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PRICE-SETTING SUPERGAMES
WITH CAPACITY CONSTRAINTS

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Abstract

This paper develops supergame theory for price setting oligopoly where firms produce perfect substitutes. Results are: First price setting supergame equilibria may support higher industry price and lower industry output than quantity setting equilibria even when there are no capacity constraints at the firm level -- contrary to the classical static results of Bertrand and Cournot.

Second, the maximum price that can be supported by trigger strategies is not monotonic as a function of the number of firms and capacity of each.

Third, properties of industry equilibrium under free entry but possible tacit collusion on price are developed. We show that entry is likely to decrease welfare if each entrant uses up resources in establishing his firm.

1. Introduction

This paper is an attempt to highlight the role of industry capacity in enforcing collusive behavior. It uses supergame theory à la (Friedman [1], Rubenstein [5], Radner [4]) et al. to achieve this objective. Unlike the existing supergame applications to oligopoly, we study price-setting supergames rather than quantity-setting supergames.

The basic insight of supergame theory for oligopoly is the following (Friedman [1]): If a market situation is repeated for an infinite number of periods, it is possible that an industry will settle at a cartel price, and the reason why each firm does not defect from the implicit cartel agreement is the future losses that it will incur when competitors retaliate against a defection.

For example, we show that if each firm produces at constant cost, that the level of cooperation in equilibrium will, in general, depend monotonically on the discount rate that firms apply to future profits and the number of firms in the industry. The reason is as follows. A higher discount rate diminishes the value of future retaliation while a larger number of firms gives, at each price, a smaller market share for each of the colluding firms. This increases the value of defection.

This situation is drastically modified when a capacity constraint at the firm level is present. The magnitude of the discount rate has the same influence as in the no capacity constraints case. A central finding of this paper is that increasing the number of firms has a different effect.

An intuitive explanation follows. On the one hand the number of firms --as in the no capacity constraints case--determines market shares at each cartel price. On the other hand, the number of firms determines total industry capacity and thus the amount by which prices may fall as a consequence of retaliation.

The amount that prices may fall if competition breaks out is an important underpinning of cartel strength. The importance of this effect on cartel maintenance can be well understood if one imagines a situation in which total industry capacity is slightly above monopoly capacity. The cartel is weak in this case. The reasoning follows. If firms collaborate at monopoly price, the immediate gain to a cheater will be proportional to the difference between his capacity and his share of monopoly output. The strongest retribution that the other firms can envisage is to produce forever at full capacity. This action will result in a per firm profit in future periods which will be up to a first approximation equal to shared monopoly profits. This is so because, by definition, shared monopoly profits <u>maximize</u> per capita profits at monopoly capacity. Hence the threat of future price competition unleashed by defection is weak. Thus firms would tend to defect from the cartel and in equilibrium one would see competition as the outcome. We argue in the sequel that this is the typical situation when total capacity is close to monopoly capacity. If the number of firms is increased, the threat that the rest of the market may impose on a single firm may be large enough to dominate any one-period profits obtained by defecting. However, as the number of firms is increased even further, cartel profits per capita diminish. Yet the one-period gain to chiseling gets large enough to outweight the threat of an outbreak of competition. The threat of any outbreak of competition becomes relatively smaller than the gains from chiselling because monopoly profits foregone per capita diminish as the number of firms increases. Furthermore, each firm may always choose to produce zero and thus obtain zero profits. Thus, in general, under capacity constraints, the number of firms has a non-monotonic effect on cartelization.

We formalize our problem by considering a fixed number of firms N each with a given capacity k and a fixed marginal cost c up to k. Thus total industry capacity is Nk. We consider a supergame in which the strategies of each firm consist of price choices at time t that are functions of past prices used by all firms.

