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Implications of Crop Yield and Revenue Insurance for Producer Hedging

Keith H. Coble, Richard G. Heifner, and Manuel Zuniga

New types of crop insurance have expanded the tools from which crop producers may choose to manage risk. Little is known regarding how these products interact with futures and options. This analysis examines optimal futures and put ratios in the presence of four alternative insurance coverages. An analytical model investigates the comparative statics of the relationship between hedging and insurance. Additional numerical analysis is conducted which incorporates futures price, basis, and yield variability. Yield insurance is found to have a positive effect on hedging levels. Revenue insurance tends to result in slightly lower hedging demand than would occur given the same level of yield insurance coverage.

Key words: crop insurance, forward pricing, optimal hedging, put options, risk, simulation

Introduction

The context in which farm program crop producers make futures marketing decisions has been dramatically altered by government policy in recent years. The 1996 Farm Bill replaced deficiency payments, which (like put options) provided farmers larger payoffs when prices were low, with fixed production flexibility payments. This has affected the risk management decision environment for program crop producers. What has been less often addressed is the nearly simultaneous and rapid evolution of governmentally subsidized insurance products. In recent years, the U.S. Department of Agriculture/Risk Management Agency (USDA/RMA) has offered crop revenue insurance and allowed private insurance firms to develop other revenue insurance products which were accepted for subsidy and reinsurance. To date, three different individual coverage revenue insurance products have been offered. The acceptance of these revenue insurance products has been dramatic. Revenue products represented approximately 50% of U.S. corn and soybean acres insured above the catastrophic level in 1999 (USDA/RMA).

Because these revenue insurance products deal with both price and yield risk, it is relevant to ask what implications these subsidized insurance products have for producer forward contracting demand. If they do affect the demand for forward contracting, then

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Coble is assistant professor, Department of Agricultural Economics, Mississippi State University; Heifner is agricultural economist with the Economic Research Service, U.S. Department of Agriculture; and Zuniga is graduate research assistant, Department of Agricultural Economics, Mississippi State University.

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in what direction and magnitude? Further, these revenue insurance designs differ in the coverage provided. Thus, it is plausible that the various insurance designs could result in substantially differing hedging demand.

This issue has two important implications. First, producers are confronted with a new decision context for risk management. The advent of revenue insurance forces producers to consider price and yield risk management in a context where insurance may potentially subsume the price risk that forward pricing tools also mitigate. Consequently, optimal producer risk management behavior may need to adapt to this new scenario. Second, the expansion of federally subsidized insurance and the introduction of new insurance designs such as revenue insurance have the potential to affect the demand for private risk management tools. This ancillary effect of such changes to farm policy has not, to our knowledge, been addressed. In a policy environment where protecting producers from risk has a common political appeal, the potential for overwhelming private risk markets with subsidized public alternatives appears real.

The above discussion prompts consideration of the joint optimization of insurance and hedging decisions. The work on optimal hedging for crop growers who face yield as well as price risk traces to McKinnon, who reported that minimum-variance hedge levels decline as yield variability increases relative to price variability. Quantifying the yield variability to obtain meaningful empirical results has proven to be difficult due to the paucity of farm-level observations. Grant and others provided empirical estimates using county yields, which typically show optimal hedge ratios of 60% or less. Miller and Kahl demonstrated that estimates based on farm-level data may deviate substantially from those based on aggregate yield data.

More recently, Lapan and Moschini have shown how optimal hedge ratios can be determined in an expected utility framework for the farmer with constant absolute risk aversion (CARA) and joint normally distributed yield, price, and basis. They provided a closed-form solution for the CARA case and examples of numerical solutions under constant relative risk aversion (CRRA). Their results revealed that utility-maximizing hedge ratios depend on risk attitudes and generally differ from minimum-variance hedge ratios.

When instruments such as insurance or put options are used to manage risk, the derivatives of revenue become discontinuous, making the search for an optimum more difficult. Poitras addressed the analytical issues associated with the use of such censuring instruments. Lapan, Moschini, and Hanson examined the joint use of futures and options in an expected utility framework. They found that only futures enter into the optimal solution when prices are unbiased and yield risk is absent. Based on findings of subsequent studies, when yield risk is present but insurance is not used, options may enter into the optimal hedge. Sakong, Hayes, and Hallam allowed for price, basis, and yield risk in an analytical model which suggested producers underhedge in futures and purchase put options. Moschini and Lapan also observed options entering the optimal portfolio, with both papers finding the optimal strategy being conditional on yield-price correlation.

Relatively few studies have addressed the combined effects of yield and price instruments. The work on combined yield and price hedging was a step in this direction. The introduction of yield futures by the Chicago Board of Trade created a yield risk market to complement the traditional price futures contract. This led to investigations of the combination of price and yield futures hedging by Li and Vukina for corn in North

Carolina; Tirupattur, Hauser, and Chaherli for soybeans in Illinois; and Heifner and Coble (1996a) for corn across the United States. In general, these papers report a complementarity between price and yield risk futures, but that yield futures tend to involve significant basis risk which is not present in an individual yield insurance product.

Previous analyses of the joint use of crop insurance and hedging are even fewer in number. Dhuyvetter and Kastens examined combinations of hedging with yield insurance and with a particular form of revenue insurance—Crop Revenue Coverage (CRC). However, they do not directly address hedging levels, but rather show comparisons of mean and variance of returns. Wang et al. investigated the joint use of hedging with either individual yield or area yield insurance. Comparisons are made between producer willingness to pay for alternative insurance designs optimized with futures and options. However, optimal hedge levels are not reported.

There are several challenges to modeling the interactions of insurance with forward pricing instruments. First, crop revenues, which are products of random yields times random prices, are potentially non-Gaussian, making mean-variance methods questionable.[1] Second, deviations in revenue distributions from normality are increased by crop insurance and put options, which add censored distributions that generally are highly skewed as components of revenue. Moreover, the underlying yield and price distributions often are non-Gaussian, thus affecting the modeling of risk instruments such as yield insurance or a futures hedge. Nelson and Preckel have given strong support to the notion that yields often appear to be non-Gaussian. When modeling the joint distribution of price times yield, care must also be given to the potential for correlation of price and yield to influence outcomes. Recent research by Hennessy, Babcock, and Hayes, and by Heifner and Coble (1996b) has reported strong indication of negative correlation between farm yield and market price for corn in major production regions.

In this study we analyze the relationship of four insurance designs to the optimal hedge ratio of a risk-averse corn producer. Specifically, the sensitivity of optimal hedge and put ratios to varying levels of insurance is shown. The following section reports an investigation of the analytical relationship between yield insurance and hedging. Numerical procedures are then used to evaluate the interaction between hedging and four alternative insurance designs. In particular, we incorporate farm-level yield information, which more accurately characterizes yield variability and its effect on optimal decisions. The analysis is replicated across four regionally diverse representative farms. This allows comparisons of how differences in yield variability and yield-price correlation affect outcomes, and shows the diversity of outcomes across regions.

