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# **Optimal forest rotation and invasive species control when damages are heterogeneous**

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This paper examines the economics of forest management in the presence of a timber-impairing invasive shrub, including the optimal forest rotation age, optimal invasion management strategy, and resulting economic outcomes for a single rotation of a plantation forest.

The case of invasive shrubs, as compared to invasive species such as diseases, insects, or other pests, is a unique ecological case for two critical reasons. First, trees are only susceptible to the impacts of invasive shrubs until a certain age (when their height exceeds that of their shrub competitor). Additionally, invaded juvenile trees experience delayed maturity, with the length of this delay being inversely proportional to the age of which they were invaded. This is in contrast to the way we typically think about the impacts of invasive species on timber production, where plots become increasingly invaded/infested as the invasion's footprint increases over time (up to the time of harvest) with affected trees losing some or all of their economic value (such as in Macpherson et al. 2016).

These key ecological differences produce dramatically different economic results. In the latter case, the increasing degradation of timber reduces the marginal benefit of waiting to harvest, resulting in an optimal rotation age that is below that of the case of no invasion, with these harvests often being referred to as “salvage” harvests (Sims, 2011). However, in the case of invasive shrubs, juvenile trees typically reach insusceptibility prior to merchantability, such that the economic damages caused by the invasion have already been fully realized by the earliest economical harvest date, eliminating the need for an earlier salvage harvest to prevent further degradation. At the same time, the heterogeneous levels of growth delay experienced across the plot as the invasion spreads results in more akin to a mixed-aged plot that is, on average, “younger” than it would otherwise be. Modeling timber production in this context requires an age-structured timber production function whose derivative over time differs from the original production function.

Despite the increasing prevalence of invasive shrubs in the deciduous forests of eastern North America (Maynard-Bean & Kaye, 2019), to the best of the authors' knowledge, there exists no economic model of impaired timber production that adequately accounts for the type of heterogeneous impairment caused by invasive shrubs. To address this gap, we present a generalizable framework that combines a Faustmann rotation model, an age-structured impaired timber production function, and an ecological model of invasion dynamics. We show how this model can be used to solve for the optimal forest rotation age in combination with the optimal invasion management strategy, and find that the presence of invasive shrubs results in an optimal rotation age that is longer, rather than shorter, than the case of no invasion. We go on to analyze how our optimal solutions change with two sets of parameters, one set that governs the impacts of impairment on forest regeneration (via both delayed growth and reduced juvenile survival), and another set that describes the population dynamics of the invasion (initial invasion level, local growth rate, and inbound dispersal rate). We show how the

optimal invasion management strategy can be mapped as a function of the site-specific invasion growth dynamics, for a known set of impairment parameters.

As an application we consider the case of glossy buckthorn, a non-native and invasive shrub in North America that uses its shade tolerance to colonize forest understory and inhibits or delays forest regeneration. Using estimates of the effect of the invasive shrub glossy buckthorn on forest regeneration (Fagan & Peart, 2014; Lee, Eisenhaure & Gaudreau, 2016), we go on to find that, if glossy buckthorn is not controlled, it has the potential to reduce the net present value of timber revenue by over 70%, with roughly 67% resulting from decreased forest density due to juvenile mortality, and the remainder resulting from the discounting of delayed timber revenue resulting from an extended rotation.

Our modeling framework has the benefit of allowing one to decompose the economic damages resulting from invasion into three distinct categories, namely those resulting from the initial invasion level, those resulting from subsequent local seed dispersal, and those resulting from off-site (external) seed dispersal. Thus, while our analysis focuses primarily on private decision-making, our model could be expanded for use in an optimal policy design setting by allowing policy-makers to compare both the internal and external benefits of invasive shrub management.

## **I. INTRODUCTION**

The impact of invasive plant species is estimated to cost the US \$15 billion per year (Anderson, 2018). At the same time, the impact of invasive shrubs on forest regeneration, and thus timber production, is a unique case among invasive species. Unlike the case of invasive insects, diseases, or other pests, the ecological evidence suggests that trees are only susceptible to the negative impacts of invasive shrubs while their height falls below that of their shrub competitors, with the effect that once trees have reached their merchantable height, they are no longer susceptible to these negative impacts. This is in contrast to an invasive insect or disease in which trees continue to be negatively impacted until they are harvested. Invasive shrubs are also unique in how they impair timber production. Unlike other forest invasives that act akin to infections that consume their host, invasive shrubs impair forest regeneration by competing for resources, specifically sunlight, by shading shorter juvenile trees. The result is that invaded juveniles have a lower survival rate and take longer to reach maturity than their uninvaded peers, with this delay increasing the earlier a juvenile tree was invaded (Frappier, Eckert, & Lee, 2003; Fagan & Peart, 2014; Lee, Eisenhaure & Gaudreau, 2016).

These key differences produce two distinct types of impaired forests. In the traditional invasive species case, timber is being continuously degraded until it is harvested, increasing the marginal cost of waiting to harvest. In contrast, by the time the shrub-invaded plot is old enough to be economically harvested, the impacts of impairment have been fully realized and there is no additional degradation. Additionally, because not all trees become invaded at the same time, the result is a forest plot

comprised of heterogeneously impaired trees, specifically a subset of trees impaired at the time of planting who face the maximum level of growth impairment, a subset of trees that were never impaired (those who reached their height threshold without becoming invaded), and a continuous sequence of subsets of trees that became impaired in each period following planting but prior to reaching insusceptibility and thus suffer some intermediary level of growth impairment. The resulting plot thus becomes analogous to a mixed-aged forest comprised of trees that are, on average, “younger” than the plot itself.

These key differences produce dramatic differences from an economic perspective. Whereas invasive diseases or insects increase the marginal cost of waiting to harvest, invasive shrubs have the opposing effect of increasing the marginal benefit of waiting to harvest due to the fact that some trees are now “less far along” the timber production function, resulting in a larger second derivative with respect to time. These crucial differences have dramatic implications in terms of both a forester’s optimal forest management and optimal invasion management decisions. For example, previous research on the effect of invasive disease on optimal rotation age has found that the optimal response to such an invasion is to *decrease* the rotation length in order to salvage the remaining timber before it is further degraded, often referred to as a “salvage harvest.” In other words, the disease increases the marginal cost of waiting an additional period to harvest, thus reducing the age at which the marginal cost and benefit align. In contrast, economic intuition suggests that the presence of an invasive shrub *increases* the optimal rotation length, a hypothesis we confirm in this paper. Likewise, because any determination of the optimal invasion management strategy implicitly relies on calculating the optimal rotation age with and without invasion control, meaning that the

optimal invasion management strategy is likely to differ between shrub and non-shrub invasive species, even in cases where the population dynamics are relatively consistent.

Despite the increasing prevalence of invasive shrubs in the deciduous forests of eastern North America (Maynard-Bean & Kaye, 2019), to the best of the authors' knowledge, there exists no generalizable model of impaired timber production that adequately accounts for the type of heterogeneous impairment caused by invasive shrubs. To address this gap, this paper provides a general framework for modeling the effects of an invasive shrub that impairs the growth of trees below a certain height on the management decisions of a timber producer. Among other results, this paper develops and examines solutions for the two key problems facing timber producers responding to the threat of invasive shrubs, namely i) how do invasion dynamics affect the optimal forest rotation age, and ii) given a set of options for managing invasion prior to (re-)planting, what is the optimal invasion management strategy?

This modeling framework combines a Faustmann single-rotation model, an age-structured impaired timber production function, and an ecological model of invasion dynamics to create an expanded Faustmann model in which the traditional timber production function is transformed as a function of the ecological characteristics of the invasive shrub (initial presence, growth, impact), but where the initial invasion level and/or invasion spread can be endogenously reduced by choosing from a discrete set of invasion management options (growth prevention and/or removal, inaction). The forest manager thus considers how each invasion management option will affect the impaired timber production function, what the optimal rotation age will be given that production function, what the resulting economic outcome (net-present value of the timber given

said rotation age minus invasion management costs) will be, and chooses the most advantageous option.

While we are hardly the first to model the impact of invasives on optimal forest/invasion management, to the best of our knowledge we are the first to answer these two inter-related questions in the specific context of invasive shrubs, whose impact on forest regeneration is unique from those of other invasive pests or disease. In addition to simultaneously solving the optimal invasion and forest management strategy, we examine how the optimal solution changes as a function of the impacts of impairment on forest regeneration (via both delayed growth and reduced juvenile survival), and the population dynamics of the invasion (initial invasion level, local growth rate, and inbound dispersal rate).

As an application, we consider the case of glossy buckthorn (*Frangula alnus* Mill.) in Sitka spruce (*Picea sitchensis*) forests. Glossy buckthorn is a non-native and invasive shrub in North America that uses its shade tolerance to colonize forest understory and inhibits or delays forest regeneration (Fagan & Peart, 2014; Lee, Eisenhaure & Gaudreau, 2016).

The paper is outlined as follows. Section II reviews the related existing literature in economics and ecology and outlines this paper's contribution. Section III presents the model, including both the biological and economic components. Section IV outlines the numerical framework used to generate our results. Section V presents the results of our analysis, with Section VI providing a discussion and conclusion.

Noteworthy findings are as follows:



- Unlike previous models (those focusing on invasive insects and pathogens rather than shrubs, e.g. Macpherson et al., 2016, Sims 2011), which find that invasion reduces the optimal rotation age, we find that for the case of invasive shrubs, invasion increases the optimal rotation age to compensate for stunted juvenile growth.
- Using ecological data from two independent studies (Fagan & Peart, 2014; Lee, Eisenhaure & Gaudreau, 2016) on the impacts of the invasive shrub glossy buckthorn on forest regeneration in the American northeast and an annual discount rate of 0.03, we find that buckthorn has the potential to reduce the net present value (NPV) of a forest plot by 71.5%. This loss in NPV is the combined result of a 2/3 reduction in the volume of merchantable timber caused by increased juvenile mortality, with the additional roughly 3.8% loss resulting from a 5-year delay in becoming merchantable, leading to a 5-year increase in the optimal rotation age.
- We show how our modeling procedure can be used to calculate the economic damages (lost timber revenue) resulting from the invasion in such a way as to allow one to decompose these damages into three distinct categories with separate policy implications: those caused by the invasion level at the time of planting, those caused by the spread of these initial invaders, and those caused by inbound invader dispersal from neighboring properties. This ability to compare the internal and external damages resulting from invasion and thus the internal and external benefits of invasion management suggests that our modeling procedure may also prove useful in an optimal policy design setting where determining the optimal invasion management subsidy, for example, requires a comparison of the internal and external benefits of invasion management.

- We use our model to optimize over a discrete bundle of integrated invasion management strategies – namely, ‘no control’, ‘prevention only’, ‘removal only’, and ‘prevention and removal’ and show how the optimal invasion management strategy changes as a function of the initial invasion level, local growth rate, and inbound dispersal rate, for a given set of impairment parameters.

In addition to serving as a template for further research, the modeling framework presented herein can serve as a valuable decision support tool for those seeking to best respond to the threat of invasive shrubs, while also providing novel insights into an ecologically unique form of forest disturbance.

## II. BACKGROUND

In this section we describe the related literature on the economics of invasive species management and position the model developed in this paper with respect to existing models in forest economics and invasion ecology.

The economics of invasive species management is a branch of bioeconomics that seeks to determine the optimal allocation of resources in combating the negative impacts of invasive species.