Thus the N firms play a "Bertrand Supergame." Hence the profit of firm j will depend on a fixed per period demand curve, on the fixed marginal cost c, and the relation between its own price p_t^j and the prices charged by the other firms. We look at "trigger strategies" of the type: $p_t^j = p^*$ if $p_{t-s}^i = p^*$ for $s=1,\ldots,t-1$, $i=1,\ldots,n$, $p_t^j = \tilde{p}$ otherwise where $\tilde{p}$ may be either a fixed price or a random variable. We require that such strategies be "perfect" in the sense that if each firm is using the strategy described above and if $p_{t-s}^i \neq p^*$ at time t for some i, then it is optimal for each firm to charge $\tilde{p}$ given that all other firms are charging $\tilde{p}$. This eliminates the possibility of empty threats.

We first examine the possibility of the monopoly price being sustained by such a trigger strategy and show that it depends, as argued above, in a non-monotonic fashion on N.

Next we find, for each N, the highest per capita profit that may be sustained in this fashion. We show that the maximum sustainable prices and profits depend in a non-monotonic fashion on the number of firms.

This finding leads us to consider games in which free-entry is allowed at a fixed cost, and thus industry capacity is an endogenous variable. The above results imply that as fixed costs are decreased equilibrium prices will first increase and later decrease. In particular, we show that government taxation of entry in such industries may increase welfare while government subsidies always lower welfare.

The models developed here assume constant marginal costs and linear demand curves. This allows us to calculate explicitly some of the equilibrium variables, and thus facilitate the comparative statics. However, the qualitative results should hold in much more general models.

The paper is organized as follows. Section 1 contains the introduction. The second section briefly develops and reviews results on static price-setting and quantity-setting games. The third section compares results generated by price-setting supergames with the more familiar results generated by quantity-setting supergames. Conditions are found that imply more tacit collusion is supported by trigger strategies in price-setting supergames than in quantity-setting supergames. This contrasts with the static case where the presence of two or more firms leads to zero profits in a price-setting game with no capacity constraints.

In the fourth section we develop price-setting supergames with capacity constraints. It is here that the non-monotonic relationship between the viability of tacit collusion and the number of firms appears. Results are also presented that relate the maximum rate of interest that supports tacit collusion at the monopoly price as a function of the number of firms. This relationship is <u>not</u> monotonically decreasing--contrary to intuition.

Finally, section 5 develops results on entry with an entry fee. In section 6 welfare results that show that it never pays to subsidize entry are presented. It is also shown that replicating the number of demanders and the number of firms has no effect on the viability of tacit collusion -- contrary to intuition.

The paper closes with a discussion of the relevance of these results to the regulation and identification of tacit collusions, conscious parallelism, and shared monopoly.

2. One-Period Quantity Setting and Price Setting Games

Use the notation and setup of Radner [4] in what follows. There are N identical firms, facing industry level inverse demand

$$P = \max\{\alpha - \beta \sum_{j=1}^{N} Q_j, 0\} \tag{2.1}$$

Take up the quantity setting game first. Profit to firm i is therefore, copying Radner,

$$p(Q_i, Q_i') \equiv PQ_i - \gamma Q_i = (\delta - \beta Q_i')Q_i - \beta Q_i^2, \quad \delta \equiv \alpha - \gamma, \; Q_i' \equiv \sum_{j \neq i} Q_j \tag{2.2}$$

Here unit cost is γ and it is assumed that $\delta > 0$.

Given Q_i' the profit-maximizing choice of i, $r(Q_i')$, is given by

$$r(Q_i') \equiv \max\{(\delta - \beta Q_i')/2\beta, 0\} \quad . \tag{2.3}$$

Maximum profit to i (if $r(Q_i') > 0$) if the rest choose Q_i' is given by

$$g(Q_i') = (\delta - \beta Q_i')^2/4\beta \tag{2.4}$$

Cournot-Nash Equilibrium (CNE) is defined by

$$r[\sum_{j \neq i} Q_j] = Q_i \; , \quad i=1,2,\ldots,N. \tag{2.5}$$

Cournot-Nash Equilibrium quantities are given by

$$Q_i = Q_N^* \equiv \delta/(N+1)\beta \; , \quad i=1,2,\ldots,N. \tag{2.6}$$

Also, CNE profits per firm are given by

$$g_N^* \equiv g(\sum_{j \neq i} Q_j) = g((N-1)Q_N^*) = \delta^2/(\beta(N+1)^2) \tag{2.7}$$

In contrast cartel output per firm, cartel profit per firm, and cartel price are given by

$$\hat{Q}_N \equiv \delta/2\beta N \; , \quad \hat{g}_N \equiv \delta^2/4\beta N \; , \quad \hat{P}_N \equiv (\alpha + \gamma)/2 > \gamma \tag{2.8}$$

respectively.

