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## Technical efficiency and farm size: a conditional analysis

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### Abstract

The relationship between technical efficiency and size might be affected by farm heterogeneity. We analyse this relationship conditional on a set of control variables. These control variables are chosen using a production model where technical efficiency is introduced as a parameter. As a result, technical efficiency affects both the input demand and the output supply of a profit maximising producer. The empirical application explores these issues using panel data of dairy farms in Spain.

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### 1. Introduction

Following the classical definition of Farrell (1957), a firm is considered to be technically efficient if it obtains the maximum attainable output given the amount of inputs and the technology used. Since technical efficiency is unobservable, it has to be estimated somehow. In the parametric approach the typical way to do this is to model inefficiency as part of the random term (Aigner et al., 1977). In the nonparametric approach, linear programming methods such as data envelopment analysis (DEA) are used to calculate an envelope of the data and the distance (inefficiency) of each observation from the frontier.

Many studies have shown that inefficiency is the rule rather than the exception (see Battese, 1992, for a survey of efficiency in agriculture). This finding is important because the main consequence of technical inefficiency is to raise production costs, making farms less competitive. Therefore, its study is interesting and has triggered the publication of many papers devoted to the measurement of efficiency.

In particular, a number of papers in this literature have analysed the relationship between efficiency/productivity and size. The extent of this research effort has prompted Townsend et al. (1998) to ask provocatively: “is another paper on the relationship between farm size and productivity necessary?” The authors convincingly argue that more research needs to be done for two reasons: the extreme relevance of the topic and some flaws in previous analysis.

The current relevance of the topic can be illustrated by three policy issues frequently analysed in

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the literature. The first policy issue is the effects of farm growth policies on efficiency. Many traditional agricultural policies have encouraged farmers to increase the size of their farms in order to lower costs and/or raise income. However, if more efficient farmers increase the size of their operations (as shown in this paper), it may be more reasonable for policy makers to implement agricultural policies designed to increase farmers' human capital in order to improve their technical efficiency.<sup>1</sup> The second policy issue is the efficiency implications of land reforms where large farms are substituted for small farms. In this case, it is important to analyse the efficiency effects of land distribution. The third policy issue is the concern in developed countries about the secular growth on farm size and the gradual disappearance of the family farm. In this case, the question is the role that the efficiency of larger farms plays in these processes.

Regarding the flaws in past research, in the present paper we analyse an issue not previously explored in the literature: the conditional versus unconditional analysis of the relationship between technical efficiency and size. The most frequent analysis of the relationship between efficiency and size is unconditional (i.e., simple correlation or univariate regression). Since farms usually face different economic environments, it seems natural to include control variables in the analysis, i.e., to make the analysis conditional on relevant farm characteristics. However, the inclusion of control variables in an ad hoc manner can be problematic because different control variables may lead to different empirical results. Hence, we propose to use a production model as a guide for choosing control variables. The differences between conditional and unconditional analysis are explored empirically using panel data for 196 Spanish dairy farms for the years 1993–1998.

The outline of the paper is as follows. In Section 2, we review the literature on technical efficiency and size. In Section 3, we present the production model. Section 4 contains the empirical model and Section 5 contains a description of the data. In Section 6, we

present the econometric estimation, and in Section 7, we discuss some conclusions.

## 2. Technical efficiency and size: a review of the literature

Many studies have analysed the relationship between technical efficiency and size, although the first wave of research dealing with this issue looked at the relationship between productivity (measured as output per acre) and size (measured by acres of land).<sup>2</sup> In an influential paper, Sen (1962) found an inverse relationship between farm size and yields per acre in Indian agriculture, giving rise to a large set of follow-up papers attempting to confirm his results or studying related issues such as the impact of technical progress on the productivity of small and large farms (Deolalikar, 1981).

While average productivity can be a relevant concept, partial productivity measures are problematic when making performance comparisons across farms. Total factor productivity (TFP) measures, which are ratios of output to input aggregates, are more appropriate for this endeavor. In a single output process, the output oriented index of technical efficiency can be interpreted as a TFP measure. In fact, the numerator in the index of technical efficiency is the observed output and the denominator is the potential output obtained using the production frontier, which can reasonably be interpreted as an input aggregator.

