INSTITUTE FOR FARM ECONOMICS
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JÓZSEF SEBESTYÉN

A SHORT SKETCH OF A MATHEMATICAL METHOD OF PLANNING OF AGRICULTURAL PRODUCTION

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Director:

FERENC ERDEI

Ordinary Member of the
Hungarian Academy of Sciences

I. Sources and Development of the Present Model

The organization of a planned economy is a tremendous task, producing ever more complicated problems day by day. As the sphere of economic activities subject to the control of State plans had been expanding, new and new contradictions arose. These contradictions were rooted partly in the economic policy, partly in the lack of planning skill. Grave difficulties emorged in agriculture too.

The consequences of this situation were already beginning to take shape when the National Planning Office started to organize its county filials and a central directing section. The main task of this Section for Territorial Planning was the development of the methods of territorial planning based on the system of territorial balances. Most of the problems claiming for an improvement of territorial planning emerged in agriculture, so agricultural balances constituted an overwhelming majority in this system.

However, only minor results c0uld be achieved by the introduction of the meth0d of territorial planning. It had many weak points because good-will and intuition could not compensate the lack of various kinds of knowledge needed to the right formulation of the problems and to the development of the method of a satisfactory solution. The application also suffered from the rigidity of the official methods of economic direction, so that when one could have made use of the experiences coming from the first experiment with territorial balances, the initiators, among them the author too, had already worked in other spheres of planning and the activity of the Section for Territorial Planning had a less militant character.

In his new sphere of activity, the author found a stimulus to develop the planning system built up only partially in the earlier years. The incertitude in the planning of yields in agriculture urged for an analysis of the results of agricultural research work and of international statistics. The discussions about the organization of the economic co-operation of the Socialist countries pointed out the importance of a territorial planning on an international scale.

So in the methodical work started with in the spring of 1953, he tried to define the balance relationships for a complex of economic units with input-output relations quantitatively known and for which a best one must be selected from a great number of possible plans. As it became clear in a short time that elementary statistical methods were insufficient, the author began to study different mathematical methods applicable in economic analysis.[x] In 1958 results of a series of analyses led to a formulation of the solution as an optimization for interdependent systems that have non-linear production functions. This solution promised to deliver useful conclusions about the price system. The distribution of production schedules appeared as the planning of an interfarm and interregional co-operation. At the interpretation of the results of analyses relating to this problem, we found that some questions of political-economic character must be answered by a research work in the field of differential rent. Based upon the results of these investigations, the development of a model valid for any price system ended in March 1960. In July 1960, a general solution of the planning problem was completed. This solution was not only independent of the price system but it promised to afford the data needed to determine a system of farm prices conforming to the optimum.

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II. Some Preliminary Explications

1. The Planning Period

It follows from the character of the task, that the planning model must involve a longer period. The solution gives not only the optimum distribution of production schedules but that of resources, among them investments too, therefore periods shorter than 7-1o years cannot be considered. On the other hand, the needs of society and the technical development can be forecast to more than 1o-15 years only with a great incertitude. Therefore a decade could be regarded to be the right planning period. During a decade, the structure of social consumption is not subject to fundamental, and therefore inpresumable, changes, on the other hand, radical changes of technology allowed by the development of productive forces, and the new productivity relations created by them, are still easy to be surveyed on the basis of the technical knowledge that qualified planners posses at the time of preparing the plan.

2. Labour as Social Cost of Production

As already montioned above, the model must give not only an optimal distribution of production schedules and resources but it must also produce the data for appropriate price centres for the various agricultural products. Starting from the Marxian statement that the value of a commodity is determined by the quantity of social labour needed to its reproduction, live and materialized labour consumed in the production of a certain commodity is quantified on the basis of the technological structure of the national economy.

This technological structure, subject to changes caused by technical development and investment policies, can be approximated by a time series of static Leontief inverses on the sector and on the national economic scale. By the use of these inverses, the live labour content of inputs of non-agricultural origin, including transport costs too, can be determined well enough. As a second step, the total labour content of final products in agriculture must be computed for each location, in different systems of farming. The optimum plan minimizes this total labour content as social cost of the net output of agriculture.

3. The Differential Rent

A unit of a certain agricultural commodity produced at different locations but on a common technical basis, claims for different quantities of social labour. This difference in social costs is caused by natural conditions that give rise to the primary element of differential rent I. A certain quantity of social labour may have different degrees of productivity on the same location if it is consumed by different structures of production. By an improvement of the structure of production we may exploit the possibilities offered by the generation of the secondary element of the differential rent I. The third form is differential rent II. which is connected with advanced technologies and extra supplies of productive forces.

The resources of society being finite, a certain quantity of final products can only be delivered by locations and technologies having different levels of productivity. But one can distribute the schedules of production over the locations in the form of suitable structures and the resources to be used on the basis of appropriate technologies, so that the average level of productivity is increased and the variance of the productivity coefficient of the different locations for each commodity is considerably decreased. That means in a model like this one in question that an optimum plan minimizes the amount of differential rent too.