A Bertrand-Nash Equilibrium (BNE) is defined exactly the same way as a CNE except that the strategy space is price not quantity. It is trivial to see that the only BNE is given by

$$\tilde{Q}_N \equiv \delta/N\beta \ , \quad \tilde{g}_N = 0 \ , \quad \tilde{P}_N \equiv \gamma \tag{2.9}$$

in this simple constant cost game. Cartel quantities for a price setting cartel are the same as those for a quantity setting cartel.

Let us pause for two comments on the computations laid out above. First, notice that $N > 1$ implies all BNE are perfectly competitive. Such a result has been known since the time of Bertrand's famous attack on Cournot and flies in the face of economic "intuition." It is not consistent with "stylized facts" such as the positive correlation between profit and market share.

Second, note well that the CNE performs much better from an intuitive point of view with perfect competition emerging as $N \to \infty$. This too has been known for a long time. Notice, however, that unlike the CNE, the division of the output among the firms is uniquely determined in the BNE. We will focus on symmetric CNE in what follows.

Turn now to repeated games.

3. Repeated Games: Quantity Setting versus Price Setting

Here we shall follow Friedman's [1] presentation. Recall that "a supergame" formed from a one-period game is just an infinite repetition of the one-period game.

$$s \equiv (Q_1, Q_2, \ldots, Q_N) \;, \quad u \equiv (P_1, \ldots, P_N) \;, \quad \pi_i(s) \equiv p(Q_i, Q_i'),$$

$$\varepsilon_i(u) \equiv \varepsilon_i(P_1, \ldots, P_N) = \text{profit to } i \text{ under price vector } u.$$

As Friedman explains, a supergame strategy for the i^{th} player is just a function of the history of the game. Define a non-cooperative equilibrium for the supergame in the obvious way. Usually there are many supergame equilibria. It is natural to study those that have a simple structure.

One class of candidate supergame equilibria are those enforced by "grim" trigger strategies. I.e., everyone stays at the cartel point unless someone chisels. If someone chisels at date t all players go, at date t-1, to the one-period game non-cooperative equilibrium and stay there forever. Such a threat is "subgame perfect" in the sense of Selten [6]. We are following Green [2] in our request for perfection of threats.

We may, following Friedman, formulate and prove the theorem that gives sufficient conditions on the data for a set of grim trigger strategies to be a non-cooperative supergame equilibrium. We need some notations and definitions.

Define the net Cournot temptation to chisel for player i by

$$CT_i \equiv p[\hat{\hat{Q}}_i, \hat{Q}'_{iN}] - \hat{g}_N - \frac{\alpha_i}{1 - \alpha_i} [\hat{g}_N - g_N^*] \tag{3.1}$$

Here $\hat{Q}'_{iN}$ denotes the cartel quantities chosen by everyone but i, $\hat{\hat{Q}}_i$ is the best reply to i to $\hat{Q}'_{iN}$, α_i is the discount factor on future profits applied by i, and "CT_i" stands for "Cournot temptation to chisel by i." Notice that if $\alpha_i \equiv \frac{1}{1 + R_i}$ then $\alpha_i/(1 - \alpha_i) = 1/R_i$, where R_i is the discount rate by i.

