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Articles in the field of agricultural economics, suitable for publication in the journal, will be welcomed.

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# Contents

# A production-function analysis to determine marginal value products and optimum intensity of farms in the Swartland

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## 1. INTRODUCTION

The main purpose of this exposition is to determine the marginal value products of resources and their services in the Swartland. With due regard to all the limitations[1)] in respect of data and methods, an attempt will also be made to determine the effect of several combinations and inputs of resources on gross income and marginal productivities. It must be stressed that these analyses are based on the measurable standards of economic efficiency and resource productivity and that they bear no relation to the immeasurable and subjective aspects of farming.

If information is available from similar studies of other geographical regions, such figures can furnish a basis for (a) inter-region, (b) intra-region and inter-farm, (c) intra-region and intra-farm, and (d) intra-farm and inter-product comparisons regarding resource productivities.[2)] Such information is of great value for individual farming decisions as well as for formulating policy on national level. From the point of view of the individual, it indicates -

(1) the gross income which can be expected if various combinations of resources are applied in a specific region; and

(2) the profit or loss involved with the transfer of such resources between production regions.

From the point of view of those charged with formulating policy, it indicates -

(1) the difference in resource productivities between farms in a given region and the difference between agricultural regions, and

(2) the reasons for the differences in resource productivities.

1) Joubert J.S.G.: An economic basis for farm planning in the Swartland. Unpublished M.Sc (Agric.) thesis, University of Stellenbosch, p.70.

2) Heady E.A. and Shaw R.: Resource returns and productivity coefficients in selected farming areas of Iowa, Montana and Alabama, Agricultural Experiment Station, Iowa State College, Research Bulletin 425, p. 336-72

## 2. THE FARM FUNCTION

The necessary data for the calculation of the farm function were obtained from 30 farmers during a cost of production survey in the Swartland area. These data relate to the 1967/68 production year and the results for this single production season were utilised.

The Cobb-Douglas function was fitted and the following regression function was calculated for the farm as a whole:

$$Y = 3.21828 \; X_1^{.52339323} \; X_2^{.19421956} \; X_3^{.46391490} \quad \ldots\ldots\ldots (1)$$

The classification of the variables (inputs as well as outputs) is as follows:

$Y$ = gross income (R) - this figure includes all sales, household consumption of farm products, as well as appreciation of livestock.

$X_1$ = land (morgen) - all arable land.

$X_2$ = labour (R) - cash wages, clothing, rations purchased, medical expenses, repairs to houses and also all other monetary benefits.

$X_3$ = variable costs (R) - all cash disbursements necessary to enable the farm business to function, labour excluded.

A variance analysis was done and this indicated that all the calculated F values are statistically significant at a 5 per cent level of probability. The coefficient of determination, $R^2$, for this function is .92, which indicates that 92 per cent of the variance in Y is due to the influence of $X_1$, $X_2$ and $X_3$ in the year concerned and the deduction can be made that the function was of good fit.

## 3. MARGINAL VALUE PRODUCTS OF RESOURCES

The marginal value products of the basic resources were calculated from the farm function[3)] and are given in Table 1.

3) The marginal value products were obtained by utilising the first partial derivative of the production function in connection with the factors concerned.

TABLE 1 - Mean inputs of the production factors and the marginal value products thereof at this input level, Swartland, 1967/68

| | Land | Labour | Variable costs |
|---|---|---|---|
| | | (R) | (R) |
| Inputs (geometric mean*) | 748.37 (morgen) | 2 379.76 | 13 369.66 |
| Marginal value product per unit** | 26.69 | 3.12 | 1.33 |

* The geometric mean of the gross income Y, was R38 039.54.

** The units of the inputs were land = 1 morgen, labour and working costs = R1.

These marginal value products have reference to the "average farm", as determined from the details procured from the 30 farming units.

### 3.1 Marginal value product of land

Areas (farmyard and odd pieces excluded) in the sample investigated, varied from 293 to 1 665 morgen, with a mean of 748 morgen. As is evident from Table 1, the marginal value product of the 748th morgen was estimated at R26.69. This proves that the average farmer in the Swartland should not pay an annual hire in excess of this figure. As far as the owner is concerned, his costs per morgen to own land (interest on land plus tax) should not exceed this figure. The average annual factor cost of land was calculated at R12.51[4)] per morgen, which indicates that the optimum utilisation of land has not yet been reached.

