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TAIL DEPENDENCE OF COMMODITY FUTURES RETURNS IN THE AGRICULTURAL AND ENERGY SECTORS

Key words: bivariate models, commodity futures returns, copula, tail dependence, ARMA-GARCH model

ABSTRACT. The goal of this research was to examine tail dependence structures between selected commodity futures returns. Tail dependence, called also extremal dependence, was evaluated for the pairs of commodities coming from the same sector (energy or agricultural). The study covers the years 2018-2023, embracing the COVID-19 pandemic and the outbreak of the Russia-Ukraine war. To achieve the goal, bivariate dynamic models were applied to percentage log returns of commodity futures. Marginal distributions were described using the ARMA-GARCH models. Joint distributions were constructed using the symmetrized Joe-Clayton copula, which allowed to model asymmetric dependence in the tails of a distribution. Time variation of the copula parameters, here equal to tail dependence coefficients, was described using the evolution equations [Patton 2006]. In the energy sector, the dependence in both tails of analyzed distributions was relatively strong, dynamic and higher in the lower tail than in the upper tail. On the contrary, the agricultural sector lacks common patterns of tail dependency. This feature of the agricultural sector creates an opportunity for investors or risk managers to build well-diversified portfolios.

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INTRODUCTION

Since the early 2000s, commodity markets have attracted much attention. Institutional investors have started to intensively trade on this market to diversify their portfolios. Their inflows have been rising and at the same time agricultural prices have increased significantly. This phenomenon, called financialization of commodity markets, have consequences on both financial and real economy, therefore understanding the processes taking place there is significant.

The literature on commodity futures markets is reach and covers wide range of topics, like relationship between stock and commodity markets [Büyüksahin et al. 2009, Aït-Youcef 2018], cross-sectional dependencies [Tang and Xiong 2012, Chen et al. 2023], relationship between position taken by agent and price dynamics [Cheng et al. 2015], risk of extreme returns [Echaust 2014] and many others. More comprehensive literature review might be found in [AïtYoucef 2018]. However, studies concerning bivariate models from one sector are still rare [Just 2019] and usually they concentrate on the average dependence between commodity prices or returns. Meanwhile, dependence in the tails of a return distribution, called also extremal dependence, is of greater importance for an investor. Simultaneously occurring extreme values cause corresponding changes in a portfolio value, which results in huge losses during periods of turbulences and huge gains during periods of upheavals.

Additionally, dependence structures observed on commodity market are time-varying, which is another reason for further studies. Commodity futures market is sensitive to global demand violations [Aït-Youcef 2018], which subsequently cause instability of dependence structures [Chen et al. 2023]. The outbreaks of the COVID-19 pandemic and the Russia-Ukraine war were certainly factors causing such disturbances. The pandemic has changed global supply chains. The war, though it primarily impacts Europe, has an impact on the entire world. Russia is one of the largest crude oil, refined products and natural gas exporters, while Ukraine is one of the largest corn and soybean exporters. After the outbreak of the war, Europe shifted away from reliance on gas from Russia and increased its efforts towards renewable energy. On the other hand, Russia increased its export to countries outside Europe. The prices in the energy sector were affected worldwide [Inacio et al. 2023]. Due to the number of possible price determinants in the agricultural markets and connectedness of these markets [Just and Echaust 2022], the prices in this sector also should be perceived worldwide.

The goal of this article is to examine extremal dependencies of futures returns between commodities from the same sector. In particular, the research investigates existence of tail dependency, its' time variation and asymmetry between the upper and lower tails. The chosen sectors are the energy and agricultural ones.

RESEARCH MATERIAL AND METHODS

To achieve the goal of the article, conditional joint distributions were modeled using the copula theory. Copulas allow to detach marginal distributions from dependency structure. Therefore, to create a bivariate distribution using the copula theory, separate models must be specified for marginal distributions and copulas. Here, the ARMA-GARCH model was used for marginal distributions and the symmetrized Joe-Clayton copula was utilized to join the marginal distributions. In general this model is called the Copula-ARMA-GARCH model. To show time variation of the copula parameters, the evolution equations were used [Patton 2006].

This paragraph is organized as follows: first the data are described, then the conditional bivariate copula model is outlined, later the ARMA-GARCH model is presented, finally the symmetrized Joe-Clayton copula and the parameters evolution equations are described.