There are generally three distinct types of economic agents, or decision makers, found in this literature: policy makers, public land managers, and private land managers (with policy makers often relying on models of public and private land managers). In this paper we focus primarily on the decision making of a private land manager, specifically a timber producer whose objective is to maximize the net present value of their forest by minimizing the private damages caused by the presence of invasive shrubs within their

forest. As such we are explicitly ignoring both non-market damages (e.g. reduced provisioning of ecosystem services, biodiversity loss), and external damages experienced by others, though our model could include such values.

Resources are generally allocated across a range of potential invasion management activities, for example invasion prevention, control, removal, and/or monitoring, as well as adjusting production decisions such as the forest rotation age. In this paper we consider two distinct but interdependent resource allocation problems. First, we model how a producer determines (or adjusts) the optimal time at which to harvest the forest (rotation age) given the presence of an invasive shrub that negatively impacts timber regeneration by delaying or inhibiting juvenile tree growth. Our producer also chooses among a discrete set of potential invasion management strategies which have the effect of removing a proportion of the invasion prior to planting and/or reducing the invasion's rate of spread. We model these two choices as being simultaneously made immediately prior to planting. The model generates population dynamics under each possible option and determines the optimal rotation age and identifies the most cost-effective strategy.

This paper does not take up the issue of invasion detection and/or monitoring, assuming instead that the producer has perfect information regarding current and potential population dynamics, i.e., invasion levels in each period (including future periods) under each potential management option. The model also assumes that the inbound seed dispersal rate is constant across time and is independent of the location, actions, and conditions of neighboring properties. As such our bioeconomic model is deterministic and does not consider decision-making under risk or uncertainty. Our

modeling approach is also spatially implicit: we model the level of invasion in each period as the percent of the total area that is invaded and is equal to the quantity of the invasive shrubs within the forest divided by the forest's carrying capacity for that shrub. We likewise take the percent of trees invaded as equal to this percent of area invaded under the assumption that trees are planted with even spacing.

Our modeling approach is most unique in that it represents the invasion features, damages, and control that are characteristic of forest invasive shrubs, which are not readily accounted for in other invasive species bioeconomic models. While invasive pathogens (e.g. chestnut blight and sudden oak death) and pests (i.e., invertebrates such as emerald ash borer and mammals such as wild hogs) typically affect timber production by affecting trees directly (e.g. reducing the quality of timber or otherwise harming trees), invasive plants typically affect timber production *indirectly* by outcompeting tree seedlings and saplings for essential resources such as land and sunlight. The effect of this reduction in available space leads to increased seedling mortality, while the reduction in available sunlight results in established saplings experiencing reduced rates of growth prior to outgrowing their invasive competition (see Introduction for more details). Thus, unlike the case of invasive pests or pathogens, the impact of an invasive shrub on the output of a mature forest is not a function of current (or even lagged) invasion levels, but rather are a function of the percentage of trees that were invaded prior to escaping impairment, as well as the distribution of when these trees became invaded. From a discrete-time perspective, the result is an impairment-class structured timber production function comprised of various sized cohorts of trees, each of which face a unique level of

impairment, which we model via a modified age-structured timber production function that also accounts for increased juvenile mortality.

Despite the increasing abundance of invasive plants, particularly in the forest of the American northeast (Maynard-Bean & Kaye, 2019), we do not know of other work on invasive species that accounts for the type of heterogeneous juvenile impairment caused by invasive shrubs into a bioeconomic optimization framework.

The closest studies that look at the impact of invasion on the optimal harvest timing are Sims (2011) and Macpherson et al. (2018). Sims (2011) examines the optimal timing of salvage harvest in response to a stochastic infestation, using a real options approach to determine the level of infestation at which a salvage harvest to recoup timber losses is triggered. More in line with our rotation age analysis is that of Macpherson et al. (2018) who look at the effect of an invasive pathogen on the optimal forest rotation by combining a Faustmann rotation model with an epidemiological compartmental model of timber impairment. While their applications are quite different, both studies find that as the damages from bioinvasion increase, the optimal harvest time decreases. A key feature of both of these models is that damages increase over time as the bioinvasion spreads such that waiting an additional year increases the total damages. In contrast, we model invasion as only impairing the production of juvenile trees such that by the time the forest has become merchantable the damages from invasion have become fully realized.

Many more studies have looked at the optimal control of bioinvasion. The general findings from this literature are:

- Invasions that are expected to spread quickly generally warrant more control.

- Invasion removal may be optimal when invasion levels are low, with the optimal control switching to slowing the spread of invasion (and eventually abandonment) as the level of invasion increases.
- Regarding the timing of control, in the face of a fast-spreading invasion, rapid invasion management is generally preferred to a “wait and see” approach, even when faced with invasion uncertainty (Sims and Finnoff 2013; Sims et al. 2016).
- However, where invasion control is irreversible, ecological uncertainty is high, and multiple landowners are affected by a spatial invasion externality, it might be socially optimal to ask the landowners first affected to adopt a wait-and-see approach (Sims, Finnoff, and Shogren 2018).

For a thorough review of the literature on the economics of bioinvasion management in forest contexts, we recommend Epanchin-Niell (2017). For additional reviews on the economics of invasive species policy and management, see Olson (2006), Gren (2008), and Finnoff et al. (2010).

From an ecological perspective, we use two independent studies looking at the effect of the invasive shrub glossy buckthorn on forest regeneration to parameterize our model, namely Fagan & Peart (2014), and Lee, Eisenhaure & Gaudreau (2016). Glossy buckthorn often grows in dense patches that crowd out sunlight, which Lee et al. refer to as thickets. They report the average height at the thicket’s tallest point across multiple sub-plots and find that glossy buckthorn generally grows to a height of approximately 10 ft, which we take to be the height at which a juvenile tree transitions to being insusceptible to additional impairment. Using their height growth data for trees grown in

the absence and presence of glossy buckthorn thickets, we estimate that the presence of glossy buckthorn at the time of planting increases the length of time required to reach this 10 ft. threshold by 5 years. Fagan & Peart studied how the presence of glossy buckthorn affects tree survival/juvenile mortality, from which we are able to calculate that trees planted and grown in the presence of glossy buckthorn have one third the chance of survival of those planted and grown in its absence, or that the presence of glossy buckthorn can increase juvenile mortality by an additional 66.6%.

### III. GENERAL MODEL

#### The Biological Model

##### *The Unimpaired Timber Production Function*

We model the volume of merchantable timber produced by an unimpaired, even-aged plantation forest at age  $t$  as

$$f(t) \tag{1}$$

For completeness, we define the volume of merchantable timber as the combined volume of all trees at or above some merchantable height, where this merchantable height is assumed to be greater than or equal to the invasive shrub's maximum height. We let the term  $T_I$  denote the age at which an unimpaired forest becomes merchantable, such that  $f(t) = 0$  for all  $t < T_I$ .

### *The Impaired Timber Production Function for a Single Impairment Class*

We model tree-level impairment from invasion as decreasing both the rate of juvenile tree<sup>6</sup> growth (via a reduction in a tree's 'effective age'<sup>7</sup>) and the probability of juvenile survival (and thus total forest density), both as a function of whether and when impairment occurs.

Letting  $a$  denote the age at which a tree's immediate environment becomes invaded<sup>8</sup>, and extrapolating back to the plot level, we can model the production of an equivalent evenly impaired<sup>9</sup> plot as

$$g(t, a) = (1 - \delta(a))f(t - \tau(a)) \quad [2]$$

where  $\tau(a)$  [Eq 3] represents a decrease in the rate of tree growth via a reduction in a tree's effective age such that a tree's effective age is equal to  $t - \tau(a)$ , while  $\delta(a)$  [Eq 4] represents the percent reduction in the probability of juvenile survival<sup>10</sup> and thus also represents the percent decrease in overall density of merchantable timber.

In order to account for the fact that trees are only susceptible to impairment from invasion when their height is below that of their invasive competition, we let  $\bar{a}$  denote the age at which the height of an unimpaired tree reaches that of the invasive plant, rendering it no longer susceptible to impairment<sup>11</sup>. We can thus describe  $\tau(a)$  and  $\delta(a)$  as follows:

$$\tau(a) \geq 0, d\tau/da \leq 0, \tau(a \geq \bar{a}) = 0 \quad [3]$$

$$\delta(a) \geq 0, d\delta/da \leq 0, \delta(a \geq \bar{a}) = 0 \quad [4]$$

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<sup>6</sup> For the purposes of this paper, we define 'juvenile' as referring to trees whose height falls below the expected height of the invasive shrub and are thus susceptible to impairment.

<sup>7</sup> Defined as the corresponding age of an equivalently sized, unimpaired tree. For example, a

<sup>8</sup> We assume that, once a tree has become invaded, it remains invaded until harvest.

<sup>9</sup> That is, all trees within the plot were invaded at the same age,  $a$ .

<sup>10</sup> Specifically, outliving impairment to become merchantable.

<sup>11</sup> Where invasive plant height is assumed to be a fixed constant.



This is to say that our impairment functions are constrained to be non-negative, and that they decrease as the age of impairment increases, reaching a value of zero once a tree has reached  $\bar{a}$ . For the sake of notational convenience, we let  $\bar{\tau}$  and  $\bar{\delta}$  represent  $\tau(\alpha = 0)$  and  $\delta(\alpha = 0)$ , or the maximum impairment levels.

### *Modeling Infestation Growth Dynamics*

Let  $y(t)$  represent the total percentage of forest area invaded, which we will refer to simply as the invasion level, in period  $t$ , where  $y_0$  denotes this invasion level at the time of planting. Because  $y(t)$  represent the total percentage of forest area invaded, it takes the constraint  $0 \leq y(t) \leq 1$ . When  $t$  is already specified and we wish to refer to the invasion level at some other point in time we follow the standard sub-script notation  $y_i = y(t = i)$ . For example,  $y_{\bar{a}}$  would denote percent area invaded at the time when unimpaired trees transitions from being a susceptible juveniles to being an insusceptible adults.

We model the invasion's spread as resulting from the product of invader seed deposits and the rate of seedling survival as

$$\dot{y}(t) = dy/dt = \sigma_0(t)z(t) \quad [5]$$

In the equation above,  $z(t)$  denotes the quantity of invader seeds deposited in  $t$  and is a function of local seed production and inbound seed deposits from offsite. The expression  $\sigma_0(t)$  denotes the probability of a invader seed germinating and surviving to adulthood and is assumed to be a function of  $t$  in at least as much as that it is bound by the constraint  $y(t) + dy(t)/dt \leq 1$ .<sup>12</sup> We impose the final constraint that  $\dot{y}(t) \geq 0$  to ensure that

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<sup>12</sup> Given  $y \in [0,1]$ , seedling survival must be equal to zero once  $y(t) = 1$ .

once a tree has become invaded, it remains invaded until harvest, allowing us to model impairment as based solely on the age at which a tree becomes invaded.