The Behavioral Model

The planting-time optimization behavior of a producer with yield insurance and the opportunity to hedge is examined. The producer is assumed to maximize expected utility according to a von Neumann-Morgenstern utility function defined over end-of-season wealth (W) and which is strictly increasing, concave, and twice continuously differentiable. For ease of illustration, the price basis (local cash price–futures price) is omitted

[1] Mean-variance analysis is strictly applicable to the crop hedging problem only if revenue is normally distributed or utility is quadratic. However, since the revenue is the product of two random variables, normality is a strong assumption. Quadratic utility implies decreasing marginal utility at some point, which generally is unrealistic.

from the analytical model such that price and yield remain as the two stochastic variables. Additionally, price and yield are assumed to be distributed bivariate normal. Later numerical simulations relax the distributional assumptions and incorporate basis risk.

Insurance has the effect of censoring the realizations of the underlying random variable. In the case of yield insurance, a yield guarantee level (y_g) is set as a percentage of the expected yield. If yield (y) falls below the guarantee level, the production shortfall is replaced.[2] Thus, the producer observes:

$$(1) \qquad \begin{aligned} & y_g \quad \text{if } y < y_g, \text{ and} \\ & y \quad \text{if } y \geq y_g. \end{aligned}$$

End-of-season wealth is then conditional on whether insurance payouts are made, and may be written as follows:

$$(2a) \qquad \text{if } y < y_g, \quad W_L = W_0 + A\left[p_1 y - C - R(y_g) + h(p_0 - p_1) + p_g(y_g - y)\right];$$

$$(2b) \qquad \text{otherwise, } W_H = W_0 + A\left[p_1 y - C - R(y_g) + h(p_0 - p_1)\right],$$

where W_H and W_L denote end-of-period wealth associated with yields above and below the yield guarantee, respectively. Initial wealth is represented by W_0 and crop acres by A. Crop prices are denoted by p. Price is subscripted by 0 to identify a known planting-time expectation of harvest price, and subscripted by 1 to denote the stochastic harvest-time price. Nonstochastic production cost is denoted as C. Crop insurance premiums, $R(y_g)$, represent the cost of purchasing insurance and are an increasing function of the yield guarantee. The quantity of the crop forward priced with a futures hedge is denoted by h. These contracts are rigidly defined as to delivery time, quality, and quantity. A growing crop can be hedged by selling futures contracts equal to a portion of the expected crop before harvest and purchasing an equal number of futures contracts later when the actual crop is sold.

Let random harvest-time price and yield be normal variates, where y is defined over the support $[\underline{y}, \bar{y}]$, and p_1 is defined over $[\underline{p}, \bar{p}]$. Assuming bivariate normality, the joint cumulative distribution is $F(y,p)$, where $[\mu_y, \mu_{p1}]$ represents the means, $[\sigma_y, \sigma_{p1}]$ the standard deviations, and ρ represents the correlation coefficient, all of which characterize the distribution.

The objective function of a producer choosing the optimal hedge level may then be written as follows:

$$(3) \qquad \max_h L = \int_{\underline{p}}^{\bar{p}} \int_{\underline{y}}^{y_g} U(W_L) f(y, p_1)\, dy dp_1 + \int_{\underline{p}}^{\bar{p}} \int_{y_g}^{\bar{y}} U(W_H) f(y, p_1)\, dy dp_1.$$

In this model, the producer's utility is evaluated as the sum of expected utility in the W_L states where yield is certain, and the W_H states where yield is random. The producer's choice variable (h) represents the quantity of production to hedge given that the producer

[2] Federal Multiple-Peril Crop Insurance (MPCI) yield shortfalls are valued at planting-time price. The MPCI price guarantee is based on internal USDA forecasts rather than directly tied to the futures markets. However, preseason futures prices are used here.

is insured. The first-order condition (after dividing through by A and assuming all derivatives exist at the optimum) is:

$$(4) \qquad L_h = \left[\int_{\underline{p}}^{\bar{p}} \int_{\underline{y}}^{y_g} U'(W_L)(p_0 - p_1) f(y, p_1)\, dy dp_1 - \int_{\underline{p}}^{\bar{p}} \int_{y_g}^{\bar{y}} U'(W_H)(p_0 - p_1) f(y, p_1)\, dy dp_1 \right].$$

The effect of a change in y_g on the optimal h is derived using Leibnitz' rule where the bound of integration is a function of the variable of interest. The effect may be written

$$(5) \qquad \frac{\partial h}{\partial y_g} = \frac{L_{hy_g}}{L_{hh}} = -\frac{1}{L_{hh}} \left[\int_{\underline{p}}^{\bar{p}} \int_{\underline{y}}^{y_g} U''(W_L) \left(p_0 - \frac{\partial R(y_g)}{\partial y_g} \right) (p_0 - p_1) f(y, p_1)\, dy dp_1 - \int_{\underline{p}}^{\bar{p}} \int_{y_g}^{\bar{y}} U''(W_H) \left(\frac{\partial R(y_g)}{\partial y_g} \right) (p_0 - p_1) f(y, p_1)\, dy dp_1 \right],$$

where $L_{hh} = \partial^2 L / \partial h^2 < 0$ by assumption. The sign of the term $\partial R(y_g)/\partial y_g$ is positive given that premiums increase with higher coverage. The derivatives of utility take the assumed signs $U' > 0$, and $U'' < 0$. Note that $(p_0 - p_1)$ is included in each term, but evaluated over a different portion of the joint distribution. A perception of bias in futures prices would affect the outcomes; however, the assumption of unbiasedness will be maintained here. Bivariate normality allows the signing of this term over differing portions of the distribution.

The right-hand side of equation (5) can be shown positive. Since L_{hh} is negative, the leading term is positive. Inside the bracket, the first term contains $U''(W_L)$ which is negative, as is $(p_0 - p_1)$, because it is evaluated over the range $y < y_g$. Negative correlation implies that $p_1 > p_0$. The term $(p_0 - \partial R(y_g)/\partial y_g)$ can be treated as a constant and interpreted as the difference in the rate of change in expected indemnity (p_0) and premium $(\partial R(y_g)/\partial y_g)$ as y_g is changed. For actuarially fair insurance, these two rates of change must be equal.

In a subsidized product such as the recently reformed U.S. crop insurance program, the increase in indemnity would exceed the increase in premium because the producer is not paying the full cost of insurance at any level of coverage. Then, in marginal terms, the producer would not pay the full change in premium resulting from increased coverage. Actuarially fair insurance will result in the first term of equation (5) falling out. An assumption of subsidized insurance will result in the first term having a positive value. The second term inside the bracket has a leading negative sign. Inside the second term, $U''(W_H)$ is negative, while $\partial R(y_g)/\partial y_g$ is again assumed positive. In this term, $(p_0 - p_1)$ is positive as it is evaluated over the range $y > y_g$ such that $E[p_1 | y > y_g] > E[p_1]$ if $\rho < 0$. The second term can also be unequivocally signed positive as the expectation within the double integral is negative, but multiplied by a leading negative sign.

Under these assumptions, yield insurance is shown to be complementary to hedging. To our knowledge, this is a result not previously reported in the literature, but is consistent with previous research which finds that hedging levels are inversely related to yield variability. Because yield insurance censors the yield distribution from below, there is a reduction in downside yield variability. However, it is not at all clear that

revenue insurance would have the same effect because it subsumes price risk as well as yield risk.