An easy way to try to account for the effect of technical efficiency on size (output) is to estimate production functions including a dummy variable for size and test whether its coefficient is significantly different from zero. For example, Bagi (1982) estimated a Cobb-Douglas production function for three groups of Indian farms, including a size dummy (based on land) both additively and interactively with the rest of the inputs. He found that, given a level of inputs, small farms produced more output than large farms.<sup>3</sup>

<sup>2</sup> The concept of size is not clear-cut (Lund and Price, 1998; Shalit and Sankar, 1987). While total output seems to be a reasonable measure of size, most studies have employed a single (quasi-fixed) input, such as land or number of cows.

<sup>3</sup> The paper by Lau and Yotopoulos (1971) is an early example of this approach using a profit function.

<sup>1</sup> Some papers have shown that farmers with low management (technical efficiency) levels have difficulties exploiting economies of size (Hubbard and Dawson, 1987; Alvarez and Arias, 2003).

Another group of papers is based on production frontiers. As a first step, a technical efficiency index is calculated. In the second step, the estimated technical efficiency index is regressed on a set of variables, including size. This second step (Timmer, 1971) is pervasive in this literature. Two recent papers (Townsend et al., 1998; Sharma et al., 1999) analyse the relationship between technical efficiency and size using DEA. In both cases, the index of technical efficiency is regressed on a measure of size. Sharma et al. (1999) use a Tobit model to accommodate the fact that they have an accumulation of observations with efficiency score equal to 1. In general, the second stage is problematic since the statistical properties of the DEA efficiency score are not well known. Townsend et al. (1998) report a negative relationship while Sharma et al. (1999) find a positive relationship.

Page (1984) estimates the production frontier using econometric methods and calculated technical efficiency for four Indian manufacturing industries. In the second stage he includes a size dummy variable (based on number of employees), but he fails to find conclusive relationships for most industries. Other papers based on production frontiers use panel data in the first step estimation, allowing for time varying technical efficiency. For example, Ahmad and Bravo-Ureta (1995) find a negative correlation between herd size and technical efficiency.

Battese and Coelli (1995) criticise the second stage and suggest a one-step model that allows the inclusion of explanatory variables in the inefficiency term. This model is widely used in the efficiency literature. Papers using this model and including a size variable in the inefficiency term are Wilson et al. (1998) and Tauer (2001). A similar approach is using stochastic frontiers with heteroskedastic disturbances for the technical efficiency error component, an approach which allows the estimated technical efficiency to depend on firm size (Yuengert, 1993).

An interesting development in this literature is Kalaitzandonakes et al. (1992). They fail to find robust results when analysing the relationship between technical efficiency and size: the results change depending on the method used to estimate technical efficiency. Therefore, they use a latent variable model with several technical efficiency indices to obtain the 'true' (latent) technical efficiency. They report a positive and significant rela-

tionship between the 'true' technical efficiency and size.

Overall, the empirical analysis in previous studies makes little reference to the underlying economic model. At the same time, unconditional analysis of the relationship between efficiency and size seems to be the norm. We consider unconditional analysis inappropriate given the usual heterogeneity of the units in the analysis (fixed inputs, etc.). For this reason, in Section 3, we propose a simple production model to analyse the relationship between technical efficiency and size and to provide a basic framework for empirical research.

### 3. Technical efficiency and size in a production model

This section relies on a microeconomic model of production where the level of technical efficiency is a parameter in the production function.<sup>4</sup> This approach, based on Lau and Yotopoulos (1971), differs from the main body of literature which, following Aigner and Chu (1968) and Aigner et al. (1977), models technical efficiency as part of the random disturbance term in a production function. The main advantage of our approach is that specifically parameterising technical efficiency in the production function allows us to analyse the role that technical efficiency plays in farm production decisions. For example, stochastic frontier analysis assumes that technical inefficiency is independent of input use (for the estimators to be unbiased). This assumption is unrealistic in many cases and, more important for our purposes, precludes the analysis of technical efficiency effects on input demands.