DATA

The chosen commodities are the components of the FTSE/CoreCommodity CRB Index, which currently consists of 19 elements. As the aim of the study is the pairwise comparison of extreme returns, only 11 commodities were chosen for further research: 4 commodities from the energy sector and 7 from the agricultural sector. However, the sum of their weights equals 77% of the whole index.

The research is based on the percentage daily log returns of the closing prices, calculated according to the formula:

$$y_{i,t} = 100 \left(\ln P_{i,t} - \ln P_{i,t-1} \right) \tag{1}$$

where $P_{i,t}$ denotes the closing price of the commodity future i, $i = 1, ..., N$, at period t, $t = 1, ..., T$.

The time scope embraces six years between 2018 and 2023, i.e. the prepandemic and pandemic periods, as well as the prewar and war in Ukraine periods. It must be noted, that on April 20, 2020, WTI crude oil futures dropped down below zero for the first time since its listing and reached minus 37.63 USD. One of the main reasons for the occurrence of this event, besides fundamental factors, was change of the market trading mechanism by Chicago Mercantile Exchange (CME) around April 15, 2020. Starting from August 31, 2020, the market mechanism was changed back by CME [Lu and Bu 2023]. Therefore, as it was the one-time extraordinary event, it was excluded from the analysis.

Table 1. Commodity futures' weights in the FTSE/CoreCommodity CRB Index and summary statistics of their percentage daily log returns (2018-2023)

Sector	Commodity name	Weight [%]	T*	Mean	St. dev.	Min	Max	Skewness	Kurtosis
Energy	Heating oil	5	1,509	0.014	2.656	-24.754	12.357	-1.145	14.022
	Natural gas	6	1,509	-0.013	4.203	-30.048	38.173	0.121	10.734
	RBOB gasoline	5	1,509	0.012	3.140	-38.535	22.397	-2.071	31.313
	WTI crude oil	23	1,508	0.011	3.567	-60.168	31.963	-3.123	71.826
Agriculture	Cocoa	5	1,509	0.051	1.777	-8.902	11.491	-0.031	4.902
	Coffee	5	1,509	0.024	2.161	-9.021	9.557	0.262	4.068
	Corn	6	1,506	0.019	1.781	-19.100	7.657	-1.806	22.534
	Cotton	5	1,509	0.003	1.869	-27.292	6.779	-2.036	33.001
	Live Cattle	6	1,508	0.023	1.224	-15.648	7.000	-1.619	26.536
	Soybeans	6	1,508	0.020	1.345	-11.092	6.426	-0.740	9.564
	Sugar	5	1,509	0.020	1.750	-8.178	10.814	0.023	5.184

* T – time series length
Source: own elaboration on the basis of [FTSE Russell 2024] and data from financial service https://finance.yahoo.com/

The names of the commodities, their weights in the index and summary statistics of the time series are presented in Table 1. Arithmetic means are close to zero for all commodities under consideration. Skewness is different from zero and kurtosis is higher than three in all cases, which suggests that probability of extreme values is higher than in the normal distribution. The source of the commodity futures prices was the "yahoo!finance" page.

CONDITIONAL BIVARIATE COPULA MODEL

The research presented in this article is based on conditional joint distributions for continuous variables described in [Patton 2006]. The description below is accommodated to the bivariate case considered in this paper.

Let's assume that $\boldsymbol{Y}_{ij} = (Y_i, Y_j)^T$, $i, j=1,\dots,N$, $i \neq j$, denote a random vector with cumulative distribution function F_{ij} and let F_i and F_j denote the marginal distributions of Y_i and Y_j accordingly. Then, according to Sklar [1959], there exists a copula C_{ij}: $[0,1]^2 \rightarrow [0,1]$ such that, for all $\boldsymbol{y}_{ij} = (y_i, y_j) \in \mathbb{R}^2$, we have $F_{ij}(\boldsymbol{y}_{ij}) = C_{ij}\{F_i(y_i), F_j(y_j)\}$.

To denote that a random vector $\boldsymbol{Y}_{ij}$ has a cumulative distribution function F_{ij} we write $\boldsymbol{Y}_{ij} \sim F_{ij} = C_{ij}\ \{F_i\ (y_i),\ F_j\ (y_j)\}$. Copula is unique for continuous random variables and it operates on the probability integral transforms of the original variables, i.e. on variables $U_i = F_i\ (y_i)$ and $U_j = F_j\ (y_j)$, where $U_i \sim U\,(0,1)$ and $U_j \sim U\,(0,1)$.