The numerical analysis investigates these other insurance designs using farm-level data and relaxing the joint normality assumptions. We turn to numerical procedures to allow investigation of the full set of yield and revenue designs under a common set of assumptions. As will be described in the next section, the alternative designs involve fundamentally different censoring of the yield, price, or revenue distribution. Further, some of the insurance designs involve mixtures of protection that add complexity to the problem. When insurance is combined with options, there is multiple censoring of various portions of the revenue distribution. These mixtures of censoring have the potential effect of limiting analytical models that require differentiability.

The Risk Management Tools Examined

In our numerical analysis, four insurance products are modeled to reflect the products that are now appearing in the crop insurance market. The crop insurance products currently offered to farmers in the United States fall into four categories depending on: (*a*) whether yield or revenue per acre is guaranteed, and (*b*) whether indemnities are paid at a price set at sign-up time or at the higher of the sign-up time and harvest-time prices. There are reasons to expect that each type of insurance has a different effect on optimal hedging levels. Revenue insurance is expected to result in lower hedge ratios than yield insurance because it partly substitutes for forward pricing in protecting against price declines. Indemnifying losses at the higher of sign-up and harvest-time prices is expected to increase the optimal hedge ratio because it increases the effect of price change on the farmer's income. One of our objectives is to quantify these differences.

Multiple-Peril Crop Insurance (MPCI) is the traditional crop yield insurance program generally available for major crops in most states. Market Value Protection (MVP) is a private product that modifies the traditional crop yield insurance design by increasing the value of lost production if prices increase during the season. Our representation of revenue insurance is a pure revenue insurance design similar to either Revenue Assurance (RA) or Income Protection (IP), which are currently offered by the RMA and available for a limited set of major crops and regions. Crop Revenue Coverage (CRC) is a revenue insurance product which provides increasing coverage when prices increase during the season. CRC is also a federally subsidized product and is the most widely offered revenue design. Thus, two of the four designs examined are yield triggered, while the remaining two are revenue triggered. A brief explanation of each instrument follows.

MPCI indemnifies yield losses when an insured acreage's yield falls below the guaranteed yield level. These losses are valued at a preseason price selected at sign-up time. The equation for MPCI may be written as follows:

$$NI_{MPCI} = \delta f_0 * \text{Max}\left[\gamma y_0 - y_1, 0\right] - R_{MPCI}, \tag{6}$$

where NI is the net return to insurance purchase, δ is the percentage of the maximum price election, f_0 is the preseason price for a harvest month futures contract, γ is the

insurance coverage level, and y_0 and y_1 are, respectively, the expected farm yield at planting and realized yield at harvest. Although RMA allows producers to select a price election that is a percentage of the preseason price, we assume that producers are insuring at 100% of the preseason expected price. Summaries of RMA data indicate that 98% of Midwestern corn producers insuring above the fully subsidized catastrophic coverage insurance choose the 100% price election. The insurance premium, R, reflects the producer paid insurance premium cost for the policy. The subscripts on NI and R denote the type of insurance evaluated.

The Market Value Protection (MVP) design, shown in equation (7), is also yield triggered. However, in this case, losses are valued at the maximum of either springtime expected price or the actual harvest-time price, f_1. Price is multiplied by 0.95 to reflect an average basis in the futures market:

$$NI_{MVP} = 0.95 * \text{Max}[f_0, f_1] * \text{Max}[\gamma y_0 - y_1, 0] - R_{MVP}. \tag{7}$$

Since 1997, three types of farm-level revenue insurance have been offered to U.S. producers—Crop Revenue Coverage (CRC), Income Protection (IP), and Revenue Assurance (RA). All three of these products insure the gross revenue of the insured crop. The products differ in rate-setting procedures and locations where they are offered. All three are reinsured and subsidized by the USDA and use harvest-month futures prices at sign-up and at harvest to compute losses. Because of similarities in design, we treat IP and RA as a single insurance type, designated as Revenue Insurance (RI). Equation (8) shows the net returns from RI. Here, shortfalls in harvest revenue ($f_1 y_1$) trigger losses rather than y_1, as in the case of yield insurance:

$$NI_{RI} = \text{Max}[\gamma f_0 y_0 - f_1 y_1, 0] - R_{RI}. \tag{8}$$

The final insurance design investigated in this study is Crop Revenue Coverage (CRC). This insurance design combines the revenue insurance protection of RI with the "upside" price protection of MVP. Ninety-five percent of the maximum of preseason price expectations or the actual harvest-time futures is used to compute the coverage:

$$NI_{CRC} = \text{Max}[0.95\gamma \text{ Max}[f_0, f_1] y_0 - f_1 y_1, 0] - R_{CRC}. \tag{9}$$

Two forms of forward pricing are modeled: futures hedging and the purchase of put options. The net return for each forward pricing strategy is denoted by NF. The net returns from futures marketing are modeled in equation (10):

$$NI_F = \alpha_F y_0 (f_0 - f_1) - R_F. \tag{10}$$

As shown, futures hedging protects against price risk on a given quantity hedged. The futures marketing hedge ratio is represented by α_F, and is the proportion of the expected yield that is protected. In this case, the cost of risk protection (R_F) reflects commissions and interest charges to carry out the hedging transaction. The returns from a put option contract are shown in equation (11):

$$NF_P = \alpha_P y_0 * \text{Max}[\gamma f_0 - f_1, 0] - R_P. \tag{11}$$

In this case, the put option ratio is denoted by α_P, and represents the proportion of the expected yield covered by options. The option strike price relative to the futures price is γ. The cost of a put option (R_P) includes the option premium, commissions, and interest charges for capital invested.

Combining forward pricing and insurance results in additional terms added to the end-of-period wealth states, which may be written:

$$W_{jk} = W_0 + A\left[p_1 y_1 - C + NI_j + NF_k\right], \tag{12}$$

where j represents the alternative insurance design and k the forward pricing alternative.

Stochastic Specification

In the model, end-of-period wealth, and thus utility, is a function of three random variables: farm yield (y_1), futures prices (f_1), and harvest-time basis (b), where $p_1 = f_1 + b$. At decision time, expected yield (y_0), current futures price for the harvest month contract (f_0), and the expected basis are assumed known. Harvest-time futures prices are generated assuming a multiplicative shock such that:

$$f_1 = f_0 \varepsilon_1, \tag{13}$$

where ε_1 is the relative futures price movement from planting to harvest-time, and is assumed to follow a log-normal distribution.[3]

Local harvest-time prices are generated as follows:

$$p_1 = f_0 \varepsilon_1 + b_0 + \varepsilon_2, \tag{14}$$

where b_0 reflects the expected harvest-time basis, and ε_2 represents deviations in the realized basis from the expected basis. Basis risk (ε_2) is assumed normally distributed. The expected futures price was set at $2.50 and price variability over the growing season at 20%, respectively, to represent typical price levels and volatility. The mean and variance for each location were calculated from differences between state average prices received by farmers, as reported by the USDA's National Agricultural Statistics Service (NASS), and monthly averages of futures settlement prices for the month of November over the years 1976–95.

Farm yield variability is represented by augmenting the potentially nonnormal county yield series with information on the difference between county yield and yield of farms in the county. This follows Miranda in assuming that farm yield variability may be treated as being composed of a systemic portion correlated with county yields and idiosyncratic individual variation. This approach is taken to augment fairly short available farm yield series with the added information available at the county level.