4. Autarchy and Co-operation

Many kinds of agricultural products can be considered both as final and intermediate products. A farm may equally be self-sufficient in these products or it may buy them as final products of other farms. The first case is connected with a diversified farming, the second means a high degree of specialization. The existence of specialization claims for a co-operation of the different locations which are mutually consumers and suppliers of commodities that are intermediate products in the process of reproduction of agriculture.

It is the task of the planners to find the best combination of autarchic and co-operative types of production in the course of the distribution over the country of the schedules. Every improvement compared to the initial situation means a better compliance with the conditions of production and a decrease in social costs.

Therefore, every activity in the model is considered in both forms: on a subsistence basis and being dependent on the interregional co-operation. Every degree of specialization /or diversification/ can be turned out as a combination of these two forms which differ in their relations to resources and in their social cost coefficients.

5. The Systems of Farming

A commodity can be produced in various systems of farming which differ in their claims for certain resources. In plant production for example, different systems are the irrigated and non-irrigated production; as to the way of supply of plant nutrients, one of the systems rests on farnyard manure, the other supplies organic materials by green manuring or by plowing down the stubbles and uses a great quantity of commercial fertilizers. One of the possible systems of farming relies mainly upon manual labour, the other is earmarked by chemical weed killing and by a high degree of mechanization. In animal husbandry, types of barns, forms of nutrition and the rate of mechanization may be mentioned as criteria for different systems.

In the course of the distribution of the production schedules the systems of farming must be assigned too, with the discrimination that these systems are based on autarchy or on an inter-regional co-operation.

6. The Technologies

Technologies are determined in a certain sense by the system of farming but it doesn't involve the quantification of the resource combination. The present model deals with optimal technologies for every system of farming. This means that the optimum combinations of resources are those which are linked with the maximum productivity of social labour on a given location. As follows from this, the optimum technologies must be determined for every location in relation to the products and systems of farming that can be considered on the location in question.

7. The Conflict of Central and Local Interests

In a planned economy, the conflicting local interests are subject to the social interest but it does not mean an antagonism. The central /social/ insterests claim for the production of a certain net output, by using up given quantitities of resources and maximizing the productivity of social labour in agriculture as a whole. On the other hand, the individual interests urge the farms to compete for the greatest advantages in the tasks of production and in the supply of resources, in order to assure the highest level of local productivity of social labour.

Our model presents the solution of this problem by giving the frames in the form of social needs, the level of resource supplies and the general objective function, and by allowing the different locations to make use of their best chances in their competition. These chances are the choice in different systems of farming, the technologies offering local maxima of labour productivity, further the possibility of choosing and combining the assignments both on an autarchic and on a co-operative basis.

III. Preparations to Planning

1. Activities and Systems of Farming on Different Locations

The first step is to determine the systems of farming and the activities that may be considered on a certain location. This means already a selection for improving the chances of the individual locations and, on the other hand, a reasonable way for decreasing the dimensions of the model.

An activity on a certain location may be considered in different systems of farming. But if an activity, for example corn production can, be considered in, say, three systems of farming on the location in question, it figures in the model in the form of 6 activities, there also being a choice between the supply of intermediate agricultural products on an autarchic and on a co-operative basis. And these activities must be related to different years. This multiplies then by the number of years considered in the model of perspective planning.

2. The Productivity Functions

Input-output relationships for each product can be quantified by the production functions. These non-linear functions relate to yields per hectare, per cow, per hen, etc. The independent variables for plant production can be ranged between two groups. The first of them involves the variables for natural conditions /indices for temperature and precipitation and one characterizing soil qualities/, in the second we find farm labour and material inputs /the latter ones may be contracted on the basis of their content of non-agricultural labour/. The production function for plant products can be generalized as follows:

$$Y_k = f_k /X_1, X_2, X_3, X_4, X_5, X_6, X_7/, \qquad /1/$$

where k characterizes the product, X_1, X_2 and X_3 are the indices of precipitation, temperature and soil, X_4 and X_5 stand for live labour and machine hours, X_6 means the quantity of fertilizers and X_7 the other material inputs.

In the production functions for animal products

$$Z_k = g_k /V_1, V_2, V_3, V_4, V_5/, \qquad /2/$$

V_1, V_2 and V_3 stand for the quantities used from digestible protein, starch and dry content, respectively, V_4 means live labour and in V_5 are grouped other material inputs.

These functions must be transformed into productivity functions, in order to make use of them in the model but this transformation differs somewhat for plant and animal products. In plant production, the dependent variable will be the quotient of the yield and of the total labour input. The variables for natural conditions remain unchanged, those representing material inputs may be considered in their contents of social labour. These new functions may be represented by the following formula:

$$P_k^{/N/} = f_k \,/X_1, X_2, X_3, M_1^{/N/}, M_2^{/N/}, M_3^{/N/}, M_4^{/N/}/, \quad /3/$$

where the dependent variable is determined by the quotient

$$P_k^{/N/} = \frac{Y_k}{\sum_{i=1}^{4} M_i^{/N/}} \quad /4/$$

and the variables $M_1^{/N/}$, $M_2^{/N/}$, $M_3^{/N/}$, $M_4^{/N/}$ stand for the agricultural live labour and for the social labour content of material inputs used in the production of a certain vegetable commodity.

