Global Trade Analysis Project
https://www.gtap.agecon.purdue.edu/

This paper is from the
GTAP Annual Conference on Global Economic Analysis
https://www.gtap.agecon.purdue.edu/events/conferences/default.asp

Distributing Incomes between Representative Households in Dynamic CGE Models: Empirical Tests of Alternative Structures

Hans Lofgren, World Bank

Martín Cicowiez, CEDLAS-Universidad Nacional de La Plata

Sherman Robinson, IFPRI

Incomplete draft

April 14, 2017

Introduction

While not much attention has been paid to the issue, a variety of approaches to the household sector have been used in dynamic CGE models based on the dominant recursive-dynamic approach. The purpose of this paper is to test the empirical implications of alternative approaches. It first reviews the approaches used, including how they differ in terms of the disaggregation of income sources, the number of representative households (RHs) and, if there is more than one RH, the treatment of demographic change and the evolution of the claims of each RH on different income flows. The SAM representation of households and their incomes summarizes the potential ability of the model to capture distributional change.

Models with multiple RHs vary in terms of their treatment of the population growth for each RH (endogenous or exogenous) and whether they link RH population growth to RH shares in different income flows. Drawing on a review of the literature, we identify three alternative treatments of income shares:

1. fixed shares: no link between (a) the shares of each RH in different incomes; and (b) the growth in its population and, typically implicitly, its labor force (which may be disaggregated according to educational attainment or other criteria), or its savings;
2. fixed per-capita shares: the shares of each RH in different income flows depend on the growth of its population but not on its shares in one or more segments of the labor force or its savings; and
3. links between (a) the shares of each RH in different income flows; and (b) its endowments of different factors, driven by its labor force growth (for labor shares), its savings (for shares in private capital rents), and its population growth (for other incomes).

Among these, 1 is the most common while 3 has been used in a few applications without being well documented or tested.[1] Alternative 2 may not have been implemented before this paper. 3 is more satisfactory on theoretical grounds since it links the size distribution of incomes to the evolution of household-specific endowments with a link to demographic change.

This discussion raises an empirical question: to what extent do different approaches generate different results? To analyze this issue, we test the impact of alternative approaches and disaggregations with the help of a fairly standard recursive-dynamic CGE model, applied to a database for Guatemala. In its most disaggregated form, the database has 10 factors (unskilled salaried labor, skilled salaried labor, unskilled non-salaried labor, skilled salaried labor, private and government capital, land, and 3 other natural resources), and 24 sectors. The RHs are disaggregated into 4 groups on the basis of location and the type of its major labor endowment: rural unskilled, rural skilled, urban unskilled, and urban skilled. Factors other than private capital grow at exogenous rates; private capital growth is a function of the initial stock, investment, and depreciation. The model is simulated for a 10-year period (2011-2021). We impose two shocks with distributional repercussions: (1) an increase in world export prices for selected agricultural sectors; and (2) the introduction of a government transfer to RHs sufficient to eliminate a uniform share of the poverty gap for each RH with an above-zero headcount poverty rate. The two shocks are implemented under the three alternative approaches outlined above. We test the sensitivity of the results to a more aggregate representation of factors. In our analysis of results, we focus on per-capita consumption (aggregate and for each RH), poverty, inequality, and the growth rates for different factor incomes. Poverty and inequality

[1] A review of the documentation of a set of major recursive-dynamic models that permit (but do not impose) multiple RHs, indicates that they have applied treatment (1). See for example Thurlow (2004, pp. 51-54 and 60), van der Mensbrugghe (2005, pp. 8-10, 26), and Decaluwé et al. (2013, pp. 17-19 and 51). (3) is used in multiple-RH applications of MAMS (Lofgren et al., 2013, p. 210). Of course, this does not mean that, in specific applications, researchers have stuck to these assumptions; however, we have not found documentation of any alternative approach.

indicators will be generated on the assumptions that the distribution within each RH group is unchanged.

The main anticipated finding is that the approaches generate significantly different results with the largest gaps between approaches 1 and 3; approach 3 dominates since the results are consistent with basic economic principles. The differences are smaller if the database is more aggregated and hence less able to capture changes in income distribution.

In outline, the paper proceeds as follows. Section 2 provides a non-technical description of GEM-Core Guatemala and the disaggregation of its current database. The paper appendix has a detailed mathematical presentation of GEM-Core.

Method and data

GEM-Core Guatemala: A CGE Model for Guatemala

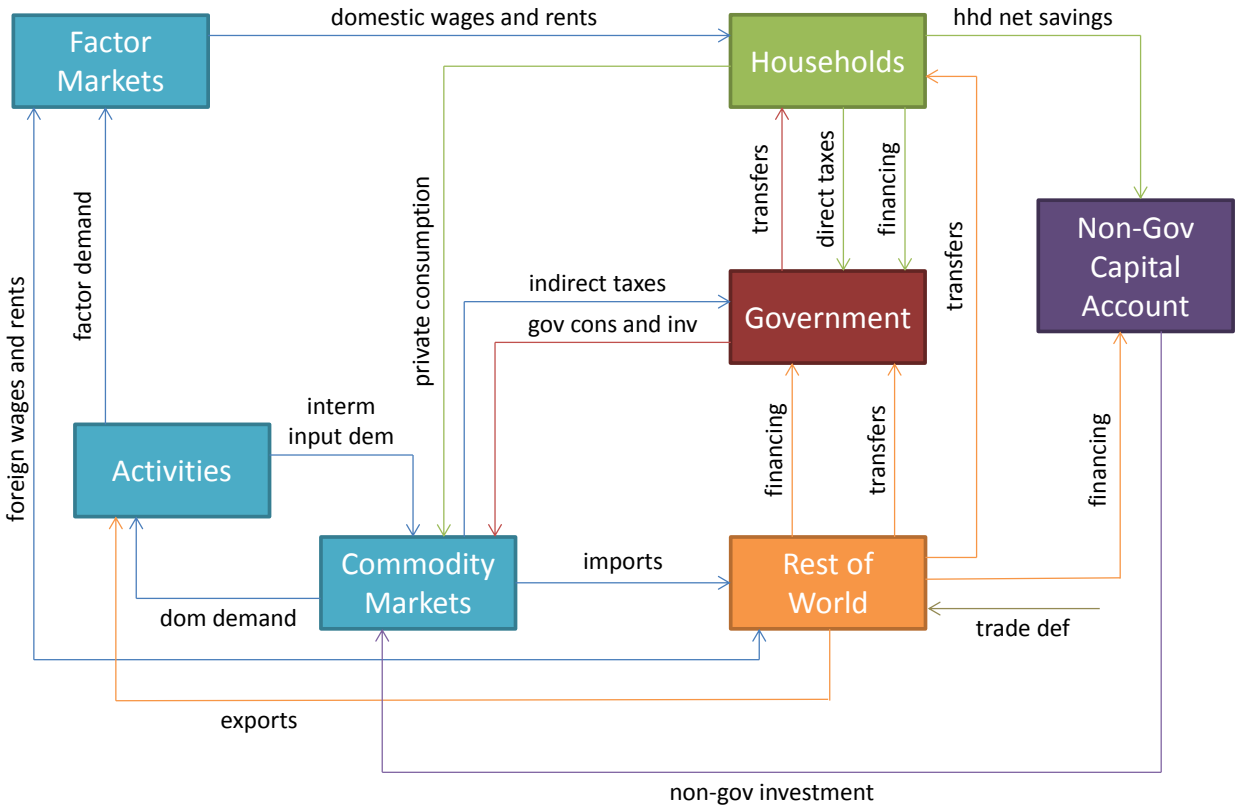
In this section, we provide a non-technical overview of GEM-Core.[2] (Appendix A has a detailed mathematical model statement.) GEM-Core is a single-country recursive-dynamic general equilibrium model designed for medium- and long-run policy analysis. It is a multi-purpose model in the sense that it can analyze policies in a wide range of areas including growth, fiscal space, and external shocks.

As indicated by Figure 2.1, which serves as the reference point for this model overview, the major building blocks of GEM-Core are activities (the entities that carry out production), commodities (activity outputs or, exceptionally, imports without domestic production; linked to markets), factors (also linked to markets), and institutions (households, enterprises, the government, and the rest of the world). Given the relatively detailed treatment of the financing

[2] GEM-Core is part of GEM suite, a set of models with a common core and extensions in selected areas. The model is a descendant of MAMS (Lofgren et al. 2013).

of private investment in GEM-Core (compared to most other CGE models), the private (non-government) capital account also has its own box. In any GEM-Core application (and dataset), most blocks are disaggregated on the basis of the SAM (Social Accounting Matrix) of the application in question.

Figure 2.1: Overview of GEM-Core



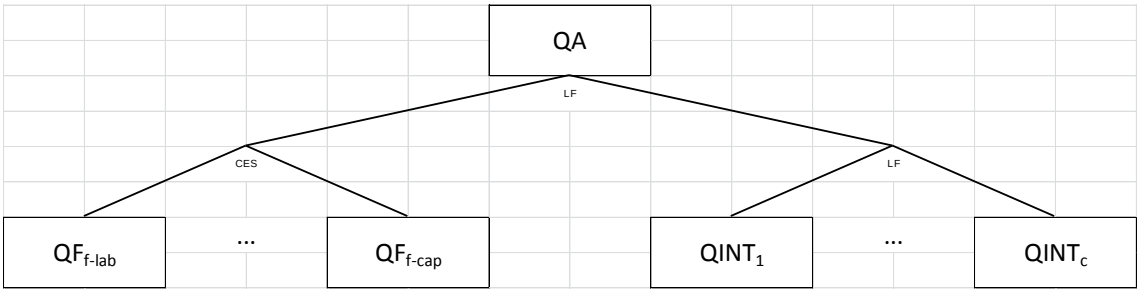
Source: Authors' elaboration.

In any single year, GEM-Core has the structure summarized in the above figure. Activities produce, selling their output at home or abroad to the rest of the world. The activities use their revenues to cover costs (of intermediate inputs, factor hiring, and taxes). Their decisions regarding factor employment, which determines the output level, are driven by profit

maximization. The shares of their outputs that are exported and sold domestically depend on relative sales prices in these two destinations.

Figure 2.2 provides additional detail on the production technology of production activities. The level (or quantity) of any activity (QA) and its output quantities (via yield coefficients) are a function of the quantities employed of factors (in this example labor and capital). Intermediate input use (QINT) is a Leontief (LF) function of the activity levels (QA).

Figure 2.2: Production factor and intermediate input demand



where *QA* = activity level, *QVA* = value added quantity, $QF_{f\text{-}lab}$ = quantity of labor employed, $QF_{f\text{-}cap}$ = quantity of capital employed, and $QINT_c$ = quantities of disaggregated intermediate input *c*.