### *The Impairment-Class Structured Timber Production Function*

Combining our single impairment-class timber production function (Eq 2) and our marginal invasion growth function (Eq 5), we can now put forth our impairment-class-structured timber production function,  $F(t, y_0, \dot{y})$ , that allows for the potential of different cohorts of trees becoming invaded as different ages and thus facing heterogeneous levels of impairment. When  $a$  is continuous, this model of timber production under class-structured juvenile impairment from a single invasive shrub takes the form

$$\begin{aligned}
 F(t, y_0, \dot{y}) &= y_0 g(t, \alpha = 0) + \int [0, \bar{a}] \dot{y}(t = u) g(t, \alpha = u) du + (1 - y(\bar{a})) f(t) \quad [6 C] \\
 &= y_0 (1 - \delta(a = 0)) f(t - \tau(a = 0)) \\
 &\quad + \int [0, \bar{a}] \dot{y}(t = u) (1 - \delta(a = u)) f(t - \tau(a = u)) du \\
 &\quad + (1 - y_{\bar{a}}) f(t)
 \end{aligned}$$

In the equation above, the first expression,  $y_0 g(t, \alpha = 0)$ , denotes the total volume of merchantable timber available in time  $t$  produced by trees that were impaired at the time of planting, i.e.  $\alpha = 0$ , while the last expression,  $(1 - y_{\bar{a}}) f(t)$ , denotes the total volume of merchantable timber available in time  $t$  produced by unimpaired trees, where  $1 - y_{\bar{a}}$  represents the percent of trees that reached  $\bar{a}$  without becoming invaded. The middle integral term combines the total volume available in time  $t$  from all trees impaired at  $0 < a < \bar{a}$  by ‘continuously summing’ the percentage of trees impaired at each  $a$ ,  $\dot{y}(t = a)$ , multiplied by its corresponding impaired production function,  $g(t, \alpha)$ .

This continuous model can be just as easily (and perhaps more intuitively) expressed in its discrete form as

$$\begin{aligned}
F(t, y_0, \dot{y}) &= y_0 g(t, \alpha = 0) + \sum [i=1, \bar{a} - 1] (y_i - y_{i-1}) g(t, \alpha = i) + (1 - y_{\bar{a}-1}) f(t) \quad [6 D] \\
&= y_0 (1 - \delta(a = 0)) f(t - \tau(a = 0)) \\
&\quad + \sum [i=1, \bar{a} - 1] (y_i - y_{i-1}) (1 - \delta(a = i)) f(t - \tau(a = i)) \\
&\quad + (1 - y_{\bar{a}-1}) f(t)
\end{aligned}$$

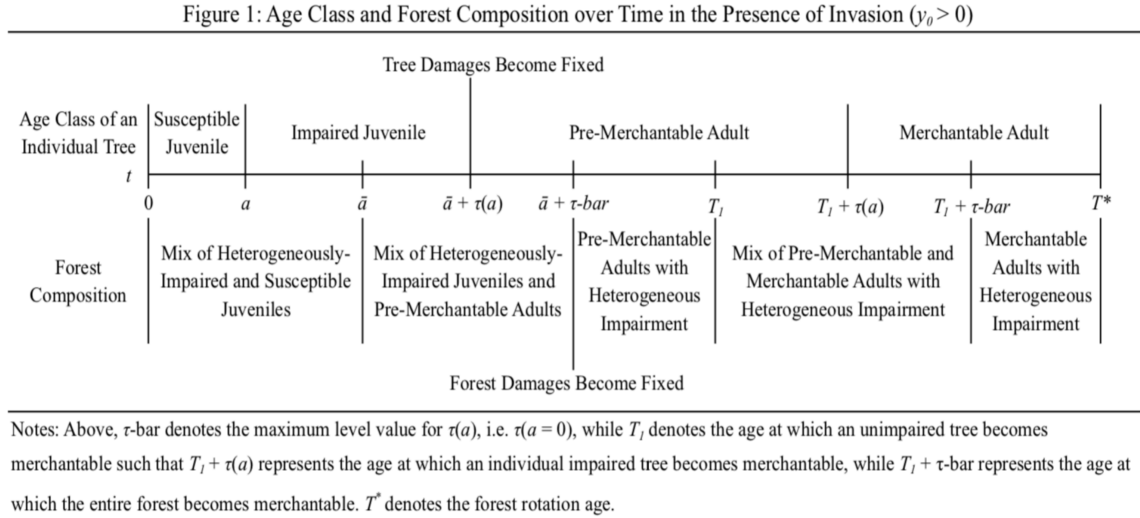
While the first part of this expression,  $y_0 g(t, \alpha = 0)$ , remains unchanged, the second and last terms have both been modified. In our final term,  $f(t)$  is now multiplied by  $1 - y_{\bar{a}-1}$  rather than  $1 - y_{\bar{a}}$ . This change is to account for the fact that in a discrete-time setting,  $\bar{a} - 1$  represents the last period in which unimpaired trees are susceptible to impairment and thus  $1 - y_{\bar{a}-1}$  represents the percentage of trees that reach adulthood (insusceptibility) without becoming impaired. This same logic also applies to the middle expression. In addition to replacing the integral with a summation, the upper bound of summation is now  $\bar{a} - 1$  instead of  $\bar{a}$  to account for the fact that those invaded at  $\alpha = \bar{a} - 1$  represent the last cohort of impaired trees. This summation expression can be expressed in its generalized expanded form as  $(y_1 - y_0) g(t, \alpha = 1) + \dots + (y_{\bar{a}-1} - y_{\bar{a}-2}) g(t, \alpha = \bar{a} - 1)$ , where the ellipses represent the timber produced by all trees impaired in the intermediate periods.

### *Tree Age Class and Total Forest Composition*

In addition to belong to an impairment class, individual trees can also be described as belonging to one of three chronological age classes depending on its effective age, namely juvenile, pre-merchantable adult, and merchantable adult. A

juvenile tree is define as having an effective age below  $\bar{a}$  (i.e.  $t - \tau(a) < \bar{a}$ ); a pre-merchantable adult is define as having an effective age at or above  $\bar{a}$ , but below  $T_l$  (i.e.  $\bar{a} \leq t - \tau(a) < T_l$ ); a merchantable adult is define as having an effective age greater than or equal to  $T_l$  (i.e.  $t - \tau(a) \geq T_l$ ).<sup>13</sup>

While the result of this age-class-structure on the total forest composition is to produce a mixed-age-class forest for certain values of  $t$ , once  $t$  reaches  $T_l + \bar{\tau}$ , the forest will have evolved into a single-age-class forest comprised entirely of merchantable adults. It is important to note that even when the forest is comprised of a single age class, it will still be comprised of trees belong to multiple impairment classes so long as  $y_0 < 1$  and  $y(t) > 0$  for some  $t \leq \bar{a}$ . For a graphical illustration of how both the age class of an individual tree, as well as the total forest composition, evolve over time see Figure 1.



<sup>13</sup> For the case where  $T_l = \bar{a}$ , all adults are merchantable and there is no longer an intermediate pre-merchantable adult stage.

## The Economic Model

### *Present Value of Timber and Optimal Rotation Age*

In their analysis of the effect of disease on the optimal forest rotation, Macpherson et al. (2016, and henceforth referred to simply as ‘Macpherson’) develop a single rotation version of the Faustmann model for an even-aged forest where the net present value (NPV) of a forest plot includes the cost of planting, the benefits of harvesting, and a payment for future land rent (which is received every year after harvesting). To ensure that any differences between our results and theirs are the result of changes in the ecological framework and not the economic framework, we use this same model with only minor cosmetic changes. The rationale behind using a single rotation Faustmann model, rather than the more traditional infinite rotation Faustmann model is that once the timber production function has been modified to include the invasion level at the time of planting,  $y_0$ , there is no longer a single optimal rotation age across all rotations due to the fact that different rotations are likely to have different values for  $y_0$  such that the optimal rotation age will differ from rotation to rotation depending on the long-term spread of the invasion.

In line with Macpherson, we assume that planting costs, timber revenue, and rent payments are constant and scale linearly with the size of the plot,  $H$ . Letting  $T$  denote the rotation length (harvest age), planting costs are equal to  $H \times C_0$ , timber revenue is equal to  $H \times p \times F(T, y_0, \hat{y})$ , and sum of future land rents are equal to  $H \int [T, \infty] A dt$ . Here,  $C_0$ , denotes the planting costs,  $p$  denotes the per-cubic-meter price of standing timber, and  $A$  denotes the annual rent payment received in each year *after* harvest, all three of which are

assumed constant and known. Letting  $\pi$  denote the forest manager's annual exponential discount factor (applied to both revenue and rents), we can write the net present value at the time of planting for an  $H$  sized forest with rotation age  $T$  as

$$NPV(T, y_0, \dot{y}) = H[-C_0 + pF(T, y_0, \dot{y})e^{-\pi T} + \int [T, \infty] Ae^{-\pi t} dt] \quad [7]$$

In order to solve for the value of  $T$  that maximizes the NPV, which we denote as  $T^*$ , we can take the first-order condition by differentiating Eq 7 with respect to  $T$ . By setting this first-order condition (FOC) equal to zero, we can express the FOC as a function of  $p$ ,  $\pi$ ,  $A$ , and  $F(T^*, y_0, \dot{y})$  as

$$dF/dT|_{T=T^*} = \pi F(T^*, y_0, \dot{y}) + A/p \quad [S1]$$

According to this optimality condition, the optimal rotation age is independent of both  $C_0$  and  $H$ . This is not surprising since planting costs are independent of  $T$  and  $NPV$  scales linearly with  $H$ . Henceforth, we therefore set  $H = 1$  and  $C_0 = 0$ .<sup>14</sup> As described by Macpherson, this condition also implies that  $T^*$  is determined by balancing the marginal gains from waiting to harvest (left-hand side) with the marginal cost of waiting (in terms of both discounted future revenue and foregone rent payments, right-hand side). We can also see that just as an increase in  $\pi$  will lead to a reduction in  $T^*$  by increasing the marginal cost of waiting to harvest, it also tells us that as  $A$  increases,  $T^*$  will (at least weakly) decrease as well, a finding that Macpherson points out is in line with previous studies (e.g. Amacher et al. 2009).

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<sup>14</sup> Planting costs are not set to zero simply because they are a sunk cost that does not factor in the harvest decision, but also to be in line with Macpherson who take the net cost of planting to be zero on the basis that the gross cost of planting is the same as the government subsidy payment available for Woodland Creation in the form of an initial planting payment (see Macpherson et al. 2016 for more details).

Macpherson goes on to show that when the timber production function is defined as an increasing, concave function, then the second derivative of Eq 7 evaluated at  $T^*$  is negative, telling us that there is a single solution for  $T^*$  and that this value corresponds to the global maximum of the NPV function, i.e. we know that  $T^*$  is indeed the rotation age that maximizes the NPV.

Before moving on, we define the expression  $NPV^*(y_0, \dot{y}(t))$  as denoting the plot's maximum net present value given the invasion level at the time of planting,  $y_0$ , and the rate of invasion spread over time,  $\dot{y}(t)$ .

### *Economic Damages from Invasion*

In order to calculate the economic damages resulting from the invasion, denoted as  $Dam(y_0, \dot{y})$ , we let  $T^U, NPV^U$  represent the optimal rotation age and the resulting net present value in the absence of invasion; specifically,  $T^U = T^*$  when  $y_0 = 0, \dot{y}(t) = 0$ ,  $NPV^U = NPV(T^U, y_0 = 0, \dot{y}(t) = 0)$ . Our damage function than thus be expressed as

$$Dam(y_0, \dot{y}(t)) = NPV^U - NPV^*(y_0, \dot{y}(t)) \quad [8]$$

which is the reduction in the plot's NPV as a result of the invasion level at the time of planting,  $y_0$ , and the rate of invasion spread over time,  $\dot{y}(t)$ . This damage function can also be thought of as a producer's willingness-to-pay (WTP) for the permanent eradication of the invasion within the plot at the time of planting.