Define the net Bertrand temptation to chisel for player i by

$$BT_i \equiv \sup_{P_i} \; \xi_i(P_i, \tilde{u}'_{iN}) - \tilde{g}_N - \frac{\alpha_i}{1 - \alpha_i}[\tilde{g}_N - \tilde{g}^*_N] \tag{3.2}$$

Here $\tilde{u}'_{iN}$ denotes the vector of cartel prices chosen by everyone but i. The supremum appears in formula (3.2) because the maximum will not be achieved, in general, for the case of perfect substitutes. We may now state

Theorem 1 (A) If $CT_i < 0$, $i=1,2,\ldots,N$ ($BT_i < 0$, $i=1,2,\ldots,N$) then the following is a non-cooperative equilibrium for the quantity (price) setting supergame. Each player i does the following. He chooses his cartel output (price) level until someone (including himself) chooses a non-cartel output (price) level. If someone chooses a non-cartel ouptut (price) level at date t, i plays his CNE (BNE) output level for all succeeding dates.

Proof: The proof follows Friedman [1].

The intuition here is simple. The difference of the first two terms in formulae (3.1), (3.2) is the maximal gain from deviance from the cartel. The last term is the discounted losses caused by triggering a return to "competition" (CNE or BNE) from one period after the cheating date to eternity. Obviously, if the one-period gain to chiseling is greating than the opportunity cost of infidelity then $CT_i > 0$ ($BT_i > 0$).

Notice also that $N > 1$ does not imply that the Bertrand "tacit collusion" collapses. In fact the Bertrand tacit collusion may be "stronger" than the Cournot tacit collusion in the sense that

$$BT_i < CT_i < 0 \ , \quad i=1,2,\ldots,N \ . \tag{3.3}$$

Let us develop this important message in detail by computing BT_i and CT_i. The point is made graphically in Figure 3.1 below. Suppose that $\alpha_i = \alpha$, $i=1,2,\ldots,N$. Then

$$\hat{Q}'_{iN} = (N-1)\delta/(2\beta N), \quad \hat{\hat{Q}}_i = r(\hat{Q}'_{iN}) \equiv r[(N-1)\delta/(2\beta N)]$$

$$= \max\{(\delta - \beta\hat{Q}'_{iN})/2\beta, 0\} = \delta(N+1)/(4\beta N) > \delta/(2\beta N), \tag{3.4}$$

for $N > 1$.

Now

$$\hat{\hat{g}}_N \equiv p[\hat{\hat{Q}}_i, \hat{Q}'_{iN}] = (\delta - \beta\hat{Q}'_{iN})\hat{\hat{Q}}_i - \beta\hat{\hat{Q}}_i^2 = [\delta(N+1)/2N]^2[1/4\beta] \tag{3.5}$$

$$\hat{\hat{g}}_N - \hat{g}_N = [\delta^2/(4\beta N)][(N+1)^2/(4N) - 1] > 0 \tag{3.6}$$

$$\hat{g}_N - g^*_N = [\delta^2/(4\beta N)][1 - (4N)/(N+1)^2] > 0 \tag{3.7}$$

$$\sup_{P_i} \xi_i(P_i, \tilde{u}'_{iN}) = N\hat{g}_N \ , \quad \tilde{g}_N = \hat{g}_N, \quad \tilde{g}^*_N = 0 \tag{3.8}$$

All of the above is obvious except possibly the last. Since the products are perfect substitutes the Bertrand deviant can shave his price and capture the whole market. Hence he can get as close as he likes to the total industry monopoly profit $N\hat{g}_N$. This explains the first term of (3.8). The per firm monopoly profit of a price setting cartel (where all charge the same price) is the same as that of a quantity setting cartel. Hence $\tilde{g}_N = \hat{g}_N$. Furthermore, BNE profit is zero for $N > 1$. Hence $\tilde{g}^*_N \equiv$ BNE profit per firm $= 0$. We may now calculate CT_i, BT_i. We have, putting $a \equiv \alpha/(1-\alpha)$,

$$CT_i = [\delta^2/(4\beta N)][(N+1)^2/(4N) - 1] - a[\delta^2/(4\beta N)][1 - (4N)/(N+1)^2] > 0 \tag{3.9}$$

if and only if (iff)

$$C(N) \equiv (N + 1)^2/4N > a \tag{3.10}$$

In contrast

$$BT_i = N\hat{g}_N - \hat{g}_N - a\,\hat{g}_N > 0\ , \tag{3.11}$$

iff

$$B(N) \equiv N - 1 > a \quad \text{iff} \quad \frac{N - 1}{N} > \alpha \tag{3.12}$$