The description of conditional joint distributions is based on some information set $\mathcal{F}_{t-1}$ and decomposes the conditional distribution of $\boldsymbol{Y}_{ijt}$ into two parts, conditional marginal distributions and the conditional copula. Assuming the following notation:

$$\boldsymbol{Y}_{\boldsymbol{ijt}}|\mathcal{F}_{t-1} \sim F_{ij}(\cdot\,|\mathcal{F}_{t-1}) \tag{2}$$

$$Y_{it}|\mathcal{F}_{t-1} \sim F_i(\cdot\,|\mathcal{F}_{t-1}) \tag{3}$$

$$Y_{jt}|\mathcal{F}_{t-1} \sim F_j(\cdot\,|\mathcal{F}_{t-1}) \tag{4}$$

conditional joint distribution is described as:

$$F_{ij}\left(\boldsymbol{y}_{\boldsymbol{ij}}|\mathcal{F}_{t-1}\right) = C_{ij}\left\{F_i(y_i|\mathcal{F}_{t-1}), F_j\left(y_j|\mathcal{F}_{t-1}\right)|\mathcal{F}_{t-1}\right\} \tag{5}$$

where the same information set is used for both marginals and for the copula.

In the research, the parameters of each marginal distribution and each copula were estimated separately, using the maximum likelihood method. In the next paragraphs, the details concerning marginal distributions and copula are given.

MARGINAL MODELS

Marginal distribution of percentage log returns of each commodity futures was described using the ARMA-GARCH model or its selected extensions. In general, marginals have the following form [Patton 2012]:

$$Y_{it} = \mu_i(\boldsymbol{Z}_{t-1}; \boldsymbol{\phi}_i) + \sigma_i(\boldsymbol{Z}_{t-1}; \boldsymbol{\phi}_i)\varepsilon_{it} \tag{6}$$

$$Y_{jt} = \mu_j(\boldsymbol{Z}_{t-1}; \boldsymbol{\phi}_j) + \sigma_j(\boldsymbol{Z}_{t-1}; \boldsymbol{\phi}_j)\varepsilon_{jt} \tag{7}$$

where:

$$\boldsymbol{Z}_{t-1} \in \mathcal{F}_{t-1} \tag{8}$$

$$\varepsilon_{it}|\mathcal{F}_{t-1} \sim F_{it} \tag{9}$$

$$\varepsilon_{jt}|\mathcal{F}_{t-1} \sim F_{jt} \tag{10}$$

$$\boldsymbol{\varepsilon}_{ijt} \equiv \left(\varepsilon_{it}, \varepsilon_{jt}\right)^T|\mathcal{F}_{t-1} \sim F_{\varepsilon ijt} = C_{ijt}\left(F_{it}, F_{jt}\right) \tag{11}$$

$\boldsymbol{\phi}_i, \boldsymbol{\phi}_j$ denote the marginal distributions parameters of the commodity i and j respectively, $i, j = 1, ..., N$, $i \neq j$ at period $t = 1, ..., T$.

The conditional time-varying means μ_i and μ_j are modeled by autoregressive moving average (ARMA). The conditional time-varying volatility σ_i and σ_j are described by one of the following models [Broda and Paolella 2020]: ARCH, GARCH, TGARCH or EGARCH. The standardized residuals ε_{it} and ε_{jt} are described by the conditional distributions F_{it} and F_{jt} respectively, with mean zero and standard deviation equal to one, chosen from the following set: the standard normal distribution (sn), the standardized Student's *t* distribution (std), the standardized skewed *t* distribution (sstd) and the standardized generalized error distribution (sged).

SYMMETRIZED JOE-CLAYTON COPULA

As the main goal of the paper is to measure dependence between extremes, the symmetrized Joe-Clayton copula was chosen, allowing for asymmetric dependence in the tails of a distribution. First the "BB7" copula [Joe 1997] is presented, as the symmetrized Joe-Clayton copula is the modification of this copula.

