuals in the relatively more improved line of production would lead to an increase in the relative supply and a decrease in the (autarky) exchange ratio for the given product. Depending of the rate of technical innovation across populations, relative productivities may change over time, and those populations that experience relatively rapid improvements in the capital structure in a certain line of production would experience an increase in the relative supply and a reduction in the (autarky) exchange ratio for the given good. However, in light of the previous discussion, any strict quantifications of the relationship between the capital structure and labour time productivity are of a highly questionable meaning.

The most important difference between this model and the H-O paradigm is that inputs other than labour *time* are heterogeneous. As opposed to the H-O model, where *countries* are endowed with homogeneous inputs, in this model, *individuals* are endowed with inputs of varying quality. Each individual uses the inputs that he or she owns in order to achieve the outcome that he or she deems most desirable. While this adds realism to the model, this is not its

[24] Here I use the Austrian interpretation of capital as a progression of steps through time.

primary role. The primary role of input heterogeneity is to provide a logical basis for an existence of market exchanges and thus – prices. Without input heterogeneity (and/or differences in preferences), there would be no exchange between individuals. Without individual exchange, there are no market prices, and without market prices, the idea of autarky price differences loses any meaning.

The purpose of this paper was not to reinvent two hundred years of macroeconomic theory. Rather, the aim was to take a small step toward demonstrating how the basic idea of individual comparative advantage can be translated from the microeconomic optimizing agent framework into the international/interregional context in the same way as it can be applied to the within-nation specialization and exchange. This was done in hope of bridging the gap in the way the concept has been analyzed within the disciplines of microeconomics and macroeconomics. Another objective was to contribute, at least a small part, to ending the long tradition of the Austrian and the "mainstream" neoclassicals' talking past each other, by translating some common ideas into a mutually understandable language. The basic advantage of this approach compared to the standard H-O model is its internal consistency and its consistency with the logic of human action.

The policy implications of the two different approaches to inter-regional coordination of production are significant. While the H-O framework lends itself well to conflicting interventionist policies of production allocation based on different interpretations of ambiguous aggregate data (Katz et al., 2008; Mussell et al., 2009; Bruneau and Schmitz, 2009), the alternative microeconomic approach, developed in this paper, acknowledges the importance of the institutional setting[25] in which individual comparative advantage is discovered and expressed. As Boettke (2001) stresses, different institutional arrangements lead to different aspects of

[25] For example, any inter-individual exchange presupposes the existence of property rights.

individual comparative advantage being identified, developed and expressed by the acting agents, which leads to social outcomes of differing desirability. Lastly, the model demonstrates that the critiques made by the labour value theorists, while correct in claiming that international exchange cannot consistently be explained using the formalist, quantitative language of the H-O model, miss their mark when claiming that capital heterogeneity precludes explanation of international exchange within the neoclassical framework.

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APPENDICES

Appendix 1: Deriving individual demand for food and clothing for an individual specializing in clothing

The individual objective function is to maximize utility given the income constraint.

$$\max U = {F_{D_i}}^{\alpha} {C_{D_i}}^{1-\alpha} \text{ s.t. } F_{D_i} \cdot P_F + C_{D_i} \cdot P_C = C_i^*(L_0)$$

The individual income is the physical quantity of clothing that is available for exchange for food. The individual is not buying clothing on the market. In fact he or she is "buying" his own clothing at the price of 1 unit of clothing per unit of clothing. Thus:

$$P_C = 1\left(\frac{C}{C}\right)$$

And the objective function becomes:

$$\max U = {F_{D_i}}^{\alpha} {C_{D_i}}^{1-\alpha} \text{ s.t. } F_{D_i} \cdot P_F + C_{D_i} = C_i^*(L_0)$$

F.O.C.:

$$U_F = \alpha {F_{D_i}}^{\alpha-1} {C_{D_i}}^{1-\alpha} - \lambda P_F = 0$$

$$U_C = (1-\alpha) {F_{D_i}}^{\alpha} {C_{D_i}}^{-\alpha} - \lambda = 0$$

$$\frac{U_F}{U_C} = \frac{\alpha}{(1-\alpha)} \frac{C_{D_i}}{F_{D_i}} = P_F$$

$$C_{D_i} = P_F \frac{(1-\alpha)}{\alpha} F_{D_i}$$

After substituting into the income constraint:

$$F_{D_i} \cdot P_F + P_F \frac{(1-\alpha)}{\alpha} F_{D_i} = C_i^*(L_0)$$

$$F_D F_i \cdot P_F \left(1 + \frac{(1-\alpha)}{\alpha}\right) = C_i^*(L_0)$$

$$F_{D_i} \cdot P_F \left(\frac{\alpha + 1 - \alpha}{\alpha}\right) = C_i^*(L_0)$$

$$F_{D_i} \frac{P_F}{\alpha} = C_i^*(L_0)$$

$$F_{D_i} = \frac{\alpha}{P_F} C_i^*(L_0)$$

The individual demand for clothing:

$$C_{D_i} = P_F \frac{(1-\alpha)}{\alpha} F_{D_i}$$

$$F_{D_i} = \frac{\alpha}{P_F} C_i^*(L_0)$$

$$C_{D_i} = P_F \frac{(1-\alpha)}{\alpha} \frac{\alpha}{P_F} C_i^*(L_0)$$

$$C_{D_i} = (1-\alpha) C_i^*(L_0)$$

Appendix 2: Deriving individual demand for food and clothing for an individual specializing in food