[3] The log-normal assumption is well accepted in commodity pricing literature (e.g., the Black-Scholes option valuation model is based on log-normality). Shapiro-Wilk and Kolmogorov-Smirnov tests of the assumption failed to reject the log-normal assumption.

NASS county yield data over the years 1956–95 were used to estimate each county yield distribution.[4] Technological trends in yields were taken into account by a linear trend estimator using weighted least squares to corrected for heteroskedasticity. The variances of farm-county yield differences were estimated by combining 1985–94 farm yield observations provided by the RMA with corresponding county yield observations and pooling all farms in the county. Farms with at least six years of actual yields during the 10-year period were used in the analysis. Farm yield variances by county were estimated as the sum of the estimated county yield variance and the average variance of farm-county yield differences for farms in the county. Omission of covariances is justified by the assumption that farm-county yield differences are, on average, uncorrelated with county yields (Miranda). Given that a representative farm for a particular county is being constructed, the acre-weighted average of all farm-county yield differences will equal zero (proof of this assumption is provided in the appendix).

Potential nonnormality of the county yield is addressed by transforming the data to approximate normality with a hyperbolic tangent transformation. This general approach avoids making specific distributional assumptions. Moss and Shonkwiler; Ramirez; and Taylor have proposed various forms of hyperbolic transformations and applied them to crop yield distributions. Our analysis follows the hyperbolic tangent transformation proposed by Taylor. The transformation to normality involves first expressing the cumulative density as a hyperbolic tangent function of linearly detrended yield, $\tilde{y}$:

$$F(\tilde{y}) = 0.5 + 0.5 * \tanh\left(\beta_0 + \beta_1\tilde{y} + \beta_2\tilde{y}^2 + \beta_3\tilde{y}^3\right), \tag{15}$$

where $F(\tilde{y})$ is the empirical cumulative density function (CDF), and the β's are estimated with maximum-likelihood procedures.

Farm yield for a farm with a mean yield equal to the county mean is modeled as $y_f = \mu_i + \varepsilon_3$, with ε_3 defined as:

$$\varepsilon_3 = \beta\left(T^{-1}F(\varepsilon_n) - \mu_i\right), \tag{16}$$

where y_f is farm yield, μ_i is the expected county yield, β is the ratio of farm yield standard deviation relative to the county yield standard deviation estimated from the variances of county yield and farm-county yield differences, T^{-1} is the inverse of the Taylor transformation, and $F(\varepsilon_n)$ is the distribution of the standard normal.

Product moment correlations are used to model relationships between the transformed random variables. The correlations between yield, futures price, and basis were estimated using transformed data over the 1975–95 period.

Methods for Estimating Expected Utility Under Alternative Strategies

Given the specified stochastic structure, the farmer's expected utility under each combination of risk management strategies is estimated using Gaussian quadrature. Gaussian quadrature generally is the preferred form of numerical integration because it gives greater accuracy for the same number of calculations, or equivalently, requires fewer

[4] Jerry Skees (Department of Agricultural Economics, the University of Kentucky) assembled the county yield observations prior to 1972.

calculations for any given level of accuracy, than trapezoidal integration or use of Simpson's rule. This advantage is obtained by relaxing the requirement that the areas summed under the function are of equal widths (Gerald and Wheatley). An n-point Gaussian quadrature will compute the integral of a function exactly if the function is a polynomial of order $2n - 1$ or less (Miranda and Fackler). Because this model contains multiple random variables, higher order quadratures are used where applicable. The accuracy of the Gaussian quadrature approximation is conditional on the number of quadrature nodes used. The GAUSS software allows a maximum of 40 quadrature nodes, and this maximum is applied to all random variables in our analysis. Bounds of plus and minus four standard deviations are imposed on the multivariate normal distribution. Gaussian quadrature is used initially, as needed, to estimate the expected value of crop sales and fair premiums for insurance and options. Then it is used to estimate expected utility for alternative scenarios.

The search for optimal hedge ratios conditional on insurance coverage is performed by using a quadratic approximation of the response of expected utility to variation in the hedge ratio. This is the starting point for a step search which changes the ratio up and down in 1% increments until an optimum is found. These evaluations are repeated across insurance coverage levels in increments of 5%. Such an approach allows examination of the effect of insurance on optimal hedging across a range of insurance coverages.

Representative Farms and Base Scenarios

Having described the general decision model and the numerical procedures used, we now detail the specific characteristics of the four representative farms. Because price variability tends to differ little among farms, and basis risk is small relative to price risk, regional differences are most apparent in yield variability and yield-price correlation. Four counties were chosen to represent farms from areas with differing levels of yield variability and yield-price correlation. Statistics for these counties are reported in table 1. Iroquois County in east central Illinois was chosen to represent the typical Corn Belt case of relatively low yield variability and yield-price correlation that is strongly negative. Douglas County in east central Kansas represents an area with relatively high yield variability and negative yield-price correlation. Lincoln County in west central Nebraska is an irrigated area with low yield variability and low yield-price correlation. Pitt County in east central North Carolina is representative of an area with high yield variability and low yield-price correlation. Statistical tests of correlation were conducted for each location and are found significantly different from zero in Kansas and Illinois.

Certainty equivalent gains are derived by calculating the certainty equivalent revenue associated with the expected utility for a particular risk protection scenario and comparing it to the certainty equivalent revenue for the scenario where no risk instruments are used. The use of certainty equivalents facilitates comparison of scenarios in a convenient money metric (Hardaker, Huirne, and Anderson). Results are estimated for two combinations of initial wealth and risk aversion using constant relative risk aversion (CRRA) utility functions.[5] Initial wealth level for a farm with 500 acres of corn is set at

[5] The power utility function is used. This function is of the form: $U = 1/(1 - r)W^{(1-r)}$ if $r \neq 1$, otherwise $U = \ln(W)$.

Table 1. Estimated Parameters for the Counties Included in the Numerical Analysis

Parameter		Counties			
		Iroquois, IL	Douglas, KS	Lincoln, NE	Pitt, NC
Basis ($/bu.)	Mean	-0.24	-0.15	-0.26	0.00
	Std. Dev.	0.14	0.17	0.17	0.20
Futures Price–Basis	Correlation	-0.28	-0.27	-0.46	0.16
County Yield–Basis	Correlation	0.18	-0.01	0.06	-0.44
Farm Price	Mean	2.24	2.33	2.20	2.52
	CV	0.22	0.21	0.21	0.23
Farm Yield	Mean	143.20	90.48	150.30	87.70
	CV	0.20	0.34	0.15	0.40
Farm Yield–Price	Correlation	-0.40*	-0.35*	-0.13	-0.17

Note: An asterisk (*) denotes statistically different from zero at the 0.10 level.

Table 2. Optimal Hedge Ratio and Certainty Equivalent Gains for the Four Representative Farms, Without Insurance

Location	CRRA = 2 Wealth = $300,000		CRRA = 4 Wealth = $300,000	
	Optimal Hedge Ratio (%)	Cert. Equiv. Gain ($)	Optimal Hedge Ratio (%)	Cert. Equiv. Gain ($)
Iroquois County, IL	22	0.14	22	0.91
Douglas County, KS	0	0.00	0	0.00
Lincoln County, NE	55	2.31	61	6.01
Pitt County, NC	32	0.48	35	0.65

$300,000. Relative risk aversion is assigned a value of 2 to represent moderate risk aversion and is set at 4 to represent high risk aversion. The certainty equivalent measures show how the individual's initial wealth and degree of risk aversion affect the gains from alternative strategies. However, the estimated certainty equivalent dollar gains are not necessarily representative because they rest on assumptions about wealth and risk aversion.

