A DYNAMIC DIFFERENTIAL DEMAND SYSTEM: AN APPLICATION OF TRANSLATION

Mark G. Brown and Jong-Ying Lee

Abstract

The differential demand system or Rotterdam model is extended to include lagged consumption through translation parameters, allowing habit and inventory effects. Applications of the model to annual U.S. expenditure and weekly juice sales data illustrate the importance of the time interval of an observation on the relative strengths of the habit inventory.

Key words: dynamic differential demand system, translation, habits, household inventories

The differential demand model or Rotterdam model, developed by Theil (1965) and Barten (1966), provides a first-order approximation of true demand. Analyses by Barnett, Byron, and Mountain show that the approximation is comparable to other popular flexible functional forms. To allow for trends in consumption and changes in tastes, a constant term is sometimes included in the Rotterdam demand specification as a rough approximation (e.g., Theil 1976; Barten 1969; Deaton; Deaton and Muellbauer). In other demand models, a common approach to allow for the impact of past consumption, in both single-equation and system specifications, has been to include lagged consumption in the model (e.g., Houthakker and Taylor; Tilley; Sexauer). In a demand system with n goods, inclusion of lagged consumption of each good results in n^2 additional responses to consider. A parsimonious approach to modeling the latter is through translating (Gorman; Pollak and Wales 1980, 1981). Translation involves adding fixed quantity levels, referred to as translation parameters, to the direct utility maximization problem or, equivalently, fixed costs to the expenditure function. Sometimes, the fixed quantity levels are also referred to as subsistence quantities but, in general, are parameters indicating preferences-in fact, the translation parameters might even be negative (Solari, Phlips, Jackson). The demand impacts of lagged consumption can be specified through the translation parameters (over time, or across individuals or households, the translation parameters need not be fixed but may vary).

In this article, the impact of lagged consumption is examined in the differential version of the translation model. In the next section, the differential demand model is extended to include lagged consumption through translation parameters. The extension includes specifications for the long-run demand responses. For illustration purposes, the model is then applied to two separate data sets: (1) U.S. Department of Commerce annual personal consumption expenditures for four groups of goods-food, alcohol, other nondurables, and services; and (2) A. C. Nielsen weekly retail sales for different types of juice. The final section includes some concluding comments.

MODEL

In this section, the translation model is briefly reviewed and then approximated, using the differential approach. The analysis includes development of long-run demand responses.

The consumer choice problem for translation can be written as

$$(1) \quad \text{maximize } u = u(q_1^*, \ldots, q_n^*)$$

$$\text{subject to } \sum p_i q_i^* = x - \sum p_i \gamma_i = x^*,$$

where subscript i indicates a particular good; $q_i^* = q_i - \gamma_i$, q_i being quantity and γ_i being the translation parameter; p_i is the price; and x is total expenditure or income. The indirect utility function and expenditure or cost function for (1) are $u = \psi(p_1, \ldots, p_n, x^*)$, and $x = \sum p_i \gamma_i + c(p_1, \ldots, p_n, u)$, respectively. The parameter γ_i is sometimes referred to as a subsistence level and x^* as supernumerary income. In the cost

Mark G. Brown is an Adjunct Associate Professor and Jong-Ying Lee is an Adjunct Professor and in the Department of Food and Resource Economics at the University of Florida at Gainsville, and both are Research Economists with the Florida Department of Citrus.

function, fixed costs $\sum p_i \gamma_i$ are added to a general cost specification.

The demand equations for (1) can be written as

$$(2)\quad q_i = \gamma_i + q_i^* (p_1, ..., p_n, x^*) .$$

To obtain the Rotterdam model for the translation specification of demand, first totally differentiate (2), i.e.,

$$(3)\quad d\,q_i = d\,\gamma_i - \frac{\partial q_i}{\partial x} \sum_j p_j\, d\gamma_j + \sum_j \frac{\partial q_i}{\partial p_j} d\,p_j + \frac{\partial q_i}{\partial x} d\,x$$

where $\frac{\partial q_i}{\partial p_j} = \frac{\partial q_i^*}{\partial p_j} - \frac{\partial q_i^*}{\partial x} \gamma_j$.