Let Y_i and Y_j be random variables, where $i, j = 1, ..., N$ and $i \neq j$. Then let's define by probability integral transform $U = F_i(Y_i)$ and $V = F_j(Y_j)$, where F_i, F_j are the marginal distributions of Y_i and Y_j respectively. The "BB7" copula, also called the Joe-Clayton copula [Patton 2006], is defined as follows:

$$C_{JC}(u,v) = 1 - \left(1 - ((1-(1-u)^{\kappa})^{-\gamma} + (1-(1-v)^{\kappa})^{-\gamma} - 1)^{-\frac{1}{\gamma}}\right)^{\frac{1}{\kappa}} \tag{12}$$

where $\kappa = 1/\log_2(2-\lambda^U)$ and $\gamma = 1/\log_2(\lambda^L)$.

The parameters $\lambda^U \in (0,1)$ and $\lambda^L \in (0,1)$ measure dependence, respectively in the upper and lower tail of a distribution. Formally, the tail dependence coefficients are defined as [Patton 2006]:

$$\lambda^L = \lim_{\varepsilon \to 0} \Pr[U \le \varepsilon | V \le \varepsilon] = \lim_{\varepsilon \to 0} \Pr[V \le \varepsilon | U \le \varepsilon] = \lim_{\varepsilon \to 0} \frac{C(\varepsilon, \varepsilon)}{\varepsilon} \tag{13}$$

$$\lambda^U = \lim_{\delta \to 1} \Pr[U > \delta | V > \delta] = \lim_{\delta \to 1} \Pr[V > \delta | U > \delta] = \lim_{\delta \to 1} \frac{(1 - 2\delta + C(\delta,\delta))}{1-\delta} \tag{14}$$

If the first limit exists, the copula C exhibits lower tail dependence if $\lambda^L \in (0,1]$ and no lower tail dependence if $\lambda^L = 0$. Respectively, if the second limit exists, the copula exhibits upper tail dependence if $\lambda^U \in (0,1]$ and no upper tail dependence if $\lambda^U = 0$.

The main advantage of the Joe-Clayton copula is the ability to model lower and upper tail dependence separately. The main disadvantage of it is the lack of symmetry even if both dependence measures have the same value. To remove this drawback, the original Joe-Clayton copula was modified and the "symmetrized Joe-Clayton" (SJC) copula was proposed [Patton 2006], defined as:

$$C_{SJC}(u,v|\lambda^U,\lambda^L) = 0{,}5 \cdot (C_{JC}(u,v|\lambda^U,\lambda^L) + C_{JC}(1-u,1-v|\lambda^L,\lambda^U) + u + v - 1) \quad (15)$$

This copula is symmetric when $\lambda^U = \lambda^L$.

It is assumed in the research that the functional form of the copula is not changing over time, but the parameters are allowed to vary. Specifically, the parameters of the SJC copula, change according to the evolution equations [Patton 2006]:

$$\lambda_t^U = \Lambda\left(\omega_U + \beta_U \lambda_{t-1}^U + \alpha_U \cdot \frac{1}{10}\sum_{j=1}^{10}\left|u_{t-j} - v_{t-j}\right|\right) \quad (16)$$

$$\lambda_t^L = \Lambda\left(\omega_L + \beta_L \lambda_{t-1}^L + \alpha_L \cdot \frac{1}{10}\sum_{j=1}^{10}\left|u_{t-j} - v_{t-j}\right|\right) \quad (17)$$

where $\Lambda(x) = (1 + e^{-x})^{-1}$ transforms the values into (0, 1).

Review of other models of time changing copulas might be found in [Doman 2011, Patton 2012].

RESEARCH RESULTS

This section presents at first results for univariate models and then for bivariate models. The bivariate models part includes estimations of constant and time-varying copulas. All calculations were conducted in the Julia programming language.

The marginal model for each commodity, i.e. the ARMA-GARCH model, was selected according to the Bayesian Information Criterion (BIC). Types of the chosen models are presented in Table 2.