Source: Authors' elaboration.

Returning to Figure 2.1, GEM-Core Guatemala includes four types of institutions: households, enterprises (excluded from the figure for simplicity), the government, and the rest of the world.[3] As shown, households earn incomes from factors, transfers from the government, and transfers from the rest of the world. These incomes are used for direct taxes, savings, and consumption. After deducting net financing of the government (which in the real world equals

[3] In fact, enterprises are not necessary in GEM-Core applications and related SAMs. In SAMs without enterprises, other institutions (most importantly households) share the full value of enterprise incomes (instead of only sharing part of these incomes) while also taking on the expenses of the enterprises (most importantly direct taxes and savings).

household lending to the government minus household interest earnings) and resources needed for changes in foreign reserves, household savings are used to finance private investment. Household consumption decisions change in response to income and price changes. By construction (and as required by the household budget constraints), the consumption value of the households equals their income net of direct taxes and savings. Enterprises are modeled like households with the main exception that they do not demand commodities.

The government gets its receipts from taxes, transfers from abroad, and net financing (borrowing net of interest payments) from households and the rest of the world. It uses these receipts for transfers to households, consumption, and investment (to provide the capital stocks required for government services).[4] To remain within its budget constraint, it either adjusts some part(s) of its spending on the basis of available receipts or mobilizes additional receipts of one type or more in order to finance its spending plans.

In Figure 2.1, imports and exports appear as a payment to/from the rest of the world. Foreign wages and rents is the only non-trade payment to the rest of the world; it is typically an exogenous projection. The non-trade payments received from the rest of the world consist of net transfers to households and government and financing; the latter term here stands for the sum of household net foreign financing (borrowing less interest) and foreign investment, net of changes in foreign reserves.[5] Total financing from the rest of the world (to the government and to the non-government capital account) is positive (negative) if the model country has a deficit (surplus) in its non-financing payments. The balance of payments clears (inflows and outflows

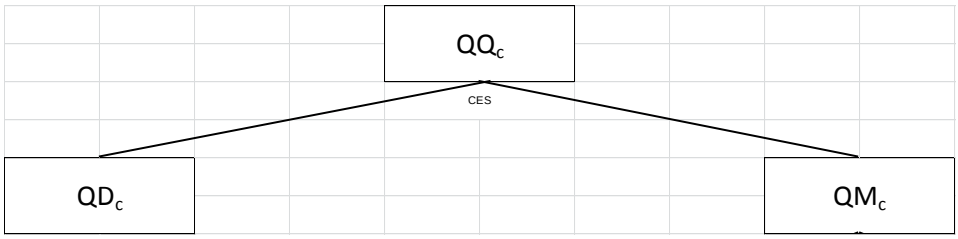
[4] The government primary deficit is defined as spending on consumption, investment, and domestic transfers minus taxes and transfers from abroad. This deficit is covered by domestic and foreign net financing.

[5] This treatment of payments from the rest of the world to the non-government capital account reflects the fact that, in the context of most applications, their major impact is to provide investment financing. In so far as they are associated with other effects (such as productivity gains associated with foreign investment), these are typically best handled via exogenous parameter adjustments.

are equalized) via adjustments in the real exchange rate (the ratio between the international and domestic price levels), influencing export and import quantities and values.

In commodity markets, flexible prices ensure balance between demands for domestic output from domestic demanders and supplies to the domestic market from domestic suppliers. The parts of domestic demands that are for imports face endogenous or exogenous world prices depending on the specific model application; under the common small-country assumption, which is followed for Guatemala's imports, prices in foreign currency are fixed.[6] On the basis of relative prices, domestic demanders decide on the split between domestic purchases and imports and, among the latter, between different sources (see Figure 2.3). Similarly, domestic suppliers (the activities) also consider relative prices when deciding on the allocation of their output between domestic supplies and exports (see Figure 2.4). For exports, we also assume that Guatemala faces exogenous world prices.

Figure 2.3: Allocation of domestic demands across alternative sources

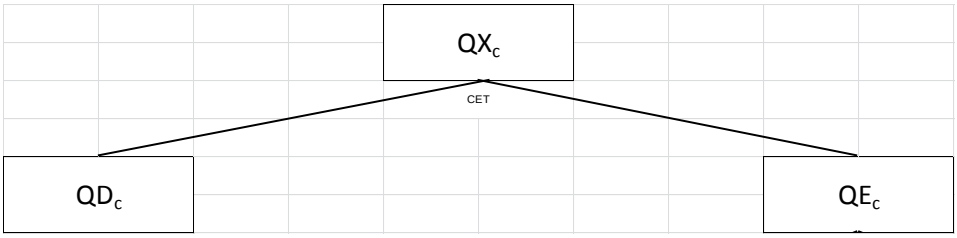


where *c* = commodities, total (national) *QQ* = composite demand, *QD* = domestic output demand, and *QM* = aggregate import demand.

Source: Authors' elaboration.

[6] Both for imports and exports, the model offers the option of endogenizing prices (in foreign currency) using constant-elasticity demand and supply functions, respectively.

Figure 2.4: Allocation of output across alternative destinations



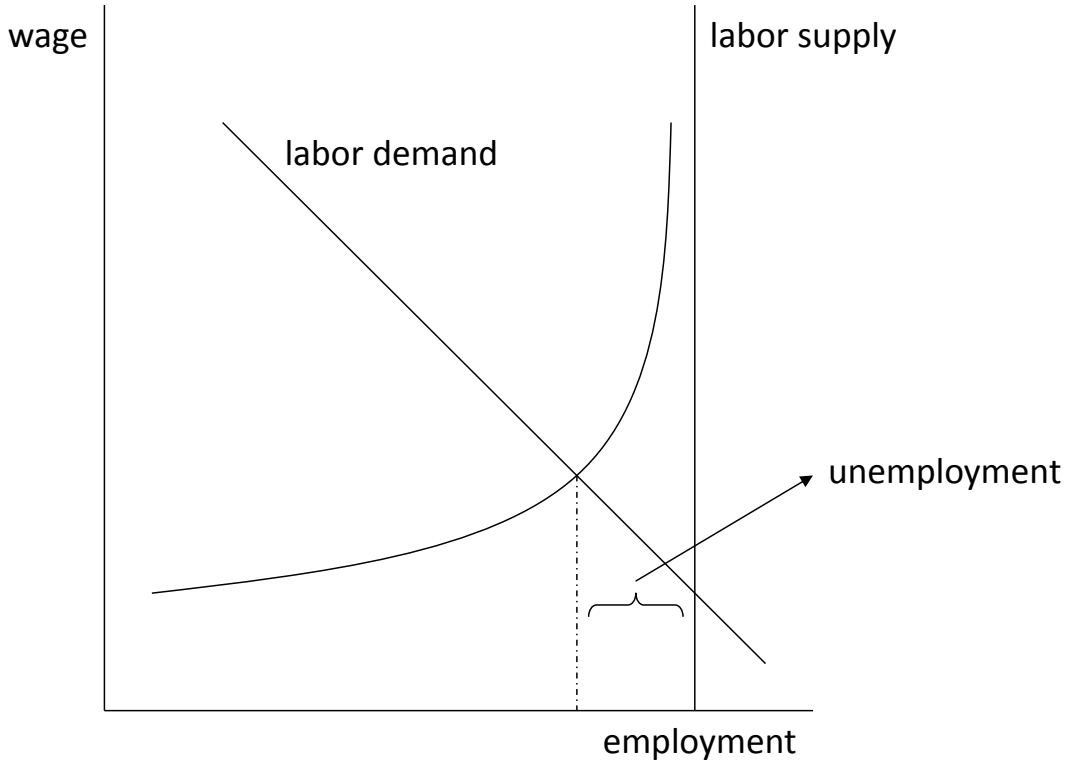
where *c* = commodities, *QX* = output, and *QE* = aggregate export supply.

Source: Authors' elaboration.

For factors with fixed total employment, markets reach balance between demands and supplies via wage (or, alternatively, rent) adjustments. Across all factors, the demand curves are downward-sloping, reflecting the responses of production activities to changes in wages. In labor markets, unemployment may be endogenous. If so, the model includes a wage curve that establishes a negative relation between the real wage and the unemployment rate or, alternatively, a positive relation between the real wage and the employment rate (see Figure 2.5). For non-labor factors, total employment is fixed.[7]

[7] The treatment for labor with a wage curve could readily be extended to other factors (capital and application-specific factors like land and other natural resources), if so letting the share of the total supply that is employed (the rate of capacity utilization) depend on the wage or rent.

Figure 2.5: Labor market specification



Source: Authors' elaboration.

The above discussion refers to the functioning of the model economy in a single year. In GEM-Core, growth over time is endogenous. The economy grows due to accumulation of capital (determined by investment and depreciation), exogenous growth in the stocks of labor and other non-capital factors (for example agricultural land), and growth in total factor productivity (TFP).[8] Apart from an exogenous component, the TFP of any production activity potentially depends on the levels of capital stocks (typically government infrastructure stocks). In addition, as noted above, the model covers a set of net financing flows: to the government from domestic non-government institutions (households and enterprises) and the rest of the world;

[8] Labor is often disaggregated by level of education or skill. Labor stock growth rates are calibrated so that the total labor stock matches what is projected considering changes in the resident population, its age structure, and the rate of labor force participation for different population groups (disaggregated by age and perhaps also by gender.

and to domestic non-government institutions from the rest of the world. On the basis of the results for any simulation, assumptions about real interest rates, and initial debt stocks, post-calculations extract the implications for the evolution of domestic and foreign debt stocks. The same applies to the evolution of the stock of foreign reserves, which is computed on the basis of the initial stock and annual changes.

As mentioned, GEM-Core is a multi-purpose model. For example, the model has a particularly rich treatment of fiscal issues, with a fine disaggregation both in terms of spending (consumption, investment, and subsidies) and resource mobilization (from taxes, foreign transfers, and domestic and foreign financing). In general terms, this makes it possible to use GEM-Core to address policies in a wide range of areas, including poverty and inequality, sectoral policies and structural transformation, labor markets, demography, and infrastructure.[9]

Data

The bulk of the dataset is for the simulation base-year – a social accounting matrix (SAM); stocks and sectoral employment levels for production factors (including different types of labor and capital), as well as a set of elasticities (for production, consumption, and trade).