### *Optimal Integrated Invasion Management*

In the economic model laid out above  $NPV$  was the result of the choice variable  $T^*$ , where  $T^*$  was a function  $y_0$  and  $\dot{y}(t)$  (along with the other parameters discussed), both of which were assumed to be *exogenously* determined. We now expand the model to

consider the case where  $y_0$  and  $\dot{y}$  are *endogenously* determined based on the producer's integrated invasion management strategy. We define a producer's integrated invasion management strategy as a combination of  $r$  and  $m$ , where  $r$  represents the pre-planting invasion removal rate (percent reduction in  $y_0$ ), while  $m$  represents a reduction in seedling survivability (percent reduction in  $\dot{y}(t)$ , *ceteris paribus*) resulting from pre-planting invasion prevention/mitigation (e.g. ground cover).

### *Modeling the Effects of Invasion Management*

Starting with the effect of invasion removal, we express the invasion level at the time of planting given  $r$ ,  $y_0(r)$ , as

$$y_0(r) = (1 - r)y, \text{ where } y, r \in [0,1], \quad [9]$$

where  $y$  represents the pre-removal invasion level and  $r$  represents the removal rate (proportional).

We model the effect of  $m$  on  $\dot{y}(t)$  in much the same way. Recall that earlier we introduced Eq 5 as  $\dot{y}(t) = dy(t)/dt = \sigma_0(t)z(t)$ , where  $\sigma_0(t)$  represented the probability of a deposited invader seed surviving to adulthood. Just as we let  $y$  denote the pre-removal invasion level above, we let the term  $\sigma(t)$  to represent the natural (pre-prevention) seedling survival rate and model the effect of invasion prevention as reducing this natural seedling survival rate by  $m \times 100\%$  such that  $\sigma_0(t) = (1 - m)\sigma(t)$ . Substituting this new expression into Eq 5, we model the invader's marginal growth rate as a function of mitigation,  $\dot{y}(t, m)$ , as

$$\dot{y}(t, m) = (1 - m)\sigma(t)z(t) \quad [5.1]$$



### *Willingness-to-Pay for Invasion Management*

We define a producer's willingness-to-pay for a given management strategy  $(r, m)$  as being equal to the corresponding reduction in economic damages, i.e. the difference between the damages incurred when  $y_0 = y$ ,  $\sigma_0(t) = \sigma(t)$  and when  $y_0 = (1 - r)y$ ,  $\sigma_0(t) = (1 - m)\sigma(t)$ . Substituting each of these two scenarios into their own damage equation (Eq 8) and subtracting the latter from the former, we can express a producer's willingness to pay for  $(r, m)$  as

$$\begin{aligned} WTP(r, m \mid \phi) = & \text{Dam}(y_0 = y, \dot{y}(t) = \sigma(t)z(t)) \\ & - \text{Dam}(y_0 = (1 - r)y, \dot{y}(t) = (1 - m)\sigma(t)z(t)) \end{aligned} \quad [10]$$

where  $\phi$  denotes  $y$ ,  $\sigma(t)$ , and  $z(t)$ .

### *Measuring Producer Welfare from Invasion Management*

We can now move on to calculating the potential producer surplus (PS) obtained from  $r$  and  $m$ , which for our purposes we define as the difference between what the producer is willing to pay for  $r$  and  $m$ , minus the actual cost.<sup>15</sup> Given some management cost function  $c(r, m)$ , we can express the corresponding potential producer surplus as

$$PS(r, m \mid \phi) = WTP(r, m \mid \phi) - c(r, m). \quad [11C]$$

### *Optimal Invasion Management Strategy*

We now move on to examine a producer's optimal integrated invasion management strategy, which we define as the combination of  $r$  and  $m$  that maximizes  $PS$ . We begin by presenting the optimal solution for the case where  $r$  and  $m$  are continuous choice

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<sup>15</sup> While this definition traditionally applies to the concept of consumer surplus, in this case the timber producer is also a consumer of invasion management. As we refer to the forester as a timber producer through the paper, we refer to the positive welfare achieved via invasion management as producer surplus rather than consumer surplus to avoid confusion.

variables, before moving on to the more realistic case where potential values of  $r$  and  $m$  are constrained to be members of a discrete choice set.

When  $r$  and  $m$  are both continuous choice variables between 0 and 1, the optimal invasion management strategy can be expressed using optimization notation as

$$(r^*, m^*) = \operatorname{argmax} \{r, m \in [0,1]\} PS(r, m \mid \phi). \quad [\text{S2.1}]$$

In reality, however,  $r$  and  $m$  are unlikely to be continuous choice variables. Rather, producers typically face a finite set of potential management strategies they must optimize across, each with its own associated cost. Given a finite choice set  $\{(r_1, m_1, c_1), \dots, (r_N, m_N, c_N)\}$  with choice index  $n$ , we can express the potential producer surplus obtained from management strategy  $n$  as

$$PS(n \mid \phi) = WTP(r_n, m_n \mid \phi) - c_n \quad [\text{11D}]$$

In this discrete choice setting, the optimal solution becomes  $n^*$ , which can be written using optimization notation as

$$n^* = \operatorname{argmax} \{n \in [N]\} PS(n \mid \phi) = WTP(r_n, m_n \mid \phi) - c_n. \quad [\text{S2.2}]$$

#### *A Conditional Formulation of the Optimal Invasion Management Strategy*

Thus far we have argued that our modeling procedure provides timber producers with a generalizable framework for determining the optimal response to the presence of an invasive shrub that impairs timber regeneration. We are well aware, however, that while our modeling procedure may be tractable to the academic/research community, for many real-world timber producers there are likely to exist significant barriers in achieving the calculation we have put forth, and that doing so will likely require the help

of an extension specialist. We therefore devote this concluding sub-section to a discussion of how our model might best be used from an extension perspective.

In the proceeding pages we have outlined how to determine the optimal invasion management strategy from a discrete set of strategies as a function of the pre-management invasion level,  $y$ , whose value depends on individual planting/harvesting schedules and is therefore likely to vary across producers, and a number of functions, the other parameters values of which are likely to be comparable for all producers in similar economic and ecological conditions (e.g.  $F(t, y_0, \dot{y})$ ,  $\sigma(y(t))$ ,  $z(y(t), \beta)$ ,  $\tau(a)$ , and  $\delta(a)$ ). All this is to say that within a homogenous cohort of timber producers, the optimal choice among a fixed set of invasion management strategies for a *single* producer will depend primarily on that producer's level of invasion as they prepare for planting,  $y$ . Thus, from an extension point of view, it might prove most useful to present a given population of producers with a decision-support tool in the form of a table that presents the corresponding optimal invasion management strategy across various ranges of  $y$ .<sup>16, 17, 18, 19</sup>

In its generalized formulation, this simplified decision rule reduces to

$$n^* = n \text{ if } y \in y_n^* \quad [\text{S2.3}]$$

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<sup>16</sup> For example, implement Strategy 1 when  $y < 0.3$ , implement Strategy 2 when  $0.3 \leq y \leq 0.7$ , and implement Strategy 3 when  $y > 0.7$ .

<sup>17</sup> With the extension specialist aiding the producer in calculating  $y$  when need be.

<sup>18</sup> For the sake of mathematical rigor, consider again the case of  $N$  discrete management options. If we let  $y_n^*$  represent the set of all values of  $y$  for which  $PS_n(y) \geq PS_{n'}(y)$  for all  $n' \neq n$ , it must be the case that the union of all  $N$  sets of  $y_n^*$  will contain all values of  $y$  on the open interval  $(0, 1)$  and therefore contain the optimal management strategy for every possible value of  $y$ .

<sup>19</sup> (In response to the previous footnote) Although this formulation allows for the possibility of one or more values of  $y$  at which the producer is indifferent between two (or more) strategies due to the necessity of the weak inequality  $PS_n(y) \geq PS_{n'}(y)$ , this should rarely be an issue in practice due to both the continuous nature of  $y$  and the fact that either strategy can be considered equally cost-effective.

## IV. NUMERICAL FRAMEWORK

In this section of the paper we provide the explicit functional forms for the equations of our general model used to obtain our numerical results.

### *The Unimpaired Timber Production Function*

$$f(t) = \begin{cases} 0 & \text{if } t < T_l \\ V(1 - e^{b(t - T_l)}) + v_l & \text{if } t \geq T_l \end{cases} \quad [1.1]$$

In the expression above,  $T_l$  represents the age at which an unimpaired plot reaches a minimum merchantable height, while  $v_l$  represents the unimpaired volume produced at that height (before which the merchantable volume is equal to zero).  $V$  represents the maximum attainable volume beyond  $v_l$ , while  $b$  is the volume-growth parameter of the unimpaired timber production function. It is also important to point out that  $v_l$  and  $b$  both also account for natural tree mortality in the absence of invasion.

### *The Impairment Functions*

#### *Modeling Growth Delay*

As discussed in the previous section,  $\tau(\alpha)$  denotes the reduction in a tree's effective-age, given  $\alpha$ . Thus, rather than reaching said merchantable height at age  $T_l$ , as is the case in the absence of impairment, said tree instead reaches its merchantable height at age  $T_l + \tau(\alpha)$ .

Letting  $\bar{\tau}$  represent the maximum potential delay, i.e. the delay experienced by trees impaired at age zero, we can model  $\tau(\alpha)$  as follows:

$$\tau(\alpha) = \bar{\tau} (\bar{a} - \alpha)/\bar{a} \text{ if } \alpha < \bar{a}; \text{ else } 0 \quad [3.1]$$

where  $(\bar{a} - \alpha)/\bar{a}$  represents the proportion of  $\bar{\tau}$  experienced by the tree such that a tree invaded at age  $\bar{a}/2$  experiences half the delay of a tree invaded at age zero. Because  $g(t, \alpha) = 0$  for all  $t$  below  $T_I + \tau(\alpha)$ , and  $T_I$  is assumed to be greater than or equal to  $\bar{a}$ , we need not worry about calculating a tree's effective age prior to reaching merchantability, allowing  $\tau(\alpha)$  to be calculated independent of  $t$ .<sup>20</sup> Because  $\tau(\alpha)$  represents a reduction in a tree's effective-age, In addition, we constrain  $\tau(\alpha)$  to be non-negative.

### *Modeling the Reduction in Juvenile Survival/Adult Density*

We now move out to modeling the reduction in tree survivability as a function of  $\alpha$ , where  $\delta(\alpha)$  represents the additional juvenile mortality induced by impairment. Since the parameters of the unimpaired timber production function,  $f(t)$ , already account for natural mortality in the absence of invasion, we let  $s$  represent the relative survival rate (per-period) of an impaired and unimpaired tree such that  $s = \text{impaired survival rate} / \text{unimpaired survival rate}$ , or the percentage of natural mortality survivors that will also survive juvenile impairment in a given period.