Regarding the SJC copula, first the results for the constant copula are presented. This is the case when no time variation in the parameters is included, which means that only one copula is estimated for the whole period for each commodity and its parameters are constant. The estimation results for the upper (λ^U) and lower (λ^L) tail dependence

Table 2. Types of ARMA-GARCH models

Sector	Commodity name	ARMA-GARCH model*
Energy	Heating oil	ARMA(1,1)-EGARCH(1, 1, 1) sstd
	Natural gas	ARMA(1,1)-EGARCH(1, 1, 1) std
	RBOB gasoline	ARMA(1,1)-TGARCH(1, 1, 1) sstd
	WTI crude oil	ARMA(1,1)-TGARCH(1, 1, 1) sstd
Agriculture	Cocoa	ARMA(1,1)-TGARCH(1, 1, 1) sstd
	Coffee	ARMA(1,2)-EGARCH(1, 1, 2) sstd
	Corn	ARMA(1,2)-EGARCH(2, 1, 2) std
	Cotton	ARMA(1,1)-TGARCH(1, 1, 1) sged
	Live cattle	ARMA(1,2)-EGARCH(1, 1, 1) std
	Soybeans	ARMA(1,1)-EGARCH(1, 1, 1) std
	Sugar	ARMA(1,1)-TGARCH(1, 1, 1) std

*std – standardized Student's *t* distribution, sstd – standardized skewed *t* distribution, sged – standardized generalized error distribution
Source: own calculations

coefficients are presented in Table 3. All combinations of commodities from the same sector are displayed in the Table 3.

Regarding the time-varying SJC copula, as the results must be presented on charts, only the outcomes for selected combinations of commodities are illustrated. In particular, three pairs of commodities from the energy sector (Figure 1) and ten pairs from the agricultural sector (Figures 2 and 3) are chosen. The COVID-19 pandemic began at the end of 2019. The beginning of the war in Ukraine took place in February 2022.

Regarding the energy sector, three pairs of commodities with both static tail dependency coefficients higher than 0.01 were chosen. Here the patterns are clearly visible. Changes are dynamic, dependency in the lower tail is higher than in the upper tail and the values of the dependency measures are high, indicating strong dependency between commodities. The pandemic and the war, especially at the beginning, had similar impact on all pairs of the commodities under consideration, though the effect of the war outbreak was much stronger. Dependence in both tails was reduced and the reduction was predominantly greater in the upper tail. Cassela C.M. Inacio et al. [2023], despite using different methodology, also observed decreased dependency between selected commodity futures from the energy sector, especially after the outbreak of the war, and to a minor extent

Table 3. Estimates of upper (λ^U) and lower (λ^L) tail dependence coefficients in constant SJC copula models

Sector	Commodity pair	λ^U	λ^L
Energy	Heating oil – Natural gas	0.005	0.031
	Heating oil – RBOB gasoline	0.498	0.629
	Heating oil – WTI crude oil	0.531	0.676
	Natural gas – RBOB gasoline	0.001	0.028
	Natural gas – WTI crude oil	0.001	0.042
	RBOB gasoline – WTI crude oil	0.521	0.668
Agriculture	Cocoa – Coffee	0.002	0.008
	Cocoa – Corn	0.001	0.001
	Cocoa – Cotton	0.001	0.003
	Cocoa – Live cattle	0.001	0.001
	Cocoa – Soybeans	0.001	0.001
	Cocoa – Sugar	0.001	0.008
	Coffee – Corn	0.035	0.032
	Coffee – Cotton	0.001	0.038
	Coffee – Live cattle	0.001	0.002
	Coffee – Soybeans	0.046	0.031
	Coffee – Sugar	0.069	0.052
	Corn – Cotton	0.015	0.069
	Corn – Live cattle	0.001	0.005
	Corn – Soybeans	0.369	0.366
	Corn – Sugar	0.010	0.069
	Cotton – Live cattle	0.001	0.001
	Cotton – Soybeans	0.020	0.102
	Cotton – Sugar	0.014	0.054
	Live cattle – Soybeans	0.001	0.024
	Live cattle – Sugar	0.001	0.009
	Soybeans – Sugar	0.021	0.064