Like other CGE models, our CGE uses a base-year SAM for Guatemala to define base-year values for the bulk of the model parameters, including production technologies, sources of commodity supplies (domestic output or imports), demand patterns (for household and government consumption, investment and exports), transfers between different institutions, and tax rates. The disaggregation of the Guatemala SAM coincides with that of the rest of the model database. As shown in Table 2.1, it is disaggregated into 24 sectors (activities and commodities) – 3 in agriculture, 1 in mining, 7 in manufacturing, and 13 in services – with each activity producing one or more commodities. The factors are split into 4 labor categories, private capital, government capital, and natural resources (4 types: agricultural land, forestry land,

[9] Poverty and inequality may be analyzed using a built-in module or top-down micro simulations. This aspect is not addressed in this paper.

fishing resources, and a natural resource used in mining). The institutions are split into 4 households, enterprises, government, and the rest of world. A set of auxiliary accounts cover the different tax instruments as well as trade and transport margins on domestic sales, imports and exports. Finally, investment is split into private, government, and change in stocks.

Table 2.1: Disaggregation of Guatemala CGE and SAM

Category - #	Item	Category - #	Item
Primary (4)	Crops, traditional	**Factors (10) -- cont.**	Capital, private
	Crops, non-traditional		Capital, government
	Livestock, forestry and fishing		Land
	Mining		Nat res, forestry
Manufacturing (7)	Food, beverages and tobacco		Nat res, fishing
	Textiles and wearing apparel		Nat res, mining
	Wood and paper	**Taxes (4)**	Tax, activities
	Petrochemical		Subsidy, activities
	Rubber, plastic, non-met min prod		Tax, value added (VAT)
	Metals, met prod and mach and eq		Tax, imports (tariffs)
	Other manufactures		Tax, commodities
Services (13)	Electricy and water		Subsidy, commodities
	Construction		Tax, income
	Trade	**Inst, current acc (3)**	Households, rural, unskilled
	Hotels and restaurants		Households, rural, skilled
	Transport and communications		Households, urban, unskilled
	Financial services		Households, urban, skilled
	Real estate		Enterprises
	Business services		Government
	Public adminsitration		Rest of the world
	Education	**Inst, capital acc (5)**	Capital acc, hhd, rural, unskilled
	Health		Capital acc, hhd, rural, skilled
	Other services		Capital acc, hhd, urban, unskilled
	Domestic services		Capital acc, hhd, urban, skilled
Distribution margins (3)	Dist marg, domestic		Capital acc, enterprises
	Dist marg, imports		Capital acc, government
	Dist marg, exports		Capital acc, rest of the world
Factors (10)	Labor, salaried unskilled	**Investment (3)**	Investment, private
	Labor, salaried skilled		Investment, government
	Labor, non-salaried unskilled		Change in stocks
	Labor, non-salaried skilled		

Source: Authors' elaboration.

On the basis of SAM data, Table 2.2 summarizes the sectoral structure of Guatemala's economy in 2011: sectoral shares in value-added, production, employment, exports and imports, as well as the split of domestic sectoral supplies between exports and domestic sales, and domestic sectoral demands between imports and domestic output. For instance, while (primary) agriculture represents a significant share of employment (around 43.4 percent), its shares of value added (VA), production, and exports are much smaller (in the range of 12-20 percent). For traditional crops (i.e., coffee, banana, and cardamom), the share of output that is exported is around 83 percent. In turn, for non-traditional crops such as cereals, some 17 percent of domestic demands are met via imports.

Table 2.2: Sectoral structure of Guatemala's economy in 2011 (percent)

Sector	VAshr	PRDshr	EMPshr	EXPshr	EXP-OUTshr	IMPshr	IMP-DEMshr
Crops, traditional	3.2	2.2	6.5	13.5	83.1	0.0	0.2
Crops, non-traditional	5.4	4.0	28.2	3.1	10.6	5.4	17.2
Livestock, forestry and fishing	3.2	3.1	8.6	3.2	14.3	0.4	2.9
Mining	3.0	2.0	1.1	11.7	78.9	0.6	28.0
Food, beverages and tobacco	11.1	14.3	6.7	15.6	14.9	9.6	14.8
Textiles and wearing apparel	2.5	4.0	5.6	13.2	44.6	8.1	44.3
Wood and paper	0.9	1.3	0.8	2.5	26.4	4.1	50.6
Petrochemical	1.4	2.3	0.5	8.2	47.7	31.6	85.6
Rubber and plastic	1.6	2.6	1.0	3.1	16.8	4.5	32.8
Metals, met prod and mach and eq	1.0	1.8	0.8	5.9	44.4	27.7	86.7
Other manufactures	0.8	1.2	0.9	1.1	12.1	2.1	31.4
Electricy and water	2.0	2.3	0.3	0.2	1.3	0.4	3.9
Construction	4.1	6.4	4.9	0.1	0.3	0.0	0.1
Trade	18.6	14.4	14.2	0.0	0.0	0.0	0.0
Hotels and restaurants	2.4	3.7	2.5	14.5	53.7	3.4	35.2
Transport and communications	8.1	8.5	2.1	2.3	3.7	0.6	2.0
Financial services	1.8	2.9	0.7	0.0	0.1	0.6	5.4
Real estate	10.3	6.7	1.5	0.0	0.0	0.0	0.0
Business services	6.0	5.6	3.1	1.7	4.1	0.7	3.6
Public adminsitration	4.2	3.8	1.8	0.0	0.0	0.0	0.0
Education	3.5	2.4	3.1	0.0	0.0	0.0	0.0
Health	2.8	2.7	1.8	0.0	0.0	0.0	0.0
Other services	1.0	1.0	0.4	0.0	0.0	0.0	0.5
Domestic services	1.0	0.6	3.0	0.0	0.0	0.0	0.0

where VAshr = value-added share (%); PRDshr = production share (%); EMPshr = share in total employment (%); EXPshr = sector share in total exports (%); EXP-OUTshr = exports as share in sector output (%); IMPshr = sector share in total imports (%); IMP-DEMshr = imports as share of domestic demand (%).

Source: Authors' calculations based on 2011 Guatemala SAM and employment data.

Table 2.3 shows the factor shares in total sectoral value added. For example, the table shows that agriculture is relatively intensive in the use of unskilled non-salaried labor; this information will be useful to analyze the results from the CGE simulations.

Table 2.3: Sectoral factor intensity in 2011 (percent)

Sector	Lab, sal unsk	Lab, sal sk	Lab, non-sal unsk	Lab, non-sal sk	Capital	Natural resources	Total
Crops, traditional	14.5	2.0	35.1	3.0	21.4	24.0	100.0
Crops, non-traditional	11.6	1.6	59.0	5.0	10.8	12.1	100.0
Livestock, forestry and fishing	14.0	1.9	46.1	3.9	22.3	11.8	100.0
Mining	17.8	9.9	4.6	0.1	40.1	27.6	100.0
Food, beverages and tobacco	10.9	16.1	15.5	20.7	36.8	0.0	100.0
Textiles and wearing apparel	32.5	23.4	22.2	5.1	16.8	0.0	100.0
Wood and paper	15.1	21.1	16.0	7.3	40.5	0.0	100.0
Petrochemical	12.2	26.0	0.6	0.2	61.0	0.0	100.0
Rubber, plastic, non-met min prod	8.7	18.6	4.5	1.9	66.3	0.0	100.0
Metals, met prod and mach and eq	20.7	19.3	3.7	3.2	53.0	0.0	100.0
Other manufactures	19.6	8.0	23.8	9.3	39.3	0.0	100.0
Electricy and water	4.6	18.3	0.0	0.6	76.5	0.0	100.0
Construction	29.9	11.4	23.9	4.9	29.9	0.0	100.0
Trade	8.1	17.4	17.9	11.0	45.6	0.0	100.0
Hotels and restaurants	14.4	20.0	22.5	9.9	33.2	0.0	100.0
Transport and communications	9.3	11.2	1.8	8.2	69.5	0.0	100.0
Financial services	6.7	53.0	0.3	2.3	37.8	0.0	100.0
Real estate	0.0	0.0	0.8	11.8	87.4	0.0	100.0
Business services	4.9	29.1	0.5	15.3	50.1	0.0	100.0
Public adminsitration	19.8	80.2	0.0	0.0	0.0	0.0	100.0
Education	4.6	81.2	0.0	1.2	13.0	0.0	100.0
Health	3.4	51.2	2.9	23.9	18.6	0.0	100.0
Other services	9.8	24.9	6.8	10.9	47.6	0.0	100.0
Domestic services	87.7	12.3	0.0	0.0	0.0	0.0	100.0
Total	11.5	20.6	14.1	9.2	42.0	2.6	100.0

Source: Authors' calculations based on 2011 Guatemala SAM.

Table 2.4 shows income sources for institutions in the GEM-Core Guatemala. For rural unskilled households, which represent 47 percent of total population, the main income source is unskilled labor, followed by remittances and transfers from government. In turn, rural skilled, urban unskilled and urban skilled households represent 4.6, 29.1 and 19.4 percent of total population, respectively.

Table 2.4: Income sources by institution in 2011 (percent)

Income source	Rural unsk hhd	Rural sk hhd	Urban unsk hhd	Urban sk hhd	Ent	Gov't	RoW
Taxes	0.00	0.00	0.00	0.00	0.00	69.85	0.00
Transfers from government	4.64	0.91	2.33	0.85	2.32	0.00	1.42
Transfers from RoW	25.37	8.46	10.17	2.92	1.54	3.17	0.00
Transfers from insdng	2.45	1.76	37.41	31.85	5.23	26.89	8.72
Imports	0.00	0.00	0.00	0.00	0.00	0.00	89.80
Labor, salaried unskilled	28.24	1.50	14.85	0.69	0.00	0.00	0.02
Labor, salaried skilled	4.11	51.50	5.17	36.29	0.00	0.00	0.03
Labor, non-salaried unskilled	34.01	2.95	17.86	0.82	0.00	0.00	0.00
Labor, non-salaried skilled	0.44	32.39	0.91	16.96	0.00	0.00	0.00
Capital	0.70	0.50	10.64	9.06	85.51	0.08	0.00
Land	0.03	0.02	0.43	0.37	3.55	0.00	0.00
Forestry res	0.00	0.00	0.01	0.01	0.07	0.00	0.00
Fishing res	0.00	0.00	0.01	0.01	0.07	0.00	0.00
Extractive res	0.01	0.01	0.21	0.18	1.71	0.00	0.00
total	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Source: Authors' calculations based on 2011 Guatemala SAM.

In addition to the SAM, our CGE model also requires (a) base-year estimates for sectoral employment levels, and (b) a set of elasticities (for production, consumption and trade). In order to estimate sectoral employment we combined employment data from the national accounts with estimates for sectoral employment shares in broad sectoral categories from the ENCOVI 2011 household survey. In turn, elasticities were given a value based on the available

evidence for comparable countries. For elasticities, the following values were used: (a) the elasticity of substitution among factors is in the 0.2–1.15 range, relatively low for primary sectors and relatively high for manufactures and services (see Narayanan et al. 2015); (b) the expenditure elasticities for household consumption were obtained from Seale et al. (2003); and (c) trade elasticities are 0.9 and 1.1 for Armington and CET elasticities, respectively. Given the uncertainty with respect to our elasticity values, in Appendix B we conduct a systematic sensitivity analysis of our simulation results with respect to their values.