Given that  $\tau(\alpha)$  represents an impaired tree's delay in reaching insusceptibility and recalling that  $\bar{a}$  represents the age at which an unimpaired tree reaches insusceptibility, the total length of time that an impaired tree will be actively impaired for will be equal to

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<sup>20</sup> This is to say that while it is not true that a tree's effective age is equal to  $t - \tau(\alpha)$  prior to achieving insusceptibility, because a tree's merchantable volume is constrained to be equal to zero prior to achieving insusceptibility, we need only concern ourselves with the growth delay experienced by merchantable timber. Furthermore, because trees are no longer susceptible to impairment by the time they reach merchantability, the reduction in said tree's effective age will remain constant over time. In other words, there is no difference between a tree's merchantable volume prior to reaching merchantability regardless of  $\tau(\alpha)$ , and once merchantability (and thus insusceptibility) has been reached,  $\tau(\alpha)$  stays constant until harvest.

$\bar{a} - \alpha + \tau(\alpha)$  when  $\alpha < \bar{a}$ ; else 0. We therefore express the total mortality caused by impairment as

$$\delta(\alpha) = 1 - s^{\bar{a} - \alpha + \tau(\alpha)} \text{ if } \alpha < \bar{a}; \text{ else } 0 \quad [4.1]$$

### *The Impaired Timber Production Function for a Single Impairment Class*

Plugging Equations 3.1 and 4.1 into Equation 2, we can rewrite  $g(t, a)$  in its fully expanded form as

$$g(t, a) = \begin{cases} s^{\bar{a} - \alpha + \tau\text{-bar} (\bar{a} - \alpha)/\bar{a}} f(t - \bar{\tau} (\bar{a} - \alpha)/\bar{a}) & \text{when } \alpha < \bar{a} \\ f(t) & \text{when } \alpha \geq \bar{a} \end{cases} \quad [2.1]$$

Before moving on, recall that earlier we defined the term  $\bar{\delta} = \delta(\alpha = 0)$ , or the maximum level of invasion-induced mortality. This can now be explicitly expressed as  $\bar{\delta} = 1 - s^{\bar{a} + \tau\text{-bar}}$ . The reason why we draw attention to the term  $\bar{\delta}$  is that given the limited data available on the impacts of many invasive shrubs, it may in fact prove easier in some cases to estimate  $\bar{\delta}$  than to estimate  $s$  directly. In such cases, once  $\bar{a}$  and  $\bar{\tau}$  have been estimated,  $s$  can be estimated as a function of  $\bar{\delta}$  as  $s = (1 - \bar{\delta})^{1/(\bar{a} + \tau\text{-bar})}$ . As such, this method could also be used to cross-validate estimates obtained from different data sets.

### *The Invasion Growth Model*

We next need to specify a functional form for  $\dot{y}$ . In Section III, we expressed the generalized form of  $\dot{y}$  as

$$\dot{y}(t, m) = (1 - m)\sigma(t)z(t) \quad [5.2]$$

where  $z(t)$  represented the total number of invader seeds deposited in the plot in time  $t$  and was a combination of local onsite seed dispersal and inbound seed dispersal from invaders on neighboring properties. This total number of deposited seeds was then multiplied by  $(1 - m)\sigma(t)$ , or the probability of a deposited seed surviving to maturity in the absence of invasion mitigation,  $\sigma(t)$ ,<sup>21</sup> multiplied by the reduction in invader seedling survival resulting from invasion mitigation,  $(1 - m)$ .

In order to present a functional form that is not only straightforward, but also contains all the components necessary for our sensitivity analysis, we make the following assumptions. First, that the average number of locally deposited seeds produced per area invaded is constant over time. Secondly, we assume that the rate of seedling survival is constant over all values of  $y(t) < 1$ , and it equal to 0 when  $y(t) = 1$ .<sup>22</sup> Lastly, in order to achieve consistent numerical solutions, we also make the simplifying assumption that the rate of inbound seed dispersal from neighboring invaders is constant over time. This framework allows us to distribute and combine terms, rewriting 5.1 as

$$\dot{y}(t, m) = (1 - m) (\beta_1 y(t) + \beta_2) \text{ when } y(t) < 1; \text{ else } 0 \quad [5.2 \text{ C}]$$

Above,  $\beta_1$  represents the quantity of locally deposited seeds that enter the invader population (in units of total area) per  $y(t)$ , making  $\beta_1$  the product of the local seed production rate per  $y(t)$  and the seed survival rate while. Similarly  $\beta_2$  represents the quantity of inbound seed that go on to enter the invader population (also in units of total area) in each period and is the product of the inbound seed rate and the seed survival

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<sup>21</sup> Which we said was equal to zero when  $y(t) = 1$ .

<sup>22</sup> This is in line with the most recent and comprehensive model of glossy buckthorn population dynamics, Swewczyk et al. (forthcoming).

rate). When  $y(t) = 1$ ,  $\dot{y}(t, m) = 0$ , which can be thought of as forcing  $\beta_I$  and  $\beta_2$  equal to zero by reducing the seed survival rate to zero.

When applying our model to a discrete-time setting,  $\dot{y}(t, m)$  can be expressed using first-difference notation as  $y_{t+1} - y_t = \min((1 - m)(y_t\beta_I + \beta_2), 1 - y_t)$ .

### *The Impairment-Class Structured Timber Production Function*

Having specified the explicit functional forms for  $g(t, \alpha)$ ,  $\tau(\alpha)$ ,  $\delta(\alpha)$ ,  $y_0(y, r)$ , and  $\dot{y}(t, m)$ , they can now be plugged into either the continuous or discrete-time version of our impairment-class-structured timber production function to obtain an explicit form for  $F(t, y_0, \dot{y} \mid r, m)$ .

In its continuous-time form,

$$\begin{aligned} F(t, y_0, \dot{y} \mid r, m) = & (1 - r)y(1 - \bar{\delta})f(t - \bar{\tau}) \\ & + \int [0, \bar{a}] \dot{y}(t = u, m)(1 - \delta(u))f(t - \tau(u)) du \\ & + (1 - y_{\bar{a}})f(t). \end{aligned} \quad [6.1 C]$$

In its discrete-time form,

$$\begin{aligned} F(t, y_0, \dot{y} \mid r, m) = & (1 - r)y(1 - \bar{\delta})f(t - \bar{\tau}) \\ & + \sum [i=1, \bar{a} - 1] (y_i - y_{i-1})(1 - \delta(i))f(t - \tau(i)) \\ & + (1 - y_{\bar{a}-1})f(t). \end{aligned} \quad [6.1 D]$$

In the discrete formulation, the total volume of timber produced is equal to the sum of the timber produced by a total of  $\bar{a} + 1$  impairment classes, each of which suffer their own unique level of impairment. For a comprehensive description of each impairment class using our baseline values of  $\bar{a}$ ,  $\bar{\tau}$ , and  $s$ , see Table 1.



Table 1: Description of Discrete Impairment Classes ( $\bar{a} = 10$ ,  $\tau\text{-bar} = 5$ ,  $s = 0.93$ )

Impairment Class	Age When Class Became Invaded ( $\alpha$ )	% of Forest Area Belonging to Class	Age When Class Reaches Adulthood	Effective Age in Adulthood ( $t - \tau(\alpha)$ )	Length of Juvenile Impairment (Years)	Juvenile Survival Rate		Density Reduction ( $\delta(\alpha)$ )
						As a function of $s$	Numerical Value	
1	0	$y_0$	15.0	$t - 5.0$	15.0	$s^{15.0}$	0.333	0.667
2	1	$y_1 - y_0$	14.5	$t - 4.5$	13.5	$s^{13.5}$	0.372	0.628
3	2	$y_2 - y_1$	14.0	$t - 4.0$	12.0	$s^{12.0}$	0.415	0.585
4	3	$y_3 - y_2$	13.5	$t - 3.5$	10.5	$s^{10.5}$	0.464	0.536
5	4	$y_4 - y_3$	13.0	$t - 3.0$	9.0	$s^{9.0}$	0.517	0.483
6	5	$y_5 - y_4$	12.5	$t - 2.5$	7.5	$s^{7.5}$	0.577	0.423
7	6	$y_6 - y_5$	12.0	$t - 2.0$	6.0	$s^{6.0}$	0.644	0.356
8	7	$y_7 - y_6$	11.5	$t - 1.5$	4.5	$s^{4.5}$	0.719	0.281
9	8	$y_8 - y_7$	11.0	$t - 1.0$	3.0	$s^{3.0}$	0.803	0.197
10	9	$y_9 - y_8$	10.5	$t - 0.5$	1.5	$s^{1.5}$	0.896	0.104
11	10	$1 - y_9$	10.0	$t$	0.0	$s^{0.0}$	1.000	0.000

Notes: Values for  $\bar{a}$ ,  $\tau\text{-bar}$ , and  $s$  are our baseline estimates on the effects of glossy buckthorn on forest regeneration and are estimated using data from Fagan & Peart (2014) and Lee, Eisenhaure & Gaudreau (2016).

## IV. NUMERICAL RESULTS

The parameter values used to obtain our numerical results are presented in Table 2. All parameters relating to unimpaired timber production are taken from Macpherson et al. (2018) and are based on a yield class 14 *Picea sitchensis* without thinning and with 2-m initial spacing.<sup>23</sup> Our baseline estimates for the impairment parameters  $\bar{a}$ ,  $\bar{\tau}$ ,  $\bar{\delta}$ , and  $s$  were estimated using data from Fagan & Peart (2014) and Lee, Eisenhaure & Gaudreau (2016) who both analyzed the effect of the invasive shrub glossy buckthorn (*Frangula alnus* Mill.) on eastern white pine forest regeneration using independent data sets. These parameter estimates, as well as alternate values considered, are presented in Table 3.

Table 2: Unimpaired Timber Production Parameters

Parameter	Definition	Value
$H$	Area of forest	1 ha
$T_I$	Age when an unimpaired forest becomes merchantable	15 years
$v_I$	Volume produced at $T_I$	43 m <sup>3</sup> ha <sup>-1</sup>
$V$	Maximum attainable volume on top of $v_I$	1500 m <sup>3</sup> ha <sup>-1</sup>
$b$	Fitted timber production parameter	-0.01933
$p$	Standing price of timber	\$22.48 m <sup>-3</sup>
$\pi$	Discount rate	0.03
$C_0$	Forest establishment (planting) cost	\$0 ha <sup>-1</sup>
$A$	Future rent payments (annually, after harvest)	\$0 ha <sup>-1</sup>

Notes: All above values come directly from Macpherson et al. (2018). The price of timber has been converted from British Pounds to US Dollars. For additional information on how these parameters were estimated and why some are set to zero, see Macpherson.

<sup>23</sup> Like Macpherson, we set the land rent parameter equal to zero in order to more clearly determine the relative effects of our impairment and invasion parameters, as well as to allow for a direct comparison of their results and ours.

Table 3: Best Estimates and Alternative Values for Impairment Parameters

Parameter	Definition	Best Estimate	Alternative Values
$\bar{a}$	The age at which an unimpaired tree reaches adulthood (insusceptibility)	10	n/a
$\bar{\tau}$	Reduction in the effective age for trees invaded at the time of planting	5	0, 10
$\bar{\delta}$	Reduction in the overall juvenile survival rate for trees invaded at the time of planting	2/3	0, 1/3
$s$	Annual net-survival rate for an impaired juvenile	0.93	1, *

Notes: As previously discussed,  $\bar{\delta}$  and  $s$  are not independent estimates as  $s = (1 - \bar{\delta})^{1/(\bar{a} + \bar{\tau})}$ . Using this formulation,  $s$  depends not only on  $\bar{\delta}$ , but also on  $\bar{a}$  and  $\bar{\tau}$  so long as  $\bar{\delta} > 0$ ). Thus, in order to analyze the effects of invader-induced juvenile mortality in a way that is independent of  $\bar{\tau}$ , we consider alternate values of  $\bar{\delta}$  rather than  $s$ . For completeness, given  $\bar{a} = 10$  and  $\bar{\delta} = 1/3$ ,  $s$  is approximately equal to 0.96, 0.973, and 0.98 for  $\bar{\tau} = 0, 5$ , and 10, respectively. Also note that  $s$  denotes the annual survival rate net of the natural survival rate.