In all locations and scenarios, the costs of using futures and at-the-money options are specified with commissions of $50 per contract and margin deposits at 8%. The farmer's interest on initial margin deposits and options premiums is assumed to have an opportunity cost of 8% over the six-month growing season. Insurance is assumed to be actuarially fair, with no administrative costs included.[6]

[6] RMA crop insurance programs are subsidized at varying levels. This would clearly influence the decision to insure and the level of insurance coverage. These decisions are made prior to the sign-up deadline which is generally prior to planting. Subsidies are omitted in this analysis to clarify the risk benefits of the instruments.

Results

The optimal hedge ratio without insurance for each of the four representative farms is examined first. Table 2 shows the optimal planting-time hedge ratio and certainty equivalent gain from hedging in each of the four county locations. The differences in the underlying yield variability and yield-price correlation result in optimal hedge ratios ranging from no hedging up to hedging 61% of the expected crop under strong risk aversion. The highest hedge ratio (Lincoln County, Nebraska) results in the location where yield-price correlation is low and yield variability is relatively low due to the predominance of irrigation in this county. Conversely, the lowest optimal hedge ratio (Douglas County, Kansas) occurs in the location where yield-price correlation is strongly negative and yield variability is relatively large. These two locations confirm that the demand for hedging is negatively correlated with yield variability and yield-price correlation, as suggested by McKinnon, and by Moschini and Lapan.

The finding that the optimal hedge ratio is relatively low in areas of high yield variability is consistent with the inverse relationship between optimal hedging ratios and yield uncertainty. Greater uncertainty of yield increases the probability that the hedged quantity will not be produced, leaving the producer with an uncovered hedge position. We suggest that the finding that hedge ratios tend to be lower in areas of stronger negative yield-price correlation is interpreted as an artifact of a "natural hedge" existing between price and yield (Miranda and Glauber). When price and yield move inversely, a producer can expect higher prices in a low yield year and vice versa. By hedging, price is locked into a particular level (ignoring basis risk). Thus, the natural hedge is eliminated.

Certainty equivalent gains, which reflect the increased producer welfare from risk reduction, are also reported in table 2. Our findings reveal a generally small gain relative to the per acre crop value. The estimates shown are based on an assumed CRRA of 2. The estimates are, of course, sensitive to the degree of risk aversion assumed. However, the greatest gain does come in Nebraska where the hedging appears most effective.

The two other locations are representative of areas where yield-price correlation and yield variability produce a mixed effect. The moderately risk-averse Iroquois County, Illinois, farm has a 22% hedge ratio (table 2) which is constrained by the strongly negative yield-price correlation in spite of a relatively low yield variability. The Pitt County, North Carolina, farm's base case hedge ratio is 32% under moderate risk aversion. Here the natural hedge does not exist to limit the optimal hedge ratio, but the relatively large yield variability appears to be a more significant factor in revenue variability.[7]

Interestingly, the hedge ratio in Douglas County, Kansas, remains at zero even under the assumption of a more risk-averse producer (table 2). In this location with a strongly negative yield-price correlation associated with highly variable yields, hedging appears the least valuable. However, given efficient market assumptions, this may appear surprising. As reported more fully later in this article, we conclude that transaction costs in this particular instance outweigh the certainty equivalent gain for this location.

[7] Given that the yield-price correlation for the Nebraska and North Carolina locations were not statistically different from zero, auxiliary models were run which assumed the yield-price correlation was zero. For both locations, the optimal hedge ratio increased approximately 10%–15% throughout the range of coverages. Otherwise, the findings are essentially the same. In particular, the relative ordering of hedge ratios across insurance designs remains unchanged.

Examination of the certainty equivalent gains reported in table 2 shows that the gain in expected utility induced by hedging is generally low, even with the strongly risk-averse case. For example, these hedge ratios are generally lower than those found by Moschini and Lapan. Transaction costs have been included here which diminish the value to hedging to the risk-averse producer. While the commission paid for a futures contract and the interest charged are small in relation to the value of the contract, when compared to the certainty equivalent gain provided by futures contracts this cost looms as a significant factor. Further, by using farm-level data, we have relatively higher yield variability in the empirical specification than has often been used previously. For example (from table 1), two of the counties have a farm-level yield coefficient of variation (CV) above 0.30 (the highest level examined by Moschini and Lapan).

The effect of various insurance designs on forward pricing hedge and put ratios was computed for insurance coverages varying from zero to 100% of expected yield in 5% increments. This permitted the examination of the potentially nonlinear response of the optimal hedge as insurance levels are varied. Although insurance coverage above the 85% leverage is not allowed in any of the programs investigated here, the analysis was carried to the 100% level to more fully reveal the relationship between a particular insurance program and the optimal hedge ratio.

Figure 1 shows the futures hedging level found for each insurance design for each of the four county locations, given CRRA = 2 and transaction costs are imposed. First, very low levels of insurance protection had little effect on the optional hedge ratio in any of the four representative farms. The Pitt County, North Carolina, farm saw the earliest change in the optimal hedge ratio at 5% insurance coverage. The Lincoln County, Nebraska, optimal hedge was unaffected by insurance coverage until coverage reached the 60% level. Given the relevant range of insurance coverage levels offered in the U.S. is 50%–75%, little change in observed hedging demand would be expected from any of the designs in the Nebraska case.

As the insurance coverage is increased for each of the four locations, a consistent relationship is found as one compares across the four insurance designs. For each of the locations, at higher coverage levels, MVP is always associated with the highest optimal hedge. MPCI is the second highest. The revenue products, CRC and RI, ranked third and fourth, respectively. The two yield insurance designs, when they do cause a change in the optimal hedge, always result in an increased optimal hedge. Thus, yield insurance designs are found to be purely complementary to hedging as was found in the analytical model. It would appear that the MVP component which indemnifies producers at the greater of preseason price or harvest-time price does provide a slightly greater optimal hedge than MPCI in most cases.

The revenue insurance designs show a more complex relationship with the optimal hedge. The "MVP-like" component of CRC results in an optimal hedge for CRC that is always equal to or greater than that of RI. In fact, CRC is always found to increase the optimal hedge over the uninsured case in three of the four locations. The Lincoln County, Nebraska, case is the exception with CRC resulting in lower than uninsured hedging levels when CRC coverage reaches 60% of expected revenue. Interestingly, results for RI reveal a nonlinear relationship with hedging. For example, in Illinois, an increase in hedging occurs over the mid-range of coverages, but as RI coverage increases, the optimal hedge ratio begins to fall. This suggests RI has the strongest