The Slutsky equation shows that $\frac{\partial q_i}{\partial p_j} = S_{ij} - q_j \frac{\partial q_i}{\partial x}$, where s_{ij} is the substitution effect. Substituting the latter expression for $\frac{\partial q_i}{\partial p_j}$ in (3), multiplying both sides of the equation by $\frac{p_i}{x}$, and using the equality $d\,a = a\,d \log a$ for variable a in general, one finds the Rotterdam model with translation

$$(4)\quad \omega_i\, d \log q_i = z_i - \mu_i \sum_j z_j + \mu_i\, d \log Q + \sum_j \pi_{ij}\, d \log p_j ,$$

where $\omega_i = \frac{p_i q_i}{x}$, the budget share for the good in question; $z_i = \frac{p_i \gamma_i}{x} d \log \gamma_i$, the log change in the translation parameter weighted by the share of total expenditure committed to the good, $\frac{p_i \gamma_i}{x}$; $\mu_i = p_i \frac{\partial q_i}{\partial x}$, the marginal propensity to consume for good i (MPC); $d \log Q = \sum \omega_j\, d \log q_j = d\, logx - \sum \omega_j\, d \log p_j$, the Divisia volume index in differential form (the Divisia volume index relationship can be straightforwardly obtained by differentiation of the budget constraint); and $\pi_{ij} = \frac{p_i p_j}{x} s_{ij}$, the Slutsky coefficient. The left-hand side variable in (4) can be viewed as the percentage change in demand weighted by the budget share–the multiplication of equation (3) by $\frac{p_i}{x}$ allows one to impose the basic restrictions of demand straightforwardly, as subsequently discussed. The difference between the usual Rotterdam model and our specification is the first two terms on the right side of equation (4) which involve changes in the translation parameters. The first term is a direct effect due to a change in the translation parameter for the good in question, while the second term is an indirect income effect due to a change in supernumerary income caused by the overall change in the translation parameters. Changes in the translation parameters can be viewed as preference changes, and the resultant direct and indirect effects lead to a re-allocation of income.

The effects of past consumption can be introduced into the model by letting the translation term z_i depend on lagged consumption. In this study, we hypothesize that

$$(5)\quad z_i = a_i\, \omega_{i,t-1}\, d \log q_{i,t-1}$$

where the subscript t has been added to indicate time, and a_i is a constant. Equation (5) states that the weighted log change in the translation parameter equals a constant times the weighted log change in lagged consumption. Pollak and Wales (1969), as well as others (e.g., Phlips; Johnson, Hassan and Green), have similarly modeled the effects of lagged consumption through the translation parameter in the linear expenditure system (LES). The LES is based on an additive or strongly separable utility function and is quite restrictive. The differential model considered here is more general and is not based on some separability assumption, although, for empirical analysis, weak or strong separability is often assumed, with attention focused on the conditional demand equations for some separable group of goods.

Past consumption typically affects demand through an amalgam of inventory and habit effects (Sexauer; Tilley). For durable goods, one usually expects inventory effects to dominate habit effects, whereas, for nondurable goods, the opposite is more likely to occur. In equation (5), the parameter a_i is normally expected to be negative (positive) when inventory (habit) effects dominate. Sexauer has further shown that the length of the time period of an observation has an important influence on the relative strengths of the inventory and habit effects. The shorter the time interval of an observation, the more likely inventory effects are to dominate habit effects. The importance of the time interval of an observation is illustrated in the next section where annual and weekly data have been used to estimate the lag parameters of (5).

The model defined by (4) and (5) can be written in matrix notation as:

$$(6)\ \ Y_t = L\,Y_{t-1} + U\,(X_t - W'_t P_t) + \Pi P_t\,,$$

where $Y_t = [\omega_{it}\, d \log q_{it}]$,
$U = [\mu_i]$,

$$L = \hat{A} - U A' \text{ with } A = [a_i] \text{ and } \hat{A} = \begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_n \end{bmatrix},$$

the diagonal of A,

$X_t = d \log x_t$,

$W_t = [\omega_{it}]$,

$P_t = [d \log p_{it}]$,

$\Pi = [\pi_{ij}]$.

The short-run demand responses are indicated by U, Π and A. The long-run demand responses are determined by succesive substitution, following the procedure suggested by Theil (1971) for determining total impacts. For convenience, set Y_{t-1} and P_t to zero in (6), so that $Y_t = U X_t$. In all subsequent periods (t+1, ...), set P_t and X_t to zero, so that $Y_{t+1} = L\,Y_t = LU\,X_t$, $Y_{t+2} = L\,Y_{t+1} = L^2\,Y_t = L^2\,U\,X_t$ .... The total impact is then:

$$(7)\ \ \sum_{k=t}^{\infty} Y_k = S\,U\,X_t$$

$$S = I + L + L^2 + L^3 + \ldots,$$

where I is the *nxn* identity matrix. Provided that all the latent roots of L are less than one in absolute value, S converges to $(I - L)^{-1}$.

