UNIVERSITY OF NEVADA, RENO

STOCHASTIC PROGRAMMING FOR OPTIMAL INVESTMENT DECISION
OF GERMINATING CRESTED WHEATGRASS ON DEEDED LAND

by

Chong S. Kim
John F. Yanagida

M.S. 139

June, 1981

Division of Agricultural & Resource Economics

Max C. Fleischmann College of Agriculture
University of Nevada, Reno
Reno, Nevada 89557
Dale W. Bohmont, Dean and Director

STOCHASTIC PROGRAMMING FOR OPTIMAL INVESTMENT DECISION OF GERMINATING CRESTED WHEATGRASS ON DEEDED LAND

by

Chong S. Kim
John F. Yanagida

M.S. 139 June, 1981

Chong S. Kim is a former Research Associate and John F. Yanagida is an Assistant Professor. Both are with the Division of Agricultural and Resource Economics, University of Nevada, Reno. The United States Department of Agriculture funded this study under Research Contract Number 58-319V-9-02711 with the Economics and Statistics Service. The authors are grateful to USDA for its support.

Summary

A multiperiod linear programming model is used to determine the acreage of deeded land for germinating crested wheatgrass. Results from a multiperiod linear programming problem based on a representative rancher in northwest Nevada indicate that ranchers can make more profits by converting all of their deeded land from native grass production to crested wheatgrass production.

However, a rancher's risk or uncertainty associated with yields for native grass and crested wheatgrass may influence his investment decisions. Therefore, multiperiod stochastic programming models are introduced and applied for a given representative rancher in northwest Nevada. The first model assumes that a rancher's risk is only associated with total crested wheatgrass production. The second model assumes that a rancher's risk is associated with yields for both native grass and crested wheatgrass. Results indicate that acreage for germinating crested wheatgrass decreases as a rancher's risk allowance increases. In the case where a rancher's risk is associated with yields in both native grass and crested wheatgrass, acreage allocated for germinating crested wheatgrass is less, for a certain level of risk allowance, than in the case where risk is associated only with crested wheatgrass production.

Introduction

Most ranch management decisions are made under uncertain conditions associated with weather, disease, production techniques, prices, and institutional arrangements. Cattle in Nevada receive most of their nutrition from grazing deeded range land, National Forest Service (NFS) land, and/or Bureau of Land Management (BLM) land during the spring and summer seasons. Production of livestock forage from native grasses on deeded rangeland in Nevada is highly vulnerable to weather conditions, particularly from low precipitation. In Nevada, average annual precipitation is relatively low compared to the national average and fluctuates considerably from year to year.

The average annual production of livestock forage on deeded rangeland can be increased by removing native grasses followed by seeding with crested wheatgrass. However, acreage of seeding desired by a given rancher depends on : the length of time for successful plant establishment to occur; the associated income foregone from nongrazing; the differences in production between native grass and crested wheatgrass for fluctuating precipitation level over time periods; and the risk of various levels of success in plant establishment. However, new stands are fairly easy to establish. If the first year's results do not appear successful, usually a good stand develops in the second or third year after seeding [Archer and Bunch (1)]. Therefore, precipitation level may be considered as a prime factor affecting a rancher's decision in germinating crested wheatgrass on deeded land. There has been increasing recognition by researchers that improved estimates of resource allocation can be obtained by the inclusion of ranchers' attitudes toward risk in determining production decisions. One method used by Nevada ranchers to protect against possible economic losses due to poor weather conditions is diversification between native grass and crested wheatgrass on deeded rangeland. This type of diversification is expected to reduce the potential economic loss induced by poor precipitation. Diversification of this form is feasible when alternative forage sources have differential responses to precipitation. Tobin (14) and others have shown that only if the decision-maker is risk averse will the optimal portfolio involve a mixture of risky prospects, and output under uncertainty tends to be smaller than when conditions are more certain.

In order to minimize risk, a rancher may prefer to diversify by removing native grass followed by seeding crested wheatgrass on this segment of deeded rangeland. Costs of this project will occur in the initial periods and positive returns will occur in later periods. Therefore the decision model chosen should include not only the rancher's attitude towards risk in production decisions, but also the present value criterion for project analysis.

Specific objectives of this paper are: (1) to estimate the optimal allocation of deeded rangeland between native grass and crested wheatgrass production to maximize multiperiod net revenue, under conditions of no risk or uncertainty and (2) to measure the effects of a rancher's attitude towards risk associated with the investment decision for multiperiod optimal allocation of deeded rangeland between native grass and crested wheatgrass.

To accomplish the first objective, a multiperiod linear programming model (MLP) is used. In an MLP model with linear utility function, all coefficients are assumed to be known with certainty. This assumption of a linear utility function implicitly implies that all decision-makers are risk-neutral in decision problems. Therefore, if a decision-maker is assumed to behave as a risk-taker or as a risk-averter, he cannot be assumed to behave solely as a profit maximizer. Therefore, a multiperiod stochastic programming (MSP) model is used to determine multiperiod optimal allocation of scarce resources under risk and uncertainty.

Multiperiod Optimization Model

Multiperiod production planning models necessitate the incorporation of time and interest rates. To simplify the problems associated with time, it is common to assume that the planning horizon consists of T discrete time periods and each time period represents one year in length. It is assumed in this paper that inputs are purchased at the beginning of each period and products are sold at the end of each period.

Another assumption is that ranchers operate in a perfectly competitive money market. In other words, the borrowing rate and lending rate are assumed to be equal.

In general, there are two possible criteria to determine the optimal level of investment. One is the present value (PV) criterion and the other is the internal rate of return (IRR) criterion. The PV criterion recommends investment opportunities with a positive present value of net income stream. The IRR criterion favors investment in opportunities that have an IRR greater or equal to the market rate of return. Both PV and IRR criteria produce the same level or optimal investment over two time periods. But several problems exist with the IRR criterion for multiperiods even when the money market is perfectly competitive and investment opportunities are independent. The IRR may not be uniqely defined and it may not exist for a multiperiod investment.[1] Therefore, the concept of present value is used in this paper. That is, the rancher is assumed to maximize the present value of his profit stream.

Suppose that a rancher removes native grass on a portion of deeded rangeland and then seeds this land with crested wheatgrass to increase future livestock forage. On this portion of deeded rangeland there is no livestock forage production for the first three years. In the first year the improved area is plowed and seeded. Assuming successful germination, growth of the young plants continues in the second and third years.

The economic question addressed in this paper is what acreage of native grass on the deeded rangeland should a rancher remove and seed crested wheatgrass to maximize present value of the profit stream, given the risk associated with establishing crested wheatgrass and the differing production levels and variability over time of crested wheatgrass and native grasses.

Let the production function for the jth output be:

$$q_j = h_j\ (x_1, x_2, \ldots, x_n),\ j = 1,2,\ldots,s \qquad (1)$$

where: q_j is the jth output,

x_i is the ith input, and

$$\frac{\partial q_j}{\partial x_i} \geq 0$$

In the above production function [equation (1)], input-output ratios are assumed to be independent of the scale of production for each input x_i.

---

[1]See Hirshleifer (7) or Cohen and Cyert (3).

In other words, the production function is characterized by fixed input coefficients. Fixed-proportions production functions are homogeneous of degree one and such functions reflect constant returns to scale. It is assumed in this paper that different fixed-proportions production processes are available and all inputs are divisible. A multiperiod production function for the planning horizon can be written in implicit form as:

$$F(q_{11},\ldots,q_{s1},q_{12},\ldots,q_{s2},x_{11},\ldots,x_{n1},x_{12},\ldots x_{n2}) \quad (2)$$

where: q_{j1} denotes the jth output produced during the first three years,

q_{j2} denotes the jth output produced during the rest of the planning horizon,

x_{i1} denotes the ith input used during the first three years, and

x_{i2} denotes the ith input required during the rest of the planning horizon.

Also, the total present value of returns is expressed as:

$$\sum_{j=1}^{s} \sum_{t=1}^{3} P_j\, q_{j1}\, (1 + \rho)^{-t} + \sum_{j=1}^{s} \sum_{t=4}^{T} P_j\, q_{j2}\, (1 + \rho)^{-t} \quad (3)$$

where; ρ is a discount rate and P_j is a price of jth product.

Similarily, the total present values of costs is expressed as:

$$\sum_{i=1}^{n} \sum_{t=0}^{2} r_{i1}\, x_{i1}\, (1 + \rho)^{-t} + \sum_{i=1}^{n} \sum_{t=3}^{T-1} r_{i2}\, x_{i2}\, (1 + \rho)^{-t} \quad (4)$$

where; r_{i1} denotes price of the ith input during the first three years, and

r_{i2} denotes price of the ith input during the rest of the planning horizon.

The Lagrangian equation for maximizing the total present value of profit stream can be written as:

$$L = \sum_{j=1}^{s} \sum_{t=1}^{3} P_j\, q_{j1}\, (1 + \rho)^{-t} + \sum_{j=1}^{s} \sum_{t=4}^{T} P_j\, q_{j2}\, (1 + \rho)^{-t}$$

$$- \sum_{i=1}^{n} \sum_{t=0}^{2} r_{i1}\, x_{i1}\, (1 + \rho)^{-t} - \sum_{i=1}^{n} \sum_{t=3}^{T-1} r_{i2}\, x_{i2}\, (1 + \rho)^{-t}$$

$$+ \lambda\, [F\, (q_{11},\, q_{21},\ldots,q_{s1},q_{12},q_{22},\ldots,q_{s2},x_{11}\;\; x_{21},\ldots,x_{n1},x_{12},\ldots,x_{n2})] \quad (5)$$

where: λ is the Lagrangian multiplier.

In the above Lagrangian equation (equation (5)), output price P_j $(j=1,\ldots,s)$ is assumed to be constant over the planning horizon, input price r_{i1} $(i=1,\ldots,n)$ is constant during the first three years, and r_{i2} $(i=1,\ldots,n)$ is constant during the rest of the planning horizon, where r_{i1} and r_{i2} may or may not be equal.

First order conditions for optimizations are:

$$\frac{\partial L}{\partial q_{j1}} = \sum_{t=1}^{3} P_j\, (1 + \rho)^{-t} + \lambda \frac{\partial F}{\partial q_{j1}} = 0 \qquad j = 1,2,\ldots,s \qquad (6)$$

$$\frac{\partial L}{\partial q_{j2}} = \sum_{t=4}^{T} P_j\, (1 + \rho)^{-t} + \lambda \frac{\partial F}{\partial q_{j2}} = 0 \qquad j = 1,2,\ldots,s \qquad (7)$$

$$\frac{\partial L}{\partial x_{i1}} = - \sum_{t=0}^{2} r_{i1}\, (1 + \rho)^{-t} + \lambda \frac{\partial F}{\partial x_{i1}} = 0 \quad i = 1,2,\ldots,n \qquad (8)$$

$$\frac{\partial L}{\partial x_{i2}} = - \sum_{t=3}^{T-1} r_{i2}\, (1 + \rho)^{-t} + \lambda \frac{\partial F}{\partial x_{i2}} = 0 \quad i = 1,2,\ldots,n \qquad (9)$$

$$\frac{\partial L}{\partial \lambda} = F\ (q_{11},\ q_{21},\ldots,q_{s1},q_{12},\ldots,q_{s2}\ x_{21},\ldots,x_{n1},x_{12},\ldots,x_{n2}) = 0 \quad (10)$$

There are 2(s + n) + 1 of optimal equations from first order conditions for optimization.

By choosing optimal equations for any two products produced during the first three years, the marginal rate of transformation for the first three years (MRT_1) can be written as:

$$MRT_1 = \frac{\partial q_{k1}}{\partial q_{j1}} = \frac{-\lambda \frac{\partial F}{\partial q_{j1}}}{-\lambda \frac{\partial F}{\partial q_{k1}}} = \frac{\sum_{t=1}^{3} P_j\ (1+\rho)^{-t}}{\sum_{t=1}^{3} P_k\ (1+\rho)^{-t}} = \frac{P_j}{P_k} \quad (11)$$

Equation (11) indicates that the marginal rate of transformation between any two products during the first three years should be equal to the ratio of product prices. Similarily, the marginal rate of transformation for the rest of the planning horizon (MRT_2) can be written as:

$$MRT_2 = \frac{\partial q_{k2}}{\partial q_{j2}} = \frac{P_j}{P_k} \quad (12)$$

Since output prices are assumed to be constant over the planning horizon, the results in equations (11) and (12) indicate that MRT_1 equals to MRT_2 for any two products over the planning horizon.

Also, the marginal rate of technical substitution between any two inputs can be obtained by choosing optimal equations for any two inputs.

$$MRTS_1 = -\frac{\partial x_{h1}}{\partial x_{i1}} = \frac{\frac{\partial F}{\partial x_{i1}}}{\frac{\partial F}{\partial x_{h1}}} = \frac{\sum_{t=0}^{2} r_{i1}\ (1+\rho)^{-t}}{\sum_{t=0}^{2} r_{h1}\ (1+\rho)^{-t}} = \frac{r_{i1}}{r_{h1}} \quad (13)$$

and,

$$MRTS_2 = -\frac{\partial x_{h2}}{\partial x_{i2}} = \frac{r_{i2}}{r_{h2}} \tag{14}$$

Equations (13) and (14) show that the marginal rate of technical substitution between any two inputs should be equal to the ratio of the input prices. Since input prices are assumed to be constant only during the first three years and the rest of the planning horizon, $MRTS_1$ and $MRTS_2$ may or may not be equal.