The individual objective function is to maximize utility given the income constraint.

$$\max U = {F_{D_i}}^{\alpha} {C_{D_i}}^{1-\alpha} \text{ s.t. } F_{D_i} \cdot P_F + C_{D_i} \cdot P_C = F_i^*(L_0)$$

The individual income is the physical quantity of food that is available for exchange for clothing. The individual is not buying food on the market. In fact he or she is "buying" his own food at the price of 1 unit of food per unit of food. Thus:

$$P_F = 1 \left(\frac{F}{F}\right)$$

And the objective function becomes:

$$\max U = {F_{D_i}}^{\alpha} {C_{D_i}}^{1-\alpha} \text{ s.t. } F_{D_i} + C_{D_i} \cdot P_C = F_i^*(L_0)$$

The F.O.C. imply:

$$U_F = \alpha {F_{D_i}}^{\alpha-1} {C_{D_i}}^{1-\alpha} - \lambda = 0$$

$$U_C = (1-\alpha) {F_{D_i}}^{\alpha} {C_{D_i}}^{-\alpha} - \lambda P_C = 0$$

$$\frac{U_F}{U_C} = \frac{\alpha}{(1-\alpha)} \frac{C_{D_i}}{F_{D_i}} = \frac{1}{P_C}$$

$$C_{D_i} = \frac{1}{P_C} \frac{(1-\alpha)}{\alpha} F_{D_i}$$

After substituting into the income constraint:

$$F_{D_i} + \frac{1}{P_C}\frac{(1-\alpha)}{\alpha}F_{D_i} \cdot P_C = F_i^*(L_0)$$

$$F_{D_i} + \frac{(1-\alpha)}{\alpha}F_{D_i} = F_i^*(L_0)$$

$$F_{D_i}\left(1 + \frac{(1-\alpha)}{\alpha}\right) = F_i^*(L_0)$$

$$F_{D_i}\left(\frac{\alpha + 1 - \alpha}{\alpha}\right) = F_i^*(L_0)$$

$$\frac{F_{D_i}}{\alpha} = F_i^*(L_0)$$

$$F_{D_i} = \alpha F_i^*(L_0)$$

The individual demand for clothing:

$$C_{D_i} = \frac{1}{P_C}\frac{(1-\alpha)}{\alpha}F_{D_i}$$

$$F_{D_i} = \alpha F_i^*(L_0)$$

$$C_{D_i} = \frac{1}{P_C}\frac{(1-\alpha)}{\alpha}\alpha F_i^*(L_0)$$

$$C_{D_i} = \frac{(1-\alpha)}{P_C}F_i^*(L_0)$$

Appendix 3: Deriving the slope and the curvature of the aggregate excess demand

Slope of the aggregate excess demand:

$$\frac{d\left[\frac{\alpha}{P_F}\int_{C^*{}_1(L_0)}^{C^*{}_n(L_0)} C^*(L_0)dC^*\right]}{dP_F} = -\frac{\alpha}{{P_F}^2}\int_{C^*{}_1(L_0)}^{C^*{}_n(L_0)} C^*(L_0)dC^* + \frac{\alpha}{P_F}(-C)^* < 0$$

Curvature of the aggregate excess demand:

$$\frac{d\left[-\frac{\alpha}{{P_F}^2}\int_{C^*{}_1(L_0)}^{C^*{}_n(L_0)} C^*(L_0)dC^* + \frac{\alpha}{P_F}(-C)^*\right]}{dP_F} = \frac{\alpha}{{P_F}^3}\int_{C^*{}_1(L_0)}^{C^*{}_n(L_0)} C^*(L_0)dC^* + \frac{\alpha}{{P_F}^2}C^* + \frac{\alpha}{{P_F}^2}C^* - \frac{\alpha}{P_F}\frac{dC^*}{dP_F}$$

$$= \frac{\alpha}{P_F}\left[\frac{1}{{P_F}^2}\int_{C^*{}_1(L_0)}^{C^*{}_n(L_0)} C^*(L_0)dC^* + \frac{2C^*}{P_F} - \frac{dC^*}{dP_F}\right] \approx \frac{\alpha}{P_F}\left[\frac{1}{{P_F}^2}\int_{C^*{}_1(L_0)}^{C^*{}_n(L_0)} C^*(L_0)dC^* + F^*\right] > 0$$

$$dP_F = d\left(\frac{C^*}{F^*}\right) \approx \frac{dC^*}{F^*}$$

$$\frac{dC^*}{dP_F} \approx \frac{dC}{\frac{dC^*}{F^*}} = F^*$$

$$P_F < \frac{C^*}{F^*}$$

$$\frac{2C^*}{P_F} > 2F^*$$