We begin by examining the effects impairment on the optimal forest rotation age, along with the subsequent economic damages. We then move on to demonstrate how our modeling framework can be used to optimize across a discrete set of invasion management strategies.

## Optimal Forest Rotation Age

### *Sensitivity to Impairment Parameters Under Static Impairment*

Figure 2

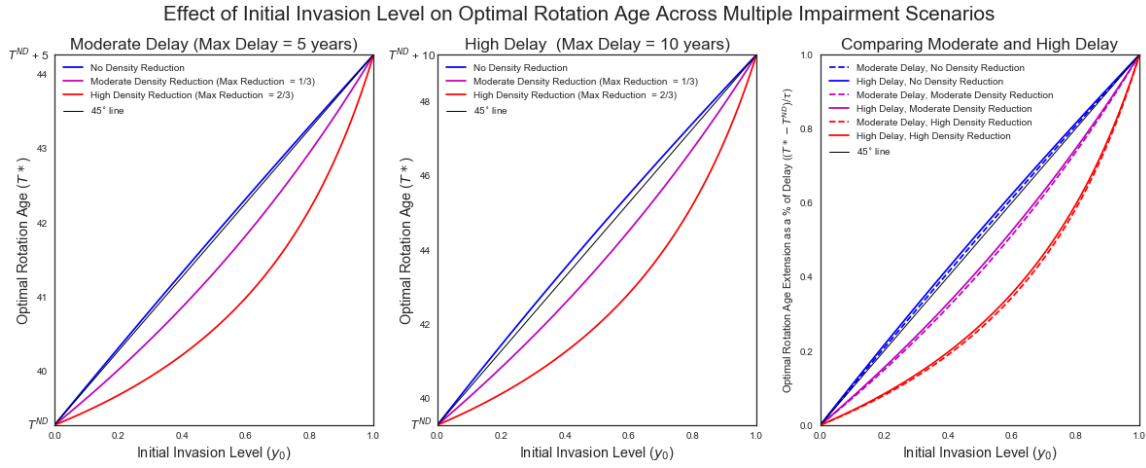


Figure 2 presents the optimal rotation age ( $T^*$ ) as a function of initial infestation level ( $y_0$ ) across six possible combinations of impairment parameters (described in terms of  $\bar{\tau}$  and  $\bar{\delta}$ ) for the case where invasion levels are assumed constant over time ( $\beta_1 = \beta_2 = 0$ , or  $y(t) = y_0$  for all  $t$ ). Though making such an assumption may be unrealistic in most cases, we do so temporarily in order to isolate the competing effects of delayed growth and reduced juvenile survivability in a way that is not compounded by the additional competing effects of  $\beta_1$  and  $\beta_2$ . The left-hand graph plots the optimal rotation age for the case where trees infested at the time of planting face a 5-year delay in reaching their merchantable height ( $\bar{\tau} = 5$ ), while the center graph plots the optimal rotation age for the case where trees infested at the time of planting face a 10-year delay in reaching their merchantable height ( $\bar{\tau} = 10$ ). For both delay scenarios, we consider three (3) potential

reductions in overall density, namely 1) trees infested at the time of planting face no reduction in density, 2) trees infested at the time of planting face a 1/3 reduction in density/survival ( $\bar{\delta} = 1/3$ ), and 3) trees infested at the time of planting face a 2/3 reduction in density/survival ( $\bar{\delta} = 2/3$ ).

While the left and center graphs compare the optimal rotation age across different density reduction values for a fixed delay parameter, the right-hand graph compares the effect of the growth delay parameter across each of the three density reduction scenarios. For an apples-to-apples comparison, we normalize this y-axis to be in units of  $(T^* - T^U)/\bar{\tau}$ , which is to say in terms of the extension in the optimal rotation age,  $T^* - T^U$ , as a fraction of the maximum delay. For all three (3) subplots we include a 45° line to aid the reader in determining the concavity/convexity of a given curve.

Starting our examining of these results with the left-hand graph, we see that when impairment reduces juvenile growth without reducing juvenile survival ( $\tau > 0, \delta = 0$ ), the extension in the optimal rotation age is concave over  $y_0$  ( $d^2T^*/dy_0^2 < 0$ ), though only barely so (as evidenced by the fact that the blue and black lines are nearly overlapping with the blue line being only slightly higher than the black 45° line for  $0 < y_0 < 1$ ). Thus, we find that in the absence of any density/survival reduction, the optimal rotation age in the case of static impairment is approximately equal to  $T^U$  plus the average delay, or  $T^* = T^U + y_0 \bar{\tau}$ .

Once we introduce a reduction in density/survival for impaired trees, however, this curve becomes convex over  $y_0$  ( $d^2T^*/dy_0^2 > 0$ ), with optimal extension in rotation age following below that of the average delay. This result makes sense intuitively due to the fact that the impaired cohort of trees will be producing less timber and thus their relative

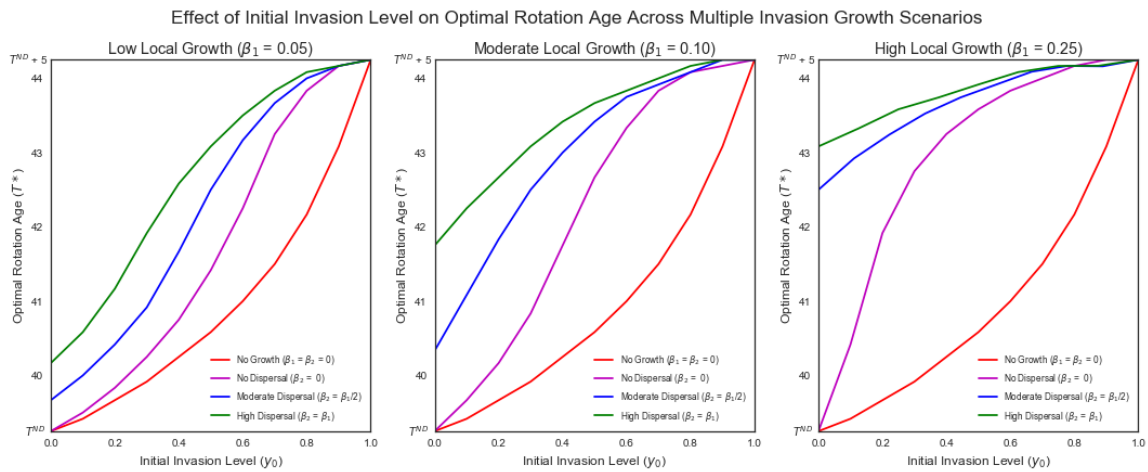
value will be reduced, i.e. less relative importance is placed on the impaired trees thus reducing their impact of the optimal rotation age.

At first glance, the center subplot looks nearly identical to the left-hand subplot, save that as  $\bar{\tau}$  has been doubled so has the range of the y-axis. However, overlaying these two subplots in units of  $(T^* - T^U)/\bar{\tau}$  allows us to see that this transformation is not entirely linear, but rather that increasing  $\tau$  in the center subplot caused  $T^*(y_0 | \tau, \delta)$  to become less convex/more concave for all three curves, though only very slightly (the result of a small Jensen's inequality). This tells us that the effect of  $\bar{\tau}$  on  $T^*$  is approximately linear such that doubling  $\bar{\tau}$  roughly doubles the additional time between  $T^U$  and  $T^*$ .

Lastly, as should be expected in the case of static impairment, across all six scenarios we observe that when  $y_0 = 0$ ,  $T^* = T^U$ , whereas when  $y_0 = 1$ ,  $T^* = T^U + \bar{\tau}$ .

### Sensitivity to Invasion Growth Parameters

Figure 3



In Figure 3 we abandon our earlier assumption of fixed invasion levels to examine the effects of increased impairment over time on optimal rotation age,  $T^*$ , as a function of initial invasion level,  $y_0$ , using our baseline estimates of  $\bar{\tau}$  and  $s$ . We consider three rates of local seed dispersal and survival,  $\beta_1$ , (from left to right 0.05, 0.10, and 0.25) in combination with three levels of inbound seed dispersal and survival,  $\beta_2$ , (0 (purple),  $\beta_1/2$  (blue), and  $\beta_1$  (green)). In each sub-plot, the lower red line serves as our baseline results from the case of static impairment (i.e.  $\beta_1 = \beta_2 = 0$ ). Whereas in Figure 2 we observed that when  $0 < y_0 < 1$ , an increase in the rate of mortality for impaired trees led to a decrease in  $T^*$  by reducing the volume produced by impaired trees and thus their relative importance, when we include the impact of increasing invasion levels over time we find that as the rate of local invasion growth ( $\beta_1$ ) increases, the optimal rotation age increases ( $dT^*/d\beta_1 > 0$ ) for all  $0 < y_0 < 1$ , with  $T^*$  increasing faster for lower levels of  $y_0$  and decreasing as  $y_0$  approaches 1. We also observe that while  $\beta_2$  has the similar effect of increasing  $T^*$ , it does so for all values of  $y_0 < 1$ . This is because the presence of inbound seed dispersal no longer means that  $T^*(y_0 = 0)$  is equal to  $T^U$ . For example, when  $\beta_1 = \beta_2 = 0.25$ , the optimal rotation solution when  $y_0 = 0$  is to extend  $T^U$  by almost 4 years (or almost 80% of the maximum delay of 5 years).

While the finding that the presence of invasive shrubs increases the optimal rotation length is intuitive given the underlying mechanics of impairment, this finding is in striking contrast to previous results found under other models of the effect of biological invasion (non-shrub) on optimal rotation length, specifically Macpherson (see Background section for more details). This is because traditional models of invasion-induced impairment model the damages caused by impairment as continuing to increase

into adulthood as the invasion continues to spread. Thus there becomes a tradeoff between increased growth and increased impairment that causes producers to harvest earlier than they otherwise would to avoid additional damages. In contrast to these models, our model accounts for the fact that the damages resulting from impairment remain fixed once all trees have become merchantable, thus eliminating this incentive to harvest early. Rather, the delay in growth has the effect of increasing the point in time when the marginal benefits from additional growth become equal to the marginal losses from additional time discounting.

The finding that the optimal response to the presence of invasive shrubs is to extend rather than reduce rotation length is important not just because adjusting harvest decisions in the face of an ecological disturbance is an essential tool in reducing the overall impact of that disturbance, but due to the fact that any determination of the optimal invasion management strategy implicitly relies on determining the optimal rotation length under each potential invasion management scenario<sup>24,25</sup>.