Notice that both formulae (3.10), (3.12) are independent of demand and cost. Although this is an artifact of the linear demand constant cost setup there is economics in it. Viz. the same demand-cost conditions that increase gains to cartelization also tend to raise the temptation to chisel. Such a correlation between gains to cartelization and gains to chiselling on the brotherhood is likely to be present in more general models.

Let us see if it takes more firms to destablize a Bertrand cartel over a Cournot cartel. Plot the LHS of (3.10), (3.12) as a function of N, treating N as a continuous variable. The graph is relevant only for integer $N > 1$. It is easy to verify that the graphs look as depicted below.

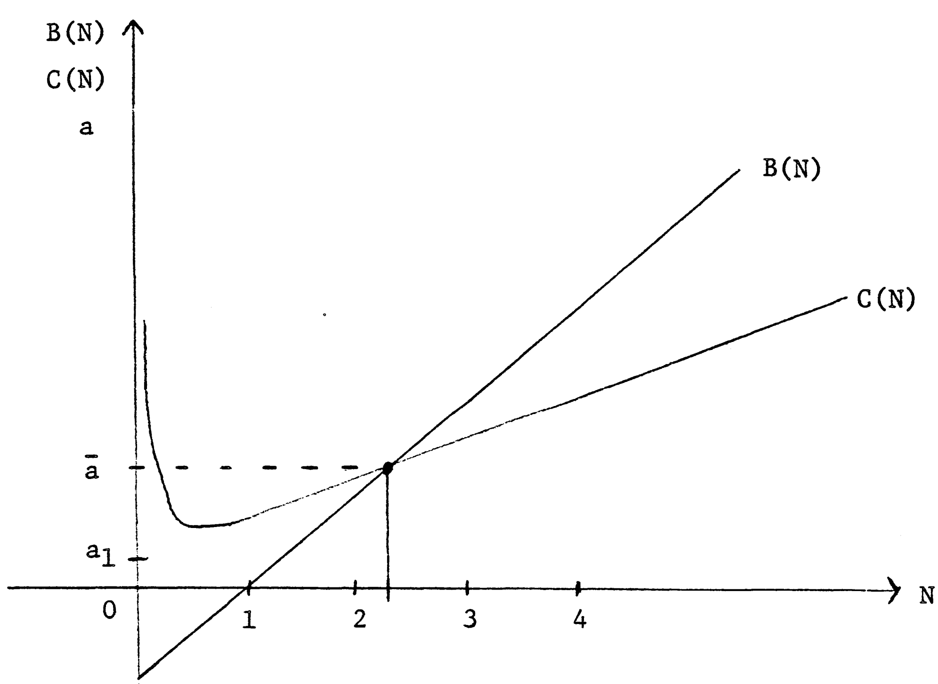


Figure 3.1

There is only one intersection $\bar{N} > 2$. We may now ask: For a given a, what is the maximal number of firms consistent with cartel stability? Note first that $a \equiv \alpha/(1 - \alpha)$ is an increasing function of α on $[0,1)$. Define $N_B(a), N_C(a)$ by

$$N_B(a) = \inf\{N \mid BT > 0, \text{ on } [N,\infty)\} \tag{3.13}$$

$$N_C(a) = \inf\{N \mid CT > 0, \text{ on } [N,\infty)\} \tag{3.14}$$

It is now easy to see from Figure 3.1 that for $a = a_1$ that the Cournot cartel is always unstable no matter how small the number of firms. Only for $a > \bar{a}$, where $\bar{a} \equiv B(\bar{N}) = C(\bar{N})$ do we get

$$N_B(a) < N_C(a) \quad . \tag{3.15}$$

The number $\bar{N} = 2.1547005$ is slightly larger than 2. We can conclude the following

<u>Proposition</u> For discount factors close enough to unity (interest rates close enough to zero) the Cournot cartel is more stable than the Bertrand cartel in the sense that $N_B(a) < N_C(a)$. But the opposite is true for large interest rates.