Our starting point is a technology represented by a production function with one variable input and one fixed input<sup>5</sup>

$$y_i = A_i f(z_i, x_i) \quad (1)$$

<sup>4</sup> Atkinson and Cornwell (1993, 1994) are examples of papers that model technical efficiency as a parameter using a dual approach.

<sup>5</sup> This simple model describes the main features of the relationship between TE and size. As shown in the Appendix, it is possible to derive similar results with several inputs under mild assumptions about the role of technical efficiency in the production function.

where subscript  $i$  denotes farms,  $y$  is production,  $f$  denotes a monotonic and concave production function,  $z$  is a fixed input,  $x$  is a variable input, and the  $A_i$  are farm-specific parameters that capture the technical efficiency of each farm. Monotonicity of the production function implies that marginal productivity of the variable input is positive ( $f_x > 0$ ) and concavity that the second partial derivative with respect to the variable input is negative ( $f_{xx} < 0$ ). What happens in this model to producers' choices when there is a change in technical efficiency? Comparative static analysis provides an answer to this question. We assume that producers maximise profits subject to a technological constraint. Since technical efficiency appears in the production function, we expect technical efficiency to appear in the input demand and output supply functions.

In a competitive industry let the short-run profit function for farm  $i$  be:

$$\pi_i = pA_i f(z_i, x_i) - wx_i \quad (2)$$

where  $p$  is output price and  $w$  represents the variable input price.<sup>6</sup>

The first-order condition for profit maximisation is

$$\frac{\partial \pi_i}{\partial x_i} = pA_i f_x(z_i, x_i) - w = 0 \quad (3)$$

Solving (3) for  $x$ , yields the input demand function:

$$x_i = x \left( z_i, \frac{pA_i}{w} \right) \quad (4)$$

which shows that the demand for inputs depends not only on prices and fixed inputs, but also on technical efficiency.

Differentiating the first-order condition in (3), the effect of technical efficiency on the demanded quantity of the variable input  $x$  is

$$\frac{\partial x_i}{\partial A_i} = - \frac{f_x(z_i, x_i)}{A_i f_{xx}(z_i, x_i)} > 0 \quad (5)$$

<sup>6</sup> A referee notes that there may be a relevant difference between firms and farms. We have not explored the implications of this distinction for our research and we are not aware of any work in this direction. However, we can think of two issues. First, farm decisions are sometimes modeled in the framework of household production theory while firm models rely on more standard production theory. Second, we expect farms to rely more on quasi-fixed inputs than the average firm in other sectors. This insight is important for our paper since the relationship between efficiency and size is conditional on quasi-fixed inputs.

Given the assumptions of monotonicity ( $f_x > 0$ ) and concavity ( $f_{xx} < 0$ ) of the production function, the derivative in (5) is positive.

Substituting (4) into (1) gives the output supply function:

$$y_i = y(A_i, z_i, p, w) \quad (6)$$

Now, differentiating Eq. (1) gives us:

$$\frac{\partial y_i}{\partial A_i} = f(z_i, x_i) + f_x(z_i, x_i) \frac{\partial x_i}{\partial A_i} > 0 \quad (7)$$

which shows that there is a positive effect of technical efficiency on size (measured by output).<sup>7</sup>

In summary, this model shows that more efficient producers buy more variable inputs (5), use them better (1), and therefore produce more output (7). If this model were the data generating process of observed data, a positive relationship would be found between technical efficiency and size, measured by output. Jovanovic (1982) finds a similar result in a model with exogenous technical efficiency affecting costs, which he uses as an essential part of his analysis of firm dynamics.<sup>8</sup> Lundvall and Battese (2000) look for empirical evidence regarding the hypothesis developed by Jovanovic and find a positive relationship between technical efficiency and size in a sample of manufacturing firms in Kenya.

For the purposes of this paper, the existence of a positive relationship between technical efficiency and size is not the only relevant result. Basic economic theory suggests that input and output prices and fixed inputs are the control variables to include in the conditional analysis of the relationship between technical efficiency and size.

#### 4. Empirical model

In this section we analyse the relationship between technical efficiency and size by estimating an empirical version of the theoretical model presented above.

<sup>7</sup> This result is valid only for a production function with decreasing returns to scale on the variable input. Under non-decreasing returns to scale, we need to move away from profit maximisation to determine optimal output. In that case, the relationship between technical efficiency and size has to be analysed in the appropriate model to explain optimal output.