## **Economic Damages**

We now move on to examine the corresponding economic damages associated with the scenarios presented in Figure 3. Figure 4 shows the economic damages incurred by the producer as a result of the invasion population dynamics. In each graph, the red shaded area depicts the damages resulting from the initial invasion level,  $y_0$ , and is equivalent to the damages incurred under the static impairment model and are therefore

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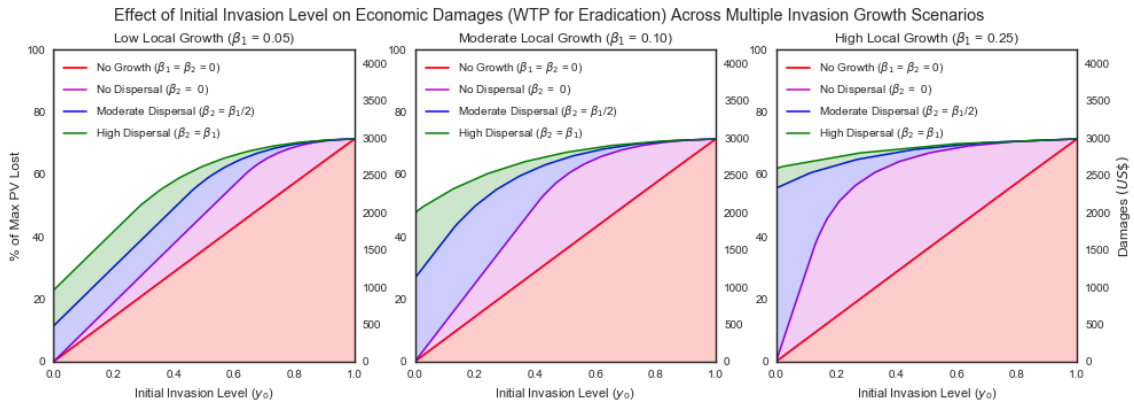
<sup>24</sup> Specifically, in order to compare one or more invasion management strategies, one must calculate NPV in each scenario, with each NPV equation taking its own  $T^*$  as a necessary input such that the solution for  $T^*$  is nested within the solution for the optimal management strategy.

<sup>25</sup> This is also true for calculating economic damages.



constant across all values of  $\beta_I$ . The purple shaded area represents the additional damages incurred as a result of strictly local invasion growth<sup>26</sup>, which can be thought of as the internalized spillover effects of  $y_0$ . The blue shaded area represents the damages incurred as a result of imported invaders in the case of medium dispersal and can be thought of as the negative externality imposed on the producer from inbound offsite invaders. The green area represents the additional damages from dispersal in the high dispersal case, such that the total damages resulting from high dispersal are equal to the combined height of the blue and green area, or the distance between the green and purple line.

Figure 4



Focusing on the externally-caused damages, we can observe that when  $y_0$  and  $\beta_I$  are relatively small, the marginal damages caused by  $\beta_2$  approximately double when  $\beta_2$  is doubled. However as  $y_0$  and/or  $\beta_I$  increase, the marginal effect of  $\beta_2$  decreases and may even become equal to zero (i.e. the points where the blue and green lines overlap). This results from the fact that when  $y_0$  and/or  $\beta_I$  are sufficiently high,  $y(t)$  will reach its upper-bound of 1 quickly enough that the effect of additional inbound seed dispersal becomes

<sup>26</sup> New invaders originating from those present in the time of planting.

minimal. Not surprisingly, increasing  $\beta_I$  has the affect of increasing damages for all  $0 < y < 1$ .

While all lines no longer share the same value at  $y_0 = 0$ , they still share the same level of maximum damages at  $y_0 = 1$ , which we can solve for explicitly in terms of % of  $NPV^U$  Lost as

$$\text{Max \% } NPV^U \text{ Lost} = 100\% \times (1 - (1 - \bar{\delta})e^{-r\bar{\tau} - bar}).$$

Using our best estimates of  $\bar{\tau}$  and  $\bar{\delta}$  for the shrub glossy buckthorn and assuming a discount rate of 0.03, we find that buckthorn has the potential to reduce the present value of a forest plot by 71.5%. This loss in NPV is the combined result of a 2/3 reduction in the volume of merchantable timber caused by increased juvenile mortality, with the additional roughly 3.8% loss resulting from a 5-year delay in becoming merchantable leading to a 5-year increase in the optimal rotation age and thus an additional five years of discounting.

What we find particularly noteworthy about Figure 4 are not its results *per se*, but rather in the ability to decouple the internal and external damages experienced by the forest manager. This is because while this paper focuses exclusively on the private decision-making of a single manager, this framework also has the potential to be used in an optimal policy-design setting as well. By expanding our model to consider multiple forest plots and including site-specific dispersal, it is possible to calculate not just the private benefits of invasion management for a specific site, but also the external (social) benefits of invasion management on that site as well. Such an ability is particularly useful for the case of optimal subsidy design since the socially optimal subsidy is the one that sets subsidies equal to the external benefits produced.

## Optimal Integrated Invasion Management Strategy

### *Optimal Strategy as a Function of $y$ Across 9 Invasion Growth Scenarios ( $\beta_1, \beta_2$ )*

In order to demonstrate how our modeling framework can be used to determine a producer's optimal invasion management strategy, we allow the producer to choose between a discrete combination of integrated management strategies, specifically 'no control', 'prevention only', 'removal only', and 'prevention and removal'. Parameter values ( $r, m, c$ ) for each strategy are outlined in Table 4. Recall that whereas in our previous analyses our results were a function of the invasion level at the time of planting,  $y_0$ , the optimal invasion management strategy is a function of the pre-management invasion level,  $y$ , where  $y_0 = y(1 - r)$ . Figure 5 presents the optimal invasion management strategy as a function  $y$  across the nine scenarios presented in Figures 3 and 4.

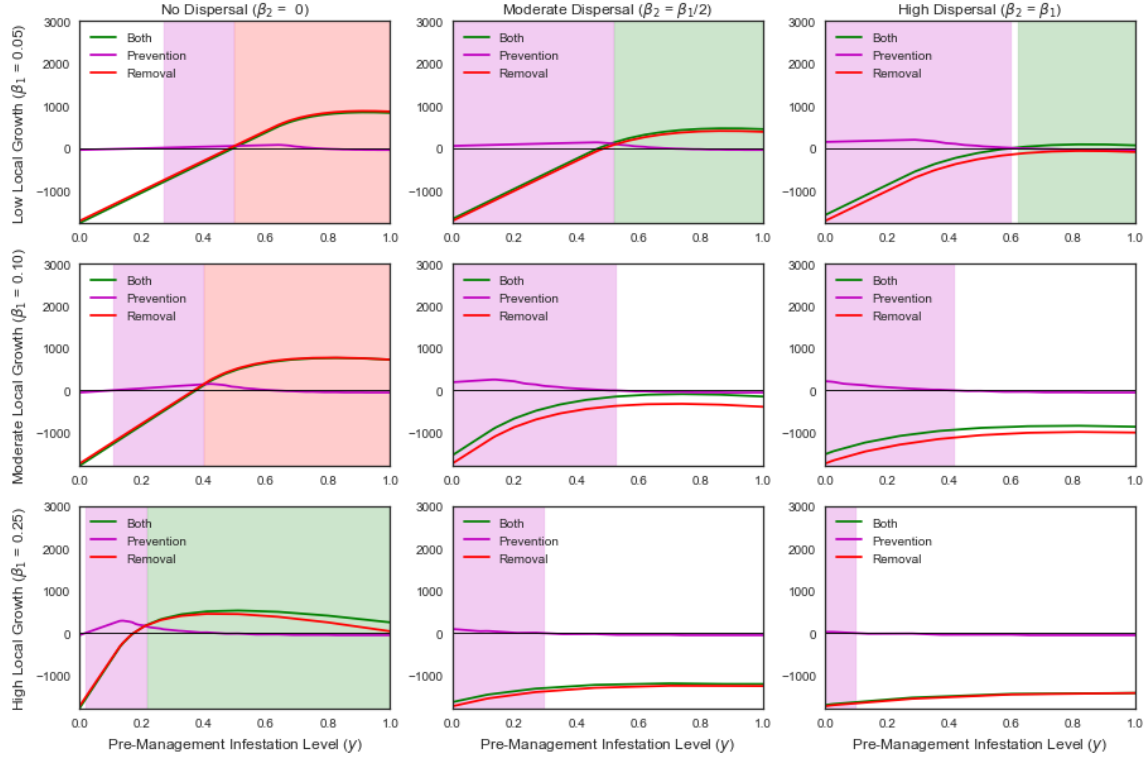
Table 4: Description of Potential Invasion Management Strategies

Management Strategy	$r$	$m$	$c$
No Control	0	0	0
Prevention Only	0	0.175	\$50 ha <sup>-1</sup>
Removal Only	0.9	0	\$1730 ha <sup>-1</sup>
Prevention and Removal	0.9	0.175	\$1780 ha <sup>-1</sup>

Notes: Removal and prevention rates sourced from Szewczyk et. al (forthcoming). Cost data sourced from Lee (2017). Costs have been converted from per acre to per hectare.

Figure 5

Effect of Pre-Management Invasion Level on Producer Surplus (US\$) and Optimal Invasive Management Strategy Across Multiple Invasion Growth Scenarios



Examining Figure 5, we observe that across all 9 scenarios, ‘prevention only’ is strictly preferred across some set of pre-management invasion levels,  $y$ . In the absence of inbound dispersal, the lower bound of this set will always be  $> 0$  due to the fact that without inbound dispersal,  $\text{Dam}(y_0 = 0) = 0$ . Once inbound dispersal is included, however, this lower bound quickly approaches 0, making invasion prevention a cost-effective strategy even when no invasion is present at the time of planting.

In terms of invasion removal, we find that as both  $\beta_1$  and  $\beta_2$  increase, the benefits of removal decrease such that it is no longer a cost-effective strategy regardless of the initial invasion level. This is shown by the lack of either a green or red shaded area in the four bottom-right graphs. This is not surprising given that when invasion growth is high,

those removed are quickly replaced, thus reducing the benefits of removal. When either  $\beta_1$  or  $\beta_2$  are low, however, there generally exists a level of  $y_0$  at which the optimal invasion management strategy switches from ‘prevention only’ to some form of removal (with or without prevention).

We find that when inbound dispersal is absent and local growth is low-moderate, ‘removal only’ is more cost-effective than ‘prevention and removal’ despite the relatively low cost of prevention. This is because in such cases removal alone is effective enough to significantly reduce the spread of invasion. However, as the rate of either local growth or inbound dispersal grows, this is no longer the case and the combination of ‘prevention and removal’ is preferred to ‘removal only’.

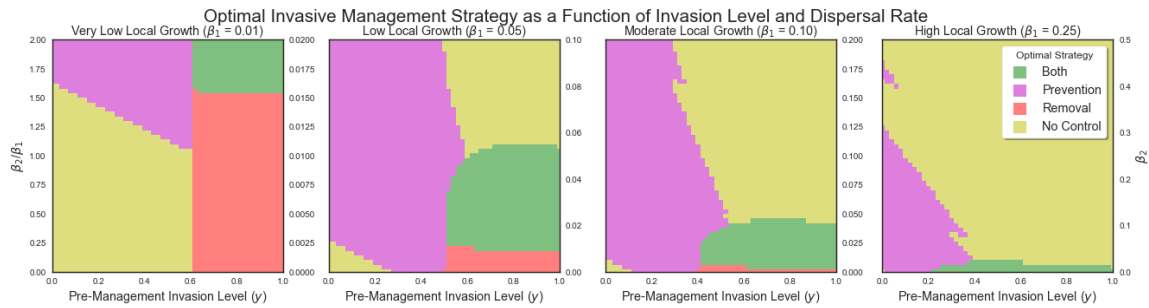
Interestingly, however, it is not always the case that the optimal strategy switches directly from prevention to some form of removal. As the Low-High scenario (top right sub-plot) demonstrates, there can exist an intermediary range of  $y$  where ‘no control’ is preferred, but below which ‘prevention only’ is preferred and above which ‘prevention and removal’ is preferred.