The marginal rate of technical substitution between any two inputs for different periods can be obtained by choosing optimal equations for any two inputs. That is,

$$MRTS_{12} = -\frac{\partial x_{n1}}{\partial x_{k2}} = \frac{\frac{\partial F}{\partial x_{k2}}}{\frac{\partial F}{\partial x_{n1}}} = \frac{\sum_{t=3}^{T-1} r_{k2}\ (1+\rho)^{-t}}{\sum_{t=0}^{2} r_{n1}\ (1+\rho)^{-t}}$$

The above equation indicates that the marginal rate of technical substitution between any two inputs for different periods should be equal to the ratio of the sums of their discounted prices.

Choosing optimal equations for any one output and one input for the first three years such as:

$$\sum_{t=1}^{3} P_j\ (1 + \rho)^{-t} + \lambda \frac{\partial F}{\partial q_{j1}} = 0$$

and,

$$-\sum_{t=0}^{2} r_{i1}\ (1 + \rho)^{-t} + \lambda \frac{\partial F}{\partial x_{i1}} = 0$$

Marginal product of x_{i1} in q_{j1} can be obtained as follows:

$$\text{Marginal product of } x_{i1} \text{ in } q_{j1} = -\frac{\partial q_{j1}}{\partial x_{i1}} = \frac{\lambda \frac{\partial F}{\partial x_{i1}}}{\lambda \frac{\partial F}{\partial q_{j1}}} = -\frac{\sum_{t=0}^{2} r_{i1} (1+\rho)^{-t}}{\sum_{t=1}^{3} P_j (1+\rho)^{-t}} \qquad (15)$$

or

$$\left(\frac{\partial q_{j1}}{\partial x_{i1}}\right)\left(\sum_{t=1}^{3} P_j (1+\rho)^{-t}\right) = \sum_{t=0}^{2} r_{i1} (1+\rho)^{-t}$$

or

$$\left(\frac{\partial q_{j1}}{\partial x_{i1}}\right)\left(P_j \sum_{t=1}^{3} (1+\rho)^{-t}\right) = r_{i1} \sum_{t=0}^{2} (1+\rho)^{-t} \qquad (16)$$

Equation (16) indicates that the sum of the discounted value of marginal product should be equal to the sum of the discounted input price.

Similarly, the marginal product of x_{i2} in q_{j2} can be written as:

$$\left(\frac{\partial q_{j2}}{\partial x_{i2}}\right) P_j \left(\sum_{t=4}^{T} (1+\rho)^{-t}\right) = r_{i2} \sum_{t=3}^{T-1} (1+\rho)^{-t} \qquad (17)$$

Also, the marginal product of x_{i1} in q_{h2} can be derived by choosing optimal equations for any one output q_{h2} and one input x_{i1}.

$$\text{This is, } \frac{\partial q_{h2}}{\partial x_{i1}} = \frac{\sum_{t=0}^{2} r_{i1} (1+\rho)^{-t}}{\sum_{t=4}^{T} P_h (1+\rho)^{-t}}$$

or

$$\frac{\partial q_{h2}}{\partial x_{i1}} P_h \sum_{t=4}^{T} (1+\rho)^{-t} = r_{i1} \sum_{t=0}^{2} (1+\rho)^{-t} \qquad (18)$$

Equation (18) indicates that the marginal product of earlier inputs in the production of output later is decreasing.

By comparing equations (16) and (18), one can derive the following results [equation (19)].

$$\frac{\partial q_{j1}}{\partial x_{i1}} P_j \sum_{t=1}^{3} (1 + \rho)^{-t} = \frac{\partial q_{j2}}{\partial x_{i1}} P_h \sum_{t=4}^{T} (1 + \rho)^{-t}$$

or

$$\frac{\partial q_{h2}}{\partial q_{j1}} = \frac{\sum_{t=1}^{3} P_j (1 + \rho)^{-t}}{\sum_{t=4}^{T} P_h (1 + \rho)^{-t}} \qquad (19)$$

From equation (19), the marginal rate of transformation between one product during the first three years and another product during the rest of the planning horizon should be equal to the ratio of the sum of discounted values of product prices.

The second-order conditions for the maximization of total present value of profit stream require that the relevant Hessian matrix must be negative definite or negative semidefinite.

$$H = \left| \begin{array}{cccc} \frac{\partial^2 L}{\partial q_{11}^2} & \frac{\partial^2 L}{\partial q_{11} \partial q_{21}} & \cdots\cdots & \frac{\partial^2 L}{\partial q_{11} \partial x_{n2}} \\ \frac{\partial^2 L}{\partial q_{21} \partial q_{11}} & \frac{\partial^2 L}{\partial q_{21}^2} & \cdots\cdots & \frac{\partial^2 L}{\partial q_{21} \partial x_n{}^2} \\ \hline \frac{\partial^2 L}{\partial x_{n2} \partial q_{11}} & \frac{\partial^2 L}{\partial x_{n2} \partial q_{21}} & \cdots\cdots & \frac{\partial^2 L}{\partial x_n{}^2{}_2} \end{array} \right|$$

This is, the Hessian matrix (i.e. H) of second order partial derivatives of the Lagrangian with respect to the q_{ij} and x_{kj} must be negative definite or negative semidefinite when evaluated at the local maximum point (q^*, x^*, λ^*) when subject to the condition that:

$$dE\ (q^*, x^*, \lambda^*) = 0 \qquad (20)$$

The Hessian matrix, H, is negative definite subject to the constraints [equation (20)] if and only if the signs of the 2 (s + n) - 2 determinants of submatrices of the [2 (s + n) + 1] x [2 (s + n) + 1] matrix obtained by bordering the Hessian matrix, H, by the Jacobian matrix of the constraint production functions are alternating. That is:

$$\begin{vmatrix} 0 & \frac{\partial F}{\partial q_{11}} & \frac{\partial F}{\partial q_{21}} \\ \frac{\partial F}{\partial q_{11}} & \frac{\partial^2 L}{\partial q_{11}^2} & \frac{\partial^2 L}{\partial q_{11} \partial q_{21}} \\ \frac{\partial F}{\partial q_{21}} & \frac{\partial^2 L}{\partial q_{21} \partial q_{11}} & \frac{\partial^2 L}{\partial q_{21}^2} \end{vmatrix} > 0, \ldots\ldots\ldots,$$

$$\begin{vmatrix} 0 & \frac{\partial F}{\partial q_{11}} & \frac{\partial F}{\partial q_{21}} & \frac{\partial F}{\partial q_{31}} \\ \frac{\partial F}{\partial q_{11}} & \frac{\partial^2 L}{\partial q_{11}^2} & \frac{\partial^2 L}{\partial q_{11} \partial q_{21}} & \frac{\partial^2 L}{\partial q_{11} \partial q_{31}} \\ \frac{\partial F}{\partial q_{21}} & \frac{\partial^2 L}{\partial q_{21} \partial q_{11}} & \frac{\partial^2 L}{\partial q_{21}^2} & \frac{\partial^2 L}{\partial q_{21} \partial q_{31}} \\ \frac{\partial F}{\partial q_{31}} & \frac{\partial^2 L}{\partial q_{31} \partial q_{11}} & \frac{\partial^2 L}{\partial q_{31} \partial q_{21}} & \frac{\partial^2 L}{\partial q_{31}^2} \end{vmatrix} < 0, \ldots\ldots,$$

$$
(-1)^{2(s+n)+1}
\overbrace{\left|\begin{array}{c:cccccccccc}
0 & \frac{\partial F}{\partial q_{11}} & --- & \frac{\partial F}{\partial q_{s1}} & \frac{\partial F}{\partial q_{12}} & --- & \frac{\partial F}{\partial q_{s2}} & \frac{\partial F}{\partial x_{11}} & --- & \frac{\partial F}{\partial x_{n2}} \\
\hdashline
\frac{\partial F}{\partial q_{11}} & \frac{\partial^2 L}{\partial q_{11}^{2}} & --- & \frac{\partial^2 L}{\partial q_{11}\partial q_{s1}} & \frac{\partial^2 L}{\partial q_{11}\partial q_{12}} & --- & \frac{\partial^2 L}{\partial q_{11}\partial q_{s2}} & \frac{\partial^2 L}{\partial q_{11}\partial x_{11}} & --- & \frac{\partial^2 L}{\partial q_{11}\partial x_{n2}} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\frac{\partial F}{\partial q_{s1}} & \frac{\partial^2 L}{\partial q_{s1}\partial q_{11}} & --- & \frac{\partial^2 L}{\partial q_{s1}^{2}} & \frac{\partial^2 L}{\partial q_{s1}\partial q12} & --- & \frac{\partial^2 L}{\partial q_{s1}\partial q_{s2}} & \frac{\partial^2 L}{\partial q_{s1}\partial x_{11}} & --- & \frac{\partial^2 L}{\partial q_{s1}\partial x_{n2}} \\
\frac{\partial F}{\partial q_{12}} & \frac{\partial^2 L}{\partial q_{12}\partial q_{11}} & --- & \frac{\partial^2 L}{\partial q_{12}\partial q_{s1}} & \frac{\partial^2 L}{\partial q_{12}^{2}} & --- & \frac{\partial^2 L}{\partial q_{12}\partial q_{s2}} & \frac{\partial^2 L}{\partial q_{12}\partial x_{11}} & --- & \frac{\partial^2 L}{\partial q_{12}\partial x_{n2}} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\frac{\partial F}{\partial q_{s2}} & \frac{\partial^2 L}{\partial q_{s2}\partial q_{11}} & --- & \frac{\partial^2 L}{\partial_{s2}\ \partial q_{s1}} & \frac{\partial^2 L}{\partial q_{s2}\partial q_{12}} & --- & \frac{\partial^2 L}{\partial q_{s2}^{2}} & \frac{\partial^2 L}{\partial q_{s2}\partial x_{11}} & --- & \frac{\partial^2 L}{\partial q_{s2}\partial x_{n2}} \\
\frac{\partial F}{\partial x_{11}} & \frac{\partial^2 L}{\partial x_{11}\partial q_{11}} & --- & \frac{\partial^2 L}{\partial x_{11}\partial q_{s1}} & \frac{\partial^2 L}{\partial x_{11}\partial q_{12}} & --- & \frac{\partial^2 L}{\partial x_{11}\partial q_{s2}} & \frac{\partial^2 L}{\partial x_{11}^{2}} & --- & \frac{\partial^2 L}{\partial x_{11}\partial x_{n2}} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\frac{\partial F}{\partial x_{n2}} & \frac{\partial^2 L}{\partial x_{n2}\partial q_{11}} & --- & \frac{\partial^2 L}{\partial x_{n2}\partial q_{s1}} & \frac{\partial^2 L}{\partial x_{n2}\partial q_{12}} & --- & \frac{\partial^2 L}{\partial x_{n2}\partial q_{s2}} & \frac{\partial^2 L}{\partial x_{n2}\partial x_{11}} & --- & \frac{\partial^2 L}{\partial x_{n2}^{2}}
\end{array}\right|}^{2(s+n)\,+\,1}
\Bigg\}\ 2(s+n)+1 \qquad (21)
$$

In this section of the multiperiod optimization model, several assumptions are made. These are: (1) output prices are constant over the planning period, (2) input price r_{i1} $(i=1,\ldots,n)$ is constant during the first three years and input prices r_{i2} $(i=1,2,\ldots,n)$ is constant during the rest of the planning horizon, where r_{i1} and r_{i2} may or may not be equal, (3) different fixed-proportions production processes are available, and (4) the discount rate is constant over the planning horizon. Under these assumptions the first order conditions for maximization of the total present value of profit stream can be summarized as follows:

(1) the marginal rate of transformation between any two outputs over the planning horizon equals the ratio of their prices;

(2) the marginal rate of technical substitution between any two inputs of the same period equals the ratio of their prices. The marginal rate of technical substitution between any two inputs of different periods equals the ratio of their discounted prices;

(3) the sum of the discounted value of marginal product equals the sum of the discounted input prices.

The second order conditions for maximization of the total present value of profit stream can also be summarized as follows:

(1) the marginal rate of transformation between outputs is increasing;

(2) the marginal rate of technical substitution between any two inputs is diminishing;

(3) the marginal product of each input increases at a decreasing rate.

A Multiperiod Linear Programming Model

If the implicit production functions [equation (2)] can be assumed to have different fixed proportions production processes and all inputs are divisible, then a multiperiod linear programming problem which is consistent with multiperiod optimization can be formulated. This multiperiod optimization problem can be written as:

$$\text{maximize: } \sum_{i=1}^{s} \sum_{t=1}^{3} P_i \, q_{i1} \, (1 + \rho)^{-t} - \sum_{j=1}^{n} \sum_{t=0}^{x} r_{j1} \, x_{j1} \, (1 + \rho)^{-t}$$

$$+ \sum_{i=1}^{s} \sum_{t=4}^{T} P_i q_{i2} \, (1 + \rho)^{-t} - \sum_{j=1}^{n} \sum_{t=3}^{T-1} r_{j2} \, x_{j2} \, (1 + \rho)^{-t}$$

$$\text{subject: } \begin{bmatrix} A_{11} & \\ --- & --- \\ & A_{22} \\ --- & --- \\ A_{31} & A_{32} \end{bmatrix} \begin{bmatrix} Q_1 \\ X_1 \\ --- \\ Q_2 \\ X_2 \end{bmatrix} \leq \begin{bmatrix} b_1 \\ \\ b_2 \\ \\ b_3 \end{bmatrix} \qquad (22)$$

and Q_i and $X_i \geq 0$ $i = 1,2$.

where A_{11} and A_{31} are submatrices of technical input-output coefficients for the first three years, A_{22} and A_{32} are submatrices of technical input-output coefficients for the rest of the planning horizon, and $Q_i' = [q_{1i}---q_{si}]$, for $i = 1,2$, $x_i' = [x_{1i} \; x_{2i} \; --- \; x_{ni}]$, for $i = 1, 2$, b_i is a vector of resource constraints (where b_1 and b_2 represent resource constraints for the first and second periods and b_3 is the transfer constraint from period one to period two), and all other variables are the same as defined previously. A rancher's multiperiod production decision problem for a representative ranch in Northwest Nevada is formulated according to equation set (22), as shown in Appendix I. The method of multiple grazing activities is used in a multiperiod simplex table to represent grazing activities on deeded and BLM lands (Garoian and Kim (4)). The acreage of deeded land for crested wheatgrass production, which maximizes multiperiod net revenues, is determined in the model.