Similarly, for convenience, let Y_{t-1} and X_t be set to zero in (6), so that $Y_t = (\Pi - UW')P_t$. Again setting P_t and X_t to zero in subsequent periods results in:

$$(8)\ \ \sum_{k=t}^{\infty} Y_k = (I - L)^{-1} (\Pi - U\,W')\,P_t\,.$$

Expressions (7) and (8) indicate that the long-run income and price responses are $(I - L)^{-1}\,U$ and $(I - L)^{-1}\,(\Pi - U\,W')$, respectively.

The short-run elasticities for (6) are:

the income elasticity $e_i = \dfrac{\mu_i}{\omega_i}$,

the compensated price elasticities $e^*_{ij} = \dfrac{\pi_{ij}}{\omega_i}$,

and the uncompensated price elasticities $e_{ij} = e^*_{ij} - \omega_j\, e_i = \dfrac{\pi_{ij}}{\omega_i} - \dfrac{\omega_j}{\omega_i}\,\mu_j$.

As the latter indicates, estimation of price and income elasticities in the Rotterdam model involves division by the budget shares. Long-run elasticities in this study are similarly estimated as $(I - L)^{-1}$ times the short-run elasticities. In the long run, the budget shares may change for a discrete income or price change; the long-run elasticity estimates here treat the changes in the budget shares as negligible.

The basic restrictions of demand require

(a) adding-up: $\sum_i \mu_i = 1$ and $\sum_i \pi_{ij} = 0$;

(b) homogeneity : $\sum_j \pi_{ij} = 0$; and

(c) symmetry: $\pi_{ij} = \pi_{ji}$.

The model thus far has been in terms of infinitesimal changes. However, for estimation we need to work with finite changes and follow the usual practice of approximating ω_{it}, $d \log p_{it}$ and $d \log q_{it}$ by $\dfrac{\omega_{i,t} + \omega_{i,t-1}}{2}$, $\log \dfrac{p_{i,t}}{p_{i,t-1}}$, and $\log \dfrac{q_{i,t}}{q_{i,t-1}}$, respectively (e.g., see Barten 1969, or Theil 1971, among others). A vector of disturbances $\varepsilon_t = [\,\varepsilon_{it}\,]$ is also added to (6) to complete the model. The disturbance vector is assumed to have a multinormal distribution with $E(\varepsilon_t) = 0$ and $E(\varepsilon_t\, \varepsilon_\alpha') = \Omega$ for $t = \alpha$, for $t \neq \alpha$. Given that the data add up by construction–the left-hand side variables ($\omega_i\, d \log q_i$) in the model sum over i to the income variable ($d \log Q$)–the disturbances sum over *i* to zero and Ω is singular.

APPLICATION

For illustration purposes, the model developed in the previous section was applied to two data sets. The first data set is comprised of annual observations on U.S. personal consumption expenditures, while the second data set is comprised of weekly observations on retail sales of different types of juice. The estimated translation lag parameters based on the annual data and those based on the weekly data illustrate how the length of the time period of an observation may affect the relative importance of habits and household inventories.

Model (6) was first applied to U.S. Department of Commerce data on personal consumption expenditures for food, alcohol, other nondurables and services.[1] Alcohol has specifically been broken out as a separate category based on the expectation that alcohol consumption is subject to habit persistence. The data are annual and the sample runs from 1934 through 1989. Prohibition of the sales of alcohol ended in December 1933. The expenditure data are measured in both actual and real (1982 = 100) dollars. Average prices were obtained by dividing actual expenditure by real expenditure for each of the four expenditure groups. Quantities were measured by real expenditures. U.S. Department of Commerce data on the U.S. population were used to put demand on a per capita basis. Food, alcohol, other nondurables and services are treated as weakly separable from durable goods and savings, and the analysis focuses on the conditional demand system for the four product categories.

Descriptive statistics for the product categories are given in Table 1. Over the period from 1934 to 1989, annual total real consumer expenditures on food, alcohol, other nondurables and services averaged $1,096 billion (base year 1982). Annual real expenditures averaged $245 billion for food, $32 billion for alcohol, $243 billion for other nondurables and $576 billion for services. On average, food accounted for 24 percent of actual total expenditures, while alcohol, other nondurables and services accounted for 4 percent, 24 percent, and 47 percent, respectively. The average price level was highest for alcohol and roughly the same for the other product categories.