<sup>8</sup> Some extensions of Jovanovic's approach can be found in Hopenhayn (1992) and Ericson and Pakes (1995).

We represent the technology with a translog production function. As is well known, this function provides a second-order approximation to the technology at the geometric mean of the sample. The translog production function is written as

$$\ln y_i = \ln A_i + \sum_j \beta_j \ln x_{ji} + \frac{1}{2} \sum_j \sum_k \beta_{jk} \ln x_{ji} \ln x_{ki} \tag{8}$$

where subscript *i* denotes farms, *y* is output, and *x* are inputs. The production function parameters are represented by the  $\beta$ 's, while the  $A_i$  are farm-specific parameters that represent the technical efficiency of each farm.

The production function in (8) cannot be estimated from cross-section data since the  $A_i$  are not identified, making the case for panel data. The fixed-effects model for the estimation of technical efficiency with panel data is written as

$$\ln y_{it} = \mu_i + \lambda_t + \sum_j \beta_j \ln x_{jit} + \frac{1}{2} \sum_j \sum_k \beta_{jk} \ln x_{jit} \ln x_{kit} + \varepsilon_{it} \tag{9}$$

where subscript *t* denotes time (years), the parameters  $\mu_i$  are the farm effects, and  $\lambda_t$  are the time effects. The random term  $\varepsilon_{it}$  is assumed to be iid (0,  $\sigma^2$ ).

The relative index of technical efficiency is computed by comparing the individual effects. In the case of a logarithmic specification, the index is (Schmidt and Sickles, 1984):

$$TE_i = \exp(\mu_i - \max \mu_j) \tag{10}$$

The index takes the value 1 for the farm with the largest individual effect, which is defined to be on the production frontier and therefore be technically efficient. The remaining farms have indices lower than 1, reflecting the existence of technical inefficiency.

The estimation of technical efficiency from the fixed effects of a panel data model has several advantages over the cross section stochastic frontier (Schmidt and Sickles, 1984). The main advantage is that it is not necessary to assume that the input demands and the level of technical efficiency are uncorrelated. This property is very important to our model, where input demands are theoretically correlated with the level of technical efficiency (see Eq. (5)). Although Schmidt and

Sickles (1984) show that the fixed effect estimator is consistent as  $t \rightarrow \infty$ , in practical terms the fixed effects estimator may be sensitive to outliers and to data noise in short panels.

There is no closed functional form for the output supply function associated with a translog production function.<sup>9</sup> However, we believe that it is reasonable to use a translog functional form for the supply function using the approximation argument as a rationale. The translog output supply function can be written as

$$\begin{aligned} \ln y_i = & \alpha_0 + \alpha_1 \ln TE_1 + \alpha_2 \ln z_i + \alpha_3 \ln p_i \\ & + \alpha_4 \ln w_i + 0.5\alpha_{11} (\ln TE_1)^2 + 0.5\alpha_{22} (\ln z_i)^2 \\ & + 0.5\alpha_{33} (\ln p_i)^2 + 0.5\alpha_{44} (\ln w_i)^2 \\ & + \alpha_{12} \ln TE_1 \ln z_i + \alpha_{13} \ln TE_1 \ln p_i \\ & + \alpha_{14} \ln TE_1 \ln w_i + \alpha_{23} \ln z_i \ln p_i \\ & + \alpha_{24} \ln z_i \ln w_i + \alpha_{34} \ln p_i \ln w_i + u_i \end{aligned} \tag{11}$$

where the  $\alpha_i$  are parameters to be estimated and  $u_i$  is a symmetric random disturbance.

Note that subscript *t* has been dropped because, since technical efficiency is time invariant, Eq. (11) is estimated using only the sixth year of the sample. In order to avoid a potential bias if the error term  $u_i$  is correlated with technical efficiency, the production function in (9) and the index of technical efficiency are estimated with just the first 5 years of the sample.

### 5. Data

This study uses technical and accounting data from a group of 196 dairy farms located in Northern Spain which are enrolled in a voluntary extension program, where farms are visited monthly by a technician who takes detailed records of the farm operations. We have data on these farms for a period of 6 years (1993–1998).