Why doesn't the Bertrand brotherhood degenerate into a war of sequential price cutting as in the static case? The answer is that a prospective Bertrand deviant faces larger losses after he is detected. The threat of reversion to the static BNE is stronger than in the Cournot case. Whilst it is true that the Bertrand chiseler can get almost the entire <u>industry</u> monopoly profit for himself for one period the price he must pay is the capitalized loss from the gale of competition he unleashes upon detection one period hence. In short, the Bertrand cartelist faces a choice between a larger one-period gain from deviance relative to his Cournot counterpart and a larger capitalized loss from Bertrand competition than Cournot competition relative to his Cournot counterpart. In general the net effect must be calculated on a case-by-case basis.

Notice from Figure 3.1 that for all integers $N > \bar{N} > 2$ the Bertrand cartel requires a larger discount factor to be stable than the Cournot cartel. So in this sense the Bertrand cartel is less stable than the Cournot cartel for each fixed N.

Armed with the above analysis the economic intuition of why a Bertrand cartel might be more stable than a Cournot cartel is clear. The Bertrand chiseler overcomes the severe cost of the Bertrand competition that he unleashes by slightly shaving the monopoly price and swiping the entire market for one period. But if the chiseler faces rapidly rising marginal cost his gain from own incremental output is rapidly choked off. Hence the gains from chiseling are likely to be smaller than the losses. The phenomenon of rising marginal cost does not loom so large in the Cournot cartelist's reckoning. All this is developed in section 4 below.

4. Bertrand Equilibria under Capacity Constraints

In this section we assume that each producer has a cost function of the type $C(x) = c \cdot x$ provided $x \leqslant k$. In the case where price is the strategic variable one must make an assumption on what happens when the lowest priced firm does not have enough capacity to satisfy demand. We adopt the assumption that the market demand curve is derived from a utility function that yields a zero income effect. This will imply that if consumers face prices $P_1,\ldots,P_n$ with $P_i < P_{i+1}$, the supply at P_i is restricted to k_i, then the contingent demand is given by

$$\rho(P_j|P_1,\ldots,P_{j-1},P_{j+1},\ldots,P_n;k_1,\ldots,k_n) = \max\{0,D(P_j) - \sum_{i=1}^{j-1} k_i\} \quad (4.1)$$

We also assume that if m different producers charge price $\bar{P}$ then each firm will sell $\frac{1}{m}$ of the contingent demand. These assumptions are the ones adopted by Levitan and Shubik [3].

In order to obtain precise results we also assume that the market demand is given by

$$Q = a - bP\ , \quad a > k, \quad \frac{a}{b} > c$$

Here Q is industry quantity. Under these conditions it is well-known that a pure strategy equilibrium may fail to exist. In fact we have the following result for static games.

Lemma 1: If $k \leqslant (a - bc)/(N + 1)$ then a pure strategy equilibrium exists and is given by $\bar{P}_i \equiv b^{-1}(a - Nk)$. If $k \geqslant \frac{a-b}{N-1}$, then $\bar{P} = C$ is the pure strategy equilibrium. Otherwise no pure strategy equilibrium exists.

Proof: First notice that at a pure strategy equilibrium all firms must be be charging the same price $\bar{P} \geq c$. Otherwise since $a > k$, the lowest price firm could raise its price without losing customers. If the lowest price firm is charging the monopoly price P^m and $k \geq a - bP^m$ then the lowest price firm is making a profit whereas all others are selling zero. Thus either $P_{min} < P^m$ or $k < a - bP^m$. In any case, the lowest price firm could raise its price and increase revenue.