As the data add-up by construction—income in the model is total consumer expenditure on the four product categories—the error covariance matrix is singular as previously indicated, and the equation for services was excluded (Barten 1969). With the errors across equations assumed to be contemporaneously correlated, the maximum likelihood procedure was used to estimate the model. A constant term was added to each equation to account for consumption trends not related to the translation lag variables. A dummy variable for World War II years was also included in the model but was found to be insignificant and dropped for further analysis. Initially, the analysis focused on the overall impacts of the constant and translation terms, and on the homogeneity and symmetry restrictions of demand theory. Following Barten (1969) and Deaton, the likelihood ratio test was used to test the latter demand restrictions and the significance of the constant and translation terms. Estimates for five model specifications-model A: Rotterdam specification (6) with constant terms but without homogeneity and symmetry imposed; model B: model A without translation lag variables; model C: model A without constant terms; model D: model A with homogeneity imposed; and model E: model A with homogeneity and symmetry imposed-were obtained using the maximum likelihood estimation computer program provided by TSP. The likelihood ratio test involves comparison of the logarithmic likelihood values for the different models. Under the null hypothesis of the restricted model, twice the difference between the maximum logarithmic likelihood value for the unrestricted model and that value for the restricted model is asymptotically distributed as a chi-square statistic with the number of degrees of freedom equal to the number of restrictions imposed.

Table 1. Descriptive Statistics for U.S. Department of Commerce Personal Consumption Expenditures, 1934 to 1989

Group	Mean Expenditure	Mean Conditional Budget Share[a]	Mean Price[b]
	Billions of 1982 $		
Food	244.788 (89.253)[c]	.242 (.039)	.460 (.332)
Alcohol	32.189 (12.622)	.041 (.013)	.532 (.306)
Other Nondurables	242.646 (112.744)	.242 (.028)	.467 (.304)
Services	575.909 (344.006)	.474 (.078)	.450 (.363)
Total	1,095.532 (556.751)		.460 (.341)

[a]For actual dollar expenditure.
[b]Consumer expenditures in actual dollars divided by consumer expenditures in 1982 dollars.
[c]Standard errors in parentheses.

[1]The U.S. Department of Commerce product categories included the following goods:
(1) Food: food purchased for off-premise consumption, purchased meals and beverages, food furnished for employees, and food produced and consumed on farms.
(2) Alcohol: for off-premise and other consumption.
(3)Nondurables: clothing and shoes, gasoline and oil, fuel oil and coal, and other.
(4) Services: housing, housing operation, transportation, medical care, and other.
For a more detailed description of goods included in product categories, see pages 106-112 in "The National Income and Product Accounts of the United States, 1929-82," U.S. Department of Commerce.

Table 2. Logarithmic Likelihood Values for Alternative Rotterdam Model Specifications for U.S. Department of Commerce Personal Consumption Expenditures

Model	Logarithmic Likelihood Value	Free Parameter
A. Unconstrained[a]	784.554	22
B. No Lags	774.524	18
C. No Intercepts	779.442	19
D. Homogeneity[a]	779.001	19
E. Homogeniety and Symmetry[a]	767.985	16

[a]Includes intercepts and translation lag variables.

Table 2 shows the logarithmic likelihood values for the different models. When models B and C are compared with model A, the results in the table indicate that both the constant and translation terms should be included in the model at the 5 percent level of significance (models B and C involved four and three restrictions, respectively; the critical value for a chi-square statistic with three (four) degrees of freedom at the 5 percent level is 7.815 (9.488)). On the other hand, comparing models D and E against model A, the results in the table indicate that both homogeneity, and homogeneity and symmetry should be rejected at the 5 percent level of significance (models D and E involved three and six restrictions, respectively; the critical value for a chi-square statistic with six degrees of freedom at the 5 percent level is 12.592). Note that symmetry would also be rejected if the homogeneity constrained model were accepted and treated as the unrestricted model for comparison. In contrast, Deaton found that for British expenditure data, symmetry could be accepted when the maintained hypothesis includes homogeneity.