The transformation into productivity functions of the production functions for animal products is more complicated. The nutrients building up the animal body being grouped as the variables V_1, V_2 and V_3 in formula /2/, can be found in a series of fodder crops, each of which consumes a different quantity of social labour in the process of production. At the time of computing the parameters of these functions, one does not know what a combination of fodders could be considered to supply these nutrients in the different years of the planning period and how many hours of social labour their production will consume. So, one must be contented with an incomplete productivity function where the dependent variable can be received from a fraction, the numerator of which is the yield or live weight per animal and the denominator involves only the sum of live labour consumed by the activity $/M_1^{/N/}/$ and of the social labour content of non-agricultural material inputs $/M_2^{/A/}/$, as follows:

$$P_k^{/A/} = \frac{Z_k}{M_1^{/A/} + M_2^{/A/}}, \qquad /5/$$

and the incomplete productivity function:

$$P_k^{/A/} = g_k\,/V_1, V_2, V_3, M_1^{/A/}, M_2^{/A/}/, \qquad /6/$$

the completion of which will be discussed later.

3. The Optimal Technologies

As it was said above, the various locations compete in the model for the schedules and resource supplies on the basis of their productivity coefficients. Therefore, one must find the local maximum of the productivity of social labour for every activity on every location. In plant production, the maximization of the productivity function gives a combination of resources to be held constant and to be used as a locally optimal technology. This takes place in the following way: if one substitutes into the function the actual values of the indices for natural conditions that are valid for a given location, and computes thereafter the maximum for the remaining variables /represented by both pure and mixed elements/, so one has the resource combination of a technology which is optimal at that location. That must be done for every location considered in the planning model.

By the maximization of the incomplete productivity functions of the activities in animal husbandry, an optimum combination of the variables enumerated in /6/ can be had which is independent from locations. As a second step, we must determine the kinds of fodder and their combination that supply the quantity of nutrients given by the maximum of the productivity function and consume the minimum of social labour on a certain location. The solution is given by the wellknown formulae:

$$\begin{aligned} Q &= \underline{\underline{c}}^{*} \, \underline{\underline{x}} \longrightarrow \min., \\ \underline{\underline{x}} &\geq \underline{\underline{0}} \\ \underline{\underline{A}} \, \underline{\underline{x}} &= \underline{\underline{v}} \end{aligned} \qquad /7/$$

where $\underline{\underline{x}}$ is the vector of feedstuffs, $\underline{\underline{c}}^{*}$ is that of labour coefficients, Q is the total labour content the optimal combination of fodder, $\underline{\underline{A}}$ is the matrix of technical coefficients and $\underline{\underline{v}}$ is the vector, the components of which are the values determined by the maximum of the incomplete productivity function for the variables V_1, V_2 and V_3. This programming procedure must take place for every location and every system of farming considered there for the different activities in animal husbandry, in order to determine local optimal technologies.

4. The Local Inverses for Intermediate Production

The continuity of production requires a certain quantity of intermediate products that should permanently stay in the process of reproduction. In our model, this fund of replacement is confined to agricultural products. On the basis of a given system of technologies defined for a complex of activities, say, for a farm or agriculture as a whole, the volume of intermediate products as a requirement for a certain net output can be determined by the use of input-output-analysis. These computations must be made for every location and system of farming.

In building up the local inverses for intermediate production, we start from the optimal technologies determined as it was described in Section III/3. These inverses are generally specified for the autarchic systems of farming and have the notation $\underline{\underline{U}}$. Another type derived from them relates to the systems of farming that depend on the interregional flow of commodities. The coefficients of one group of these matrices represent the quantities of products required from other locations, in order to satisfy a final demand represented by a vector of units $\underline{\underline{1}}$. We denote this matrix by $\underline{\underline{U}}''$. The difference of the matrices $\underline{\underline{U}}$ and $\underline{\underline{U}}''$, denoted by $\underline{\underline{U}}'$, represents the other group of matrices relating to the systems of farming, depending on interregional co-operation. The elements of $\underline{\underline{U}}'$ have the same meaning for the systems of farming based on co-operation as those of $\underline{\underline{U}}$ for the autarchic systems.

5. The Final Production in Economic Units of Different Degrees of Aggregation

The matrices $\underline{\underline{U}}$ and $\underline{\underline{U}}'$ are used for the determination of gross output, i.e. the concrete program of crop production and that of animal husbandry. In the formula

$$\underline{\underline{p}}_n^{/t/} = \underline{\underline{U}}_n \underline{\underline{x}}_n^{/t/} + \underline{\underline{U}}'_n \underline{\underline{x}}'^{/t/}_n , \qquad /8/$$

$\underline{\underline{p}}_n^{/t/}$ is the vector of gross output of the n-th location in the t-th year, $\underline{\underline{x}}_n^{/t/}$ and $\underline{\underline{x}}'^{/t/}_n$ are the vectors of net output relating to the two basic types of the systems of farming.