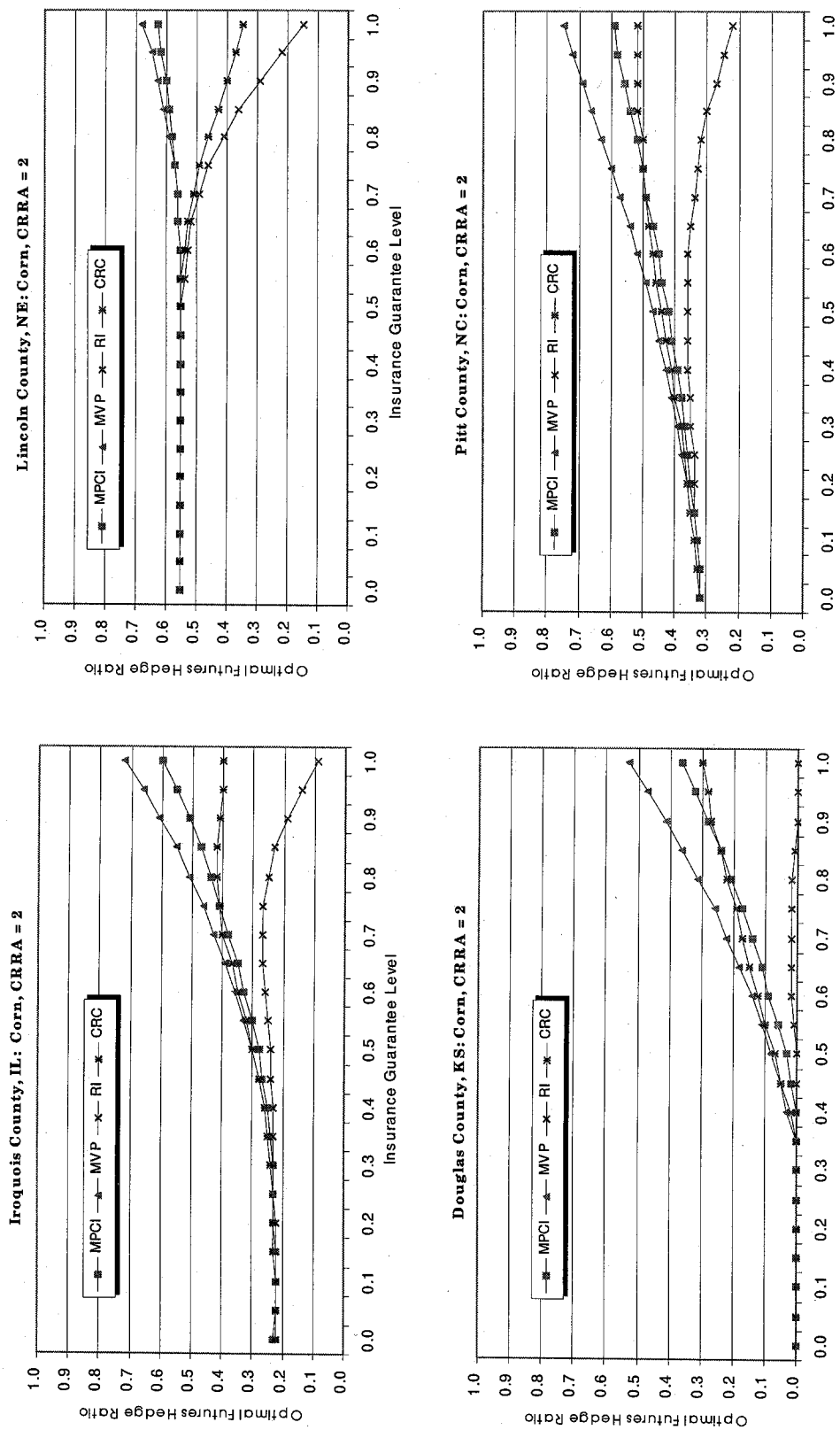

Figure 1. Optimal futures hedge ratios for alternative insurance designs with transaction costs

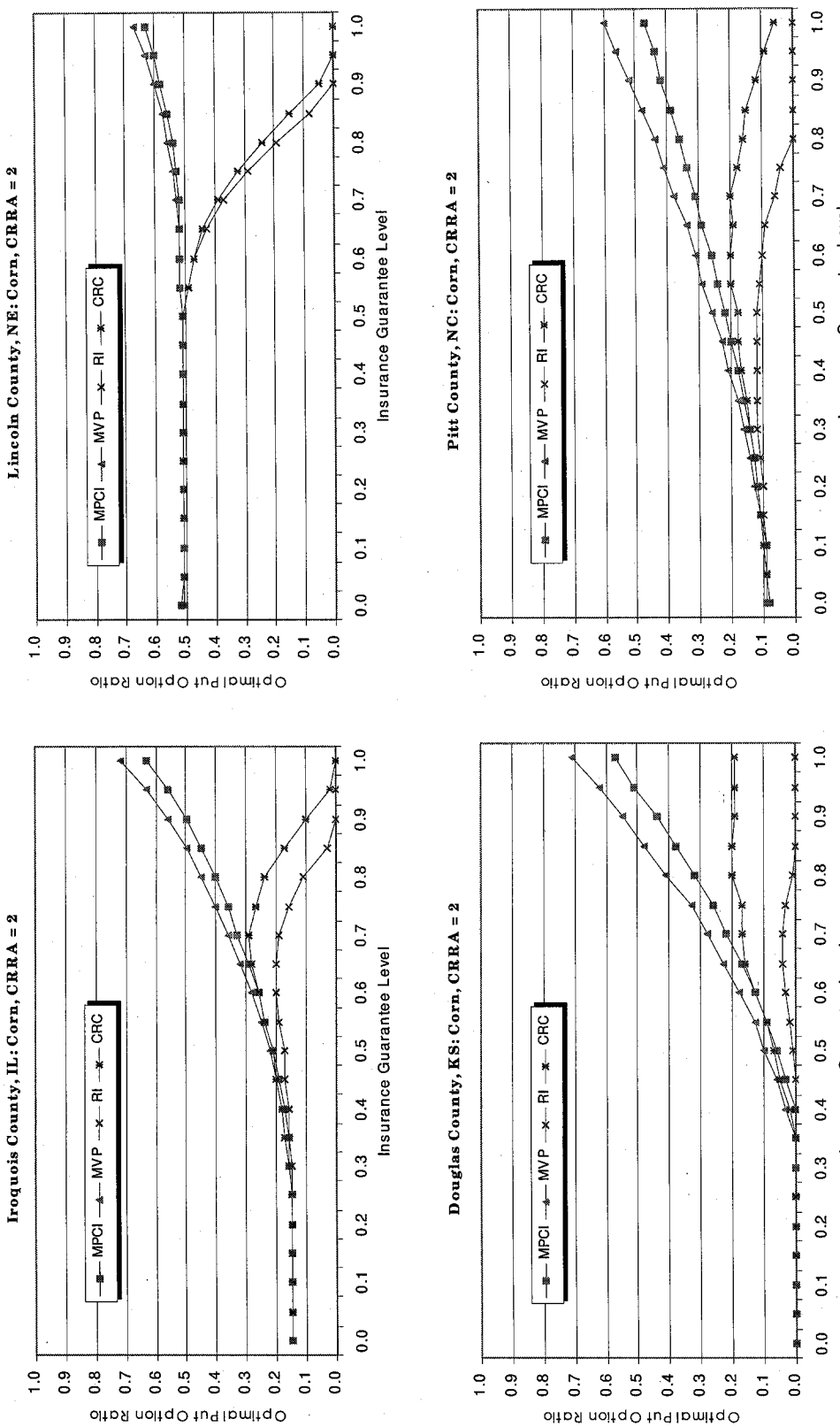

Figure 2. Optimal put hedge ratios for alternative insurance designs with transaction costs

substitution effect on hedging of the four insurance designs. In contrast, these results for the two revenue insurance products indicate there is a mixed effect of revenue insurance on the optimal hedge. As the level of RI is increased, it appears it is increasingly a substitute for the price protection of a hedge. It is also clear that the upside price component of CRC makes it more complementary to hedging since this component is like a futures call—i.e., it increases the insurance payout when prices increase.