Simulations

Scenarios

[in progress]

Results and Analysis

[in progress]

Concluding remarks

References

Appendix: GEM-Core mathematical statement

This appendix presents a mathematical statement of GEM-Core, showing the relationships that, together with the database, determine the results of model simulations. A good understanding

of the structure of the model and its database is needed to well understand simulation results. The appendix is divided into two subsections, notation (A.1) and equations (A.2). Throughout, the presentation is organized around a set of tables.

A.1. Notation

Table A.1.1 explains notational principles, designed to make it easy to understand the statement. Tables A.1.2-A.1.5 define model sets, variables, Latin-letter parameters, Greek-letter parameters, respectively. In each of these tables, the items are arranged alphabetically. Given that this model is dynamic, a time index is part of the domains of all variables and the parameters that are most likely to change over time.

Table A.1.1. Notational principles

Items	Notation	Example
Sets	Lower-case Latin letters as subscripts to variables and parameters	see the following rows
Endogenous variables	Upper-case Latin letters (without a bar)*	QG_c
Exogenous variables**	Upper-case Latin letters with a bar*	$\overline{QFS}_f$
Parameters**	Lower-case Latin letters* or lower-case Greek letters (with or without superscripts)	$ica_{c,a}$; ρ_c^q

*The names of Latin letter variables and parameters that refer to prices, quantities, and factor wages (rents) start with *P*, *Q*, and *WF*, respectively.
**The distinction between exogenous variables and parameters is that the latter always have exogenous values whereas the former under alternative assumptions may be endogenous.

Table A.1.2. Sets

Name	Description
$a \in A$	activities (production sectors or industries)
$c \in C$	commodities (i.e., goods and services)
$c \in CD \left(\subset C \right)$	commodities with domestic sales of domestic output
$c \in CE \left(\subset C \right)$	exported commodities
$c \in CM \left(\subset C \right)$	imported commodities

$c \in CT (\subset C)$	transactions commodities (services paid under distribution margins)
$d \in D$ $(INSD \cup FCAP \cup A \cup TAC)$	domestic demanders (or demand types): institutions (for consumption), investment by capital type, activities, transactions (distribution margins)
$f \in F$	factors
$f \in FVA (\subset F)$	factors that earn value added (in SAM)
$f \in FCAP (\subset F)$	capital factors
$f \in FCAPG (\subset FCAP, \not\subset FVA)$	gov't capital factors (do not earn value-added)
$f \in FCAPNG (\subset FCAP, \subset FVA)$	non-gov't capital factors (earn value-added)
$f \in FLAB (\subset FVA)$	labor factors (earn value-added)
$f \in FOTH (\subset FVA,$ $\not\subset FLAB, \not\subset FCAP)$	other factors (earn value-added; not capital or labor)
$f \in FUEND (\subset FVA)$	factors with endogenous unemployment rate
$i \in INS$	institutions
$i \in INSD (\subset INS)$	domestic institutions
$i \in INSDNG (\subset INSD)$	domestic non-government institutions
$i \in INSNG (\subset INS)$	non-gov't institutions (rest of world and elements in *INSDNG*)
$h \in H (\subset INSDNG)$	households
$t \in T$	time periods (simulation years)
$t \in TMIN$	base period (first simulation year)
$tac \in TAC$	transactions (distribution) types (domestic, import, export)
$tacd \in TACD (\subset TAC)$	transactions (distribution) for domestic sales
$tace \in TACE (\subset TAC)$	transactions (distribution) for exports
$tace \in TACM (\subset TAC)$	transactions (distribution) for imports

Table A.1.3. Variables

Name	Description
CPI_t	consumer price index
$DKA_{f,a,t}$	change in capital stock *f* allocated to activity *a*
$DKINS_{i,f,t}$	investment by institution *i* (in *INS*) in capital stock *f*
DPI_t	domestic producer price index (PDS-based)

EG_t	total current government expenditure
$EH_{h,t}$	consumption expenditure for household *h*
EXR_t	exchange rate (local currency per unit of foreign currency
$INV_{i,t}$	value of investment (including stock change) for institution *i* (in *INSNG*)
$INVG_t$	value of investment (including stock change) for government
$MPS_{i,t}$	marginal propensity to save for domestic non-government institution *i* (in *INSDNG*)
$MPSSCAL_t$	*MPS* scaling factor
$NFFG_t$	net foreign financing of government (FCU)
$NFF_{i,t}$	net foreign financing for non-government institution *i* (in *INSDNG*) (FCU)
$PA_{a,t}$	output price for activity *a*
$PDD_{c,t}$	demand price for commodity *c* (in *C*) produced and sold domestically
$PDS_{c,t}$	supply price for commodity *c* (in *C*) produced and sold domestically
$PE_{c,t}$	price for export of *c* (in *C*) (LCU) (net of export taxes and distribution margin)
$PK_{f,t}$	price (per unit of) of capital stock *f*
$PM_{c,t}$	price for import of *c* (in *C*) (LCU) (includes import tariffs and distribution margin)
$PQD_{c,d,t}$	composite commodity price for *c* (in *C*) for domestic demander (type) *d* (in *D*) [includes commodity subsidies, all taxes (including VAT and sales tax), and distribution margins]
$PQS_{c,,t}$	composite commodity price for *c* (includes import tariffs and distribution margins but not sales tax, commodity subsidies, or VAT)
$PX_{c,t}$	producer price for commodity *c*
$PVA_{a,t}$	value-added price for activity a
$QA_{a,t}$	level of activity *a*
$QD_{c,t}$	quantity sold domestically of domestic output *c*
$QE_{c,t}$	quantity of exports of commodity *c* (in *C*)
$QF_{f,a,t}$	quantity demanded of factor *f* by activity *a*
$QFINS_{i,f,t}$	endowment of institution *i* (in *INSD)* of factor *f*
$QG_{c,t}$	quantity of government consumption of commodity *c*
$QGSCAL_t$	government consumption scaling factor
$QH_{c,h,t}$	quantity consumed of commodity *c* by household *h*
$QINT_{c,a,t}$	quantity of commodity *c* as intermediate input to activity *a*

$QINV_{c,t}$	quantity of investment demand for commodity *c* (investment by source)
$QINVSCAL_t$	investment scaling factor
$QM_{c,t}$	quantity of imports of commodity *c* (in *C*)
$QQ_{c,t}$	quantity of composite demand (and supply) of commodity *c* (in C)
$QT_{c,t}$	quantity of trade and transport services demand for commodity *c* (in C)
$QX_{c,t}$	quantity of domestic output of commodity *c* (in C)
$SAVF_t$	foreign savings (FCU)
$SAVG_t$	government savings
$SAV_{i,t}$	savings of domestic non-government institution *i* (in *INSDNG*)
$SHIF_{i,f,t}$	share for institution *i* (in *INSD*) in the income of factor *f*
$TFP_{a,t}$	total factor productivity for activity *a*
$TFPSCAL_t$	scaling of total factor productivity
$TRII_{i,i',t}$	transfers to institution i (in *INS*) from domestic non-government institution *i'* (in *INSDNG*)
$UERAT_{f,t}$	unemployment rate for factor *f*
$WALRAS_t$	variable check on Walras' law (which is satisfied if value is zero)
$WF_{f,t}$	economywide wage of factor *f*
$WFAVG_{f,t}$	average wage for factor *f* (in *FCAPNG*)
$WFDIST_{f,a,t}$	wage distortion factor for factor *f* in activity *a*
$YF_{f,t}$	income of factor *f*
YG_t	government current revenue
$YI_{i,t}$	income of (domestic non-government) institution *i* (in *INSDNG*)
$YIF_{i,f,t}$	income of institution *i* (in *INSD*) from factor *f*

Table A.1.4. Latin letter parameters

Name	Description
$capcomp_{c,f}$	quantity of commodity *c* per unit of new capital stock *f*
$cwts_{c,h}$	weight of commodity *c* in consumption basked of household *h*
$depr_{f,t}$	rate of depreciation for capital stock *f*
drf_t	change in foreign reserves (FCU)
$dwts_c$	weight of commodity *c* in the DPI (PDS-based producer price index)
$ica_{c,a}$	quantity of intermediate input *c* per unit of activity *a*
$icd_{c,c'}$	input of *c* for trade and transportation per unit of commodity *c'* produced and sold domestically

$ice_{c,c'}$	transactions input of *c* per unit of commodity *c*' export
$icm_{c,c'}$	transactions input of *c* per unit of commodity *c*' imports
$invshr_{f,i,t}$	share for capital stock *f* in investment spending of institution *i* (in *INSNG*)
$mpsb_{i,t}$	baseline marginal propensity to save for domestic non-gov't institution *i* (in *INSDNG*)
$ndfg_t$	net domestic financing to government (indexed to numéraire) (FCU)
$nff_{i,t}$	net foreign financing to institution *i* (in *INSD*) (FCU)
$pop_{ac,t}$	population of *ac* (household *h* in *H* or country total)
$pwe_{c,t}$	export price for commodity *c* (in foreign currency)
$pwm_{c,t}$	import price for commodity *c* (in foreign currency)
$qdstk_{c,i,t}$	change in stock (inventories) of *c* for institution *i* (in *INSD*)
$qfinsb_{i,f,t}$	endowment for institution *i* (in *INSD*) of factor *f* (in *FOTH*)
$qgb_{c,t}$	baseline quantity of government consumption of commodity *c*
$qg01_{c,t}$	0-1 parameter turning on-off potential scaling of gov consumption of *c*
$qinvb_c$	base-year quantity of investment (GFCF) demand for *c*
$shii_{i,i'}$	share of institution *i* (in *INS*) in the income (net of direct taxes and savings) of domestic non-gov't institution *i'* (in *INSDNG*)
$sub_{c,d,t}$	rate of subsidy on commodity *c* (in *C)* for demander *d* (in *D*)
$ta_{a,t}$	rate of tax on gross output value for activity *a*
$te_{c,t}$	rate of tax on commodity *c*
$tf_{f,t}$	rate of direct tax on factor *f*
$tfpb_{a,t}$	exogenous component of TFP for activity *a*
$tm_{c,t}$	rate of import tariff on commodity *c*
$tq_{c,t}$	rate of sales tax on commodity *c*
$trnsfr_{ac,i,t}$	transfers from institution i (gov't or rest of world) to *ac* [where *ac* is institution *i* (in *INS*) or factor *f* (in *F*)](LCU if from gov't; FCU if from rest of world)
$tva_{c,d,t}$	rate of value-added tax on commodity *c* (in *C*) for demander *d* (in *D*)
$ty_{i,t}$	rate of direct tax on domestic non-gov't institution *i* (in *INSDNG*)
$ueratb_{f,t}$	exogenous unemployment rate for factor *f* (not in *FUEND*)
wfb_f	exogenous economywide wage term for activity-specific factors
$wfdistb_{f,a}$	exogenous activity-specific wage term for mobile factors