<sup>9</sup> If the technology is represented by a Cobb-Douglas production function the associated supply function is also Cobb-Douglas. This property, known as auto-duality, is one reason for the popularity of the Cobb-Douglas production function. Unfortunately, the translog function does not share this convenient property.

The variables used in the estimation of the production frontier are

Milk	Milk production (l)
Labour	Number of man-equivalent units
Cows	Number of milking cows
Feedstuffs	Total amount of feedstuffs fed to the dairy cows (kg) <sup>a</sup>
Land	Hectares of land devoted to pasture and crops
Roughage	Expenses incurred in producing roughage on the farm (euros) <sup>b</sup>

<sup>a</sup> Since farms have different replacement rates, feedstuffs have been adjusted to include only concentrates given to milking cows.

<sup>b</sup> Includes expenses for fertiliser, hired machinery and labour, seeds and sprays, silage additives and plastics, and the depreciation of the machinery.

Table 1 shows some descriptive statistics of the variables. The coefficients of variation are large, indicating heterogeneity in the production decisions.

Table 1  
Descriptive statistics of the data

Variable	Mean	Coefficient of variation	Minimum	Maximum
Milk	150,266	0.66	18,749	727,281
Labour	1.71	0.31	1	3.5
Cows	24.31	0.47	4.6	82.3
Feedstuffs	67,984	0.76	5,736	376,852
Land	13.85	0.44	3.7	39
Roughage	4,025	0.92	18.3	36,887

## 6. Estimation and results

As discussed earlier we estimate the production function (9) (using the within-group estimator) and the technical efficiency index using data from the first 5 years in the sample. Then, we use the sixth year of data to estimate the relationship between technical efficiency and size in Eq. (11). The estimate of technical efficiency is a random variable correlated with the disturbance of the production function in Eq. (9), which in turn may be correlated with the random disturbance of the supply equation. We try to avoid this

Table 2  
Estimates of the parameters of the production function (1993–1997)

Variable	Estimate	t-ratio
Labour	0.045	1.23
Cows	0.686	18.0
Feedstuffs	0.183	11.5
Land	0.039	2.24
Roughage	0.028	2.70
Labour × labour	-0.120	-0.49
Cows × cows	-0.005	-0.02
Feedstuffs × feedstuffs	0.106	2.81
Land × land	0.001	0.02
Roughage × roughage	0.011	0.58
Labour × cows	0.313	2.97
Labour × feedstuffs	-0.018	-0.38
Labour × land	-0.069	-1.02
Labour × roughage	-0.091	-2.60
Cows × feedstuffs	-0.087	-1.02
Cows × land	-0.035	-0.46
Cows × roughage	0.032	0.78
Feedstuffs × land	-0.039	-1.08
Feedstuffs × roughage	0.030	1.63
Land × roughage	0.014	0.56
D94	0.041	5.56
D95	0.077	9.55
D96	0.100	10.2
D97	0.112	10.8

$R^2 = 0.98$ .

potential bias problem by splitting the sample in two periods (5 + 1).<sup>10</sup>

The translog production function is estimated with each variable in the original data divided by its geometric mean. In this way, the first-order coefficients are output elasticities evaluated at the geometric mean of the sample. The results of the estimation of the production function can be seen in Table 2 (the fixed effects are not shown).<sup>11</sup> The output elasticities evaluated at the sample geometric mean are positive and significantly different from zero at conventional levels of significance, except for labour.<sup>12</sup> The standard errors are computed using a variance-covariance matrix robust to heteroskedasticity of unknown form.

<sup>10</sup> The use of the estimated technical efficiency as an explanatory variable may cause an error in variables problem. However, we believe that the solution to this problem is beyond the scope of the present paper.

<sup>11</sup> All models were estimated using LIMDEP 7.0 (Greene, 1995).

<sup>12</sup> The result that labour is not significant is not unusual in production functions using dairy farm data. See, for example, Ahmad and Bravo-Ureta (1995).

The null hypothesis of no correlation between the individual effects and the regressors was rejected using a Hausman test. This result shows that the input demands are correlated with technical efficiency (individual effects), as our theoretical model predicts.