The gross output of a greater economic unit, say, that of a country, is simply the sum of the local vectors $\underline{\underline{p}}_n^{/t/}$:

$$\underline{\underline{p}}^{/t/} = \sum_{n=1}^{N} \underline{\underline{p}}_n^{/t/}. \qquad /9/$$

Contrary to this, the final production of the unit of a higher degree is less than the sum of the vectors $\underline{\underline{x}}_n^{/t/}$ and $\underline{\underline{x}}_n^{,/t/}$, because the systems of farming that depend on the interrogional flows of commodities require their "imports" from the net output of other locations. Therefore, the "imported" quantities must be subtracted by the use of the matrix $\underline{\underline{U}}''$:

$$\underline{\underline{x}}^{/t/} = \sum_{n=1}^{N} \underline{\underline{x}}_n^{/t/} + \sum_{n=1}^{N} \underline{\underline{x}}_n^{,/t/} - \sum_{n=1}^{N} \underline{\underline{U}}_n'' \underline{\underline{x}}_n^{,/t/}, \qquad /1o/$$

6. The Technological Matrices

These matrices also relate to different locations and systems of farming. Their coefficients are derived from the values belonging to the maxima of the productivity functions. These matrices $\underline{\underline{T}}_n$ are connected with gross production: their components express the quantities of the different resources that are needed to produce a unit of gross output of the activities considered. /It must be mentioned here that for some resources, like labour for example, the matrices $\underline{\underline{T}}_n$ have several rows indicating the distribution over time of the requirements./ . The planning model being built up for the distribution over locations of a final demand in agricultural products of the sectors other than agriculture of national economy, the matrices $\underline{\underline{T}}_n$ must also be transformed into ones reflecting the requirements of final production. This transformation has to take place for both the autarchic and co-operative types of systems of farming:

$$\underline{\underline{A}} = \underline{\underline{T}}\,\underline{\underline{U}}$$
$$\underline{\underline{A}}' = \underline{\underline{T}}\,\underline{\underline{U}}' \qquad /11/$$

where $\underline{\underline{A}}$ and $\underline{\underline{A}}'$ donote the technological matrices, relating to the net output produced as the autarchic or co-operative basis.

7. The Relation of Activities to Old and New Capacities

The process of production takes place within the frames of the existing capacities but these frames are widened by investments year by year. Therefore we must introduce special activities. These activities require supplies from the resources for investments /e.g. building material for a new barn of cows/ but they do not claim for existing capacities in the year of entering into production and for the later years, they add the new capacities to the old ones. An example in Section IV/4 will give further explanations in an algebraic form as the balances of resources will be discussed.

If there is a considerable difference between the qualities of the new capacity produced by investment and the old one, then this new capacity is linked with a separate activity or even with an other system of farming.

8. The Obsolation of Resources

The planning model must also reflect the wear and tear in both its physical and "moral" sense. The rate of obsolation must be considered as a technico-economic normative to be established for each type of capacities by the central planning organs. As to the breeding stock, a rational policy of sorting out must be worked out based on productivity considerations and natural replacement. It follows from this, that a certain capacity existing in the t-th year represents less and less in later years of the planning period. If this capacity, say, tractors, is denoted by f_t, there exist the following inequalities:

$$f_t > f_{t+1} > f_{t+2} > \dots > f_{t+n} \qquad /12/$$

9. The Social Labour Content of Final Products

The social labour consumed by the production of goods used as inputs in agriculture can be determined by input-output analysis on the national and industrial scale. If M_{ni} denotes the social labour content of these different inputs of non-agricultural origin used for a unit of gross products of the i-th agricultural commodity on a given location and in the optimal technology belonging to a certain system of farming, further t_i means the direct agricultural labour, the labour costs, denoted by m_i, of a unit of gross product of the i-th commodity on a given location and in a particular system of farming can be determined by the following formula:

$$m_i = t_i + \sum_{n=1}^{N} M_{ni} \qquad /13/$$

For the first commodities produced in autarchic systems of farming on a certain location, the vector of social labour content is given by

$$\underline{\underline{c}}^{*} = \underline{\underline{m}}^{*} \underline{\underline{U}} \qquad /14/$$

In the systems of farming depending on interregional co-operation, the labour vector $\underline{\underline{m}}^{*}$ multiplied by $\underline{\underline{U}}'$ allows the determination of only a part of the real labour consumption. The labour costs attached to the intermediate commodities delivered by other locations are still unknown at the time of the planning preparations because their origin can only be determined by solving a transportation problem supplementing the solution of the production problem. Theoretically it is possible that any location could supply its final products to be used on a particular location as intermediate commodities. Thus, the interregional commodity flows bear a charge of social costs of production valid at their location of origin and that of transport costs. If we consider the possibility of transports of goods from any location to any location and we compute the frequencies relating to a series of cost levels /classes/ established for both production and transport costs, the maximum probability of being charged with belongs to the levels of costs represented by their modes. So, if $\underline{\underline{q}}^{*}$ means the sum of the social labour contant of the modal production and transport costs, then we can quantify the labour costs of final products for the systen of farming of the co-operative type as follows:

$$\underline{\underline{c}}'^{*} = \underline{\underline{m}}^{*}\underline{\underline{U}}' + \underline{\underline{q}}^{*}\underline{\underline{U}}'' \qquad /15/$$

IV. The Description of the Planning Model

The main task of the plan for the national economy is to meet the aggregate demands of society in a form maximizing the efficiency of social labour. From this follows the duty for agriculture too that it has to meet a demand, increasing in its volume and changing in its structure in the course of time, that aggregates the consumption of population, the needs in raw materials of industry, the claims by foreign trade and by the building up of appropiate state reserves, further the uninterrupted functioning of the productive apparatus must be guaranteed for the repetition /in the measures needed/ of the process of production. This task having to be fulfilled on the basis of the most efficient use of resources, we face a constrained extremum where the lower bounds of production are given by the sum of the final demand planned and of the volume of intermediate products consumed by the productive apparatus; the upper bounds are represented by the state of forces of production in agriculture and by the system of technological possibilities belonging to these resources. The measure of efficiency in resource use is given by the productivity of social labour consumed in order to produce a certain quantity of agricultural final commodities.