Results obtained from a multiperiod linear programming problem are shown in Appendix II. Results show that a rancher can obtain maximum present value of the future profit stream, $2,548,393.00, by germinating crested wheatgrass on all 20,000 acres of deeded land.

Since yields in crested wheatgrass vary depending mainly upon rainfall during the spring months and reserve soil moisture, a rancher's attitude toward uncertain weather and consequently, yields may affect his decision on the investment problem. Therefore, nonsequential multi-period stochastic programming models are introduced in the next section to incorporate a rancher's risk or uncertainty in investment decision analysis.

A Nonsequential Multiperiod Stochastic Programming

In a multiperiod linear programming (MLP) model, all technical input-output coefficients and resource constraints are assumed to be known with certainty. One of the common problems in application of MLP is the difficulty in determining the proper values of technical input-output coefficients and resource constraints.

In stochastic programming, risk is represented by allowing a small probability for violating each constraint. Generally, two types of nonsequential stochastic (chance-constrained) programming problems can be identified. In the first case, all of the technical input-output coefficients are known constants, so that only some or all of the resource constraints are random variables. For the second case, some or all of the technical input-output coefficients are also random variables. Both cases will be studied in this section.

Case A: A multiperiod stochastic programming (MSP) problem under assumption that some or all of resource constraints are random variables.

In the previous section, results from MLP problems indicated that a rancher can obtain maximum values for the objective function by converting all 20,000 acres of deeded land from native grass production to crested wheatgrass production. However, yields in crested wheatgrass vary from year to year depending mainly on weather conditions. Therefore, a rancher's risk or uncertainty on an investment decision problem may be associated with fluctuating total crested wheatgrass production.

The MSP model for determining the optimal number of deeded acres that should be converted from native grass production to crested wheatgrass production can be formulated as follows:

$$\text{maximize: } \sum_{i=1}^{s}\left(\sum_{t=1}^{3} P_i\, q_{i1}\,(1+\rho)^{-t}\right) - \sum_{j=1}^{n}\left(\sum_{t=0}^{2} r_{j1}\, x_{j1}\,(1+\rho)^{-t}\right)$$

$$+ \sum_{i=1}^{s}\left(\sum_{t=4}^{T} P_i\, q_{i2}\,(1+\rho)^{-t}\right) - \sum_{j=1}^{n}\left(\sum_{t=3}^{T-1} r_{j2}\, x_{j2}\,(1+\rho)^{-t}\right)$$

subject to:

$$\begin{bmatrix} A_{11} & \\ & A^*_{22} \\ A_{31} & A_{32} \end{bmatrix} \begin{bmatrix} Q_1 \\ X_1 \\ Q_2 \\ X_2 \end{bmatrix} \leq \begin{bmatrix} b_1 \\ b_2^* \\ b_3 \end{bmatrix}$$

$$\mathbb{P}\left[a_{22}\, X_2 \leq B_2\right] \geq \alpha \qquad (23)$$

where $\mathbb{P}$ is a probability operator, α is a specified probability level between zero and one, $A_{22} = \begin{bmatrix} A_{22}^* \\ a_{22} \end{bmatrix}$ where a_{22} is a row vector which is associated with forage production on deeded land, $b_2 = \begin{bmatrix} b_2^* \\ B_2 \end{bmatrix}$ where B_2 is a total amounts of forage available from deeded land, and all other variables are the same as previously defined. Probabilities, α and $(1-\alpha)$, are interpreted as the decision-maker's confidence and risk allowance level, respectively. It is further assumed that the distribution of B_2 in equation (23) is assumed to be normally distributed with mean $E(B_2)$ and variance σ_B^2. Equation (23) indicates the probability that the total amount of forage required, B_2, is greater than or equal to forage production on deeded land, $a_{22}x_2$. This probability should be greater than or equal to the decision-maker's confidence level α. The probability constraint (23) can be converted properly into linear programming constraints, so that the simplex method can be applied to solve the problem. If the random variable B_2 has a normal probability distribution with mean $E(B)$ and variance σ_B^2, then its probability density function (pdf) is given by:

$$f(B) = \left(\frac{1}{\sigma_B \sqrt{2\Pi}}\right) \exp - \left(\frac{[B - E(B)]^2}{2\sigma_B^{\,2}}\right), -\infty < B < \infty$$

The random variable B can be converted to a standardized random variable K by subtracting its mean, E(B), and dividing by its standard deviation, σ_B. That is,

$$K = \frac{B - E(B)}{\sigma_B}$$

and the pdf of K is given by

$$f(k) = \frac{1}{\sqrt{2\Pi}} \exp\left(-\frac{k^2}{2}\right) \qquad -\infty < k < \infty$$

Since both the mean and standard deviation of K are fixed as zero and one, respectively, we can use a probability table to determine the area (probability) under any portion of the normal distribution. The probability table contains cumulative probability values of the standardized normal distribution. The relationship between the pdf of K, f(k), and the cumulative density function (cdf) of K, F(k), is shown graphically in Figure 1.

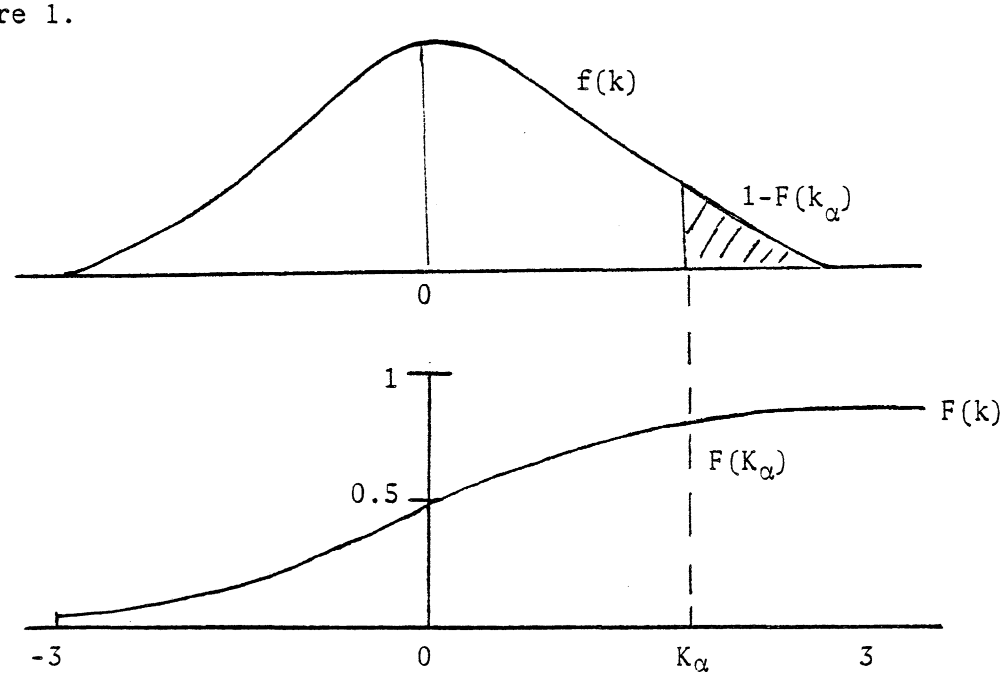

Figure 1. The pdf and the cdf of K

Now the probability constraint [equation (23)] can be written by the standardized form as follows:

$$\mathbb{P}\left(\frac{a_{22}x_2 - E(B)}{\sigma_B} \leq \frac{B - E(B)}{\sigma_B}\right) \geq \alpha \tag{24}$$

Equation (24) can also be written as,

$$\mathbb{P}\left(K \geq \frac{a_{22}x_2 - E(B)}{\sigma_B}\right) \geq \alpha \tag{25}$$

if and only if, $\frac{a_{22}\ x_2 - E(B)}{\sigma_B} \leq K_\alpha$ where $\alpha = \int_{K_\alpha}^{\infty} \frac{1}{\sqrt{2\Pi}} e^{-\frac{k^2}{2}}\, d\, k$

This implies that $\mathbb{P}\left[a_{22}x_2 \leq B\right] \geq \alpha$

$$\text{if and only if } \frac{a_{22}x_2 - E(B)}{\sigma_B} \leq K_\alpha \tag{26}$$

$$\text{That is, } a_{22}x_2 \leq E(B) + K_\alpha\ \sigma_B \tag{27}$$

Euqation (27) can be illustrated graphically as shown in Figure 2.

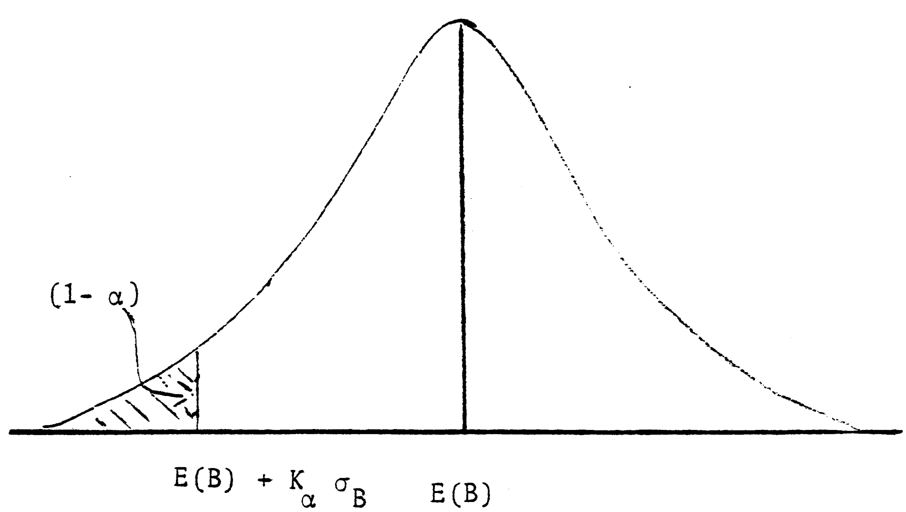

Figure 2. Probability Density Function of B

The stochastic constraint expressed by equation (27) is equivalent to the deterministic linear constraint. Therefore, equation (27) can be inserted into the nonsequential stochastic programming model to replace the probability constraint. Unfortunately, data for yields of native grass and crested wheatgrass which grow in Nevada are not available. However, Mitchell and Garrett (10) have estimated average yields per acre of native grass and crested wheatgrass to be 50 pounds and 500 pounds, respectively. Information about means and standard deviations in yields of native grass and crested wheatgrass were taken from a Canadian study (Smoliak (12)) and are shown in Table 1.

Table 1. The Means and Standard Deviations in Yield/Acre of Native Grass and Crested Wheatgrass in Alberta, Canada.

	Native Grass	Crested Wheatgrass
Mean (lbs.)	393	832
Standard Deviation (lbs.)	145.11	257.44

To approximate variances in yields of native grass and crested wheatgrass in Nevada, standard deviations shown in Table 1 are scaled down using means as weights. The results are shown in Table 2.

Table 2. Means and Estimated Standard Deviations in Yields of Native Grass and Crested Wheatgrass in Nevada.

	Native Grass	Crested Wheatgrass
Mean (lbs.)	50	500
Standard Deviation (lbs.)	18.46	154.71
Coefficient of variation	2.71	3.21

Using data shown in Table 2, results obtained from a static multiperiod stochastic programming problem at different α-levels are given in Table 6 through Table 10, Appendix II. These results are also summarized in Table 3 and Figures 3 and 4.

Table 3. Values of Objective Function and Acreages for Germinating Crested Wheatgrass at Different α levels (Case A).

α-level (Confidence level)	0.95	0.90	0.85	0.80	0.75	MLP SOL.
value of objective function (1,000 $)	2,011.0	2,130.5	2,208.8	2,270.9	2,328.0	2,548.4
acreage for germinating crested wheatgrass	9,820	12,079	13,564	14,740	15,825	20,000

Results in Table 3 indicate that a rancher's acreage allocation for crested wheatgrass production increases at a decreasing rate as the risk allowance level increases. For instance, α = 0.85 implies that the resource constraints on deeded land will be met 85 percent of the time (risk allowance of 15 percent) generating an objective function value of $2,208,800.00 with 13,564 acres planted in crested wheatgrass.