Next, notice that if all firms are charging $\bar{P}$ the contingent demand facing the sales of the first firm are given by

$$\begin{aligned} s(P) &= \min\{a - bP,\ k\}, \text{if } P < \bar{P} \\ &= N^{-1}(a - b\bar{P}), \text{if } P = \bar{P} \\ &= \min\{a - bP - (N - 1)k, k\} \text{ if } P > \bar{P}\ . \end{aligned}$$

Furthermore, notice that if $Nk < a - b\bar{P}$, then for P close to $\bar{P}$, $a - bP - (N - 1)k > k$. Thus $s(P) = k$ for $\bar{P} < c < \bar{P} + \delta$ for some $\delta > 0$. Since $\bar{P} \geq c$, the first firm may increase profits by raising price. Thus we have $Nk \geq a - b\bar{P}$. If $Nk > a - b\bar{P}$ then for $P < \bar{P}$, $s(P) > N^{-1}(a - b\bar{P}) + \gamma$ for some $\gamma > 0$. Thus if $\bar{P} > c$, the first firm may again increase profits by lowering prices. Thus we have either $Nk = a - b\bar{P}$ or $Nk > a - b\bar{P}$ and $\bar{P} = c$. If $Nk = a - b\bar{P}$ then

$$s(P) = a - bP - (N - 1)k, \text{for } P > \bar{P}$$

Notice that $(\bar{P} - c)s(\bar{P}) \geq (P - c)s(P)$ for $P \geq \bar{P}$ if and only if $k \leq \frac{a - bc}{N + 1}$.

On the other hand if $\bar{P} = c$ is an equilibrium we must have $(N - 1)k \geq a - bc$. For otherwise when charging a higher price the first firm would still have customers and thus make a profit. Thus we conclude that in order for a pure strategy equilibrium to exist we must have either $k \leq \frac{a - bc}{N + 1}$ and $\bar{P} = b^{-1}(a - Nk)$ or $(N - 1)k \geq a - bc$. It is obvious that in the later case $\bar{P} = c$ is in fact an equilibrium. In the former case we already know that no firm will wish to raise its price. On the other hand since $Nk = a - b\bar{P}$ it is already selling at capacity and thus will lose by cutting prices.

Q.E.D.

For $\frac{a - bc}{N + 1} < k < \frac{a - bc}{N - 1}$ we must look for mixed strategy equilibria. We will concentrate on the symmetric case, i.e., we shall look for a distribution $\phi(P)$ which is the best response for any firm if all other firms are using the same distribution $\phi(P)$. As in Levitan and Shubik [3], who treat the two player case when c=0 and b=1, we may quickly derive properties of $\phi(P)$.

Clearly $\phi(P) = 0$ for $P < c$. Also $\phi(P) = 1$ for $P > \frac{a}{b}$. Thus let

$$P_e \equiv \inf\{P | \phi(P) > 0\}$$

and

$$P_N \equiv \sup\{P | \phi(P) < 1\} .$$

We may state

<u>Lemma 2</u>: A mixed strategy equilibrium exists when $\frac{a - bc}{N + 1} \leqslant k \leqslant \frac{a - bc}{N - 1}$. It is given by

$$\phi(P) = \left(\frac{k - \bar{\pi}/P}{bP + Nk - a}\right)^{1/(N-1)} \text{for } P_e \leqslant P \leqslant P_N ,$$

$$\phi(P) = 0 \text{ if } P < P_e , \quad \phi(P) = 1 \text{ for } P > P_N \text{ where,}$$

$$P_e = c + \frac{(a - (N - 1)k - bc)^2}{4bk} \geqslant \frac{a - Nk}{b} ,$$

$$P_N = \frac{a - (N - 1)k + bc}{2b} \leqslant \frac{a - (N - 1)k}{b} , \quad \bar{\pi} = \frac{(a - (N - 1)k - bc)^2}{4b}$$

[1] Proof: Suppose all but the first firm are using the mixed strategy defined by ϕ. Then if the first firm charges P its expected profits are given by

(a) $\pi(P) = \max[0, (a - bP - (N - 1)k)] (P - c)$, if $P > P_N$

(b) $\pi(P) = k(P - c)$, if $P < P_e$

(c) $\pi(P) = [(1 - \phi^{N-1}(P))k(P - c) + \phi^{N-1}(P)[a - bP - (N - 1)k](P - c)$,

if $P_e \leqslant P \leqslant P_N$.