Table A.1.5. Greek letter parameters

Name	Description
$\alpha_{i,t}^{sav}$	intercept in savings function for institution *i* (in *INSDNG*)

$\beta_{c,h}$	share parameter in LES function for household consumption of commodity *c*
$\gamma_{c,h}^{min}$	minimum quantity in LES function for household consumption of commodity *c*
δ_c^{dd}	share parameter for domestic purchases in Armington function for commodity *c* (top of nest)
δ_c^{ds}	share parameter for domestic sales in CET function for commodity *c* (top of nest)
δ_c^{e}	share parameter for exports in CET function for aggregated commodity *c* (in *C*) (top of nest)
δ_c^{m}	share parameter for imports in Armington function for commodity *c* (top of nest)
$\delta_{f,a}^{va}$	share parameter for factor *f* in CES VA function for activity *a*
$\eta_{a,f}^{tfp}$	elasticity of TFP in activity *a* with respect to gov't capital stock *f*
η_f^{wf}	elasticity of wage for factor *f* (in *FUEND*) with respect to unemployment rate
$\theta_{a,c}$	yield of output *c* per unit of activity *a*
κ_f	sensitivity of the allocation of new capital for *f* (in *FCAPNG*) across activities (in *A*) to current deviations of activity capital rents from the economywide average
ρ_c^q	exponent in Armington function for commodity *c*
ρ_a^{va}	exponent in CES VA function for activity *a*
ρ_c^x	exponent in CET function for commodity *c* (top of nest)
σ_c^q	elasticity of substitution between supplies of domestic output and imports in Armington function for *c* (top of nest)
σ_a^{va}	elasticity of substitution between factors in CES VA function of activity *a*
σ_c^x	elasticity of transformation between domestic sales and exports in CET function for *c* (top of nest)
φ_c^q	shift parameter in Armington function in which domestic sales and imports of commodity *c* (in *C*) are aggregated to composite supply (top of nest)
φ_a^{va}	shift parameter for CES VA function of activity *a*
φ_c^x	shift parameter in CET function for commodity *c* (top of nest)

A.2. Equations

The equations are split into four blocks:

1. Production and factors;
2. Domestic and aggregate foreign trade;
3. Current accounts of domestic institutions

4. Investment, system constraints, and numéraire.

Each section of the presentation covers one block and has its equations stated in one table. In model simulations, it is possible to choose among alternative assumptions for (i) payments linking the government, domestic non-government institutions, and the rest of the world; and (ii) the equilibrating mechanisms (the closures) for macro balances, factor markets, and markets for exports and imports. The assumptions used in the Guatemala simulations of this paper are presented above in Section 3.[10] In this appendix, we apply the following set of relatively simple assumptions:

- Government budget: The government balance is cleared by adjustments in government investment in the context of rule-based or exogenous levels for other government payments (including exogenous values for tax rates, quantities of government consumption, and foreign and domestic financing).
- Savings-investment: The level of domestically financed private investment is determined by the level of financing from domestic non-government institutions, for which the marginal propensities to save are fixed. Government investment is financed as part of the government budget.
- Balance of payments: The balance is cleared by adjustments in the real exchange rate, which influence export and import quantities and values; other items in the balance of payments (including transfers, foreign investment, and net foreign financing) are exogenous or determined by other rules.
- Factor markets:
    - Private capital is activity-specific (not mobile across activities), with an activity-specific market-clearing wage.

[10] The User Guide that accompanies ISIM-GEM-Core provides full details on these and other model features.

- Other factors (including labor) are mobile across activities; unemployment is endogenous for selected factors (typically labor).

- Foreign markets for exports and imports. Both world export and import prices are exogenous (i.e., the small country assumption).

A.2.1. Production and factors

The equations in this block are found in Table A.2.1. They cover the determination of production by sector, demands for factors and intermediates, TFP, factor wages (or rents), unemployment, and factor incomes.

The activity levels (*QA*), which drive the level of commodity production by each activity, is a CES function of factor employment, scaled to account for the contribution of intermediate inputs (PRD1). Factor demands (*QF*) are a function of the parameters of the production function, wages, and the price of value added (i.e., the payment to factors per unit of the activity), in a setting where the producers maximize profits while taking prices and wages as given (PRD2). *TFP* by activity is a function of an exogenous trend parameter, a scaling parameter (which typically is endogenous for the base simulation but otherwise exogenous), and changes in government capital stocks (PRD3). In this equation, the impact of government capital stocks is captured by a constant-elasticity function of the product of the ratios between current and base-year values of the different stocks.

Other variables related to production are determined by activity levels, other parameters, and prices. Intermediate demands (*QINT*) are a Leontief fixed-coefficient function of activity levels (PRD4). Likewise, commodity output levels (*QX*) are driven by activity levels multiplied by fixed yield coefficients (θ), summed over all relevant activities (PRD5). Depending on the values of the yield coefficients, any commodity may be produced by more than one activity and any activity may produce more than one commodity. The value-added price (*PVA*), which appeared above in the factor demand functions (PRD2), is defined as the price (or revenue) per unit of an activity (*PA*) net of activity taxes and the intermediate input cost per activity unit (PRD6). For

any activity, *PA* is the product of yields and unit producer prices, summed over all outputs (PRD7).

The treatment of factor markets is rich, making it possible for the analyst to select among alternative assumptions with regard to mobility, unemployment, and supply growth. In this mathematical statement, we assume that (i) private capital his fully employed and activity-specific (with endogenous allocations of private capital created by new investment); and (ii) that other factors (labor and natural resources, if any) are mobile and may or may not have endogenous unemployment (depending on a set definition). Other configurations are possible.

Table A.2.1 shows the treatment of the markets for factors other than private capital, which is treated in Table A.2.5 given its links to investment and its special treatment of mobility. For non-labor factors, the unemployment (excess-capacity) rates are fixed (PRD8). For labor, wages are determined by a "wage curve", which is a function of the base-year wage and the ratios between current and base-year values for the CPI (the numéraire, which in practice does not change) and the unemployment rate, *UERAT*, which is endogenous and raised to a negative elasticity (PRD9). For all factors, the activity-specific wage term (*WFDIST*) is fixed (PRD10) and, irrespective of whether unemployment is endogenous or not, the factor market equilibrium conditions state that total employment equals total supplies adjusted for unemployment (or excess capacity) (PRD11).

Given the above-stated treatment, the factor market equilibrium conditions (PRD11) are cleared via adjustments in the economywide wage variable (*WF*). For factors no in the set *FUEND*, the quantities supplied for employment (the RHS of PRD11) are fixed in any given time period; given this, the full adjustment burden falls on the LHS and the quantities demanded (defined in equation PRD2). For factors in *FUEND* (often labor), the adjustment is shared between the demand and supply sides. For example, for the case of excess demand (in PRD11, LHS is larger than RHS), an increase in *WF* would simultaneously (i) reduce *QF* and the LHS value of PRD11 (via PRD2); and (ii) reduce the unemployment rate (*UERAT* via PRD8), thereby raising the RHS value of PRD11.

Irrespective of the treatment of the markets, the total income for each factor (*YF*), also including private capital, is the product of the two wage terms and quantities employed, summed over all activities, plus net factor transfers (or income) from abroad, adjusted for the exchange rate (PRD12).

Table A.2.1. Equations for production and factors

PRD-1	$QA_{a,t} = TFP_{a,t} \cdot \varphi_a^{va} \left(\sum_f \delta_{f,a}^{va} \cdot QF_{f,a,t}^{-\rho_a^{va}} \right)^{\frac{-1}{\rho_a^{va}}}$	$a \in A, t \in T$	Value added
PRD-2	$QF_{f,a,t} = \left(\frac{PVA_{a,t}}{WF_{f,t} \cdot WFDIST_{f,a,t}} \right)^{\sigma_a^{va}} \cdot \left(\delta_{f,a}^{va} \right)^{\sigma_a^{va}} \left(TFP_{a,t} \cdot \varphi_a^{va} \right)^{\sigma_a^{va}-1} QA_{a,t}$	$f \in FVA$ $a \in A$ $t \in T$	Factor demands
PRD-3	$TFP_{a,t} = tfpb_{a,t} \cdot \overline{TFPSCAL_t} \cdot \prod_{f \in FCAPG} \left(\frac{QFINS_{gov,f,t}}{QFINS_{gov,f}^{00}} \right)^{\eta_{a,f}^{tfp}}$	$a \in A, t \in T$	Total factor productivity
PRD-4	$QINT_{c,a,t} = ica_{c,a} \cdot QA_{a,t}$	$c \in C, a \in A$ $t \in T$	Intermediate demands
PRD-5	$QX_{c,t} = \sum_{a \in A} \theta_{a,c} \cdot QA_{a,t}$	$c \in C, t \in T$	Output
PRD-6	$PVA_{a,t} = PA_{a,t}\left(1 - ta_{a,t}\right) - \sum_{c \in C} PQD_{c,a,t} \cdot ica_{c,a}$	$a \in A, t \in T$	Value-added price
PRD-7	$PA_{a,t} = \sum_{c \in C} \theta_{a,c} \cdot PX_{c,t}$	$a \in A, t \in T$	Activity price
PRD-8	$UERAT_{f,t} = ueratb_{f,t}$	$f \in FVA, f \notin FUEND$ $f \notin FCAPNG$	Exogenous unemployment rates
PRD-9	$WF_{f,t} = WF_f^{00} \left(\frac{\overline{CPI_t}}{CPI^{00}} \right) \left(\frac{UERAT_{f,t}}{UERAT_f^{00}} \right)^{\eta_f^{wf}}$	$f \in FUEND$ $t \in T$	Wage curve
PRD-10	$WFDIST_{f,a,t} = wfdistb_{f,a}$	$f \in FVA$ $f \notin FCAPNG$	Exogenous activity-specific wage term for mobile factors
PRD-11	$\sum_{a \in A} QF_{f,a,t} = \left(1 - UERAT_{f,t}\right) \cdot \sum_{i \in INSD} QFINS_{i,f,t}$	$f \in FVA, t \in T$ $f \notin FCAPNG$	Factor markets
PRD-12	$YF_{f,t} = \sum_{a \in A} WF_{f,t} \cdot WFDIST_{f,a,t} \cdot QF_{f,a,t} + trnsfr_{f,row,t} \cdot EXR_t$	$f \in FVA, t \in T$	Factor income

A.2.2. Domestic and aggregate foreign trade

Table A.2.2 covers the allocation of domestic commodity demands between imports and domestic output and the allocation of domestic output between exports and domestic sales. Equations TRD1-TRD3 are related to prices. In TRD1, the export price received by producers, *PE*, is defined as the world export price, transformed into domestic currency via the exchange rate and adjusted for export taxes and the transactions (trade and transport) cost per unit of exports; the unit transactions cost is defined as the product of an input coefficient (*ice*) and the input price, summed over all inputs. In analogous fashion, equation TRD2 defines the domestic currency import price for demanders, *PM*, on the basis of the world import price, the exchange rate, and import tariffs, in this case with the unit transactions cost added to the price. In both equations, it is assumed that the modeled economy is small; thus, world prices for exports and imports (*pwe* and *pwm*) are exogenous. Equation TRD3 links the demander and supplier prices for domestic output sold domestically, *PDD* and *PDS*: the demander price is defined as the supplier price plus the transactions cost per unit of domestically sold output; as will be discussed below, either of these prices can be seen as the market-clearing price for this category of outputs (cf. equation INV3).