Maximum likelihood estimates for model A are shown in Table 3. The equations fit satisfactorily given they are in first differences.[2] The parameter estimates in Table 3 show that none of the translation lag variables are significant individually, although as a group the translation lag variables are significant based on the likelihood ratio test; likewise, the constant terms are significant as a group but insignificant individually. The translation lag parameter estimates for food, alcohol, and services are positive, suggesting dominance of habit persistence, while the translation estimate for other nondurables is negative, suggesting dominance of inventory effects. The estimates for the MPCs are all positive and twice or greater in size than their corresponding standard error estimates. The MPCs for food, other nondurables and services are all near .30 while the MPC for alcohol is .10. The own-price Slutsky coefficient estimate for food is negative and twice its standard error, but the other own-price estimates are insignificant. The cross-price estimates indicate significant substitution relationships between food and alcohol, and other nondurables and food; however, a number

Table 3. Maximum Likelihood Estimates for the Rotterdam Model with Translation Parameters Dependent on Changes in Lagged Consumption for U.S. Department of Commerce Personal Consumption Expenditures[a]

	Parameter						
				Price			
Product Group	Intercept	Translation Lag a_i	MPC μ_i	π_{i1}	π_{i2}	π_{i3}	π_{i4}
Food	-.002	.077	.311	-.106	.068	.074	-.011
	(.003)[b]	(.106)	(.068)	(.028)	(.027)	(.029)	(.050)
Alcohol	-.001	.276	.097	.003	$7\text{x}10^{-5}$	.013	-.010
	(.001)	(.239)	(.018)	(.012)	(.013)	(.013)	(.018)
Other Nondurables	.002	-.076	.307	.055	-.045	-.056	-.013
	(.002)	(.138)	(.042)	(.022)	(.030)	(.037)	(.043)
Services	.001	.340	.284	.049	-.023	-.031	.034
	(.004)	(.293)	(.075)	(.042)	(.047)	(.046)	(.082)

[a]Model defined by equation (6) with intercepts.
[b]Asymptotic standard errors in parentheses.

[2]The coefficients of determination (R^2s) for food, alcohol, other nondurables and services were .84, .80, .86, and .65, respectively; note that as the four demand equations are estimated jointly as a system, the R^2s have not been maximized. The Durbin-Watson (DW) statistics for food, alcohol, other nondurables and services were 1.63,2.18, 1.81, and 1.65, respectively. For demand systems obeying the adding-up property, the DW statistics for the individual equations are not precise measures of autocorrelation (Bewley).

Table 4. Selected Short-run and Long-run Elasticity Estimates for the Rotterdam Model with Translation Parameters Dependent on Changes in Lagged Consumption for U.S. Department of Commerce Personal Consumption Expenditures[a,b]

Product Group	Income		Own-Price			
			Compensated		Uncompensated	
	SR[c]	LR[d]	SR	LR	SR	LR
Food	1.286 (.280)[e]	1.173 (.332)	-.438 (.114)	-.491 (.140)	-.750 (.130)	-.775 (.158)
Alcohol	2.351 (.442)	2.734 (.630)	.002 (.308)	.011 (.466)	-.095 (.303)	-.102 (.459)
Other Nondurables	1.269 (.174)	.993 (.180)	-.232 (.151)	-.215 (.161)	-.539 (.146)	-.455 (.155)
Services	.599 (.157)	.764 (.214)	.071 (.173)	.097 (.233)	-.213 (.186)	-.266 (.250)

[a]Model defined by equation (6) with intercepts.
[b]Evaluated at budget share sample means: .242 for food; .041 for alcohol; .242 for other nondurables; and .474 for services.
[c]Short run.
[d]Long run.
[e]Asymptotic standard errors in parentheses.

of the cross-price estimates have relatively large standard errors and are insignificant.

Selected short-run and long-run elasticity estimates for the model are shown in Table 4. To avoid overburdening the reader with results, only the income elasticities and uncompensated and compensated own-price elasticities are given. The elasticities are estimated at sample mean budget shares. Long-run income and price responses previously discussed $(I - L)^{-1} U$ and $(I - L)^{-1} (\Pi - U W')$ can be obtained by multiplying the long-run elasticity estimates in Table 4 by the mean budget shares noted in the table.

All of the income elasticity estimates are significantly positive. In the long run, with relatively larger translation parameter estimates for services and alcohol, the income elasticity estimates for food and other nondurables decrease, while the estimates for alcohol and services increase. The income elasticity estimate for food only decreases slightly from 1.3 in the short run to 1.2 in the long run. The latter estimates are higher than one might expect for food and this higher level may be due to the inclusion of food away from home, along with food at home, in the food category. For alcohol, the income elasticity estimate is 2.4 in the short run and 2.7 in the long run (luxury type responses); for service, the estimate is .6 in the short run and .8 in the long run (necessity type responses). The income elasticity estimates for other nondurables are similar to those for food.

All of the uncompensated own-price elasticity estimates are negative and less than one in absolute value, indicating inelastic demands. Given the parameter estimates in Table 3, the compensated estimates for alcohol and services are positive but insignificant. Generally, the short-run compensated and uncompensated own-price elasticity estimates, as well as the income elasticity estimates, do not differ very much from the corresponding long-run estimates, as might be expected given the insignificance of the individual translation parameters.