Source: own calculations

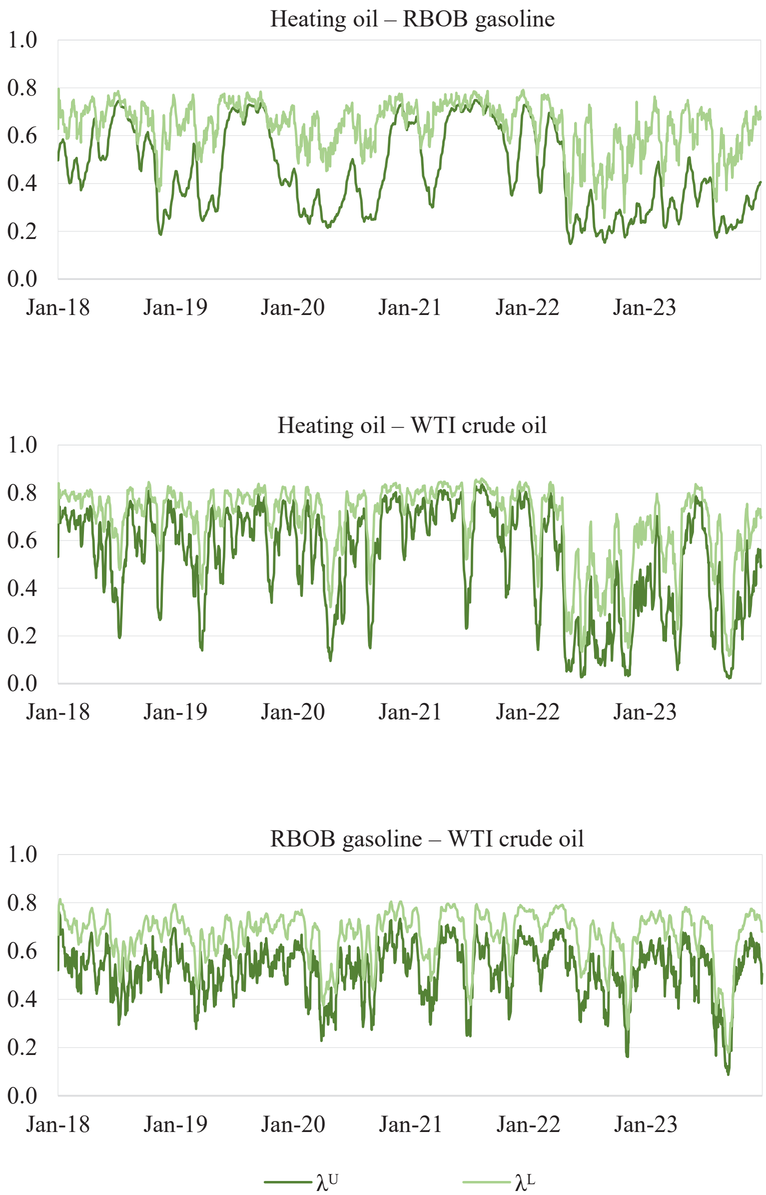

Figure 1. Dynamic upper (λ^U) and lower (λ^L) tail dependence coefficients for selected pairs of commodities from the energy sector in the years 2018-2023