We calculate technical efficiency indices for each farm using (10). The average technical efficiency is 0.7, with a minimum value of 0.34 and a maximum of 1.0 (by construction). We use the technical efficiency index as an explanatory variable in the supply equation, where land is the measure of quasi-fixed inputs and the price of feedstuffs is the price of the variable input. We estimate the translog supply function by ordinary least squares after dividing each variable by its geometric mean. Again, this procedure permits direct interpretation of the first-order coefficients as elasticities evaluated at the geometric mean of the sample. The results of the estimation are in Table 3.

The coefficient of technical efficiency is positive and significantly different from zero at conventional levels of significance. This result is consistent with the theoretical result in (7). In other words, we find empirical evidence in favour of the hypothesis that technical efficiency and size are positively correlated. The coefficients of the other explanatory variables have the expected sign and are significant, except for the coef-

ficient on the price of feedstuffs which is not significantly different from zero.

On the other hand, the simple regression between technical efficiency and size measured by output yields a coefficient for technical efficiency of 2.57, significant at the 1% level. In this case, we have a stronger positive unconditional relationship between technical efficiency and size.

Both the conditional and unconditional results should be interpreted with care. The conditional results depend on the set of control variables used in the analysis. Basic economic theory suggests that input and output prices plus fixed inputs must be used as control variables. However, more sophisticated models (e.g. a dynamic model of production) could suggest the inclusion of a different set of control variables. In the unconditional analysis, results depend not only on the basic relationship between technical efficiency and size but on the correlation between included variables (technical efficiency) and excluded variables (fixed inputs and input and output prices in our case).<sup>13</sup>

The use of a translog functional form to explore the conditional relationship between technical efficiency and size permits further analysis of the effects of the conditioning variables. In particular, the effect of technical efficiency on size can be represented by the following derivative:

$$\frac{\partial \ln y_i}{\partial \ln TE_i} = \alpha_1 + \alpha_{11} \ln TE_i + \alpha_{12} \ln z_i + \alpha_{13} \ln p_i + \alpha_{14} \ln w_i \quad (12)$$

This says that the effect of technical efficiency on size depends on the level of technical efficiency, fixed inputs and the prices of output and variable inputs. Using the results in Table 3, we find that this effect is smaller for more efficient farms ( $\alpha_{11} < 0$ ) and larger for farms that pay more for feedstuffs ( $\alpha_{14} > 0$ ). The fixed input and the output price do not affect significantly the analysed relationship.

In Table 4, we present the correlations between the elasticity of output with respect to technical efficiency and some farm characteristics.

The correlations in Table 4 indicate that the value of the elasticity of output with respect to technical

Table 3  
Relationship between size and technical efficiency (supply function)

Variable	Estimate	t-ratio
Constant	11.861	345.10
TE	1.452	8.48
Land	0.591	9.44
Pmilk	2.520	6.92
Pfeed	-0.114	-0.36
TE × TE	-2.163	-1.82
Land × Land	-0.421	-1.55
Pmilk × Pmilk	8.200	3.06
Pfeed × Pfeed	-6.145	-2.16
TE × Land	0.260	0.53
TE × Pmilk	1.447	0.87
TE × Pfeed	2.987	1.42
Land × Pmilk	1.251	1.23
Land × Pfeed	-0.177	-0.21
Pmilk × Pfeed	-7.365	-1.61

$R^2 = 0.82$ .

<sup>13</sup> This is the classic problem of omitted variables in regression analysis described by Griliches (1957).



Table 4

Correlation between farm characteristics and the elasticity of output with respect to technical efficiency

	Correlation coefficient
Technical efficiency	−0.65
Land	0.08
Milk per cow	−0.53
Milk per kg of feed	0.19
Milk per hectare of land	−0.43
Feed per cow (kg)	−0.44

efficiency is lower for farms with higher values of technical efficiency, suggesting that inefficient farms have a larger return on improvements in technical efficiency. The negative correlation of the elasticity with milk per cow, milk per hectare of land, and feed per cow, together with the positive correlation of the elasticity with milk per kg of bought feed indicate that more intensive farms have lower values of the elasticity.