Overall, application of the translation model to U.S. expenditure data yielded rather weak results. The high level of aggregation may be masking demand relationships and the time period studied may be too long to assume constancy of the income and price coefficients of the model. Nevertheless, the annual data suggest that dominance of habit persistence in consumption as might be expected based on the work by Sexauer. The translation model is next applied to weekly data to illustrate further how shortening the observation time interval may result in dominance of inventory effects.

The study of weekly data focused on the demand for five types of juice–three types of orange juice (OJ), apple juice (AJ) and remaining pure juice (RJ). The different types of OJ are (1) ready-to-serve chilled OJ not made from concentrate (COJ-NFC), (2) ready-to-serve chilled OJ made from concentrate (COJ-FC), and (3) other OJ, primarily frozen concentrate with a small amount of canned juice included. COJ-NFC includes fresh-squeezed OJ which, along with COJ-FC, has experienced substantial growth in recent years.

The data were obtained from A. C. Nielsen Co. and include 200 weekly observations for the period from the week ending November 14, 1987, through September 7, 1991. The Nielsen data include dollar and gallon retail sales in outlets with annual sales of $4 million or more. Prices were calculated by dividing dollars by gallons, and gallons were divided by the U.S. population to obtain per capita gallon sales. We assume that juices are weakly separable from other goods and apply the theory of rational random behavior (Theil 1975-76, 1980) to estimate a conditional demand system for the five juices.

Descriptive statistics for juice sales are given in Table 5. For the period studied, total juice sales averaged 21 million gallons per week with RJ, COJ-FC, other OJ, AJ, and COJ-NFC accounting on average for 25 percent, 25 percent, 23 percent, 15 percent, and 12 percent of the sales, respectively. The highest average price was for COJ-NFC while the lowest average price was for AJ.

With the same methodology used to study the annual U.S. expenditure data, five juice demand model specifications were estimated. Likelihood ratio tests for models A through E previously defined, except without constants in homogeneity-constrained model D and homogeneity-and symmetry-constrained model E, were made. The logarithmic likelihood values in Table 6 show that for this particular data set, the translation lag variables as a group are significant at the 5 percent level while the intercept terms are not (models B and C involve five and four degrees of freedom, respectively; the critical value for a chi-square statistic with four (five) degrees of freedom at the 5 percent level of significance is 9.488 (11.071)). Comparing models D and E against model C also indicates that the homogeneity- and homogeneity-and-symmetry-constrained models are not acceptable at the 5 percent level of significance (models D and E involve four and ten degrees of freedom; the critical value for a chi-square statistic with ten degrees of freedom at the 5 percent level is 18.307). Note that, in contrast to our results for annual U.S. expenditure data, but in agreement with Deaton's results for British expenditure data, symmetry would be accepted if the maintained hypothesis included homogeneity (model E against model D).

Maximum likelihood estimates for model C are shown in Table 7. The equations fit well with 29 out of 35 parameter estimates having values twice or greater in size than their corresponding standard error estimates.[3] All of the translation lag parameter estimates were negative, indicating dominance of inventory effects as expected (Sexauer). The size of the translation lag estimates for the three orange-juice products were also more than two times greater than their corresponding standard errors. All of the MPCs were positive and significant, ranging in value from .10 for COJ-NFC to .27 for other OJ and RJ. Likewise, all own-price Slutsky coefficients were two or more times greater than their corresponding standard errors, and negative as predicted by theory. Most of the cross-price estimates were also significant, and all were positive, indicating substitute relationships as might be expected for closely related competing products.

Table 5. Descriptive Statistics for Weekly Retail Juice Sales, Week Ending November 14, 1987, through September 7, 1991.

Juice	Mean Sales	Mean Conditional Budget Share	Mean Price
	million SSE gallons		$ per SSE gallon
Orange:			
COJ-NFC	1.893 (.391)[a]	.120 (.022)	5.180 (.381)
COJ-FC	5.351 (.521)	.252 (.009)	3.849 (.368)
Other	5.590 (.668)	.230 (.020)	3.352 (.323)
Apple	3.982 (.404)	.145 (.010)	2.969 (.177)
Remaining	4.550 (.306)	.253 (.013)	4.511 (.183)
Total	21.366 (1.336)		3.806 (.241)

[a]Standard errors in parentheses.