The solution of this problem may be given by a mathematical programming procedure. By its nature, it would require a quadratic form of the objective function and a procedure of stochastic programming. However, it is treated here as an acceptable approximation that can be had by the use of the non-stochastic linear programming method.

1. The Final Demand in Agricultural Products

The tasks of production to be assigned to agriculture must be determined starting from the reproduction process of national economy /taking also into consideration the connections with the process of reproduction on the international scale/. This occurs by using the Leontief inverse of national economy, valid for the k-th section covering 2-3 years of the planning period / $\underline{\underline{R}}_k$ /, and the vector of final demand planned for a particular year / $\underline{y}_t$ / :

$$\underline{\underline{R}}_k \, \underline{y}t = \underline{\underline{r}}_t \qquad /16/$$

$$\underline{\underline{z}}_t \in \underline{\underline{r}}_t$$

where $\underline{\underline{r}}_t$ stands for the gross output belonging to the vector $\underline{\underline{y}}_t$ /$\underline{\underline{z}}_t$ containing the agricultural components of the vector $\underline{\underline{r}}_t$ of social product/. If we deduce from $\underline{z}_t$ the quantities of intermediate products, so we get a vector $\underline{\underline{d}}_t$ that represents the final demand in agricultural products in the t-th year.

It must be noted that the particular variants of the final demand on the national economic scale generate different vectors of final demand in agricultural products and result in different optimum solutions of the planning problem.

2. The Objective Function

The objective may be fixed in two forms: the first maximizes the productivity of social labour /subject to the constraints to be discussed later/, the second minimizes the quantity of social labour consumed in different industries in order to produce the final commodities quantified by $\underline{\underline{d}}$. By their nature, both functions are non-linear. As we remember the determination of optimal technologies linked with the maximum local productivity of social labour, there were only variables for technological and natural conditions although productivity depends on the volume of production too. This simplification of the productivity functions could be compensated to a certain extent by assuming a quadratic objective function. However, in this model, taking its dimensions into account, the aouthor considered a linear function, or, as an alternative, the quotient of two linear functions as a first approach. These two types of objective function can be described by the following formulae:

$$C = \underline{\underline{c}}^{*} \underline{\underline{x}} + \underline{\underline{c}}'^{*} \underline{\underline{x}}' \longrightarrow \text{min.} \qquad /17/$$

$$P = \frac{\underline{\underline{l}}^{*} \underline{\underline{x}} + \underline{\underline{l}}'^{*} \underline{\underline{x}}'}{\underline{\underline{c}}^{*} \underline{\underline{x}} + \underline{\underline{c}}'^{*} \underline{\underline{x}}'} \longrightarrow \text{max.} \qquad /18/$$

Another problem in connection with the objective function: should the minimum or maximum relate to the entire planning period or only to the ending year, or, in other form, should it have non-zero coefficients for each year of the period, or for the last one? Indeed, the second form may also be justified in certain situations of the planning of development but the author prefers the first form, particularly because the optimum solution of the problem must also serve for a basis of the formation of producers' prices of the planning period.

3. The Balance System of Products

This system consists of as many rows as the number of components of $\underline{d}$ multiplied by the number of years considered in the planning period. The right side of the balance system is represented by $\underline{\underline{d}}$ as a minimum of final production /$\underline{\underline{d}}$ is minimum if every commodity on every location is produced in autarchic systems of farming, i.e. if there is no interregional co-operation/. So the selection of supply and demand in the balance-system is given by the formula:

$$\underline{\underline{S}}\underline{\underline{x}} + \underline{\underline{S}}'\underline{\underline{x}}' \geq \underline{\underline{d}} \qquad /18/$$

It was already mentioned that the vectors $\underline{\underline{x}}$ and $\underline{\underline{x}}'$ represent the activities producing final commodities in autarchic and co-operative systems of farming, respectively. $\underline{\underline{S}}$ means the supply matrix of the autarchic systems. Really, it is equal to the unit matrix $\underline{\underline{E}}$ as its components are:

$$s_{ij} = 1 \text{ if } i = j,$$

$$s_{ij} = 0 \text{ if } i \neq j. \qquad /2o/$$

This means that one unit of production in the activities denoted by $\underline{\underline{x}}$ is equally one unit in the contribution to $\underline{\underline{d}}$:

$$\underline{\underline{S}}\,\underline{\underline{x}} = \underline{\underline{E}}\,\underline{\underline{x}} = \underline{\underline{x}} \qquad /21/$$