## 7. Conclusions

We analyse the relationship between technical efficiency and size in the framework of a simple production model. In this theoretical model, output supply (a measure of size) is a function of input prices, output prices, and quasi-fixed inputs, but output supply is also related to the level of technical efficiency. In other words, simple economic theory suggests the importance of a conditional analysis of the relationship between technical efficiency and size. The theoretical model predicts a positive relationship between technical efficiency and size.

In our empirical application we find a positive and significant relationship between technical efficiency and size when controlling for the effects of output prices, input prices, and quasi-fixed inputs, as suggested by theory. The unconditional relationship between technical efficiency and size is positive as well but stronger than the conditional one. More sophisticated economic models may suggest a different set of explanatory variables which may in turn give different results. Nevertheless, the theoretical and empirical results in the paper show that conditional analysis is a part of the efficiency-size puzzle.

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## Appendix

The starting point of this model is a technology represented by a production function.

$$y = f(A, z, x) \quad (\text{A.1})$$

where  $y$  is production,  $f$  denotes a concave production function,  $z$  is vector of fixed inputs,  $x$  is a vector of variable inputs, and  $A$  is a vector of farm-specific parameters that captures the technical efficiency of each farm.

The role of  $A$  in the production function is made more explicit with the following two assumptions:

$$\frac{\partial f(A, z, x)}{\partial A} > 0 \quad (\text{A.2a})$$

$$\frac{\partial^2 f(A, z, x)}{\partial x_i \partial A} > 0 \quad (\text{A.2b})$$

In words, holding inputs constant, technical efficiency increases production (A.2a) and increases the marginal product of inputs (A.2b). Finally, we assume that inputs in the production process are normal (Takayama, 1993, p. 190).

In a competitive industry the profit function for farm  $i$  can be written as

$$\Pi(A, z, w, p) = \max_{x,y} \{py - wx \mid y = f(A, z, x)\} \quad (\text{A.3})$$

where  $p$  is output price and  $w$  is the input price vector.

The associated Lagrangean and FOC for profit maximisation can be written as

$$\begin{aligned} L &= py - wx + \lambda(f(A, z, x) - y) \\ \frac{\partial L}{\partial y} &= p - \lambda = 0 \\ \frac{\partial L}{\partial x_i} &= -w_i + \lambda \frac{\partial f(A, z, x)}{\partial x_i} = 0 \\ \frac{\partial L}{\partial \lambda} &= f(A, z, x) - y = 0 \end{aligned} \quad (\text{A.4})$$

Using the envelope theorem it is easy to prove:

$$\frac{\partial \Pi(A, z, w, p)}{\partial A} = p \frac{\partial f(A, z, x)}{\partial A} \quad (\text{A.5})$$

Differentiating (A.5) with respect to output price we have

$$\begin{aligned} \frac{\partial \Pi(A, z, w, p)}{\partial A \partial p} &= \frac{\partial f(A, z, x)}{\partial A} \\ &+ p \sum_i \frac{\partial^2 f(A, z, x)}{\partial A \partial x_i} \frac{\partial x_i}{\partial p} > 0 \end{aligned} \quad (\text{A.6})$$

The expression in (A.6) is positive using assumptions (A.2a) and (A.2b) plus input normality  $\partial x_i / \partial p > 0$ .<sup>14</sup>

As a result, using Hotelling's lemma we have

$$\begin{aligned} \frac{\partial y(A, z, w, p)}{\partial A} &= \frac{\partial(\partial \Pi(A, z, w, p) / \partial p)}{\partial A} \\ &= \frac{\partial \Pi(A, z, w, p)}{\partial p \partial A} > 0 \end{aligned} \quad (\text{A.7})$$

Therefore, in a production process with multiple fixed and variable inputs there is a direct positive relationship between technical efficiency ( $A$ ) and size measured by output ( $y$ ).

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<sup>14</sup> Input normality is defined in a cost minimising setting as  $\partial x_i / \partial y > 0$ . However, under profit maximisation  $\partial y / \partial p = \partial^2 \Pi / \partial p^2 > 0$  (convexity of profit function). Therefore,  $\partial x_i / \partial p = (\partial x_i / \partial y)(\partial y / \partial p) > 0$ .

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