The commodity demand, *QQ*, is a CES aggregation of imports and domestic purchases, named the Armington function after its originator (TRD4); *QQ* is referred to as a "composite" demand given that it is met from different sources. Equation TRD5 defines the composite demands for commodities that (as opposed to those covered by TRD4) do not have both imports and domestic purchases.

For commodities with both sources, domestic demanders are assumed to minimize the cost of any composite demand quantity subject to the Armington function and subject to the relative prices. The first-order conditions (FOCs) are made up of the Armington function itself (TRD4), and an equation that specifies the optimal demand ratio (*QM/QD*) as a function of the ratio between the prices of domestic output and imports (*PDD/PM*) (TRD6). The composite price *PQS* is implicitly defined by TRD7 given that the other variables in this equation are determined by other relationships. At the composite commodity level, a distinction is made between *PQS* and *PQD*. As shown by TRD8, the distinction is that *PQD* (the price actually paid by domestic

demanders) is adjusted to account for sales taxes, value-added taxes, and subsidies; given that both value-added taxes and subsidies always or often have different rates for different demander categories, *PQD* is disaggregated along this additional dimension, captured by the index *d*.

Turning to the production side, a constant-elasticity-of-transformation (CET) function defines the frontier for allocations of domestic output (*QX*, defined in the preceding section) between exports and domestic sales (*QE* and *QD*, respectively) (TRD9) for outputs that, according to base data, have non-zero values for both destinations. Equation TRD10 defines the equivalent of this transformation for outputs with only domestic sales or only exports.

For outputs with both destinations, producers are assumed to maximize the revenue of any output quantity subject to the CET function and relative prices. The FOCs are made up of the CET function and an equation that specifies the optimal supply ratio (*QE/QD*) as a function of the ratio between the prices of exports and domestic sales (*PE/PDS*) (TRD11)**.** The final equation in this block implicitly defines the average producer output price, *PX*, as a weighted average of the prices received for domestic sales and exports. (In section A.2.1, *PX* influences production decisions and revenues.)

Table A.2.2. Equations for domestic and aggregate foreign trade

TRD-1	$PE_{c,t} = \left(1 - te_{c,t}\right) EXR_t \cdot pwe_{c,t} - \sum_{c' \in CT} \sum_{tace \in TACE} PQD_{c',tace,t} ice_{c',c}$	$c \in CE$ $t \in T$	Export price
TRD-2	$PM_{c,t} = \left(1 + tm_{c,t}\right) EXR_t \cdot pwm_{c,t} + \sum_{c' \in CT} \sum_{tacm \in TACM} PQD_{c,tacm,t} \cdot icm_{c',c}$	$c \in CM$ $t \in T$	Import price
TRD-3	$PDD_{c,t} = PDS_{c,t} + \sum_{c' \in CT} \sum_{tacd \in TACD} PQD_{c,tacd,t} icd_{c',c}$	$c \in C$ $t \in T$	Domestic demand price for domestic output
TRD-4	$QQ_{c,t} = \varphi_c^q \left(\delta_c^m \cdot QM_{c,t}^{-\rho_c^q} + \delta_c^{dd} \cdot QD_{c,t}^{-\rho_c^q} \right)^{-\frac{1}{\rho_c^q}}$	$c \in CM \cap CD$ $t \in T$	Composite demand if use of imports

			and domestic output
TRD-5	$QQ_{c,t} = QM_{c,t} + QD_{c,t}$	$(c \in CM \cap c \notin CD) \cup (c \in CD \cap c \notin CM), t \in T$	Composite demand if not use of both imports and domestic output
TRD-6	$\frac{QM_{c,t}}{QD_{c,t}} = \left(\frac{PDD_{c,t}}{PM_{c,t}} \frac{\delta_c^m}{\delta_c^{dd}} \right)^{\frac{1}{1+\rho_c^q}}$	$c \in CM \cap CD$ $t \in T$	Import-domestic demand ratio
TRD-7	$PQS_{c,t} \cdot QQ_{c,t} = \left(PDD_{c,t} \cdot QD_{c,t} + PM_{c,t} \cdot QM_{c,t} \right)$	$c \in C$ $t \in T$	Composite demand price
TRD-8	$PQD_{c,d,t} = PQS_{c,t} \left(1+tq_{c,t}\right)\left(1-sub_{c,d,t}\right)\left(1+tva_{c,d,t}\right)$	$c \in C, d \in D$ $t \in T$	Adjusted composite demand price
TRD-9	$QX_{c,t} = \varphi_c^x \left(\delta_c^e \cdot QE_{c,t}^{\rho_c^x} + \delta_c^{ds} \cdot QD_{c,t}^{\rho_c^x} \right)^{\frac{1}{\rho_c^x}}$	$c \in CE \cap CD$ $t \in T$	Output transfor-mation if both exports and domestic sales
TRD-10	$QX_{c,t} = QE_{c,t} + QD_{c,t}$	$(c \in CE \cap c \notin CD) \cup (c \in CD \cap c \notin CE), t \in T$	Output transforma tion if not both exports and domestic sales
TRD-11	$\frac{QE_{c,t}}{QD_{c,t}} = \left(\frac{PE_{c,t}}{PDS_{c,t}} \frac{\delta_c^{ds}}{\delta_c^e} \right)^{\frac{1}{\rho_c^x - 1}}$	$c \in CE \cap CD$ $t \in T$	Export-domestic sales ratio
TRD-12	$PX_{c,t} \cdot QX_{c,t} = PDS_{c,t} \cdot QD_{c,t} + PE_{c,t} \cdot QE_{c,t}$	$c \in C$ $t \in T$	Producer output price

A.2.3. Current payments by domestic institutions

This equation block explains payments that are part of the current accounts of domestic institutions, i.e. current incomes and spending for households, the government and enterprises. In the model and its database, it is necessary to include at least one household and, in practice,

models applied to countries in this world invariably have a government. Enterprises are optional. Even though the model and the database can handle multiple representative households, this mathematical statement assumes for simplicity that there is only one household. The sets for institutions distinguish between *INSD* (all domestic institutions), *INSDNG* (all non-government domestic institutions, i.e. households and enterprises), and *H* (households, which may include "non-profit institutions in service of households"). Enterprises differ from households in that they do not consume.

On the income side, the shares of domestic institutions in factor incomes, *SHIF*, are defined on the basis of their stock (or endowment) shares (INS1); the stocks (*QFINS*) are defined below in Section A.2.4. The factor incomes of domestic institutions, *YIF*, are a function of these shares, factor incomes (*YF*) net of direct taxes, and exogenous payments of factor incomes to the outside world (INS2). (*YF* was defined Section A.2.1.) Using this information, the total incomes of domestic non-government institutions, *YI*, are the sum of factor incomes, transfers from the government (indexed to the numéraire, in this case the CPI), transfers from abroad, and transfers from other institutions in *INSDNG* (INS3). (Government incomes are defined in a separate equation.)

The values for consumption and transfer spending by domestic non-government institutions are defined after deducting payments for direct taxes and savings. The mathematical statement treats direct tax rates as exogenous (policy-determined) but demonstrates alternative treatments for savings rates. The marginal propensity to save, *MPS*, is the product of an institution-specific rate (which may change over time) and a scaling parameter (INS4). If the latter is flexible, then total savings are adjusted endogenously in the context of restrictions on the total quantity or value of private investment financed by domestic non-government institutions. Here, the scaling parameter is fixed, meaning that investment spending must be flexible. (This is discussed in Section A.2.4.) Institution-specific savings values, *SAV*, are a linear function of *MPS* and income net of direct taxes, with an optional (non-zero) intercept, which is indexed to the numéraire (INS5). The presence of an intercept is essential when base-year data indicate that some household groups have negative savings – without a separate (and negative) intercept and the related assumption that marginal and average savings rates differ, higher

incomes would in this setting reduce savings further below zero. Transfers from institutions in *INSDNG* to other institutions (in *INS*), *TRII*, are fixed shares of their incomes net of direct taxes and savings (INS6). For households, consumption spending, *EH*, is defined as income net of direct taxes, savings, and transfers to other institutions (INS7). Household consumption demands, *QH*, are a function of population, prices, and total spending (EH) (INS8); the demand functions are derived from the maximization of a Stone-Geary utility function subject to the total spending and prices. It is referred to as a linear expenditure system (LES) since spending on any commodity (the product of price and quantity) is a linear function of *EH* – this is evident if one multiplies both sides of INS8 by the price variable (*PQD*).

The remaining equations in this block define current government receipts and spending. Government receipts, *YG*, are the sum of tax revenues, domestic and foreign transfers, and factor incomes (INS9). Domestic transfers are exogenous and indexed to the numéraire; foreign transfers are exogenous in FCU. The taxes are made up of direct taxes on institutions and factors; domestic indirect taxes on sales, value-added, and activity revenues; export taxes; and import tariffs. To make the mathematical statement more easily digestible, value-added taxes (VATs) (*YTAXVAT*) are defined in a separate equation (INS10). Whether a given tax is part of an application depends on the database. As indicated, the VAT rates are disaggregated by commodity demanded, demander, and time period.

Current government spending, *EG*, is the sum of spending on consumption, domestic transfers, transfers abroad, and subsidies (INS11). The quantities of government consumption, *QG*, are defined on the basis of a trend term (*qgb*) that may be scaled selectively (by commodity and time period) (INS12); the impact of a given value for the scaling variable *QGSCAL* depends on the level of the parameter *qg01* – as indicated by its name, we propose that it be set at values between 0 and 1. In the current mathematical statement, *QGSCAL* is exogenous; if it were endogenous, it could be used to clear the government budget. Subsidy spending, *SUBCT*, is also defined in a separate equation (INS13); subsidy rates are similar to VAT rates in that they are disaggregated by commodity, demander and time period.