In this section, it has been assumed that total amounts of forage from crested wheatgrass production on deeded land are normally distributed (i.e., a resource constraint coefficient is assumed to be normally distributed)

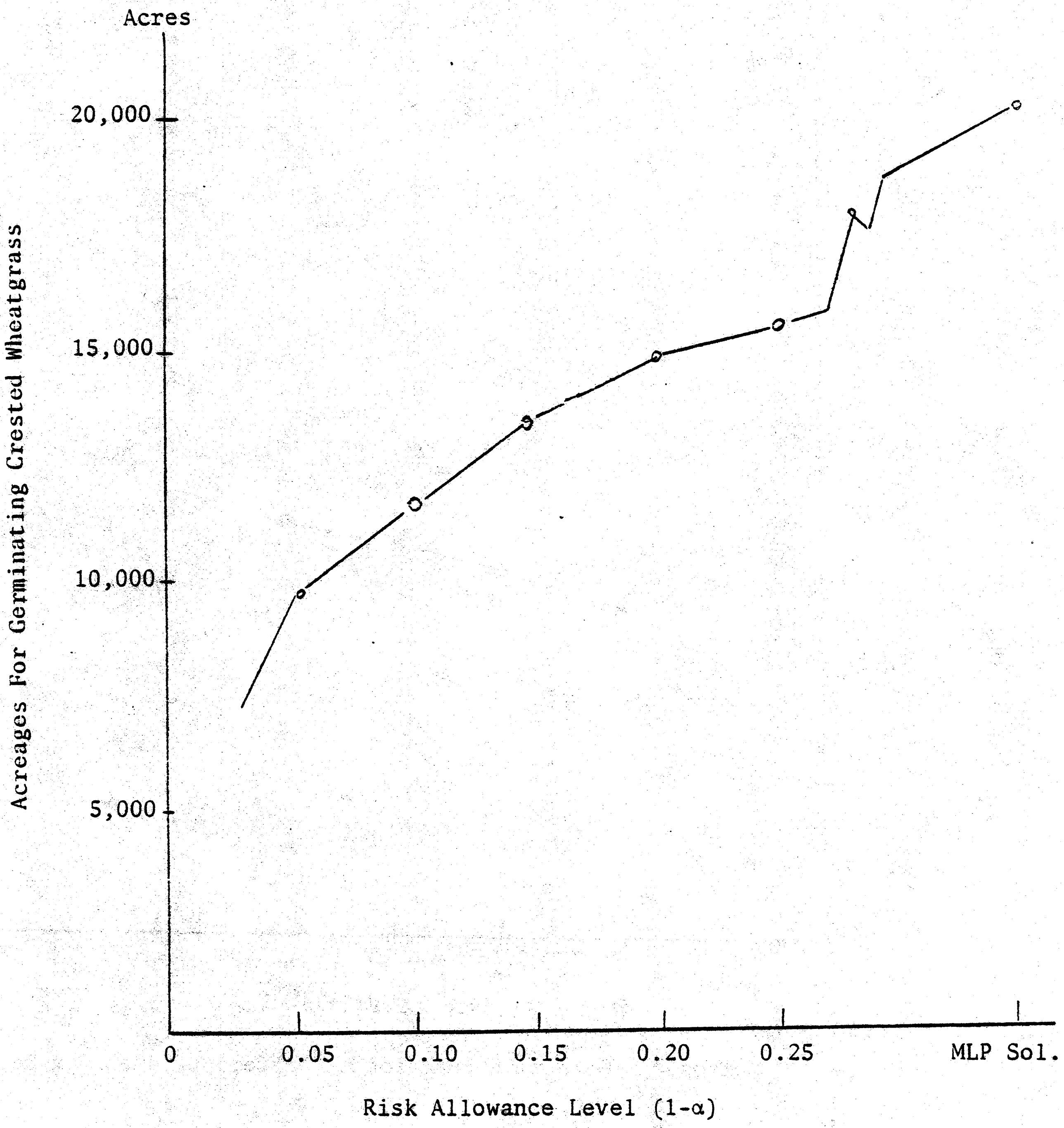

Figure 3. Acreages for Germinating Crested Wheatgrass at Different Levels of Risk Allowance.

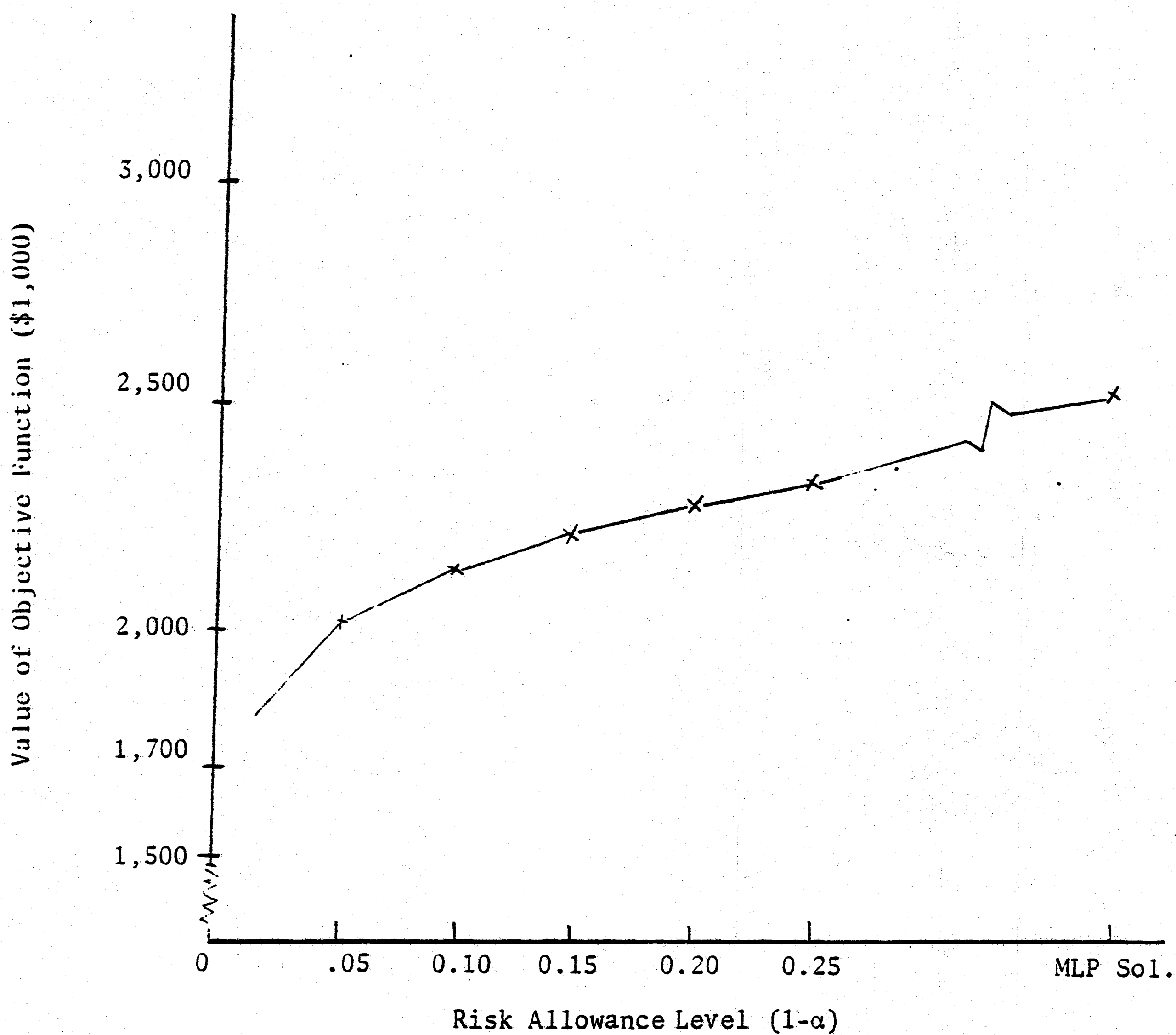

Figure 4. Values of Objective Function at Different Levels of Risk Variance

and yields of crested wheatgrass per acre (i.e., technical input-output coefficient) are assumed to be known constants. However, various weather conditions affect not only crested wheatgrass production, but also native grass production on deeded land. In Table 2, means and estimated standard deviations of yield are 50 and 18.46, respectively, for native grass and 500 and 154.71, respectively, for crested wheatgrass. Ratios of mean values to respective standard deviations (coefficients of variation) are 2.71 and 3.21 for native grass and crested wheatgrass, respectively. These ratios indicate that yields of native grass are more fluctuating than crested wheatgrass. Therefore, ranchers may germinate crested wheatgrass not only to increase forage production, but also to avoid relatively high risks associated with native grass production. Next, variations in yields of both native grass and crested wheatgrass are considered as principal sources of risk involved in a rancher's decision problem.

Case B: The technical input-output coefficients associated with native grass and crested wheatgrass productions are random variables.

Consider the probabilistic constraint such that:

$$\mathbb{P}\left[a_{22}x_2 \leq B_2 \right] = 1-\alpha \qquad (28)$$

In the previous section, the technical input-output coefficients, a_{22}, were assumed to be known constant.

Suppose that $a_{22}x_2$ is normally distributed with mean vector $E(a_{22})x_2$ and variance-covariance matrix $x_2'Wx_2$, where W is the variance-covariance matrix of a_{22}. Equation (28) shows the probability that the total forage production on deeded land, $a_{22}x_2$, is less than or equal to the amount of total forage required, B_2. This probability should be equal to the risk allowance level 1-α. The probabilistic equation (28) can be written as:

$$\mathbb{P}\left[\frac{a_{22}x_2 - E(a_{22})x_2}{(x_2'Wx_2)^{\frac{1}{2}}} \le \frac{B_2 - E(a_{22})x_2}{(x_2'Wx_2)^{\frac{1}{2}}}\right] = 1-\alpha$$

or

$$\mathbb{P}\left[Z \le \frac{B_2 - E(a_{22})x_2}{(x_2'Wx_2)^{\frac{1}{2}}}\right] = 1-\alpha$$

where: $$Z = \frac{a_{22}x_2 - E(a_{22})x_2}{(x_2'Wx_2)^{\frac{1}{2}}}$$

if and only if $$\frac{B_2 - E(a_{22})x_2}{(x_2'Wx_2)^{\frac{1}{2}}} = K_{1-\alpha}$$

Therefore, $$\mathbb{P}\left[a_{22}x_2 \le B_2\right] = 1-\alpha$$

with $$E(a_{22})x_2 + K_{1-\alpha}(x_2'Wx_2)^{\frac{1}{2}} = B_2 \qquad (29)$$

Therefore, the probabilistic constraint [equation (28)] can be replaced with the nonlinear constraint [equation (29)]. However, equation (29) includes a quadratic form and therefore, it cannot be solved by simplex method. However, the nonlinear constraint [equation (29)] can be approximated under a certain condition.

Consider the following relation such that:

$$(x'Wx)^{\frac{1}{2}} = \left(\sum_i^n x_i\ \sigma_i^2 + \sum_{\substack{i\\ i \neq j}}\sum_s x_i x_j\ \sigma_{ij}\right)^{\frac{1}{2}}$$

$$\le \left(\sum_i^n x_i\ \sigma_i^2 + \sum_{\substack{i\\ i \neq j}}\sum_j x_i x_j\ \sigma_i \sigma_j\right)^{\frac{1}{2}} \quad \text{from the Cauchy-Schwarz inequality}$$

$$= \Sigma x_i \sigma_i \qquad (30)$$

By replacing $(x'Wx)^{\frac{1}{2}}$ in equation (29) with $\Sigma x_i\sigma_i$, we have the linearized equation, $E(a_{22})x_2 + K_{1-\alpha} \sum_i x_i\sigma_i = B_2$ (31)

Equation (31) has been applied by Rahman and Bender (11) in their study of least-cost feed mixes. However, as Chen (2) noted, it is allowed for use in cases where random variables are highly correlated to each other.

In equation (30), $\Sigma x_i\sigma_i$ overestimates $(x'Wx)^{\frac{1}{2}}$. However, bias is reduced as the correlation between a_i and a_j increases. In cases where a_i and a_j are perfectly correlated, the linearized equation (31) and the nonlinear equation (29) are identical. Even though data for yields in native grass and crested wheatgrass on the same field in Nevada are not available, they may be highly correlated to each other. Therefore, bias generated from using the linearized constraint [equation (31)] instead of the nonlinear constraint [equation (29)] would be minimal.

The MSP model described in this section is formulated as follows:

$$\text{Maximize:} \quad \sum_{i=1}^{s}\left(\sum_{t=1}^{3} P_i q_{i1}\,(1+\rho)^{-t}\right) - \sum_{j=1}^{n}\left(\sum_{t=0}^{2} r_{j1}x_{j1}\,(1+\rho)^{-t}\right) + \sum_{i=1}^{s}\left(\sum_{t=4}^{T} P_i q_{i2}\,(1+\rho)^{-t}\right) - \sum_{j=1}^{n}\left(\sum_{t=3}^{T-1} r_{j2}x_{j2}\,(1+\rho)^{-t}\right)$$

$$\text{Subject to:} \quad \begin{bmatrix} A_{11} & \\ & A_{22}{}^* \\ A_{31} & A_{32} \end{bmatrix} \begin{bmatrix} Q_1 \\ X_1 \\ Q_2 \\ X_2 \end{bmatrix} \le \begin{bmatrix} b_1 \\ b_2{}^* \\ b_3 \end{bmatrix}$$

$$E(a_{22})x_2 + K_{1-\alpha} \sum_i x_i\sigma_i = B_2$$

Results shown in Table 4 are obtained from applying the above MSP model by allowing α-level to vary. Results are also given in Appendix IV in detail.

Table 4. Values of Objective Function and Acreages for Germinating Crested Wheatgrass at Different α-levels (Case B).