It is not so in the case of $\underline{\underline{S}}'$ which is connected with the systems of farming that depend on the co-operation between different locations consuming as intermediate commodities a certain part of the final products of other locations. In order to express a contribution to $\underline{\underline{d}}$, and at the same time the demands from it, $\underline{\underline{S}}'$ was constructed as a difference of two matrices /here still ignoring time lead/:

$$\underline{\underline{S}}' = \underline{\underline{E}} - \underline{\underline{U}}'', \text{ where} \qquad /22/$$

$$s'_{ij} = 1 \text{ if } i = j,$$

$$s'_{ij} = u''_{ij} \leq 0 \text{ if } i \neq j.$$

Therefore, the net contribution to $\underline{\underline{d}}$ of the activities $\underline{\underline{x}}'$ is given by

$$\underline{\underline{S}}'\underline{\underline{x}}' = / \underline{\underline{E}} - \underline{\underline{U}}'' / \; \underline{\underline{x}}' = \underline{\underline{x}}' - \underline{\underline{U}}'' \underline{\underline{x}}' \qquad /23/$$

where the product $\underline{\underline{U}}''\underline{\underline{x}}'$ represents the quantities by which the final production on a certain number of locations must be increased if we want a group of locations to contribute to $\underline{\underline{d}}$ depending on interregional commodity flows, in order to raise the national level of productivity.

The supply matrices $\underline{\underline{S}}$ and $\underline{\underline{S}}'$ are composed of submatrices relating to year, location and system of farming. /For the sake of simplicity, we ignore the existence of the time lead in the components of $\underline{\underline{U}}''$. This will be discussed later./ Likewise, the vector $\underline{\underline{d}}$ also consists of subvectors $\underline{\underline{d}}_t$. Therefore, the balance system for two years, two locations and two systems of farming /the first element of the subscripts refers to the year, the second to the location and the third to the system of farming/ can be built up as follows /omitting the unit matrices $\underline{\underline{S}} = \underline{\underline{E}}$ /:

$$\underline{\underline{x}}_{111} + \underline{\underline{x}}_{112} + \underline{\underline{S}}'_{111} \underline{\underline{x}}'_{111} + \underline{\underline{S}}'_{112} \underline{\underline{x}}'_{112} + \underline{\underline{x}}_{121} + \\ + \underline{\underline{x}}_{122} + \underline{\underline{S}}'_{121} \underline{\underline{x}}'_{121} + \underline{\underline{S}}'_{122} \underline{\underline{x}}'_{122} \geqq \underline{\underline{d}}_1$$

/24/

$$\underline{\underline{x}}_{211} + \underline{\underline{x}}_{212} + \underline{\underline{S}}'_{211} \underline{\underline{x}}'_{211} + \underline{\underline{S}}'_{212} \underline{\underline{x}}'_{212} + \\ + \underline{\underline{x}}_{221} + \underline{\underline{x}}_{222} + \underline{\underline{S}}'_{221} \underline{\underline{x}}'_{221} + \underline{\underline{S}}'_{222} \underline{\underline{x}}'_{222} \geqq \underline{\underline{d}}_2$$

It is well known that one part of the intermediate commodities consumed in a certain year was produced in the preceding year. So we must have the balance system to reflect this inter-year flow of agricultural products.

The matrix $\underline{\underline{U}}''$ can be assumed to be composed of two matrices:

$$\underline{\underline{U}}'' = \underline{\underline{W}} + \underline{\underline{W}}' \qquad /25/$$

where $\underline{\underline{W}}$ represents the quantities demanded from the present year's final production of other locations, while $\underline{\underline{W}}'$ quantifies the demands that fall upon the final production of the preceding year. From this follows that for the first year of the planning period, a central stock of intermediate products must be assumed as a special type of resources, and in the last year this special stock must be established by including $\mathbf{U}''$, as we have no knowledge about the volume of production in the first year of the next planning period. Thus, if we transform in this sense the balance system of products of the planning period covering two years which was described in /24/, we have the following inequalities:

$$\mathbf{W}'_{111}\,\mathbf{x}'_{111} + \mathbf{W}'_{112}\,\mathbf{x}'_{112} + \mathbf{W}'_{121}\,\mathbf{x}'_{121} + \mathbf{W}'_{122}\,\mathbf{x}'_{122} \leqq \mathbf{b}_{w}$$

$$\mathbf{x}_{111} + \mathbf{x}_{112} + /\mathbf{E}-\mathbf{W}_{111}/\,\mathbf{x}'_{111} + /\mathbf{E}-\mathbf{W}_{112}/\,\mathbf{x}'_{112} +$$

$$+ \mathbf{x}_{121} + \mathbf{x}_{122} + /\mathbf{E}-\mathbf{W}_{121}/\,\mathbf{x}'_{121} + /\mathbf{E}-\mathbf{W}_{122}/\,\mathbf{x}'_{122} -$$

$$- \mathbf{W}'_{211}\,\mathbf{x}'_{211} - \mathbf{W}'_{212}\,\mathbf{x}'_{212} - \mathbf{W}'_{221}\,\mathbf{x}'_{221} -$$

$$- \mathbf{W}'_{222}\,\mathbf{x}'_{222} \geqq \mathbf{d}_{1} \qquad /26/$$

$$\mathbf{x}_{211} + \mathbf{x}_{212} + /\mathbf{E}-\mathbf{U}''_{211}/\,\mathbf{x}'_{211} + /\mathbf{E}-\mathbf{U}''_{212}/\,\mathbf{x}'_{212} +$$

$$+ \mathbf{x}_{221} + \mathbf{x}_{222} + /\mathbf{E}-\mathbf{U}''_{221}/\,\mathbf{x}'_{221} + /\mathbf{E}-\mathbf{U}''_{222}/\,\mathbf{x}'_{222} \geqq \mathbf{d}_{2}.$$