The relationship between insurance coverage and at-the-money put option ratios is explored in figure 2. Analyzed over the same range of insurance coverages as for futures, the put option percentages tend to follow a similar pattern. As observed when comparing between the hedge and put ratios, in general, the put ratio is higher. It appears the higher option ratios occur because options hedgers are not subject to such large losses in low yield–high price years as are futures hedgers who may have to buy back their contracts at a high price.

As for hedge ratios, the put ratios compared across insurance designs show that when differences appear, MVP results in the highest put ratio, followed by MPCI, CRC, and RI, respectively. The effects of purchasing insurance on the optimal put do not become pronounced until higher levels of coverage. In Nebraska, there is no change until insurance coverage is above 50%. A different relationship is observed between CRC and put option levels than was found in the relationship of CRC and futures hedging. In figure 2, it can be seen that CRC tends to be more competitive with puts than with hedging. For example, in Illinois, increasing CRC tends to increase futures hedging. However, it causes reduced put percentages at higher levels of coverage. This likely results from CRC being a lower-bounding activity and thus competes more directly with puts, which are also lower-bounding. In contrast, futures are lower- and upper-bounding. In other words, the upside price protection provided by CRC is similar to a call option in that the payoff increases as the price rises. This complements a futures hedge given a net position similar to a synthetic put. Such strong complementarity is absent when CRC is combined with a put option.

To further investigate the implications of transaction costs, which were posited as a factor causing the zero hedge ratio in the Douglas County, Kansas, case, the analysis in figures 1 and 2 was replicated with all transaction costs removed. This entailed dropping the commission charges for futures and options contracts and eliminating the interest charge on margin accounts. Results of these replications are shown in figures 3 and 4.

In general, the elimination of transaction costs raised the optimal hedge ratio from 10% to 15% in all four locations. In particular, the Douglas County, Kansas, location has a positive hedge ratio, albeit the lowest of the four locations. Examination of figure 3 as compared to figure 1 shows that the pattern of optimal hedges remains very similar throughout the range of insurance coverage levels. Figure 4 reports the same analysis for optimal put ratios. As with futures, removal of the transaction costs does not alter the ranking for the various insurance products except to shift put ratios upward by roughly 25% on average. Thus, we find that transaction costs can have a nontrivial influence on optimal hedge and put ratios. Even though transaction costs are relatively small, they are sufficient to offset the certainty equivalent gain.

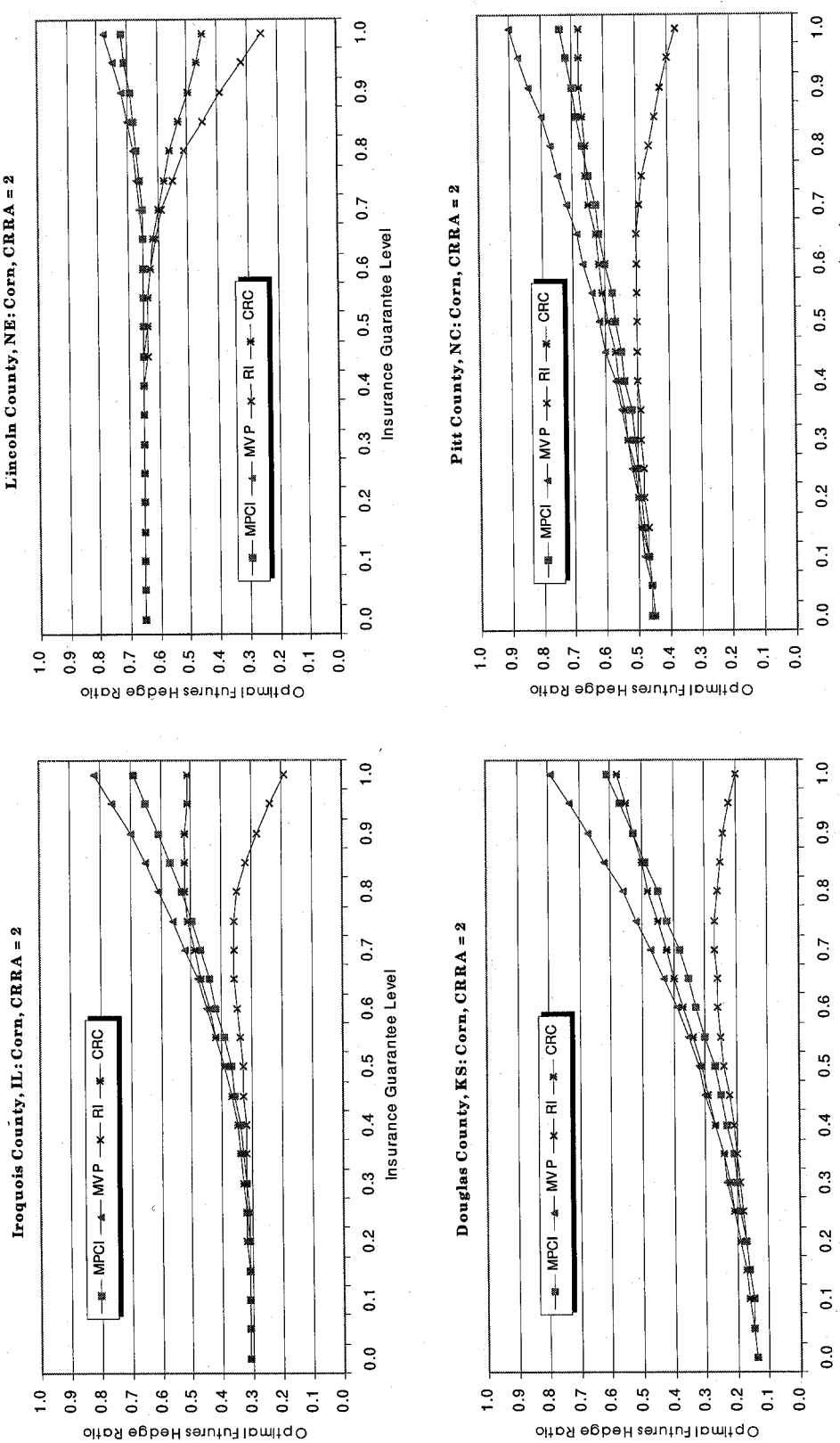

Figure 3. Optimal futures hedge ratios for alternative insurance designs with no transaction costs