α-level	0.95	0.90	0.85	0.80	0.75	MLP SOL.
value of objective function (1,000$)	2,151.1	2,214.2	2,261.8	2,303.6	2,345.8	2,548.4
acreage for germinating crested wheatgrass	12,470	13,667	14,568	15,361	16,160	20,000

Results in Tables 3 and 4 are also depicted in Figure 5.

Results in Table 4 also indicate that acreage allocated for germinating crested wheatgrass increase at a decreasing rate as the risk allowance levels increase. For example, α=0.85 implies that the resource constraints on deeded land will be met 85% of the time (risk allowance of 15%) generating an objective function value of $2,261,800 with 14,568 acres planted in crested wheatgrass. In Figure 5, acreages allocated for crested wheatgrass germination in Case B are always higher for given levels of risk allowance than in Case A. It should be noted that the results are obtained from linear approximation of the quadratic form. As long as yields in crested wheatgrass and native grass are not perfectly correlated, bias exists. Since the linear approximation expressed by equation (30) overestimates the real variance, this bias contributes to the differences in acreage allocation for crested wheatgrass germination between Case A and Case B. However, differences between Cases A and B decrease as the risk allowance level increases. As explained earlier, a rancher may germinate crested wheatgrass on deeded land not only to increase forage production, but also to avoid the relatively high risk associated with native grass production.

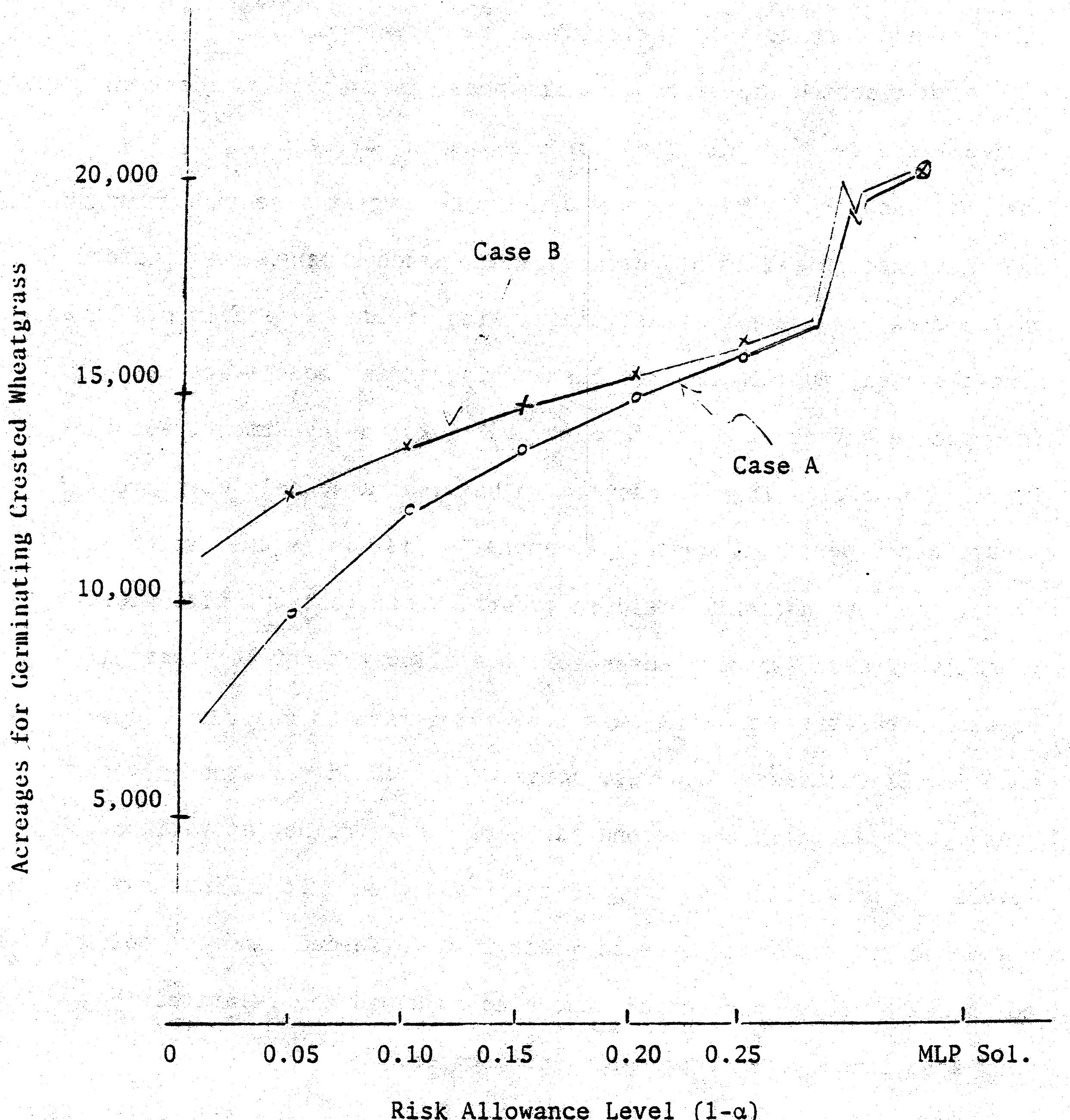

Figure 5. Acreages for Germinating Crested Wheatgrass at Different Levels of Risk Allowance and at Different Assumptions

Conclusion

Most ranchers in Nevada rely on forages from deeded and BLM lands during the grazing season. Since there exists increasing restrictions on BLM grazing allowances, ranchers may be willing to improve their deeded land to meet their forage requirements.

A multiperiod linear programming model is used to estimate an optimal acreage of deeded land for germinating crested wheatgrass. Results show that a rancher may obtain the largest objective function value from germinating all of his deeded land. Since a rancher's attitude toward risk or uncertainty associated with yields may affect his investment decision, multiperiod stochastic programming models are used to incorporate a rancher's attitude toward risks in investment decisions. First, a rancher's risk is assumed to be associated only with crested wheatgrass production. Second, a rancher's risk is assumed to be associated with not only yield in crested wheatgrass but also yield in native grass. In both cases acreages allocated for germinating crested wheatgrass increase at a decreasing rate as the risk allowance levels increase. However, acreages allocated for crested wheatgrass germination in the second case are always higher at various levels of risk allowance than in the first case. It indicates that a rancher may germinate crested wheatgrass on deeded land not only to increase forage production, but also to avoid the relatively high risk associated with native grass production.

APPENDIX I
(MLP MODEL)

Submatrix A_{11}

S1: March 15 - April 30
S2: May 1 - September 14
S3: September 15 - November 14
S4: November 15 - November 30
S5: December 1 - March 14

		Unit	Sign	RHS	R \ C	Raise alfalfa	Al. aftermath S3	Al. aftermath S4	Buy alfalfa	Sell alfalfa	Feed Al. S1	Feed Al. S2	Feed Al. S3	Feed Al. S4	Feed Al. S5	Raise Grass Hay	Grass After. S3	Grass After. S4	Feed Gr. S1	Feed Gr. S2	Feed Gr. S3	Feed Gr. S4	Feed Gr. S5	Meadow S3	Meadow S4
						a			b	c						d									
						1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Alfalfa land		Acre	≤	200	1	1																			
Alfalfa aftermath		AUM	≤	0	2	-.75	1	1																	
Alfalfa transfer		Ton	≤	0	3	-3.63			-1	1	1	1	1	1	1										
Grass hay land		Acre	≤	2,000	4											1									
Grass hay aftermath		AUM	≤	0	5											-.75	1	1							
Grass hay transfer		Ton	≤	0	6											-1.33			1	1	1	1	1		
Meadow pasture		Acre	≤	700	7																			1	1
Total deeded land		Acre	≤	20,000	8																				
Native grass transfer		AUM	≤	0	9																				
Forage transfer Season 1 and season 2	.65-S1	AUM	≤	0	10						-3								-3						
	.65-S2	AUM	≤	0	11							-3								-3					
	.8-S2	AUM	≤	0	12																				
	.9-S1	AUM	≤	0	13																				
	.9-S2	AUM	≤	0	14																				
	1 -S1	AUM	≤	0	15																				
	1 -S2	AUM	≤	0	16																				
	1.25-S1	AUM	≤	0	17																				
	1.25-S2	AUM	≤	0	18																				
	1.3 -S1	AUM	≤	0	19																				
	1.3 -S2	AUM	≤	0	20																				
Total BLM allowance		AUM	≤	4,000	21																				
Bull requirement		Head	≤	0	22																				
Bull requirement		Head	≤	0	23																				
Forage Transfer	S-3	AUM	≤	0	24		-.75						-3				-.75				-3			-.75	
	S-4	AUM	≤	0	25			-.75						-3				-.75				-3			-.75
	S-5	AUM	≤	0	26										-3								-3		
Alfalfa requirement		Ton	≤	0	27										-5								1		
Steer calf (1-8 mo.)		Head	≤	0	28																				
Heifer calf (1-8 mo.)		Head	≤	0	29																				
Weaner (9-12 mo.)		Head	≤	0	30																				
Yearling (13-17 mo.)		Head	≤	0	31																				
Yearling (18-24 mo.)		Head	≤	0	32																				
Replacement (25-32 mo.)		Head	≤	0	33																				
Bull requirement		Head	≤	0	34																				
Replace cull cow		Head	≤	0	35																				
Bull transfer		Head	≤	0	36																				
Pregnancy		Head	≤	0	37																				
Sell steer calf		Head	≤	0	38																				
Sell heifer calf		Head	<	0	39																				
Sell cull cow		Head	≥	0	40																				
Sell cull bull		Head	≤	0	41																				

Submatrix A_{11}

S1: March 15 - April 30
S2: May 1 - September 14
S3: September 15 - November 14
S4: November 15 - November 30
S5: December 1 - March 14

		Unit	Sign	RHS	R \ C	Native Grass	Grazing on Deeded Land											
							0.65	.65	.8	.9	.9	1	1	1.25	1.25	1.3	1.3	
							S1	S2	S2	S1	S2	S1	S2	S1	S2	S1	S2	S3
						21	22	23	24	25	26	27	28	29	30	31	32	33
Alfalfa land		Acre	≤	200	1													
Alfalfa aftermath		AUM	≤	0	2													
Alfalfa transfer		Ton	≤	0	3													
Grass hay land		Acre	≤	2,000	4													
Grass hay aftermath		AUM	≤	0	5													
Grass hay transfer		Ton	≤	0	6													
Meadow pasture		Acre	≤	700	7													
Total deeded land		Acre	≤	20,000	8	1												
Native grass transfer		AUM	≤	0	9	- .0625	.65	.65	.8	.9	.9	1	1	1.25	1.25	1.3	1.3	.0625
Forage transfer Season 1 and season 2	.65-S1	AUM	≤	0	10		-.65											
	.65-S2	AUM	≤	0	11			-.65										
	. 8-S2	AUM	≤	0	12				-.8									
	. 9-S1	AUM	≤	0	13					-.9								
	. 9-S2	AUM	≤	0	14						-.9							
	1 -S1	AUM	≤	0	15							-1						
	1 -S2	AUM	≤	0	16								-1					
	1.25-S1	AUM	≤	0	17									-1.25				
	1.25-S2	AUM	≤	0	18										-1.25			
	1.3 -S1	AUM	≤	0	19											-1.3		
	1.3 -S2	AUM	≤	0	20												-1.3	
Total BLM allowance		AUM	≤	4,000	21													
Bull requirement		Head	≤	0	22						1		1		1		-20	
Bull requirement		Head	≤	0	23													
Forage Transfer	S-3	AUM	≤	0	24													
	S-4	AUM	≤	0	25													-.0625
	S-5	AUM	≤	0	26													
Alfalfa requirement		Ton	≤	0	27													
Steer calf (1-8 mo.)		Head	≤	0	28													
Heifer calf (1-8 mo.)		Head	≤	0	29													
Weaner (9-12 mo.)		Head	≤	0	30													
Yearling (13-17 mo.)		Head	≤	0	31													
Yearling (18-24 mo.)		Head	≤	0	32													
Replacement (25-32 mo.)		Head	≤	0	33													
Bull requirement		Head	≤	0	34													
Replace cull cow		Head	≤	0	35													
Bull transfer		Head	≤	0	36													
Pregnancy		Head	≤	0	37													
Sell steer calf		Head	≤	0	38													
Sell heifer calf		Head	≤	0	39													
Sell cull cow		Head	≤	0	40													
Sell cull bull		Head	≤	0	41													