TABLE 2 - Marginal value product of land in the Swartland with other resources constant at their geometric averages, 1967/68

| Land | Marginal value product of land |
|---|---|
| (morgen) | (R) |
| 374 | 37.16 |
| 561 | 30.63 |
| 748 | 26.73 |
| 1 122 | 22.01 |
| 1 496 | 19.19 |

It is clear from Table 2 that farmers with relatively small farms, can increase their income appreciably, by purchasing or hiring additional land.

The productivity of a resource is usually determined by the quantities of the other resources combined therewith. This is further illustrated in Table 3, in which the other resources are held constant at two different levels.

TABLE 3 - Marginal value product of land in the Swartland, with labour and variable costs constant at two different levels, 1967/68

| Land | Other resources constant at - | |
|---|---|---|
| | Half of the geometric mean | Twice the geometric mean |
| (morgen) | (R) | (R) |
| 374 | 23.55 | 58.63 |
| 561 | 19.41 | 48.33 |
| 748 | 16.94 | 42.18 |
| 1 122 | 13.95 | 34.73 |
| 1 496 | 12.16 | 30.28 |

It will be observed from Table 3 that, with the other resources constant at one half of the geometric mean, the marginal value product of, for example, the 374th morgen, is R23.55. The productivity is, however, increased to R58.63, if the other resources are maintained constant at a level equivalent to twice the geometric mean.

### 3.2 Marginal value product of labour

According to Table 1, the marginal value product of labour is fairly high, viz. R3.12. It is clear that, with a factor price of R1.00 as in the case of land, the utilisation of labour as an input factor, is below the optimum. In his study of East Griqualand, Kassier[5)] attributed this higher productivity of labour to the monopsonistic position of the operator, coupled with the fact that labourers probably prefer security rather than high wages.

As far as the Swartland is concerned, the aforementioned also applies since coloured labourers receive a relatively low remuneration as compared to the industrial sector.

Another important reason is the high measure of mechanisation. The grain industry, which is by far the largest contributor to the gross income, is highly mechanised as far as tillage and harvesting implements are concerned.

Table 4 shows that the marginal value product of labour is a function of the amount of labour employed.

Table 5 shows how the productivity of labour is influenced by the combination therewith of different quantities of other resources.

On a farm which expends R2 379 on labour and, with land and variable costs equal to one half of the geometric mean, the 2 379th rand will contribute R4.61 more to the gross income, if land and working costs are increased to twice that of the geometric mean.

4) The factor price of land is a combination of the interest on the value of 1 morgen, calculated at 7 per cent plus divisional council tax.

5) Kassier W.E.: A production function study of marginal returns and optimum intensity on East Griqualand farms. S. Afr. J. Econ., Volume 34 No. 2 June 1966, p. 161.

TABLE 4 - Marginal value product of labour, with land and variable costs constant at the geometric mean, Swartland, 1967/68

| Labour | Marginal value product of labour |
|---|---|
| (R) | (R) |
| 599 | 9.47 |
| 1 189 | 5.45 |
| 1 788 | 3.92 |
| 2 379 | 3.12 |
| 3 568 | 2.25 |
| 4 758 | 1.78 |
| 5 357 | 1.62 |

TABLE 5 - Marginal value product of labour, with land and variable costs constant at two different levels, Swartland, 1967/68

| Labour | Other resources constant at - | |
|---|---|---|
| | Half of the geometric mean | Twice the geometric mean |
| (R) | (R) | (R) |
| 599 | 4.78 | 18.77 |
| 1 189 | 2.75 | 10.80 |
| 1 788 | 1.98 | 7.78 |
| 2 379 | 1.57 | 6.18 |
| 3 568 | 1.13 | 4.46 |
| 4 758 | .90 | 3.53 |
| 5 357 | .82 | 3.21 |

3.3 Marginal value product of variable costs

The marginal value product of variable costs of the average farm, with average resource utilisation, is R1.33. Since the factor price here, as in the case of labour, is R1.00, it is clear that the optimum utilisation of variable costs, has not yet been attained. It must also be borne in mind that variable costs consist of a collection of various cost items and it may happen that fertilizers, for example, have a relatively high marginal value product, while the marginal value product of fuel may be lower. In Table 6 the marginal value product of variable costs is indicated as a function of the amount of variable costs used.