Source: own calculations

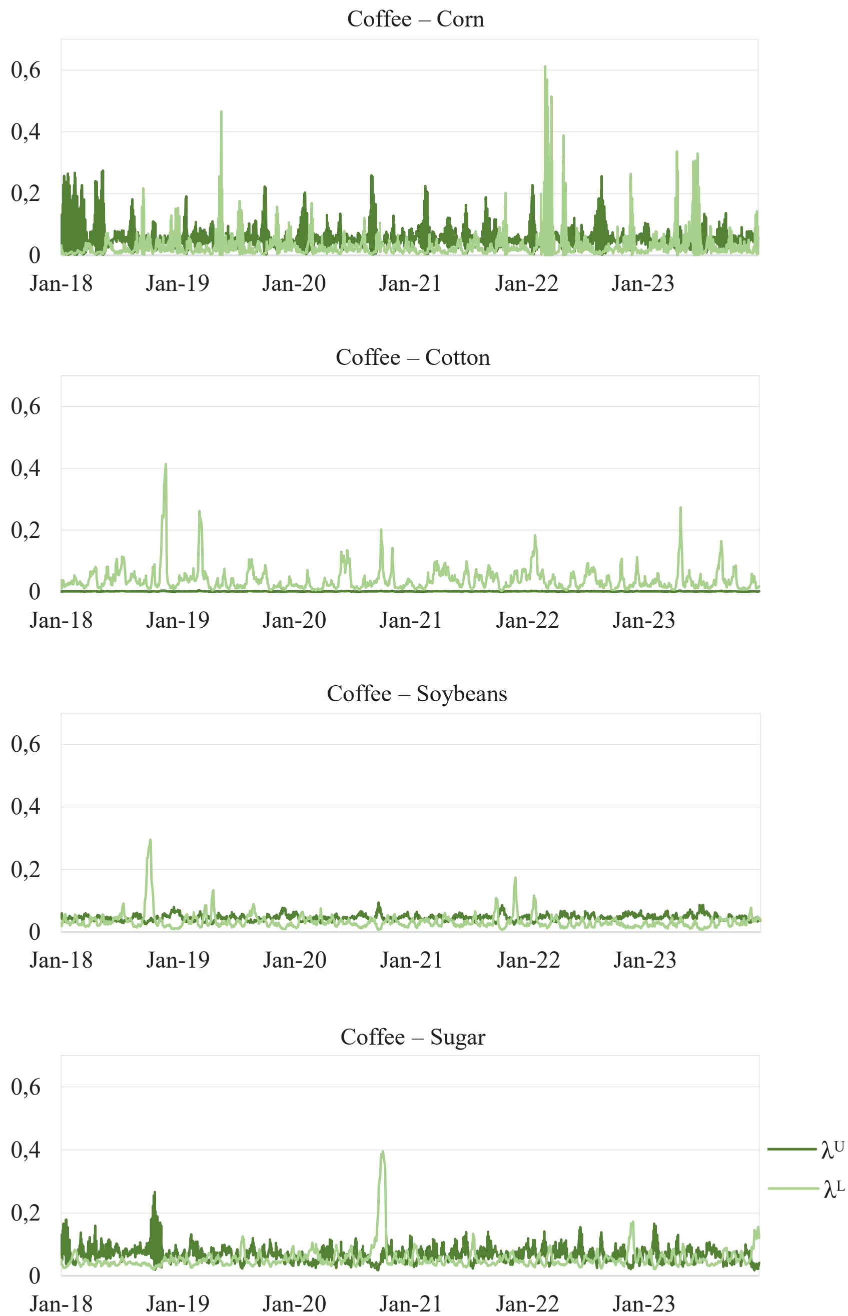

Figure 2. Dynamic upper (λ^U) and lower (λ^L) tail dependence coefficients for selected pairs of commodities from the agricultural sector in the years 2018-2023 (part 1)

Source: own calculations

Figure 3. Dynamic upper (λ^U) and lower (λ^L) tail dependence coefficients for selected pairs of commodities from the agricultural sector in the years 2018-2023 (part 2)

Source: own calculations

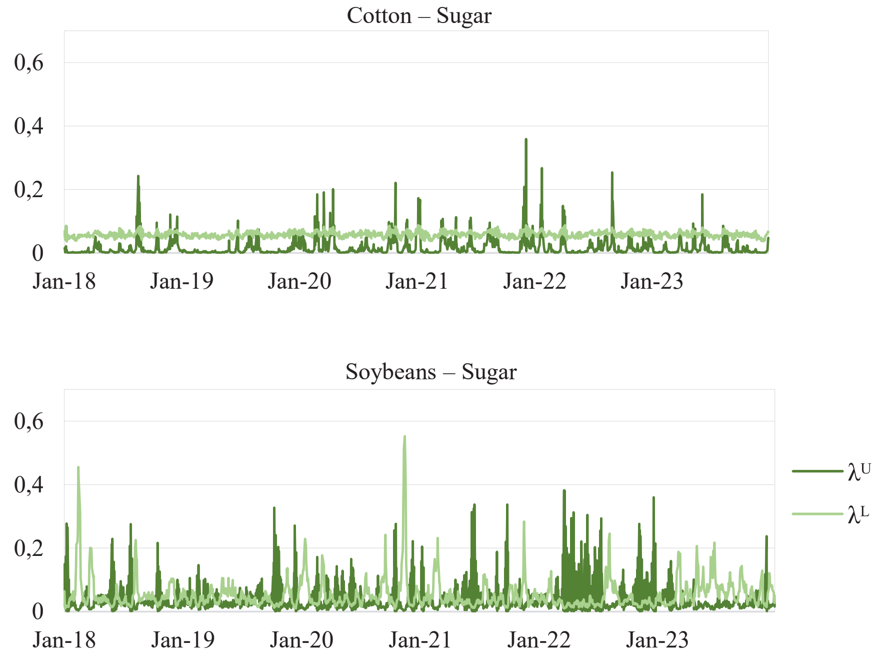

Figure 3. Dynamic upper (λ^U) and lower (λ^L) tail dependence coefficients for selected pairs of commodities from the agricultural sector in the years 2018-2023 (part 3)
Source: own calculations

also after the outbreak of the COVID-19 pandemic. The aforementioned authors suggest three reasons for this phenomenon: increased market volatility, increased uncertainty and the shortages of the products.