Submatrix A_{11} S1: March 15 - April 30 S2: May 1 - September 14 S3: September 15 - November 14 S4: November 15 - November 30 S5: December 1 - March 14						Crested Wheatgrass	Grazing on BLM March 15 - September 14)										
							0.65	.65	.8	.9	.9	1	1	1.25	1.25	1.3	1.3
							S1	S2	S2	S1	S2	S1	S2	S1	S2	S1	S2
						e	f	g	h	i	j	k	l	m	n	o	p
		Unit	Sign	RHS	R \ C	34	35	36	37	38	39	40	41	42	43	44	45
Alfalfa land		Acre	≤	200	1												
Alfalfa aftermath		AUM	≤	0	2												
Alfalfa transfer		Ton	≤	0	3												
Grass hay land		Acre	≤	2,000	4												
Grass hay aftermath		AUM	≤	0	5												
Grass hay transfer		Ton	≤	0	6												
Meadow pasture		Acre	≤	700	7												
Total deeded land		Acre	≤	20,000	8	1											
Native grass transfer		AUM	≤	0	9												
Forage transfer Season 1 and season 2	.65-S1	AUM	≤	0	10		-.65										
	.65-S2	AUM	≤	0	11			-.65									
	. 8-S2	AUM	≤	0	12				-.8								
	. 9-S1	AUM	≤	0	13					-.9							
	. 9-S2	AUM	≤	0	14						-.9						
	1 -S1	AUM	≤	0	15							-1					
	1 -S2	AUM	≤	0	16								-1				
	1.25-S1	AUM	≤	0	17									-1.25			
	1.25-S2	AUM	≤	0	18										-1.25		
	1.3 -S1	AUM	≤	0	19											-1.3	
	1.3 -S2	AUM	≤	0	20												-1.3
Total BLM allowance		AUM	≤	4,000	21		1	1	1	1	1	1	1	1	1	1	1
Bull requirement		Head	≤	0	22												
Bull requirement		Head	≤	0	23						1		1		1		-20
Forage Transfer	S-3	AUM	≤	0	24												
	S-4	AUM	≤	0	25												
	S-5	AUM	≤	0	26												
Alfalfa requirement		Ton	≤	0	27												
Steer calf (1-8 mo.)		Head	≤	0	28												
Heifer calf (1-8 mo.)		Head	≤	0	29												
Weaner (9-12 mo.)		Head	≤	0	30												
Yearling (13-17 mo.)		Head	≤	0	31												
Yearling (18-24 mo.)		Head	≤	0	32												
Replacement (25-32 mo.)		Head	≤	0	33												
Bull requirement		Head	≤	0	34												
Replace cull cow		Head	≤	0	35												
Bull transfer		Head	≤	0	36												
Pregnancy		Head	≤	0	37												
Sell steer calf		Head	≤	0	38												
Sell heifer calf		Head	≤	0	39												
Sell cull cow		Head	≤	0	40												
Sell cull bull		Head	≤	0	41												

Submatrix A_{11} S1: March 15 - April 30 S2: May 1 - September 14 S3: September 15 - November 14 S4: November 15 - November 30 S5: December 1 - March 14					Cow w/calf	Cow w/o calf	Steer calf	Heifer calf	Weaner (9-12)	Yearling (13-17)	Yearling (18-24)	Replacement (25-32)	Bull	Sell steer calf	Sell heifer calf	Sell cull cow	Sell cull bull	Buy bull
					q	r								s	t	u	v	w
	Unit	Sign	RHS	R \ C	46	47	48	49	50	51	52	53	54	55	56	57	58	59
Alfalfa land	Acre	≤	200	1														
Alfalfa aftermath	AUM	≤	0	2														
Alfalfa transfer	Ton	≤	0	3														
Grass hay land	Acre	≤	2,000	4														
Grass hay aftermath	AUM	≤	0	5														
Grass hay transfer	Ton	≤	0	6														
Meadow pasture	Acre	≤	700	7														
Total deeded land	Acre	≤	20,000	8														
Native grass transfer	AUM	≤	0	9														
Forage transfer Season 1 and season 2: .65-S1	AUM	≤	0	10						.975								
.65-S2	AUM	≤	0	11						2.275								
. 8-S2	AUM	≤	0	12							.8							
. 9-S1	AUM	≤	0	13								1.35						
. 9-S2	AUM	≤	0	14								4.05						
1 -S1	AUM	≤	0	15		1.5												
1 -S2	AUM	≤	0	16		4.5												
1.25-S1	AUM	≤	0	17	1.825													
1.25-S2	AUM	≤	0	18	6.125													
1.3 -S1	AUM	≤	0	19									1.95					
1.3 -S2	AUM	≤	0	20									5.85					
Total BLM allowance	AUM	≤	4,000	21														
Bull requirement	Head	≤	0	22														
Bull requirement	Head	≤	0	23														
Forage Transfer S-3	AUM	≤	0	24	2.0	2.0	0.5	0.5			1.6	1.8	2.6					
S-4	AUM	≤	0	25	0.5	0.5			0.25		.4		.65					
S-5	AUM	≤	0	26	3.5	3.5			1.75		2.8		4.55					
Alfalfa requirement	Ton	≤	0	27														
Steer calf (1-8 mo.)	Head	≤	0	28	- .49		1											
Heifer calf (1-8 mo.)	Head	≤	0	29	- .49			1										
Weaner (9-12 mo.)	Head	≤	0	30				- .99	1									
Yearling (13-17 mo.)	Head	≤	0	31					- .985	1								
Yearling (18-24 mo.)	Head	≤	0	32						- .99	1							
Replacement (25-32 mo.)	Head	≤	0	33							- .99	1						
Bull requirement	Head	≤	0	34	1	1						1	-20					
Replace cull cow	Head	≤	0	35		.5						- .8						
Bull transfer	Head	≤	0	36									.25					-1
Pregnancy	Head	≤	0	37	26	- .74												
Sell steer calf	Head	≤	0	38			-4							1				
Sell heifer calf	Head	≤	0	39				-3.5							1			
Sell cull cow	Head	≤	0	40		-5						-1.6				1		
Sell cull bull	Head	≤	0	41									- 3.25				1.02	

Submatrix A_{22}

S1: March 15 - April 30
S2: May 1 - Sept. 14
S3: Sept. 15 - Nov. 14
S4: Nov. 15 - Nov. 30
S5: Dec. 1 - March 14

		Unit	Sign	RHS	R \ C	Raise alfalfa	Alf. aftermath S5	Alf. aftermath S4	Buy alfalfa	Sell alfalfa	Feed Alf. S1	Feed Alf. S2	Feed Alf. S3	Feed Alf. S4	Feed Alf. S5	Raise grass hay	Grass hay after.S5	Grass hay after.S4	Feed grass hay S1	Feed grass hay S2	Feed grass hay S3
						A			B	C						D					
						60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Alfalfa land		Acre	<	200	42	1															
Alfalfa aftermath		AUM	<	0	43	- .75	1	1													
Alfalfa transfer		Ton	<	0	44	-3.63			-1	1	1	1	1	1	1						
Grass hay land		Acre	<	2,000	45											1					
Grass hay aftermath		AUM	<	0	46											- .75	1	1			
Grass hay transfer		Ton	<	0	47											-1.33			1	1	1
Meadow pasture		Acre	<	700	48																
Total deeded land		Acre	<	20,000	49																
Native grass transfer		AUM	<	0	50																
Crested wheatgrass transfer		AUM	<	0	51																
Forage transfer Season 1 and Season 2	.65-S1	AUM	<	0	52						-3								-3		
	.65-S2	AUM	<	0	53							-3								-3	
	.8 -S2	AUM	<	0	54																
	.9 -S1	AUM	<	0	55																
	.9 -S2	AUM	<	0	56																
	1.0 -S1	AUM	<	0	57																
	1.0 -S2	AUM	<	0	58																
	1.25-S1	AUM	<	0	59																
	1.25-S2	AUM	<	0	60																
	1.3 -S1	AUM	<	0	61																
	1.3 -S2	AUM	<	0	62																
Total BLM allowance		AUM	<	4,000	63																
Bull requirement		Head	<	0	64																
Bull requirement		Head	<	0	65																
Bull requirement		Head	<	0	66																
Forage transfer	S-3	AUM	<	0	67		-.75						-3				-.75				-3
	S-4	AUM	<	0	68			-.75						-3				-.75			
	S-5	AUM	<	0	69										-3						
Alfalfa requirement		Ton	<	0	70										-5						
Steer calf (1-8 mo.)		Head	<	0	71																
Heifer calf (1-8 mo.)		Head	<	0	72																
Weaner (9-12 mo.)		Head	<	0	73																
Yearling (13-17 mo.)		Head	<	0	74																
Yearling (18-24 mo.)		Head	<	0	75																
Replacement (25-32 mo.)		Head	<	0	76																
Bull requirement		Head	<	0	77																
Replace cull cow		Head	<	0	78																
Bull transfer		Head	<	0	79																
Pregnancy		Head	<	0	80																
Sell steer calf		Head	<	0	81																
Sell Heifer calf		Head	<	0	82																
Sell cull cow		Head	<	0	83																
Sell cull bull		Head	<	0	84																

Submatrix A_{22} S1: March 15 - April 30 S2: May 1 - Sept. 14 S3: Sept. 15 - Nov. 14 S4: Nov. 15 - Nov. 30 S5: Dec. 1 - March 14					Feed grass hay S4	Feed grass hay S5	Meadow S5	Meadow S4	Native grass	Grazing on Deeded Land .65 S1	.65 S2	.8 S2	.9 S1	.9 S2	1 S1	1 S2	1.25 S1	1.25 S2	1.3 S1	1.3 S2	625 S3
	Unit	Sign	RHS	R \ C	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92
Alfalfa land	Acre	≤	200	42																	
Alfalfa aftermath	AUM	≤	0	43																	
Alfalfa transfer	Ton	≤	0	44																	
Grass hay land	Acre	≤	2,000	45																	
Grass hay aftermath	AUM	≤	0	46																	
Grass hay transfer	Ton	≤	0	47	1	1															
Meadow pasture	Acre	≤	700	48			1	1													
Total deeded land	Acre	≤	20,000	49					1												
Native grass transfer	AUM	≤	0	50					-.0625	.65	.65	.8	.9	.9	1	1	1.25	1.25	1.3	1.3	.0625
Crested wheatgrass transfer	AUM	≤	0	51																	
Forage transfer Season 1 and Season 2 .65-S1	AUM	≤	0	52						-.65											
.65-S2	AUM	≤	0	53							-.65										
.8 -S2	AUM	≤	0	54								-.8									
.9 -S1	AUM	≤	0	55									-.9								
.9 -S2	AUM	≤	0	56										-.9							
1.0 -S1	AUM	≤	0	57											-1						
1.0 -S2	AUM	≤	0	58												-1					
1.25-S1	AUM	≤	0	59													-1.25				
1.25-S2	AUM	≤	0	60														-1.25			
1.3 -S1	AUM	≤	0	61															-1.3		
1.3 -S2	AUM	≤	0	62																-1.3	
Total BLM allowance	AUM	≤	4,000	63																	
Bull requirement	Head	≤	0	64										1		1		1		-20	
Bull requirement	Head	≤	0	65																	
Bull requirement	Head	≤	0	66																	
Forage transfer S-3	AUM	≤	0	67			-.75														.0625
S-4	AUM	≤	0	68	-3			-.75													
S-5	AUM	≤	0	69		-3															
Alfalfa requirement	Ton	≤	0	70																	
Steer calf (1-8 mo.)	Head	≤	0	71		1															
Heifer calf (1-8 mo.)	Head	≤	0	72																	
Weaner (9-12 mo.)	Head	≤	0	73																	
Yearling (13-17 mo.)	Head	≤	0	74																	
Yearling (18-24 mo.)	Head	≤	0	75																	
Replacement (25-32 mo.)	Head	≤	0	76																	
Bull requirement	Head	≤	0	77																	
Replace cull cow	Head	≤	0	78																	
Bull transfer	Head	≤	0	79																	
Pregnancy	Head	≤	0	80																	
Sell steer calf	Head	≤	0	81																	
Sell Heifer calf	Head	≤	0	82																	
Sell cull cow	Head	≤	0	83																	
Sell cull bull	Head	≤	0	84																	

Submatrix A_{ij}

S1: March 15 - April 30
S2: May 1 - Sept. 14
S3: Sept. 15 - Nov. 14
S4: Nov. 15 - Nov. 30
S5: Dec. 1 - March 14

					Crested wheatgrass	Grazing Crested Wheatgrass											
						.65 S1	.65 S2	.8 S2	.9 S1	.9 S2	1 S1	1 S2	1.25 S1	1.25 S2	1.3 S1	1.3 S2	.625 S3
	Unit	Sign	RHS	R \ C	93	94	95	96	97	98	99	100	101	102	103	104	105
Alfalfa land	Acre	≤	200	42													
Alfalfa aftermath	AUM	≤	0	43													
Alfalfa transfer	Ton	≤	0	44													
Grass hay land	Acre	≤	2,000	45													
Grass hay aftermath	AUM	≤	0	46													
Grass hay transfer	Ton	≤	0	47													
Meadow pasture	Acre	≤	700	48													
Total deeded land	Acre	≤	20,000	49	1												
Native grass transfer	AUM	≤	0	50													
Crested wheatgrass transfer	AUM	≤	0	51	-.625	.65	.65	.8	.9	.9	1	1	1.25	1.25	1.3	1.3	.625
Forage transfer, Season 1 and Season 2: .65-S1	AUM	≤	0	52		-.65											
.65-S2	AUM	≤	0	53			-.65										
.8 -S2	AUM	≤	0	54				-.8									
.9 -S1	AUM	≤	0	55					-.9								
.9 -S2	AUM	≤	0	56						-.9							
1.0 -S1	AUM	≤	0	57							-1						
1.0 -S2	AUM	≤	0	58								-1					
1.25-S1	AUM	≤	0	59									-1.25				
1.25-S2	AUM	≤	0	60										-1.25			
1.3 -S1	AUM	≤	0	61											-1.3		
1.3 -S2	AUM	≤	0	62												-1.3	
Total BLM allowance	AUM	≤	4,000	63													
Bull requirement	Head	≤	0	64													
Bull requirement	Head	≤	0	65						1		1		1		-20	
Bull requirement	Head	≤	0	66													
Forage transfer S-3	AUM	≤	0	67													.625
Forage transfer S-4	AUM	≤	0	68													
Forage transfer S-5	AUM	≤	0	69													
Alfalfa requirement	Ton	≤	0	70													
Steer calf (1-8 mo.)	Head	≤	0	71													
Heifer calf (1-8 mo.)	Head	≤	0	72													
Weaner (9-12 mo.)	Head	≤	0	73													
Yearling (13-17 mo.)	Head	≤	0	74													
Yearling (18-24 mo.)	Head	≤	0	75													
Replacement (25-32 mo.)	Head	≤	0	76													
Bull requirement	Head	≤	0	77													
Replace cull cow	Head	≤	0	78													
Bull transfer	Head	≤	0	79													
Pregnancy	Head	≤	0	80													
Sell steer calf	Head	≤	0	81													
Sell Heifer calf	Head	≤	0	82													
Sell cull cow	Head	≤	0	83													
Sell cull bull	Head	≤	0	84													