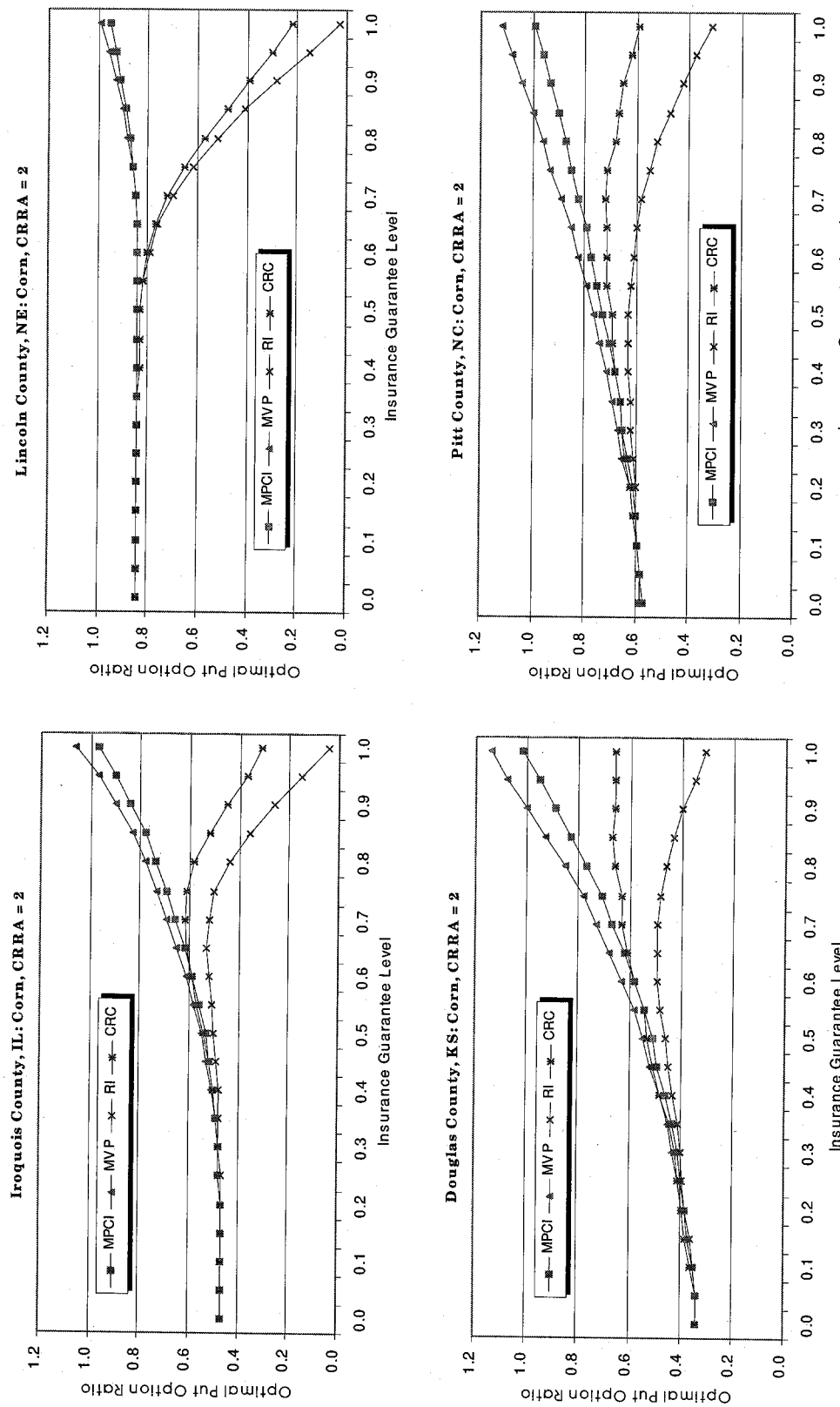

Figure 4. Optimal put hedge ratios for alternative insurance designs with no transaction costs

Conclusions

The proliferation of new insurance products greatly changes the context in which hedging decisions are made. This study was conducted to provide analysis of optimal futures and put hedging levels when producers have yield, price, and revenue risk management markets available to them. The analytical results show an unambiguous positive relation between crop yield insurance and the optimal quantity hedged. Numerical estimates are provided to allow for nonnormality in yields and show the magnitude of the effects of alternative insurance designs on optimal hedges under different cropping conditions.

In general, revenue insurance tends to result in lower hedging demand than would occur given the same level of yield insurance coverage. However, the differences tend to be small (no more than 10%) over the relevant range of insurance coverage. This study also finds a consistent pattern that the upside price protection afforded by MVP and CRC designs tends to be more complementary to hedging than the RI design. To the extent that producers would switch from yield insurance to revenue insurance, there would be a decline in the demand for hedging. Pure revenue insurance designs are shown to have a strong substitution effect on hedging. Substantial nonlinearities in these relations are observed. In general, as insurance levels increase above the 70% coverage level, the effect on hedging increases rapidly.

Because some of the insurance tools examined in this study are so new to producers and are sufficiently distinct in their design, producers at this point may have difficulty evaluating the decisions modeled here. One might expect that as producers become more familiar with the implications of these alternatives, there will be an evolution in how producers utilize the combinations of insurance and forward pricing instruments.

There appear to be several natural extensions to this work. Obviously, other crops and regions may be examined. Possibilities of further risk reductions by combining insurance with the joint use of futures and options (or combinations of options at different strike prices) deserve exploration in light of the Sakong, Hayes, and Hallam, and Moschini and Lapan results.

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Appendix: Proof that the County Average Farm Yield Variance Equals the County Yield Variance Plus the Average Variance of Farm-County Yield Differences

County average yield in year t is:

$$C_t = \frac{\sum_{i=1}^{N} a_i Y_{it}}{A}, \tag{A1}$$

where Y_{it} is a random variable denoting yield on farm i, and the constants a_i, A, and N represent acreage on farm i, the sum of acreages (a_i) in the county, and the number of farms in the county, respectively. Let

(A2) $$d_{it} = y_{it} - c_t$$

be the farm-county yield difference for farm i, where lower-case letters represent deviations from means. The variance of yield on farm i is:

(A3) $$\sigma_i^2 = \frac{\sum_{t=1}^{T} y_{it}^2}{T} = \frac{\sum_{t=1}^{T} (c_t + d_{it})^2}{T} = \frac{\sum_{t=1}^{T} \left(c_t^2 + d_{it}^2 + 2c_t d_{it}\right)}{T},$$

where T is the number of years observed. The weighted average of these farm yield variances for the county is:

(A4) $$\sigma^2 = \frac{\sum_{i=1}^{N} a_i \sum_{t=1}^{T} \left(c_t^2 + d_{it}^2 + 2c_t d_{it}\right)}{AT}.$$

Equation (A4) may be rewritten as:

(A5) $$\sigma^2 = \sigma_c^2 + \frac{\sum_{i=1}^{N} a_i \sigma_{di}^2}{A} + \frac{2 \sum_{i=1}^{N} a_i \sum_{t=1}^{T} c_t d_{it}}{AT},$$

where σ_c^2 is the variance of county yield, and σ_{di}^2 is the variance of farm-county yield differences for farm i. Rearranging the summations in the third term shows that it equals zero:

(A6) $$\frac{2 \sum_{t=1}^{T} c_t \sum_{i=1}^{N} a_i d_{it}}{AT} = 0,$$

because each year's weighted sum of farm-county yield deviations equals zero:

(A7) $$\sum_{i=1}^{N} a_i d_{it} = 0.$$

Thus, the average farm yield variance for the county equals the county yield variance plus the average variance of farm-county yield differences:

(A8) $$\sigma^2 = \sigma_c^2 + \frac{\sum_{i=1}^{N} a_i \sigma_{di}^2}{A}.$$