4. The Balance System of Resources

The inequality

$$\mathbf{A}\mathbf{x} + \mathbf{A}'\mathbf{x}' \leqq \mathbf{b} \qquad /27/$$

consists of a number of subsystems that relate to resources that can only be exploited on particular loccations and of a subsystem grouping the resources that can be considered as a central capacity and can be distributed over the country following the a location of the production schedules. The relation of production to local and central resources can be represented as follows /for two locations and a single year/:

$$
\begin{aligned}
&\underline{\underline{A}}_1 \underline{\underline{x}}_1 + \underline{\underline{A}}_1' \underline{\underline{x}}_1' && && \leq \underline{\underline{b}}_1 \\
& && \underline{\underline{A}}_2 \underline{\underline{x}}_2 + \underline{\underline{A}}_2' \underline{\underline{x}}_2' && \leq \underline{\underline{b}}_2 \\
&\underline{\underline{A}}_{c1} \underline{\underline{x}}_1 + \underline{\underline{A}}_{c1}' \underline{\underline{x}}_1' && + \underline{\underline{A}}_{c2} \underline{\underline{x}}_2 + \underline{\underline{A}}_{c2}' \underline{\underline{x}}_2' && \geq \underline{\underline{b}}_c
\end{aligned}
\qquad /28/
$$

The products $\underline{\underline{A}}_i \underline{\underline{x}}_i$ and $\underline{\underline{A}}_i' \underline{\underline{x}}_i'$ represent the needs for local resources $\underline{\underline{b}}_i$, while $\underline{\underline{A}}_{ci} \underline{\underline{x}}_i$ stand for the claims directed to the central resources $\underline{\underline{b}}_c$. Naturally, for each year of the planning period exists such a system described by /28./.

The balance system of resources also involves the increase in capacities by investments. This can be represented by the example of a single activity. Let us denote by x_1, x_2, x_3 and x_4 dairy cows in the first, second, third and fourth year based upon existing machinery f_1, f_2, f_3 and f_4, and by x_2^b and x_3^b the same activity with new implements requiring investment from the central resources b_{ct} in the first or second year, respectively:

$$
\begin{aligned}
&a_1 x_1 && \leqq f_1 \\
&g_1 x_2^b && \leqq b_{c1} \\
&a_2 x_2 && \leqq f_2 \\
&g_2 x_3^b && \leqq b_{c2} \qquad /29/ \\
&a_3 x_3 - a_{3.2}^b x_2^b && \leqq f_3 \\
&a_4 x_4 - a_{4.2}^b x_2^b - a_{4.3}^b x_3^b && \leqq f_4
\end{aligned}
$$

The different notation of the coefficients $a_{t.i}^b$ alludes to the consideration of obsolation not only for the resources existing in the first year of the planning period but for their increments too:

$$a_{3.2}^b \geqq a_{y.2}^b > a_{5.2}^b > \dots > a_{n.2}^b \qquad /30/$$

On the other hand, tehcnical development can also be represented by the numerical values of the coefficients g_t if $a_{t.i}^b$ are standardized, and conversely:

$$
\begin{aligned}
&g_2 < g_1 \quad \text{if} \quad a_{3.2}^b = a_{4.3}^b \\
&a_{3.2}^b < a_{4.3}^b \quad \text{if} \quad g_1 = g_2
\end{aligned}
\qquad /31/
$$

For the purposes of the model, the variation in g_t with standardized $a_{t.i}^b$ seems to be preferable.

5. The Gross Output Belonging to the Optimal Solution

As it was already pointed out earlier, the gross output can be had by multiplying the matrices for intermediate products by the vectors of net output.

The gross output of the whole country belonging to the optimum solution is given as a product of the matrices $\underline{\underline{U}}$ and $\underline{\underline{U}}'$ and the vectors of final production $\underline{\underline{x}}$ and $\underline{\underline{x}}'$ taken from the final /optimal/ tableau of computation, for particular years as

$$\underline{\underline{p}}_t = \underline{\underline{U}}_t \underline{\underline{x}}_t + \underline{\underline{U}}_t' \underline{\underline{x}}_t' \qquad /32/$$

and for the planning period as a whole:

$$\underline{\underline{p}} = \underline{\underline{U}}\,\underline{\underline{x}} + \underline{\underline{U}}'\,\underline{\underline{x}}' \qquad /33/$$

6. The Program of Transports

The optimum solution of the problem of the distribution over locations, systems of farming and technologies of the schedules of production and the resources must not mean that the tasks of the agricultural planning administration were fulfilled. Immense quantities of products and materials must be transported and this transportation activity must also be optimally organized. The main problem is the organization of the interregional flow of intermediate products claimed for by the activities $\underline{\underline{x}}'$

found in the optimum program vector.