Submatrix A_{22}

S1: March 15 - April 30
S2: May 1 - Sept. 14
S3: Sept. 15 - Nov. 14
S4: Nov. 15 - Nov. 30
S5: Dec. 1 - March 14

		Unit	Sign	RHS	R \ C	Grazing on BLM Land (Mar. 15 - Sept. 14) .65 S1	.65 S2	.8 S2	.9 S1	.9 S2	1 S1	1 S2	1.25 S1	1.25 S2	1.3 S1	1.3 S2	Cow w. calf	Cow w/o calf	Steer calf	Heifer calf
						E	F	G	H	I	J	K	L	M	N	O	P	Q		
						106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
Alfalfa land		Acre	≤	200	42															
Alfalfa aftermath		AUM	≤	0	43															
Alfalfa transfer		Ton	≤	0	44															
Grass hay land		Acre	≤	2,000	45															
Grass hay aftermath		AUM	≤	0	46															
Grass hay transfer		Ton	≤	0	47															
Meadow pasture		Acre	≤	700	48															
Total deeded land		Acre	≤	20,000	49															
Native grass transfer		AUM	≤	0	50															
Crested wheatgrass transfer		AUM	≤	0	51															
	.65-S1	AUM	≤	0	52	-.65														
	.65-S2	AUM	≤	0	53		-.65													
	.8 -S2	AUM	≤	0	54			-.8												
Forage	.9 -S1	AUM	≤	0	55				-.9											
transfer	.9 -S2	AUM	≤	0	56					-.9										
	1.0 -S1	AUM	≤	0	57						-1							1.5		
Season 1	1.0 -S2	AUM	≤	0	58							-1						4.5		
and	1.25-S1	AUM	≤	0	59								-1.25				1.875			
Season 2	1.25-S2	AUM	≤	0	60									-1.25			6.125			
	1.3 -S1	AUM	≤	0	61										-1.3					
	1.3 -S2	AUM	≤	0	62											-1.3				
Total BLM allowance		AUM	≤	4,000	63	1	1	1	1	1	1	1	1	1	1	1				
Bull requirement		Head	≤	0	64															
Bull requirement		Head	≤	0	65															
Bull requirement		Head	≤	0	66					1		1		1		-20				
Forage	S-3	AUM	≤	0	67												2.0	2.0	.5	.5
transfer	S-4	AUM	≤	0	68												.5	.5		
	S-5	AUM	≤	0	69												3.5	3.5		
Alfalfa requirement		Ton	≤	0	70															
Steer calf (1-8 mo.)		Head	≤	0	71												- .49		1	
Heifer calf (1-8 mo.)		Head	≤	0	72												- .49			1
Weaner (9-12 mo.)		Head	≤	0	73															- .99
Yearling (13-17 mo.)		Head	≤	0	74															
Yearling (18-24 mo.)		Head	≤	0	75															
Replacement (25-32 mo.)		Head	≤	0	76															
Bull requirement		Head	≤	0	77												1	1		
Replace cull cow		Head	≤	0	78													.5		
Bull transfer		Head	≤	0	79															
Pregnancy		Head	≤	0	80												.26	-.71		
Sell steer calf		Head	≤	0	81														-4	
Sell Heifer calf		Head	≤	0	82															-3.5
Sell cull cow		Head	≤	0	83													-5		
Sell cull bull		Head	≤	0	84															

Submatrix A_{22}

S1: March 15 - April 30
S2: May 1 - Sept. 14
S3: Sept. 15 - Nov. 14
S4: Nov. 15 - Nov. 30
S5: Dec. 1 - March 14

		Unit	Sign	RHS	R \ C	Weaner (9-12)	Yearling (13-17)	Yearling (18-24)	Replacement (25-32)	Bull	Sell steer calf	Sell heifer calf	Sell cull cow	Sell cull bull	Buy bull
											R	S	T	U	V
						121	122	123	124	125	126	127	128	129	130
Alfalfa land		Acre	≤	200	42										
Alfalfa aftermath		AUM	≤	0	43										
Alfalfa transfer		Ton	≤	0	44										
Grass hay land		Acre	≤	2,000	45										
Grass hay aftermath		AUM	≤	0	46										
Grass hay transfer		Ton	≤	0	47										
Meadow pasture		Acre	≤	700	48										
Total deeded land		Acre	≤	20,000	49										
Native grass transfer		AUM	≤	0	50										
Crested wheatgrass transfer		AUM	≤	0	51										
Forage transfer Season 1 and Season 2	.65-S1	AUM	≤	0	52		.975								
	.65-S2	AUM	≤	0	53		2.275								
	.8 -S2	AUM	≤	0	54			.8							
	.9 -S1	AUM	≤	0	55				1.35						
	.9 -S2	AUM	≤	0	56				4.05						
	1.0 -S1	AUM	≤	0	57										
	1.0 -S2	AUM	≤	0	58										
	1.25-S1	AUM	≤	0	59										
	1.25-S2	AUM	≤	0	60										
	1.3 -S1	AUM	≤	0	61					1.95					
	1.3 -S2	AUM	≤	0	62					5.85					
Total BLM allowance		AUM	≤	4,000	63										
Bull requirement		Head	≤	0	64										
Bull requirement		Head	≤	0	65										
Bull requirement		Head	≤	0	66										
Forage transfer	S-3	AUM	≤	0	67			1.6	1.8	2.6					
	S-4	AUM	≤	0	68	.25		.4		.65					
	S-5	AUM	≤	0	69	1.75		2.8		4.55					
Alfalfa requirement		Ton	≤	0	70										
Steer calf (1-8 mo.)		Head	≤	0	71										
Heifer calf (1-8 mo.)		Head	≤	0	72										
Weaner (9-12 mo.)		Head	≤	0	73	1									
Yearling (13-17 mo.)		Head	≤	0	74	-.985	1								
Yearling (18-24 mo.)		Head	≤	0	75		-.99	1							
Replacement (25-32 mo.)		Head	≤	0	76			-.99	1						
Bull requirement		Head	≤	0	77				1	-20					
Replace cull cow		Head	≤	0	78				-.8						
Bull transfer		Head	≤	0	79					.25					-1
Pregnancy		Head	≤	0	80										
Sell steer calf		Head	≤	0	81						1				
Sell Heifer calf		Head	≤	0	82							1			
Sell cull cow		Head	≤	0	83				-1.6				1		
Sell cull bull		Head	≤	0	84					-3.25				1.02	

Submatrices A_{31} and A_{32}					A_{31}		A_{32}	
					Native Grass	Crested Wheatgrass	Native Grass	Crested Wheatgrass
	Unit	Sign	RHS	R \ C	-- 21 --	34 --	-- 80 --	93
Native grass requirement	Acre	=	0	85	1		-1	
Crested wheatgrass requirement	Acre	=	0	86		1		-1

Assumptions Made for MLP Simplex Table

(1) Acreage

Acreage for alfalfa production = 200 acres

Acreage for grass hay production = 2,000 acres

Acreage for meadow pasture = 700 acres

Total deeded land = 20,000 acres

Total BLM grazing allowance = 4,000 acres

(2) Yield

alfalfa production = 3.63 tons/acre

Grass hay production = 1.33 tons/acre

Native grass production = 50 lbs./acre

Crested wheatgrass production = 500 lbs./acre (1 AUM = 800 lbs.)

Alfalfa aftermath = 0.75 AUM/acre

Grass hay aftermath = 0.75 AUM/acre

Meadow pasture = 0.75 AUM/acre

(3) AUM

Alfalfa = 3 AUM/ton

Grass hay = 3 AUM/ton

Spring calf (5-8 mo.) = 0.25 AUM

Weaner (9-12 mo.) = 0.5 AUM

Yearling (13-17 mo.) = 0.65 AUM

Yearling (18-24 mo.) = 0.8 AUM

Replacement (25-32 mo.) = 0.9 AUM

Cow = 1 AUM

Cow with calf (3 mo.) = 1.25 AUM

Bull = 1.3 AUM

(4) Cost and return[a/]

Raise alfalfa = $200.2/acre

---

[a] Figures are obtained from Torell et al. (15) and are readjusted with proper price indexes.

Raise grass hay = \$30.99/acre

Germinating crested wheatgrass = \$23.43/acre

BLM grazing = \$2.36/AUM

Buy alfalfa = \$111/ton

Sell alfalfa = \$97/ton

Raise cow = \$90.15/Head

Buy bull = \$1,500/Head

Sell steer = \$79.65/cwt

Sell heifer = \$67.68/cwt

Sell cull cow = \$43.60/cwt

Sell cull bull = \$43.60/cwt

(5) Discount rate[b] = 3%

Investments during the first 3 years: $I\left(1 + \frac{1}{(1+r)} + \frac{1}{(1+r)^2}\right) = 2.91\ I$

Investments during the rest 27 years: $I\left(\frac{1}{(1+r)^3} + \ldots\ldots + \frac{1}{(1+r)^{29}}\right) = 17.27\ I$

Returns during the first 3 years: $R\left(\frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3}\right) = 2.83\ R$

Returns during the rest 27 years: $R\left(\frac{1}{(1+r)^4} + \ldots\ldots + \frac{1}{(1+r)^{30}}\right) = 16.77\ R$

(6) a = \$582.58

b = \$323.01

c = \$274.51

d = \$148.38

e = \$ 23.43

f though P = \$6.87

q = \$262.34

r = \$262.34

s = \$225.41

t = \$191.53

u = \$123.39

v = \$123.39

w = \$1091.25 (Based on one fourth of \$1,500/head).

---

[b]Idowu, Yanagida, and Norman (8)

(7) A = $3,457.45

B = $1,916.97

C = $1,626.69

D = $ 800.60

E through O = $40.76

P = $1,556.89

Q = $1,556.89

R = $1,335.73

S = $1,134.99

T = $ 731.17

U = $ 731.17

V = $6,476.25

(8) BLM grazing season is assumed to begin April 1 and end September 15. According to the data collected by Torell et al. (15), average turnout date on BLM summer range was 95 (i.e., April 3) with a standard deviation of 28 and the mean turnoff date was 258 (September 14) with a standard deviation of 72. Torell et al. assumed that BLM grazing season begins on April 1 and ends on September 1. In their reasoning, they stated that the turnoff date is considerably more variant than the turnout date and therefore it would not significantly alter the results by assuming the BLM grazing season begins on April 1 and ends on September 1. However, these assertions are not correct. Since variance (and/or covariance) is variant for scale factors, the ratios between means and variances are compared for the turnout and turnoff date.

$\frac{E}{\sigma} = \frac{95}{28} = 3.39$ for the turnout date.

$\frac{E}{\sigma} = \frac{258}{72} = 3.58$ for the turnoff date.

These results indicate that the turnout date is widely dispersed compared to the turnoff date. Therefore, the BLM grazing season in this study is assumed to cover the period from March 15 to September 14.