Concerning the agricultural sector, ten pairs of commodities were chosen. As might be noticed in constant SJC models (Table 3), Cocoa and Live cattle commodity futures exhibit low dependency with other commodity futures, therefore they were excluded from further analysis. From the remaining commodities, only the ones with at least one constant tail dependence coefficient higher than 0.01 were investigated.

The tail dependency coefficients structures presented on the charts are very different. For example, the bivariate distribution of Coffee – Cotton futures exhibits moderate dependency in the lower tail and almost no dependency in the upper tail. The joint distribution of the Corn – Cotton futures has dynamically changing both tail dependence coefficients from 0 to more than 0.6. In the joint distribution of Corn – Soybeans futures, the tail dependency in both tails has stabilized around 0.4, with very slight changes in the lower tail and more dynamic changes in the upper tail. The pandemic and the Russia-Ukraine war had an impact on the tail dependency coefficients only in several cases. Just after the outbreak of the war, Coffee – Corn and Corn – Sugar lower dependency coefficients have risen sharply and the upper dependency coefficient of Soybeans – Sugar has gone up. Just after the pandemic outbreak, Corn – Cotton lower dependency coefficient has risen. Corn – Soybeans upper dependency coefficient has fallen down just after the two events. As Ukraine is a significant exporter of corn, soybean and to a lesser extent also sugar, some of these movements might be interpreted on this ground. However, further research is needed to give exact explanations.

SUMMARY

Deep understanding of the processes in commodity market is crucial to make rational decisions in both financial and real economy. On the one hand, commodity market allows investors to diversify their portfolios and reduce the risk. On the other hand, food and energy prices affect producers and consumers decisions. This research aims to complement existing studies in the field of bivariate tail dependence. The added value is the application of the symmetrized Joe-Clayton copula and letting the parameters change according to the evolution equations [Patton 2006]. In particular, the study investigates existence, time variation and asymmetry of extremal dependencies between the selected commodity futures originating from the same sector. It reveals that the patterns of fluctuations are substantially different between the energy and agricultural sectors. Moreover, lack of homogeneity in the tail dependency patterns seems to be the characteristic of the agricultural sector. This feature should be taken into account when modeling tail dependency in higher dimensions, which is the direction for further research.

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ZALEŻNOŚCI W OGONACH ROZKŁADÓW STÓP ZWROTU Z TOWAROWYCH KONTRAKTÓW FUTURES NA RYNKACH ROLNYM I ENERGETYCZNYM

Słowa kluczowe: modele dwuwymiarowe, stopy zwrotu z towarowych kontraktów futures, kopule, zależności w ogonach rozkładów, model ARMA-GARCH

ABSTRAKT. Celem badania było zweryfikowanie zależności w ogonach rozkładów stóp zwrotu z wybranych towarowych kontraktów futures. Zależność w ogonach rozkładów, zwana również zależnością ekstremalną, została oceniona dla par towarów pochodzących z tego samego sektora (energetycznego lub rolnego). Badaniem objęto lata 2018-2023, a więc okres pandemii COVID-19 oraz wybuchu wojny na Ukrainie. Do modelowania procentowych logarytmicznych stóp zwrotu z towarowych kontraktów futures zastosowano dwuwymiarowe dynamiczne modele. Rozkłady brzegowe opisano za pomocą modeli ARMA-GARCH. Rozkłady łączne konstruowano za pomocą symetryzowanej kopuli Joego i Claytona, co pozwoliło na modelowanie asymetrycznych zależności w ogonach rozkładu. Zmienność w czasie parametrów kopuli, tutaj równych współczynnikom zależności w ogonach rozkładu, opisano za pomocą specyfikacji dynamiki wektora parametrów [Patton 2006]. W sektorze energetycznym zależności w obu ogonach analizowanych rozkładów były relatywnie silne, dynamiczne i większe w ogonie dolnym niż w górnym. Natomiast w sektorze rolniczym nie odnaleziono wspólnych wzorców w strukturach zależności pomiędzy ogonami rozkładów. Ta cecha sektora rolniczego umożliwia inwestorom i zarządzającym ryzykiem efektywne dywersyfikowanie portfeli.

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