The optimal program of transports must be separately computed for each commodity. The objective function involves social labour consumed by the transportation activity. The solution is given by the well-known programming method.

7. The Complete Model

The allocation of production and resources may take place on the basis of these formulae:

$$C = \underline{\underline{c}}^{*} \underline{\underline{x}} + \underline{\underline{c}}'^{*} \underline{\underline{x}}' \longrightarrow \text{min.}, \quad \text{or:}$$

$$P = \frac{\underline{\underline{l}}^{*} \underline{\underline{x}} + \underline{\underline{l}}^{*} \underline{\underline{x}}'}{\underline{\underline{c}}^{*} \underline{\underline{x}} + \underline{\underline{c}}'^{*} \underline{\underline{x}}'} \longrightarrow \text{max.}$$

$$\underline{\underline{x}}, \underline{\underline{x}}' \geq \underline{\underline{0}}$$

$$\underline{\underline{S}}\, \underline{\underline{x}} + \underline{\underline{S}}'\, \underline{\underline{x}}' \geq \underline{\underline{d}}$$

$$\underline{\underline{A}}\, \underline{\underline{x}} + \underline{\underline{A}}'\, \underline{\underline{x}} \leq \underline{\underline{b}}$$

$$\underline{\underline{p}} = \underline{\underline{U}}\, \underline{\underline{x}} + \underline{\underline{U}}'\, \underline{\underline{x}}'$$

The transportation problem may be solved as follows:

$$K = \sum_{i=1}^{n} \sum_{j=1}^{n} k_{ij} x_{ij} \longrightarrow \text{min.}$$

$$x_{ij} \geq 0 \qquad /35/$$

$$\sum_{j=1}^{n} x_{ij} = fi \qquad /i = 1, 2, 3, \ldots, n/$$

$$\sum_{i=1}^{n} x_{ij} = r_j \qquad /j = 1, 2, 3, \ldots, n/$$

k_{ij} means the social costs of transportation, f_i stands for the supplying locations and r_j for the demanding ones.

8. The Stochastic Nature of the Model

The production and productivity functions applied in the preparations to planning have a stochastic nature, therefore the extrema of such functions, and the characteristics of the optimal technologies determined by them, follow a certain probability distribution, the normality of which can only be assumed. All of the coefficients being computed starting from these values, the whole model turns to take a stochastic character. Stochastic programming could give a more appropriate solution which can only be approximated by the present model.

9. The Gradual Way of Solution

It is clear that such an enormous problem can not be solved directly, therefore a gradual way of solution is proposed. The locations can be integrated into aggregate economic units of different levels. It seems to be enough to consider 5 levels of integration for such a country like ours. The coefficients for the higher integrations can be computed as averages weighted by the production area of the locations to be integrated, while the local resources are added up. The solution for the higher units determines the starting situation for the lower ones, concerning both resources and final demand.

V. Applications of the Model

1. Optimal Decisions in Planning

By the use of this model, the various types of decisions to be made in working out the plans can be optimized. The best technologies can be selected for the activities at each location and the possibilities offered by the differences in productivity can be exploited by an optimal distribution of production and resources. The gradual solutions give the best regional plans too.

The solution of the transportation problem determines the flow of commodities between co-operating locations. Besides, the flow of goods between industry and agriculture may be organized this way, such as the distribution of materials and the determination of supply zones for centres of consumption or for plants processing agricultural products.

2. The effects on national economic plans

The social costs of production were initially determined on the basis of a situation in which the location of agricultural production was not optimal. By maximizing the productivity of social labour consumed in agriculture, the cost structure of national economy is changed. The effects of this change can be felt by each sector of national economy but not equally, the cuts of costs being the sharpest in industries processing agricultural products. This may result in severe cuts in prices paid by consumers.

Another effect on national economic plans of the rise in the productivity of social labour consumed in agriculture may be a decision to increase the resource supplies for agriculture or to increase the investments in industries producing materials and implements used in agriculture.

3. Producers' Prices and Taxation

By the use of such a model, one can approximate the real social costs on each location for each commodity. At the same time, these social costs reflect a situation which is most advantageous for the society. So we can build up a price system based upon the labour hours per unit of product on locations that are marginal for different commodities. This means, first, the fixation of prices in relation to wages, second, the fixation of proportions between prices. Such a price system can be held constant during the planning period.

The local costs per unit of each product being provided, the differential rent can be determined. Thus, the expropriation by State of differential revenues arising from differential rent may be considered the right way of agricultural taxation.

4. The International Division of Labour

This model can be used not only in the planning of the agricultural production of a single country but it offers many advantages on the international scale too. For a decision in the problem of division of labour between a certain number of countries, one could not find a better criterion than productivity of social labour. In this case, regions or lands would be called locations and the maximization of the productivity of social labour for the space covered by these countries could be considered as an objective, subject to constraints as it was already described. Naturally, only the production of the most important commodities should be distributed this way and the classification of resources as local and central funds must also be differently interpreted.

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