TABLE 6 - Marginal value product of variable costs, with land and labour constant at the geometric mean, Swartland, 1967/68

| Variable costs | Marginal value product of variable costs |
|---|---|
| (R) | (R) |
| 4 100 | 2.50 |
| 6 680 | 1.92 |
| 10 020 | 1.54 |
| 13 360 | 1.33 |
| 20 040 | 1.07 |
| 22 597 | 1.00 |
| 26 720 | 0.91 |
| 27 900 | 0.89 |

The annual utilisation of variable costs fluctuated between R4 110.79 and R27 808.32 for the 30 farms, with the mean amounting to R13 360.66. It is evident from Table 6 that, with the utilisation of the 22 597th rand, the factor price is equivalent to the marginal yield, provided the other resources are maintained constant at the geometric mean.

## 4. OPTIMUM INTENSITY

The sum of the regression coefficients $b_1$, $b_2$ and $b_3$ indicates the returns to scale. The sum of the elasticities of the input factors in the farm function amounts to 1.1815277, which exceeds 1.0 and which means that no long term optimum can be calculated[6]. The function therefore indicates that there are increasing returns to scale and that the average farm is still in the irrational zone 1. If the average farmer already operates succesfully in the present situation, he will realise an even larger income if all production factors are increased, provided management does not become a limiting factor.

Although the function indicates increasing returns to scale for all farms in the Swartland, the possibility of decreasing returns to scale in the case of very large farms cannot be ruled out.

Where the sum total of the regression coefficient is below 1, the optimum combination of the input factor and the optimum intensity are determined simultaneously from the production function. We have seen, however, that the sum of the regression coefficients exceeds 1.0 and that a long run optimum can therefore not be calculated. The following exposition will illustrate clearly that the results are irrational, as the following model is applicable only if the sum of the production elasticities lies between 1.0 and 0. Given the production function:

$y = a x_i^{b_i} (i = 1, 2, 3)$ .................. (2)

and the profit function $yPy - \Sigma x_i Px_i = Z$ .. (3)

where $Py$ = price per unit output (in this case R1.00) and $Px_i$ = the price per unit of input $x_i$. It is now necessary to find static values (if any) for comparison 3, subject to $\log y - \log a - \Sigma b_i \log x_i = 0$ ......... (4)

By applying Lagrange multipliers, we regard the following static values of

$z = yPy - \Sigma x_i Px_i - \lambda(\log y - \log a - \Sigma b_i \log x_i)$ ......................... (5)

We desire

$$\frac{dz}{dy} = Py - \frac{\lambda}{y} = 0 \quad (6)$$

$$\frac{dz}{dx_i} = -Px_i + \frac{\lambda b_i}{x_i} = 0$$

and

$$\frac{dz}{d\lambda} = \log y - \log a - \Sigma b_i \log x_i = 0 \quad (7)$$

6) The computer routine, by which the problem was solved, failed to provide the information necessary to test the significance of the variation from 1. It will now be assumed that increasing returns to scale prevail.

The solution of these comparisons is indicated in Table 7.

TABLE 7 - Irrational optimum intensity and optimum combination of land, labour and variable costs, Swartland, 1967/68

| | Gross income | Land | Labour | Capital |
|---|---|---|---|---|
| | (R) | morgen | (R) | (R) |
| Present | 38 039.54 | 748.37 | 2 379.76 | 13 360.66 |
| Optimum | 619.28 | 25.91 | 120.28 | 287.29 |

Since this function indicates increasing returns to scale, the function will be convex to the horizontal co-ordinates and the price relationship line will be tangential to the function at its lowest point. Thus the figures indicated in Table 7 represent a minimum and not an optimum combination.

## 5. CONCLUSION

From this exposition it is evident that farmers in the Swartland will realise growing profits if the farming activities are expanded. In particular the purchase or leasing of additional land is envisaged here, coupled with an increase in the labour force and the investment of additional capital.[7] It is again emphasised that management was not taken into consideration, but only measurable standards.

It must be borne in mind, however, that the usefulness of these marginal value products for long term planning, is limited by the fact that 1967 was a favourable year as far as climatic conditions are concerned. It is also difficult to introduce reliable variations into the equations to accommodate years with different climatic conditions.

It is clear that, with given product and factor prices, the marginal productivity will vary considerably with a higher or lower rainfall. Moreover, there exists no reason to believewe that there is a linear relationship between production and rainfall.

Physical input-output data over a number of years should be available if reliable estimates of marginal value product over the long run are to be made for purposes of resource allocation and farm planning.

7) On the assumption that prices will remain on the existing level.