Table 6. Logarithmic Likelihood for Alternative Rotterdam Model Specification for Weekly Retail Juice Sales

Model	Logarithmic Likelihood Value	Free Parameters
A. Unconstrained[a]	3,277.50	33
B. No Lags	3,254.64	28
C. No Intercepts	3,276.29	29
D. Homogeneity[b]	3,244.44	25
E. Homogeneity and Symmetry	3,242.08	19

[a]Includes intercepts and translation lag variables.
[b]Includes translation lag variables; without intercepts.

[3]The R^2s (DW)s for COJ-NFC, COJ-FC, and other OJ, AJ, and RJ were .89 (2.61), .88 (2.62), .90 (2.24), .87 (1.93), and .89 (1.79), respectively.

Table 7. Maximum Likelihood Estimates for the Rotterdam Model with Translation Parameters Dependent on Changes in Lagged Sales for Weekly Retail Juice Sales[a]

Juice	Translation Lag	MPC	Price				
	a_i	μ_i	π_{i1}	π_{i2}	π_{i3}	π_{i4}	π_{i5}
Orange:							
COJ-FC	-.164	.099	-.228	.101	.066	.055	.124
	(.035)[b]	(.015)	(.010)	(.026)	(.029)	(.031)	(.052)
COJ-NFC	-.126	.224	.105	-.314	.069	.085	.188
	(.038)	(.014)	(.009)	(.020)	(.031)	(.033)	(.043)
Other	-.133	.270	.055	.076	-.279	.079	.033
	(.041)	(.011)	(.007)	(.017)	(.021)	(.028)	(.039)
Apple	-.009	.141	.023	.041	.033	-.295	10^{-6}x6
	(.042)	(.008)	(.006)	(.013)	(.019)	(.019)	(.027)
Remaining	-.040	.265	.045	.097	.111	.076	-.345
	(.049)	(.010)	(.009)	(.019)	(.023)	(.025)	(.033)

[a]Model defined by equation (6).
[b]Asymptotic standard errors in parentheses.

Table 8. Selected Short-run and Long-run Estimates for the Rotterdam Model with Translation Parameters Dependent on Changes in Lagged Sales for Weekly Retail Juice Sales[a,b]

Juice	Income		Own-Price Compensated		Own-Price Uncompensated	
	SR[c]	LR[d]	SR	LR	SR	LR
Orange:						
COJ-NFC	.822	.770	-1.897	-1.639	-1.996	-1.732
	(.125)[e]	(.100)	(.083)	(.072)	(.078)	(.068)
COJ-FC	.890	.861	-1.247	-1.115	-1.472	-1.332
	(.055)	(.048)	(.079)	(.071)	(.081)	(.072)
Other	1.177	1.133	-1.217	-1.087	-1.488	-1.347
	(.048)	(.043)	(.092)	(.082)	(.093)	(.083)
Apple	.973	1.050	-2.034	-1.987	-2.175	-2.140
	(.054)	(.053)	(.132)	(.128)	(.130)	(.126)
Remaining	1.049	1.099	-1.366	-1.281	-1.631	-1.559
	(.040)	(.037)	(.132)	(.123)	(.133)	(.124)

[a]Model defined by equation 6.
[b]Evaluated at budget share sample means: .120 for COJ-NFC; .252 for COJ-FC; .229 for other OJ; .145 for AJ; and .253 for RJ.
[c]Short run.
[d]Long run.
[e]Asymptotic standard errors in parentheses.

Table 8 shows the short-run and long-run income and own-price elasticities for the conditional juice demands. For this study, there are few differences between the short-run and long-run elasticity estimates in general, as the translation parameter estimates are relatively small in value. All juice categories were relatively sensitive to price with the long-run uncompensated own-price elasticities ranging from -2.1 for AJ to -1.3 for COJ-FC and other OJ. The long-run income (total juice expenditure) elasticities were 1.1 for other OJ, AJ, and RJ, .9 for COJ-FC, and .8 for COJ-NFC.

Overall, the results based on the annual and weekly data illustrate the importance of the observation time dimension in estimating the effects of past consumption on demand. Although the effects of past consumption were not strong in either of these particular applications, the relative strength of habits and inventories in each model was as expected.

CONCLUDING COMMENTS

A parsimonious approach to include habit and inventory effects in the Rotterdam model is through translation terms dependent on lagged consumption. The differential model provides an approximation of demand comparable to other flexible functional forms, and translation and other extensions of the differential model that relax the assumption of constancy of the model coefficients offer additional flexibility. In a recent study by Theil et al., the MPC was specified as a varying parameter, equal to the value of the APC or budget share at each point in the sample, plus a constant. The latter study also discusses other extensions allowing the basic parameters of the Rotterdam model to be functions of income and prices. As both the present study and the Theil et al. study suggest, the Rotterdam model might be made even more realistic by choosing appropriate parameter specifications.