### *Optimal Strategy as a Function of $y$ , $\beta_1$ , and $\beta_2$*

In order to get a more comprehensive view of the effect of our invader population parameters on the optimal management strategy, we move on to plotting the optimal management strategy as a function of both  $y$  and  $\beta_2$  across the three local growth scenarios included above, plus an additional ‘very low local growth’ scenario ( $\beta_1 = 0.01$ ). These results are presented in Figure 6. In all four sub-plots, the x-axis represents the pre-management invasion level,  $y$ , with the left y-axis displaying  $\beta_2/\beta_1$  from 0 to 2, and the

right y-axis displaying  $\beta_2$ . In each sub-plot the purple shaded area represents the set of points where ‘prevention alone’ is preferred, the red shaded area represents the set of points where ‘removal alone’ is preferred, the green shaded area represents the set of points where ‘prevention and removal’ is preferred, and the yellow shaded area represents the set of points where ‘no control’ is preferred.

Figure 6

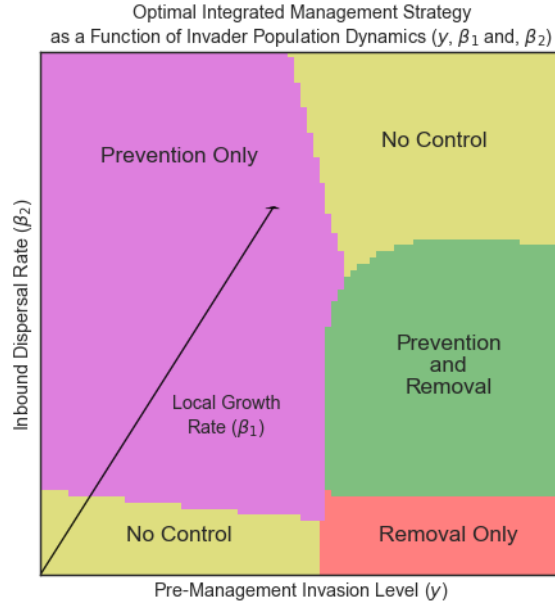


One of the first things we observe when looking at Figure 6 is that our purple, red, and green areas appear to be contiguous regions, whereas this is not the case for the yellow area (no control). This should be expected as these two distinct yellow regions represent two diametric outcomes. Specifically, while both regions represents cases where none of the potential reductions in damages exceed their corresponding cost, the bottom-left yellow region represents cases where the damages from invasion are low enough that they do not justify the cost of control. In contrast, the top-right yellow region represents cases where the spread of the invasion is so aggressive that none of the available forms of management are able to adequately contain the population and reduce their damages enough to justify its cost. In other words, the former represents cases where it is too early for control to be cost-effective, with the latter representing cases where it is too late for control to be cost-effective.

In comparing our four sub-plots, it would almost appear as if our sub-plots were overlapping puzzle pieces of some larger plot. Put another way, the plane shown in each sub-plot looks as if it could be a close-up snapshot of some larger general plane or super-plot. For example, it would appear as if a good section of the top right of sub-plot 1 overlaps with a good part of the bottom left of sub-plot 2. The same is true for sub-plots 2 and 3, and sub-plots 3 and 4, respectively.

By expanding the axes in the left sub-plot, we can plot an approximation of this ‘super-plot’, shown in Figure 7. While our x and y-axis still represent the pre-management invasion level and the rate of inbound invader seed dispersal, respectively, they no longer represent specific values. This is because as  $\beta_I$  increases, the position of corresponding pairs of  $(y, \beta_2)$  move as well such that the axis values (including the position of the origin) are themselves a function of  $\beta_I$ . This does not prevent us from observing how the optimal management strategy changes as a function of  $y$  and  $\beta_2$ .

Figure 7



By observing the approximate position of each sub-plot from Figure 6 within Figure 7 and examining how the position of corresponding points move as  $\beta_I$  increases, we find that as  $\beta_I$  increases, the point denoting the optimal management strategy generally moves steeply up and to the right.<sup>27</sup> One particularly illustrative example of this upward diagonal travel is the case of  $y = 0.45$ ,  $\beta_2 = 0$ . In this scenario the optimal strategy actually crosses through all four management options as  $\beta_I$  increases. As can be observed in Figure 5, when  $\beta_I = 0.01$ , the optimal strategy is ‘no control’. As  $\beta_I$  moves from 0.01 to 0.05, the optimal strategy moves up and to the right to become ‘prevention only’. As  $\beta_I$  moves from 0.05 to 0.10, the optimal strategy moves up and to the right again to become ‘removal only’. Finally, as  $\beta_I$  moves from 0.10 to 0.25, the optimal strategy moves up and to the right one last time to become ‘prevention and removal’.

<sup>27</sup> It is worth noting that no matter how large  $\beta_I$  gets, ‘no control’ will remain optimal for the case  $y = 0$ ,  $\beta_2 = 0$ .



Having demonstrated our ability to map how the optimal strategy changes not only as a function of  $y$  and  $\beta_2$ , but also  $\beta_1$ , it is possible for us to include the approximate effect of  $\beta_1$  in Figure 6. This is done by essentially overlaying a diagonal  $\beta_1$ -axis on top of our 2-dimensional plane, which we draw as a  $>1$ -sloped arrow-headed line from the origin.

While one way to interpret this line is as shifting the optimal strategy up and to the right, this line also has a dual interpretation. Recalling earlier that we said the units of the x and y-axis, including the origin, were actually functions of  $\beta_1$ , one can just as easily think of the effect of increasing  $\beta_1$  as shifting the origin up and to the right such that some portion of the bottom and left of our generalized plane of possible scenarios are no longer possible.<sup>28,29</sup> This idea of shifting the origin up and to the right (and adjusting the scale) as  $\beta_1$  increases rather than shifting the location of the optimal strategy directly can be seen by comparing the four sub-plots of Figure 6.

To conclude, Figure 7 provides a generalized template for examining the effects of multiple invader population dynamics parameters on the optimal integrated invasive shrub management strategy for a given set of impairment parameters.

## VI. CONCLUSION AND DISCUSSION

At its core, this paper provides a framework for answering the question of what is the most cost-effective response to the presence of an invasive shrub that reduces and delays timber growth within a plantation forest.

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<sup>28</sup> See previous footnote.

<sup>29</sup> With the axis-units being scaled if necessary.

We began by explaining how the case of invasive shrubs is unique from more traditional invasive species, and as such requires a unique modeling approach. Unlike the case of invasive insects or disease, the ecological evidence suggests that trees are only susceptible to the negative impacts of invasive shrubs while their height falls below that of their shrub competitors, with the effect that once trees have reached their merchantable height, they are no longer susceptible to these negative impacts. This is in contrast to an invasive insect or disease in which trees continue to become increasingly negatively impacted until they are harvested.

We developed a generalizable model for simultaneously determining both the optimal invasion management strategy (prevention and/or removal) and the optimal forest rotation age that combines a Faustmann single-rotation model, an age-structured impaired timber production function, and an ecological model of invasion dynamics. The model might also be described as an expanded Faustmann model in which the traditional timber production function is transformed as a function of the ecological characteristics of the invasive shrub (initial presence, growth, impact), but where the initial invasion level and/or invasion spread can be endogenously reduced by choosing from a discrete set of invasion management options (growth prevention and/or removal, inaction). In this setup, the forest manager considers how each invasion management option will affect the impaired timber production function, what the optimal rotation age will be given that production function, as well as the economic outcome (net-present value of the timber given said rotation age minus invasion management costs), before choosing the most advantageous option.

We explored the results of our model using parameter estimates of the effect of the invasive shrub glossy buckthorn on timber regeneration (increased juvenile mortality, delayed growth) as a baseline, examining how our optimal solutions change as an effect of not just this parameter set, but also as a function of the dynamics of the invasion (invasion level at the time of planting, local growth rate, inbound dispersal rate).

We provided a tractable approach for determining the optimal invasion management strategy as a function of a forester's site-specific invasion dynamics, for a known set of impairment parameters. We also showed that the optimal response to the presence of an invasive shrub is to increase, rather than decrease, the forest rotation age. This result is in contrast to previous results obtained under models of non-shrub invasives (Macpherson et al., 2018; Sims, 2011), and we believe that these opposing findings are evidence in favor of the implicit hypothesis of this paper - that the unique effects of invasive shrubs on timber production requires its own unique modeling framework.

To that point, this work has two implicit objections, in addition to answering the explicit research question. These were to demonstrate the need for a shrub-specific model of forest invasive species management, and to develop such a model in as generalizable a way as feasible given the level of complexity and number of moving parts. As such, we consider this work to be a starting framework for researchers interested in working on the economics of forest invasive shrub management.

One avenue of expansion that would be valuable in applying such a model to a public policy setting but is beyond the scope of this paper would be to expand our framework to be spatially explicit, rather than implicit, and expanding the number of foresters. As stated previously, our modeling framework allows one to decompose the

economic damages resulting from invasion into three distinct categories, namely those resulting from the initial invasion level, those resulting from subsequent local seed dispersal, and those resulting from off-site (external) seed dispersal. The introduction of additional foresters whose invasion management decisions endogenously affect inbound dispersal rates according to their location and who are additionally affected by the management decisions of others would allow policymakers to quantify the external benefits derived from invasive shrub management and design policies accordingly. As we know from the fundamentals of environmental economics, the socially optimal subsidy level for a transaction that produces an external benefit to others is equal to the value of that benefit. Thus a spatially explicit, multi-forester version of our model could allow policy makers to determine the optimal invasion management subsidy based on the ecological characteristics of a specific invasion and location, as well as the population.

Additional expansions to the model that might prove valuable include allowing for stochastic, rather than deterministic, invasion growth; considering solutions under an infinite, rather than a single, rotation, where the invasion management decisions in one rotation affect timber production in following rotations; expanding the invasion population model to allow for delayed population and seedbank dynamics to account for the facts that not all shrub seeds germinate immediately after being deposited but remain viable in a seedbank, that shrub removal is unlikely to affect the size of the seedbank, and that newly germinated shrubs may need to reach a certain age before they begin to impair/compete with juvenile trees (Jardine and Sandricho, 2018).

Despite its potential for expansion, we believe that the model as presented has the potential to serve as a valuable decision support tool for foresters and extension workers

seeking advice on how best to respond to the threat of invasive shrubs. However, in order for our model to provide case-specific recommendations, economists will have to work closely with forest ecologists/biologists to obtain accurate parameter estimates regarding the population dynamics of the specific invasive shrub in question, as well as how said shrub impairs timber production. This is in addition to quantifying the cost and effectiveness of potential invasion management strategies, which has the potential to pose its own set of challenges (Jardine and Sandricho, 2018).

It is our hope that this work can serve as a template both for those who wish to study the impacts of invasive shrubs on forest management, as well as those who wish to use bioeconomics to better model the specific mechanisms by which specific invasive species affect timber production and invasion management.

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