APPENDIX II

(MLP)

Table 5. Results Obtained from a Multiperiod Linear Programming Problem

ITEM	UNIT	FIRST 3 YRS.	REST 27 YRS.
Sell alfalfa	Ton	597	367
Feed alfalfa	Ton	129	359
Grass hay production	Acre	763	1,351
Meadow hay grazing	Acre	700	700
Native grass production	Acre	0	0
Crested wheatgrass production	Acre	0	20,000
BLM grazing	AUM	4,000	4,000
Raise steer (1-8 mo.)	Head	186	519
Raise Heifer (1-8 mo.)	Head	186	519
Yearling (9-12 mo.)	Head	86	241
Yearling (13-17 mo.)	Head	85	237
Yearling 18-24 mo.)	Head	84	235
Replacement (25-32 mo.)	Head	83	233
Raise cow w/calf	Head	380	1,059
Raise cow w/o calf	Head	133	372
Raise bull	Head	30	83
Sell steer	CWT	744	2,076
Sell heifer	CWT	651	1,816
Sell cull cow	CWT	800	2,233
Sell cull bull	CWT	95	265
Buy bull	Head	7	21
Value of objective function	$	2,548,393.00	

APPENDIX III

MSP (CASE-A)

Table 6. Results from a Multiperiod Stochastic Programming Problem with $\alpha = 0.75$

ITEM	UNIT	FIRST 3 YRS.	REST 27 YRS.
Sell alfalfa	Ton	597	413
Feed alfalfa	Ton	129	313
Grass hay production	Acre	706	1,176
Meadow hay grazing	Acre	700	700
Native grass production	Acre	4,177	4,177
Crested wheatgrass production	Acre	15,823	15,823
BLM grazing	AUM	4,000	4,000
Raise steer (1-8 mo.)	Head	186	452
Raise Heifer (1-8 mo.)	Head	186	452
Yearling (9-12 mo.)	Head	86	210
Yearling (13-17 mo.)	Head	85	207
Yearling 18-24 mo.)	Head	84	205
Replacement (25-32 mo.)	Head	83	202
Raise cow w/calf	Head	380	922
Raise cow w/o calf	Head	133	324
Raise bull	Head	30	72
Sell steer	CWT	744	1,807
Sell heifer	CWT	651	1,581
Sell cull cow	CWT	800	1,944
Sell cull bull	CWT	95	231
Buy bull	Head	7	18
Value of objective function	$	2,328,002.00	

Table 7. Results from a Multiperiod Stochastic Programming Problem with α=0.80

ITEM	UNIT	FIRST 3 YRS.	REST 27 YRS.
Sell alfalfa	Ton	597	425
Feed alfalfa	Ton	129	301
Grass hay production	Acre	691	1130
Meadow hay grazing	Acre	700	700
Native grass production	Acre	5,260	5,260
Crested wheatgrass production	Acre	14,740	14,740
BLM grazing	AUM	4,000	4,000
Raise steer (1-8 mo.)	Head	186	434
Raise Heifer (1-8 mo.)	Head	186	434
Yearling (9-12 mo.)	Head	86	202
Yearling (13-17 mo.)	Head	85	199
Yearling 18-24 mo.)	Head	84	197
Replacement (25-32 mo.)	Head	83	195
Raise cow w/calf	Head	380	886
Raise cow w/o calf	Head	133	311
Raise bull	Head	30	70
Sell steer	CWT	744	1,737
Sell heifer	CWT	651	1,520
Sell cull cow	CWT	800	1,869
Sell cull bull	CWT	95	222
Buy bull	Head	7	17
Value of objective function	$	2,270,863.00	

Table 8. Results from a Multiperiod Stochastic Programming Problem with α=0.85

ITEM	UNIT	FIRST 3 YRS.	REST 27 YRS.
Sell alfalfa	Ton	597	438
Feed alfalfa	Ton	129	288
Grass hay production	Acre	675	1,081
Meadow hay grazing	Acre	700	700
Native grass production	Acre	6,436	6,436
Crested wheatgrass production	Acre	13,564	13,564
BLM grazing	AUM	4,000	4,000
Raise steer (1-8 mo.)	Head	186	415
Raise Heifer (1-8 mo.)	Head	186	415
Yearling (9-12 mo.)	Head	86	193
Yearling (13-17 mo.)	Head	85	190
Yearling 18-24 mo.)	Head	84	188
Replacement (25-32 mo.)	Head	83	186
Raise cow w/calf	Head	380	848
Raise cow w/o calf	Head	133	298
Raise bull	Head	30	67
Sell steer	CWT	744	1,662
Sell heifer	CWT	651	1,454
Sell cull cow	CWT	800	1,787
Sell cull bull	CWT	95	212
Buy bull	Head	7	17
Value of objective function	$	2,208,827.00	

Table 9. Results from a Multiperiod Stochastic Programming Problem with $\alpha=0.90$

ITEM	UNIT	FIRST 3 YRS.	REST 27 YRS.
Sell alfalfa	Ton	597	455
Feed alfalfa	Ton	129	271
Grass hay production	Acre	655	1,019
Meadow hay grazing	Acre	700	700
Native grass production	Acre	7,921	7,921
Crested wheatgrass production	Acre	12,079	12,079
BLM grazing	AUM	4,000	4,000
Raise steer (1-8 mo.)	Head	186	392
Raise Heifer (1-8 mo.)	Head	186	392
Yearling (9-12 mo.)	Head	86	182
Yearling (13-17 mo.)	Head	85	179
Yearling 18-24 mo.)	Head	84	177
Replacement (25-32 mo.)	Head	83	175
Raise cow w/calf	Head	380	799
Raise cow w/o calf	Head	133	281
Raise bull	Head	30	63
Sell steer	CWT	744	1,566
Sell heifer	CWT	651	1,370
Sell cull cow	CWT	800	1,684
Sell cull bull	CWT	95	200
Buy bull	Head	7	16
Value of objective function	$	2,130,465.00	

Table 10. Results from a Multiperiod Stochastic Programming Problem with α=0.95

ITEM	UNIT	FIRST 3 YRS.	REST 27 YRS.
Sell alfalfa	Ton	597	480
Feed alfalfa	Ton	129	246
Grass hay production	Acre	624	924
Meadow hay grazing	Acre	700	700
Native grass production	Acre	10,180	10,180
Crested wheatgrass production	Acre	9,820	9,820
BLM grazing	AUM	4,000	4,000
Raise steer (1-8 mo.)	Head	186	355
Raise Heifer (1-8 mo.)	Head	186	355
Yearling (9-12 mo.)	Head	86	165
Yearling (13-17 mo.)	Head	85	162
Yearling 18-24 mo.)	Head	84	161
Replacement (25-32 mo.)	Head	83	159
Raise cow w/calf	Head	380	725
Raise cow w/o calf	Head	133	255
Raise bull	Head	30	57
Sell steer	CWT	744	1,421
Sell heifer	CWT	651	1,243
Sell cull cow	CWT	800	1,528
Sell cull bull	CWT	95	181
Buy bull	Head	7	14
Value of objective function	$	2,011,229.00	

APPENDIX IV

MSP (CASE-B)

Table 11. Results from a Multiperiod Stochastic Programming Problem with α=0.95

ITEM	UNIT	FIRST 3 YRS.	REST 27 YRS.
Sell alfalfa	Ton	597	451
Feed alfalfa	Ton	129	275
Grass hay production	Acre	660	1,035
Meadow hay grazing	Acre	700	700
Native grass production	Acre	7,530	7,530
Crested wheatgrass production	Acre	12,470	12,470
BLM grazing	AUM	4,000	4,000
Raise steer (1-8 mo.)	Head	186	398
Raise Heifer (1-8 mo.)	Head	186	398
Yearling (9-12 mo.)	Head	86	185
Yearling (13-17 mo.)	Head	85	182
Yearling 18-24 mo.)	Head	84	180
Replacement (25-32 mo.)	Head	83	178
Raise cow w/calf	Head	380	812
Raise cow w/o calf	Head	133	285
Raise bull	Head	30	64
Sell steer	CWT	744	1,591
Sell heifer	CWT	651	1,392
Sell cull cow	CWT	800	1,712
Sell cull bull	CWT	95	203
Buy bull	Head	7	16
Value of objective function	$	2,151,119.00	

Table 12. Results from a Multiperiod Stochastic Programming Problem with $\alpha=0.90$

ITEM	UNIT	FIRST 3 YRS.	REST 27 YRS.
Sell alfalfa	Ton	597	437
Feed alfalfa	Ton	129	289
Grass hay production	Acre	660	1,086
Meadow hay grazing	Acre	700	700
Native grass production	Acre	6,333	6,333
Crested wheatgrass production	Acre	13,667	13,667
BLM grazing	AUM	4,000	4,000
Raise steer (1-8 mo.)	Head	186	417
Raise Heifer (1-8 mo.)	Head	186	417
Yearling (9-12 mo.)	Head	86	194
Yearling (13-17 mo.)	Head	85	191
Yearling 18-24 mo.)	Head	84	189
Replacement (25-32 mo.)	Head	83	187
Raise cow w/calf	Head	380	851
Raise cow w/o calf	Head	133	299
Raise bull	Head	30	67
Sell steer	CWT	744	1,668
Sell heifer	CWT	651	1,460
Sell cull cow	CWT	800	1,794
Sell cull bull	CWT	95	213
Buy bull	Head	7	17
Value of objective function	$	2,214,240.00	

Table 13. Results from a Multiperiod Stochastic Programming Problem with α=0.85

ITEM	UNIT	FIRST 3 YRS.	REST 27 YRS.
Sell alfalfa	Ton	597	427
Feed alfalfa	Ton	129	299
Grass hay production	Acre	660	1,123
Meadow hay grazing	Acre	700	700
Native grass production	Acre	5,432	5,432
Crested wheatgrass production	Acre	14,568	14,568
BLM grazing	AUM	4,000	4,000
Raise steer (1-8 mo.)	Head	186	432
Raise Heifer (1-8 mo.)	Head	186	432
Yearling (9-12 mo.)	Head	86	200
Yearling (13-17 mo.)	Head	85	197
Yearling 18-24 mo.)	Head	84	195
Replacement (25-32 mo.)	Head	83	193
Raise cow w/calf	Head	380	881
Raise cow w/o calf	Head	133	309
Raise bull	Head	30	69
Sell steer	CWT	744	1,726
Sell heifer	CWT	651	1,510
Sell cull cow	CWT	800	1,857
Sell cull bull	CWT	95	220
Buy bull	Head	7	17
Value of objective function	$	2,261,773.00	

Table 14. Results from a Multiperiod Stochastic Programming Problem with $\alpha=0.8$

ITEM	UNIT	FIRST 3 YRS.	REST 27 YRS.
Sell alfalfa	Ton	597	418
Feed alfalfa	Ton	129	308
Grass hay production	Acre	700	1,156
Meadow hay grazing	Acre	700	700
Native grass production	Acre	4,639	4,639
Crested wheatgrass production	Acre	15,361	15,361
BLM grazing	AUM	4,000	4,000
Raise steer (1-8 mo.)	Head	186	444
Raise Heifer (1-8 mo.)	Head	186	444
Yearling (9-12 mo.)	Head	86	206
Yearling (13-17 mo.)	Head	85	203
Yearling 18-24 mo.)	Head	84	201
Replacement (25-32 mo.)	Head	83	199
Raise cow w/calf	Head	380	908
Raise cow w/o calf	Head	133	319
Raise bull	Head	30	71
Sell steer	CWT	744	1,777
Sell heifer	CWT	651	1,555
Sell cull cow	CWT	800	1,912
Sell cull bull	CWT	95	227
Buy bull	Head	7	18
Value of objective function	$	2,303,618.00	

Table 15. Results from a Multiperiod Stochastic Programming Problem with $\alpha=0.75$

ITEM	UNIT	FIRST 3 YRS.	REST 27 YRS.
Sell alfalfa	Ton	597	409
Feed alfalfa	Ton	129	317
Grass hay production	Acre	711	1,190
Meadow hay grazing	Acre	700	700
Native grass production	Acre	3,840	3,840
Crested wheatgrass production	Acre	16,160	16,160
BLM grazing	AUM	4,000	4,000
Raise steer (1-8 mo.)	Head	186	457
Raise Heifer (1-8 mo.)	Head	186	457
Yearling (9-12 mo.)	Head	86	212
Yearling (13-17 mo.)	Head	85	209
Yearling 18-24 mo.)	Head	84	207
Replacement (25-32 mo.)	Head	83	205
Raise cow w/calf	Head	380	933
Raise cow w/o calf	Head	133	328
Raise bull	Head	30	73
Sell steer	CWT	744	1,829
Sell heifer	CWT	651	1,600
Sell cull cow	CWT	800	1,967
Sell cull bull	CWT	95	234
Buy bull	Head	7	18
Value of objective function	$	2,345,766.00	

References

(1) Archer, S.G. and C.E. Bunch, The American Grass Book, Norman: University Oklahoma Press, 1953.

(2) Chen, Joyce T., "Quadratic Programming for Least-Cost Feed Formulations under Probabilistic Protein Constraints," Am. J. Agr. Econ. 55 (2): 175-183, 1973.

(3) Cohen, K.L. and R.M. Cyert, Theory of the Firm: Resource Allocation in a Market Economy, Second Edition, Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1975.

(4) Garoian, Lee and C.S. Kim, "A General Procedure For Incorporating BLM and FS Grazing Activities in Linear Programming Models," Agricultural Experiment Station, University of Nevada, Reno, 1981.

(5) Henderson, J.M. and R.E. Quandt, Microeconomics Theory: A Mathematical Approach, Third Edition, New York: McGraw-Hill Book Co., 1980.

(6) Hillier, F.S. and G.T. Lieberman, Introduction to Operation Research, Third Edition, San Francisco: Holden-Day, Inc., 1980.

(7) Hirschleifer, J., "On the Theory of Optimal Investment Decision, J. of Political Economy, Vol. 66: 329-352, Aug., 1958.

(8) Idowu, Chris A., John F. Yanagida, and Alfred W. Norman, "Economics of Heat Recovery System Use In The Dairy Parlor," M.S. 123, Agricultural Experiment Station, University of Nevada, Reno, 1980.

(9) Kim, C.S., R.L. Shane, and J.F. Yanagida, "Alternative Approaches to Risk and Uncertainty in Evaluating Optimal Firm Behavior," M.S. 130, Agricultural Experiment Station, University of Nevada, Reno, 1981.

(10) Mitchell, B. and J.R. Garrett, "Characteristics of the Range Cattle Industry, Region III, Northeastern Nevada," B 42, Agricultural Experiment Station, University of Nevada, Reno, 1977.

(11) Rahman, S.A. and F.E. Bender, "Linear Programming Approximation of Least-Cost Feed Mixes with Probability Restrictions," Am. J. Agr. Econ. 53 (4): 612-618, 1971.

(12) Smoliak, S., "Grazing Studies on Native Range, Crested Wheatgrass, and Russian Wildrye Pastures," J. of Range Management, Vol. 21, No. 1, 1968.

(13) Taha, H.A., Operation Research, Second Edition, New York: MacMillan Publishing Co., Inc., 1976.

(14) Tobin, J., The Theory of Portfolio Selection, New York: St. Martin's Press, 1965.

(15) Torell, A., et al., "Economic Impact of BLM Grazing Allotment Reductions on Humboldt County, Nevada," Agricultural Experiment Station, University of Nevada, Reno, 1980.