The results of this study also show the importance of the time dimension in observing habit and inventory effects in systems of demand equations. The shorter the observation time interval, the more likely it is that inventories will dominate habits. Important government and business decisions are often influenced by period-to-period changes in consumer expenditures or sales, and understanding the dynamics underlying the changes in demand can be helpful in making more informed decisions.

REFERENCES

Barnett, W. A. "On the Flexibility of the Rotterdam Model: A First Empirical Look." *Eur. Econ. Rev.,* 24(1984):285-289.

Barten, A. P. "Theorie en empire van een volledig stelsel van vraagvergelijkingen." PhD dissertation, University of Rotterdam, 1966.

Barten, A. P. "Maximum Likelihood Estimation of a Complete System of Demand Equations." *Eur. Econ. Rev.,* 1(1969):7-73.

Bewley, R. *Allocation Models: Specification, Estimation and Applications.* Cambridge, MA: Ballinger Publishing Co., 1986.

Byron, R. P. "On the Flexibility of the Rotterdam Model." *Eur. Econ. Rev.,* 24(1984):273-283.

Deaton, A. S. "The Analysis of Consumer Demand in the United Kingdom, 1900-1970." *Econometrica,* 42(1974):341-367.

Deaton, A., and J. Muellbauer. *Economics and Consumer Behavior.* Cambridge, MA: Cambridge University Press, 1980.

Gorman, W. M. "Tricks with Utility Functions." In Essays in Economic Analysis, M. J. Artis and A. R. Nobay, eds., Cambridge: Cambridge University Press, 1970.

Houthakker, H., and L. D. Taylor. *Consumer Demand in the United States, 1929-1970.* Cambridge, MA: Harvard University Press, 1970.

Jackson, L. R. "Hierarchic Demand and the Engel Curve." *Rev. of Econ. and Stat.,* 66(1984):8-15.

Johnson, S. R., Z. A. Hassan, and R. D. Green. *Demand System Estimation: Methods and Applications.* Ames: The Iowa State University Press, 1984.

Mountain, D. C. "The Rotterdam Model: An Approximation in Variable Space." *Econometrica* , 56(1988): 477-484.

Phlips, L. *Applied Consumption Analysis.* Amsterdam: North Holland Publishing Co., 1974.

Pollak, R. A., and T. J. Wales. "Estimation of the Linear Expenditure System." *Econometrica,* 37(1969):611-628.

Pollak, R. A., and T. J. Wales. "Comparison of the Quadratic Expenditure System and Translog Demand Systems with Alternative Specifications of Demographic Effects." *Econometrica,* 48(1980):595-611.

Pollak, R. A., and T. J. Wales. "Demographic Variables in Demand Analysis." *Econometrica* ,49(1981):1533-1551.

Sexauer, B. "The Role of Habits and Stocks in Consumer Expenditure." *Quart. J. Econ.,* 91(1979):127-142.

Solari, L. *Thérie des choix et fonctions de consummation semi-agrégées.* Geneva: Librarie Droz, 1971.

Theil, H. "The Information Approach to Demand Analysis." *Econometrica*, 33(1965):67-87.

Theil, H. *Principles of Econometrics.* New York: John Wiley & Sons, Inc., 1971.

Theil, H. *Theory and Measurement of Consumer Demand*, Vol. I and II. Amsterdam: North Holland, 1975, 1976.

Theil, H. *The System-Wide Approach to Microeconomics.* Chicago: University of Chicago Press, 1980.

Theil, H., Ching-Fan Chung, and J. L. Seale, Jr. *International Evidence on Consumption Patterns.* Greenwich, CT: JAI Press, Inc., 1989.

Tilley, D. S. "Importance of Understanding Consumption Dynamics in Market Recovery Periods." *So. J. Agr. Econ.*, 2(1979):41-46.

U.S. Department of Commerce, Bureau of Economic Analysis. "The National Income and Product Accounts of the United States, 1929-82." *Supplement to the Survey of Current Business.* Washington, D.C.: September, 1986.

U.S. Department of Commerce, Bureau of Economic Analysis. "Business Statistics, 1961-88." *Supplement to the Survey of Current Business.* Washington, D.C.: December, 1989.

U.S. Department of Commerce, Bureau of Economic Analysis. *Survey of Current Business.* Washington, D.C.: 70.9 (1990).