Table A.2.3. Equations for current payments by domestic institutions

INS-1	$SHIF_{i,f,t} = \frac{QFINS_{i,f,t}}{\sum_{i' \in INSD} QFINS_{i',f,t}}$	$i \in INSD$ $f \in FVA$ $t \in T$	Shares of factor incomes to domestic institutions
INS-2	$YIF_{i,f,t} = SHIF_{i,f,t}\left(\left(1-tf_{f,t}\right)YF_{f,t} - EXR_t \cdot trnsfr_{row,f,t}\right)$	$i \in INSD$ $f \in FVA$ $t \in T$	Factor income to domestic institutions
INS-3	$YI_{i,t} = \sum_{f \in FVA} YIF_{i,f,t} + trnsfr_{i,gov,t} \cdot \overline{CPI}_t$ $+trnsfr_{i,row,t} \cdot EXR_t + \sum_{i' \in INSDNG} TRII_{i,i',t}$	$i \in INSDNG$ $t \in T$	Non-gov't institution income
INS-4	$MPS_{i,t} = mpsb_{i,t} \cdot \overline{MPSSCAL_t}$	$i \in INSDNG$ $t \in T$	Marginal propensity to save
INS-5	$SAV_{i,t} = \alpha_{i,t}^{sav} \cdot \overline{CPI} + MPS_{i,t}\left(1-ty_{i,t}\right)YI_{i,t}$	$i \in INSDNG$ $t \in T$	Non-gov't institution savings
INS-6	$TRII_{i,i'} = shii_{i,i'}\left(\left(1-ty_{i',t}\right)YI_{i',t} - SAV_{i',t}\right)$	$i \in INS$ $i' \in INSDNG$ $t \in T$	Institutional transfers
INS-7	$EH_{h,t} = \left(1-ty_{h,t}\right)YI_{h,t} - SAV_{h,t} - \sum_{i \in INS} TRII_{i,h,t}$	$h \in H$ $t \in T$	Household consumption expenditure
INS-8	$QH_{c,h,t} = pop_{h,t}\left(\gamma_{c,h,t}^{min} + \frac{\beta_{c,h}\left(\frac{EH_{h,t}}{pop_{h,t}} - \sum_{c' \in C} PQD_{c',h,t} \cdot \gamma_{c',h,t}^{min}\right)}{PQD_{c,h,t}}\right)$	$c \in C$ $h \in H$ $t \in T$	Household consumption demand
INS-9	$YG_t = \sum_{i \in INSDNG} ty_{i,t} \cdot YI_{i,t} + \sum_{f \in F} tf_{f,t} \cdot YF_{f,t}$ $+\sum_{c \in C} tq_{c,t} \cdot PQS_{c,t} \cdot QQ_{c,t} + YTAXVAT_t$ $+\sum_{a \in A} ta_{a,t} \cdot PA_{a,t} \cdot QA_{a,t}$ $+\sum_{c \in C} te_{c,t} \cdot pwe_{c,t} \cdot QE_{c,t} \cdot EXR_t$ $+\sum_{c \in C} tm_{c,t} \cdot pwm_{c,t} \cdot QM_{c,t} \cdot EXR_t$ $+trnsfr_{gov,row,t} \cdot EXR_t$ $+\sum_{i \in INSDNG} TRII_{gov,i,t} + \sum_{f \in F} YIF_{gov,f,t}$	$t \in T$	Government current receipts

INS-10	$YTAXVAT_t =$ $\sum_{c\in C}\sum_{a\in A}\left(1-sub_{c,a,t}\right)\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot tva_{c,a,t}\cdot QINT_{c,a,t}$ $+\sum_{c\in C}\sum_{h\in H}\left(1-sub_{c,h,t}\right)\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot tva_{c,a,t}\cdot QH_{c,h,t}$ $+\sum_{c\in C}\left(1-sub_{c,gov,t}\right)\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot tva_{c,gov,t}\cdot QG_{c,t}$ $+\sum_{c\in C}\sum_{f\in FCAP}\left(\left(1-sub_{c,f,t}\right)\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot tva_{c,f,t}\right.$ $\left.\cdot capcomp_{c,f}\sum_{i\in INS}DKINS_{i,f,t}\right)$ $+\sum_{c\in C}\sum_{i\in INS}\left(1-sub_{c,dstk,t}\right)\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot tva_{c,dstk,t}\cdot qdstk_{c,i,t}$ $+\sum_{c\in C}\sum_{c'\in CDIS}\left(1-sub_{c,tacm,t}\right)\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot tva_{c,tacm,t}\cdot icm_{c,c',r}\cdot QMR_{c',r,t}$ $+\sum_{c\in C}\sum_{c'\in CDIS}\left(1-sub_{c,tace,t}\right)\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot tva_{c,tace,t}\cdot ice_{c,c',r}\cdot QER_{c',r,t}$ $+\sum_{c\in C}\sum_{c'\in C}\left(1-sub_{c,tacd,t}\right)\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot tva_{c,tacd,t}\cdot icd_{c,c'}\cdot QD_{c',r,t}$	$t\in T$	VAT revenue
INS-11	$EG_t=\sum_{c\in C}PQD_{c,gov,t}\cdot QG_{c,t}+\sum_{i\in INSDNG}trnsfr_{i,gov,t}\cdot\overline{CPI}_t$ $+trnsfr_{row,gov,t}\cdot EXR_t+SUBCT_t$	$t\in T$	Government expenditure
INS-12	$QG_{c,t}=qgb_{c,t}\left(1+qg01_{c,t}\cdot\overline{QGSCAL}_t\right)$	$c\in C$ $t\in T$	Government consumption
INS-13	$SUBCT_t =$ $\sum_{c\in C}\sum_{a\in A}sub_{c,a,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot QINT_{c,a,t}$ $+\sum_{c\in C}\sum_{h\in H}sub_{c,h,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot QH_{c,h,t}$ $+\sum_{c\in C}sub_{c,gov,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot QG_{c,t}$ $+\sum_{c\in C}\sum_{f\in FCAP}sub_{c,f,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot capcomp_{c,f}\sum_{i\in INS}DKINS_{i,f,t}$ $+\sum_{c\in C}\sum_{i\in INS}sub_{c,dstk,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot qdstk_{c,i,t}$ $+\sum_{c\in C}\sum_{c'\in CDIS}sub_{c,tacm,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot icm_{c,c',r}\cdot QMR_{c',r,t}$ $+\sum_{c\in C}\sum_{c'\in CDIS}sub_{c,tace,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot ice_{c,c',r}\cdot QER_{c',r,t}$ $+\sum_{c\in C}\sum_{c'\in C}sub_{c,tacd,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot icd_{c,c'}\cdot QD_{c',r,t}$	$t\in T$	Commodity subsidy

A.2.4. Investment, system constraints, and numéraire

This block covers investment spending by different institutions, how it is financed, and how the new capital that is generated feeds into the economy. The specification of investment and its

financing makes it possible to specify the remaining system constraints, the markets for private capital factors, commodity balances, and the balance of payments.[11] (In Sections A.2.1 and A.2.3, we covered the markets for other factors and the government balance, respectively.) In addition, we here specify the numéraire, needed for CGE models like GEM-Core.

For the government, investment spending (or gross capital formation), *INVG*, is defined as the sum of government savings (the difference between current receipts and spending), domestic net financing (indexed to the numéraire), and foreign net financing (exogenous in FCU) (INV1). The sum of the two financing terms is the government primary deficit. These two terms are referred to as net financing items since they represent the difference between new borrowing and interest payments; the latter do not appear explicitly in the model.[12] For domestic non-government institutions, the corresponding variable, *INV*, is the sum of own savings and net financing from abroad minus claims on investment funding to finance the government and add to foreign reserves (INV2). Before translating investment spending into quantities of new capital, it is necessary to specify prices; in equation INV3, the unit prices of new capital stocks, *PK*, are defined as the product of the price of commodity *c* and the matrix of capital composition coefficients (which shows the quantities of commodities *c* used as inputs per unit of any new capital stock *f*) summed over all *c*.

The next three equations define investment quantities by destination (by type of capital stock) by government and non-government institutions and investment quantities by source (the use of commodity inputs in the production of new capital). For the government, the quantity of new capital stock *f*, $DKINS_{gov,f,t}$, is defined as investment spending net of spending on new

[11] The term system constraints refers to constraints that are not perceived by individual actors (like household budget constraints) but which the economic system as a whole nevertheless most respect (like a labor market constraint which says that the quantity employed equals the labor force net of unemployment).

[12] Given simulation results for government and private net financing (domestic and foreign) and with the help of additional data on initial debt stocks, and real interest rates by time period, it is straightforward to compute debt stocks, which may be expressed in relation to other simulation results (like GDP or export values).

inventory (gross fixed capital formation or GFCF), multiplied by the spending share for *f*, and divided by *PK* to transform into stock quantities (INV4). In the corresponding equation for non-government investment (INV5), GFCF is the sum of (a) investment net of stock change spending for institutions in INSDNG; and (b) foreign investment (the value of which is exogenous in FCU). GFCF is allocated across different capital stocks (if more than one) in fixed spending shares and transformed into quantities of new capital by dividing by *PK*.[13] Final investment demands (i.e. investment quantities defined by the source of inputs into the construction of new capital), *QINV*, are defined as the product of the capital composition matrix and investment by capital stock, summed over all capital stocks (INV6).

For any capital stock, the endowments held by domestic institutions (government and non-government), *QFINS*, are defined as the sum of (a) the stock held in the previous year net of depreciation; and (b) new investment in the previous year (INV7). For the stocks of other factors, endowments are defined exogenously (INV8). The values for endowments were used in Section A.2.1 to define the supply sides of markets for non-capital factors and, in Section A.2.3, to define the distributional shares for factor incomes.

The allocation of private capital stocks across activities responds to relative capital rents.[14] As an input to the formulation used, the average wage of private capital stock *f*, *WFAVG*, is defined as total rent to *f* divided by total employment of *f* (INV9). In equation INV10, the allocation of new private capital stock *f* to an activity *a*, *DKA*, is defined as the product of (a) an allocation based on current activity shares (i.e. total new investment in *f* times the current share of *a* in the use of *f*); and (b) an adjustment term that is above (below) unity if the wage of capital stock *f* in *a* is above (below) the economywide average, assuming a positive value for the parameter κ

[13] From our experience, in most applications of models of this, it is preferable to have one type of private capital with constrained mobility. However, the option of multiple stocks may be useful if, for example, the model should mimic the allocation of foreign investment to specific sectors like mining.

[14] This approach that is presented may have been first developed in Dervis et al. (1982, pp. 175-178). Our treatment deviates in one respect: for simplicity, we use capital rents by activity instead of profit rates.

(Greek kappa; $\kappa \geq 0$). κ plays a crucial role in this formulation: the higher its value, the stronger the sensitivity of the allocation of new capital to differences in capital rents; if it is zero, the allocation of stock *f* does not change over time and if it is too high, capital rents may oscillate.[15]

Total employment of capital stock *f* in activity *a* in period *t*, $QF_{f,a,t}$, is defined as the stock installed in *t-1*, $QF_{f,a,t-1}$, net of depreciation, plus the quantity of new investment in stock *f* in *t-1* allocated to *a* , $DKA_{f,a,t-1}$ (INV11). This last equation may be seen as defining a set of activity-specific markets for capital stock *f* in which the quantity supplied (the right-hand side) is fixed within any period *t* (determined by past decisions) while the quantity demanded (the left-hand side) is determined by profit-maximization (cf. Section A.2.1). A wage variable defined over *f* and *a* is needed to clear this market. Accordingly, among the two wage variables that apply to any factor ($WF_{f,t}$ *and* $WFDIST_{f,a,t}$), equation INV12 fixes the economywide variable *WF* while leaving the activity-specific variable *WFDIST* flexible. The simulated values for the product of the two variables show the scarcity value of private capital stocks by activity. In sum, for private capital, it is assumed that installed stocks cannot be reallocated while the analyst controls the extent to which the allocation of new capital will shift toward sectors with relatively high capital rents.

---

[15] With reference to INV10, by definition, $\sum_{a \in A} DKA_{f,a,t} = \sum_{i \in INSNG} DKINS_{f,i,t}$ for $f \in FCAPNG$ and $t \in T$.

This can be shown as follows: For simplicity, replacing $\sum_{i \in INSNG} DKINS_{f,i,t}$ by DKI and $WF_{f,t} \cdot WFDIST_{f,a,t}$ by WFA_a, suppressing remaining *f* and *t* subscripts, and noting the definition of the average wage,

$$DKA_a = DKI \cdot \frac{QF_a}{\sum_{a' \in A} QF_{a'}} \cdot \left(1 + \kappa\left(\frac{WFA_a}{\overline{WF}} - 1\right)\right); \sum_{a \in A} DKA_a = DKI \cdot \sum_{a \in A}\left(\frac{QF_a}{\sum_{a' \in A} QF_{a'}} \cdot \left(1 + \kappa\left(\frac{WFA_a}{\overline{WF}} - 1\right)\right)\right)$$

$$= DKI\left(\frac{\sum_{a \in A} QF_a}{\sum_{a' \in A} QF_{a'}} + \kappa \cdot \sum_{a \in A}\left(\frac{WFA_a}{\overline{WF}} \frac{QF_a}{\sum_{a' \in A} QF_{a'}}\right) - \frac{\sum_{a \in A} QF_a}{\sum_{a' \in A} QF_{a'}}\right) = DKI\left(1 + \kappa\left(\frac{\overline{WF}}{\overline{WF}} - 1\right)\right) = DKI$$

For each domestic commodity, the demand side is now complete. The equation INV13 defines total composite demand for any commodity, *QQ*, as the sum of consumption, investment (fixed capital formation and stock changes), intermediate demands, and demands for trade and transportation services (due to domestic and foreign trade). As specified in Section A.2.2, these demands generate demands for domestic output and/or imports. The markets for domestic output sold domestically are cleared by the linked variables *PDD* and *PDS*; to exemplify, for the case of excess demand, increases in both price variables would simultaneously reduce domestic demands for domestic outputs and increase the quantities of output sold domestically (raising the total output level by raising profitability and raising the share of output sold domestically).

The statement of investment financing completes the flows in the balance of payments, which is expressed in FCU. Equations INV14 and INV15 state the current and capital accounts, respectively, with foreign savings, *SAVF*, as the linking variable. In the current account balance, inflows are due to exports, and transfers from abroad while outflows are caused by imports, transfers from domestic non-government institutions, and factor incomes. The variable *SAVF* measures the current-account deficit; if outflows (the right-hand side) are larger (smaller) than inflows (the left-hand side), foreign savings are positive (negative).[16] In the capital account balance, the current account deficit is financed by net foreign financing to government and non-government institutions and foreign investment, net of increases of foreign reserves. By influencing export and import quantities in opposite directions, raising or reducing the trade balance in FCU, adjustments in the exchange rate, *EXR*, clear the balance of payments, making sure that the level of foreign savings matches the level that is financed on the right-hand side of the capital account.

As a manifestation of Walras' law, in a CGE model like the one presented above, one equation should be removed to assure equality between the number of variables and independent

[16] The variable *SAVF* deviates from the definition of foreign savings given that interest payments are an implicit part of the capital account instead of being part of the current account. The variable *SAVF* could more accurately be referred to as the primary deficit of the nation.

equations; it is possible to check that the omitted equation holds in a post-calculation. Here we opt for the alternative of instead inserting one variable, named WALRAS, into one equation. Hence, the presence of *WALRAS* in the capital account of the balance of payments. In the absence of errors, the solution value for *WALRAS* should be (very close to) zero.

Finally, a well-specified CGE model like GEM-Core is homogeneous of degree zero in prices, meaning that only relative prices matter and that, if one set of relative prices solves the model, then any multiple of this set of prices would also solve the model (scaling all domestic prices and payments) without any influence on quantities. To anchor the price level, a price or price index, referred to as the numéraire, needs to be fixed, with the consequence that all other prices may be seen as measured relative to this numéraire. In this mathematical statement, the consumer price index, *CPI*, is the numéraire. Here, equation INV16 defines the CPI, which is fixed, on the basis of the base-year weights of household consumption payments by commodity and household type in total household consumption.[17]

Table A.2.4. Equations for investment, system constraints, and numéraire

INV-1	$INVG_t = (YG_t - EG_t) + ndfg_t \cdot \overline{CPI}_t + nff_{gov,t} \cdot EXR_t$	$t \in T$	Gov't primary deficit, investment value, and financing
INV-2	$INV_{i,t} = SAV_{i,t} + nff_{i,t} \cdot EXR_t - \left(\frac{SAV_{i,t}}{\sum_{i' \in INSDNG} SAV_{i',t}} \right) \left(ndfg_t \cdot \overline{CPI}_t + drf_t \cdot EXR_t \right)$	$i \in INSDNG$ $t \in T$	Non-gov't investment value and its financing
INV-3	$PK_{f,t} = \sum_{c \in C} PQD_{c,f,t} \cdot capcomp_{c,f}$	$f \in FCAP$ $t \in T$	Price of new capital

[17] As an alternative to CPI, the domestic producer price index (DPI) may serve as numéraire. In addition, it is often used as the denominator in the definition of the price-level-deflated (PLD) real exchange rate (REXR). Algebraically, with time subscripts omitted, $DPI = \sum_{c \in C} PDS_c \cdot dwts_c$ and $REXR = EXR / DPI$.

INV-4	$$DKINS_{gov,f,t} = \frac{invshr_{f,gov,t} \cdot \left(INVG_t - \sum_{c \in C} PQD_{c,gov,t} \cdot qdstk_{c,gov,t} \right)}{PK_{f,t}}$$	$f \in FCAPG$ $t \in T$	Gov't investment by government capital stock
INV-5	$$DKINS_{i,f,t} = \frac{invshr_{f,i,t}}{PK_{f,t}} \cdot \left(\left(INV_{i,t} - \sum_{c \in C} PQD_{c,i,t} \cdot qdstk_{c,i,t} \right)\Big\vert_{i \in INSDNG} + \left(invf_{i,t} \cdot EXR_t \right)\Big\vert_{i \in INSROW} \right)$$	$f \in FCAPNG$ $i \in INSNG$ $t \in T$	Non-gov't investment by private capital stock
INV-6	$$QINV_{c,t} = \sum_{i \in INS} \sum_{f \in FCAP} capcomp_{c,f} \cdot DKINS_{i,f,t}$$	$c \in C$ $t \in T$	Real investment demand (by source)
INV-7	$$QFINS_{i,f,t} = QFINS_{i,f,t-1}\left(1 - depr_{f,t-1}\right) + DKINS_{i,f,t-1}$$	$i \in INSD$ $f \in FCAP$ $t \in T$ $t \notin TMIN$	Accumulation of capital by domestic institutions
INV-8	$$QFINS_{i,f,t} = qfinsb_{i,f,t}$$	$i \in INSD$ $f \in FOTH$ $t \in T$	Exogenous institutional endowments for other factors
INV-9	$$WFAVG_{f,t} = \frac{\sum_{a \in A} WF_{f,a,t} \cdot WFDIST_{f,a,t} \cdot QF_{f,a,t}}{\sum_{a \in A} QF_{f,a,t}}$$	$f \in FCAPNG$ $t \in T$	Average wage (rent) by private capital stock
INV-10	$$DKA_{f,a,t} = \left(\sum_{i \in INSNG} DKINS_{f,i,t} \right) \left(\frac{QF_{f,a,t}}{\sum_{a' \in A} QF_{f,a',t}} \right) \cdot \left(1 + \kappa_f \left(\frac{WF_{f,t} \cdot WFDIST_{f,a,t}}{WFAVG_{f,t}} - 1 \right) \right)$$	$f \in FCAPNG$ $a \in A$ $t \in T$	Allocation of new private capital by activity
INV-11	$$QF_{f,a,t} = QF_{f,a,t-1}\left(1 - depr_{f,t-1}\right) + DKA_{f,t-1}$$	$f \in FCAPNG$ $a \in A, t \in T$ $t \notin TMIN$	Accumulation of private capital by activity
INV-12	$$WF_{f,t} = wfb_f$$	$f \in FCAPNG$	Exogenous economy-wide wage term for private capital

INV-13	$QQ_{c,t} = \sum_{h \in H} QH_{c,h,t} + QG_{c,t} + QINV_{c,t} + \sum_{i \in INSD} qdstk_{c,i,t} + \sum_{a \in A} QINT_{c,a,t} + QT_{c,t}$	$c \in C$ $t \in T$	Commodity balance
INV-14	$\sum_{c \in C} pwe_{c,t} \cdot QE_{c,t} + \sum_{ac \in INSD \cup F} trnsfr_{ac,row,t} + SAVF_t = \sum_{c \in C} pwm_{c,t} \cdot QM_{c,t} + \frac{\sum_{i \in INSDNG} TRII_{row,i,t}}{EXR_t} + \sum_{f \in F} trnsfr_{row,f,t}$	$t \in T$	Current account of balance of payments
INV-15	$SAVF_t = \sum_{i \in INSD} nff_{i,t} + invf_t - drf_t + WALRAS_t$	$t \in T$	Capital account of balance of payments
INV-16	$\sum_{c \in C} \sum_{h \in H} PQD_{c,h,t} \cdot cwts_{c,h} = \overline{CPI_t}$	$t \in T$	Consumer price index