We first show that (c) holds. Notice that with probability $\phi^{N-1}(P)$, the first firm is the highest priced. However, since $P \geqslant P_e \geqslant \frac{a - Nk}{b}$ we have $a - bP - (N - 1)k \leqslant k$. Also, since $P \leqslant P_N \leqslant \frac{a - (N - 1)k}{b}$, we have $a - bP - (N - 1)k \geqslant 0$. Also with probability $(1 - \phi^{N-1}(P))$ the first firm is not the highest priced. Since $P \leqslant P_N \leqslant \frac{a - (N - 1)k}{b}$, if firm one has the j^{th} lowest price, $j < N$, its contingent demand is $a - bP - (j - 1)k \geqslant (N - 1)k - (j - 1)k > k$ for $j \leqslant N - 1$. This proves (c). Now (a) and (b) follow immediately, since for $P > P_N$ firm one will surely be the highest priced and thus will sell below capacity. Similarly if $P < P_e$, firm one is not the highest priced and thus will sell k.

Also for $P_e \leqslant P \leqslant P_N$, $\pi(P) = \bar{\pi}$, and for $P < P_e$, $\pi(P) < k(P_e - c) = \bar{\pi}$. Also, $\frac{d}{dP}(a - bP - (N - 1)k)(P - c)\Big|_{P=P_N} = 0$. Thus $\pi(P) < \pi(P_N)$ for $P > P_N$.

Hence $\phi(P)$ is a best response for firm one.

Q.E.D.

We are now in a position to investigate the impact of capacity constraints on the stability of Bertrand and Cournot tacit collusion. The concept of critical discount factor is useful in this regard.

Critical Discount Factors

If $k = \infty$ then the unique equilibrium of the static Bertrand game is given by $P = c$. Thus cheating would be profitable iff

$$\frac{(a - bc)^2}{4b} - \frac{(a - bc)^2}{4bN} > \gamma \frac{(a - bc)^2}{4bN}$$

or

$$N - 1 > \gamma, \quad \text{where } \gamma \equiv \frac{\alpha}{1 - \alpha} .$$

Thus the critical discount factor, $\bar{\gamma}$, is defined by $\bar{\gamma} = N - 1$. Notice $\gamma \in [0,\infty)$ and γ is increasing with α.

Now if $k \geqslant \frac{a - bc}{2}$ the reasoning is exactly the same as above since a cheater would not sell more than $\frac{a - bc}{2}$, which is demand at monopoly price.

But if $\frac{a - bc}{N - 1} < k < \frac{a - bc}{2}$ (which can only occur, of course, if $N > 3$), cheating would be profitable iff

$$\left(\frac{a - bc}{2b}\right)k - \frac{(a - bc)^2}{4bN} = \gamma \frac{(a - bc)^2}{4bN} \qquad \text{or,}$$

$$k - \frac{a - bc}{2N} = \gamma \left(\frac{a - bc}{2N}\right) \quad \text{or,}$$

$$\bar{\gamma} = \frac{2kN}{a - bc} - 1 < N - 1 .$$

So, as expected, the critical discount factor $\bar{\gamma}$ goes down with reduced k, i.e., the system gains stability.

On the other hand since $k \geqslant \frac{a - bc}{N - 1}$ we must have, if cheating is not to occur, that

$$\gamma \geqslant \frac{2kN}{a - bc} - 1 \geqslant \frac{2N}{N - 1} - 1 = \frac{N + 1}{N - 1} .$$

The computations are depicted graphically in Figure 4.1 below. We remark parenthetically that the mixed strategy zone looms much larger in the graphs than it does in reality. The graphs are not drawn to scale.

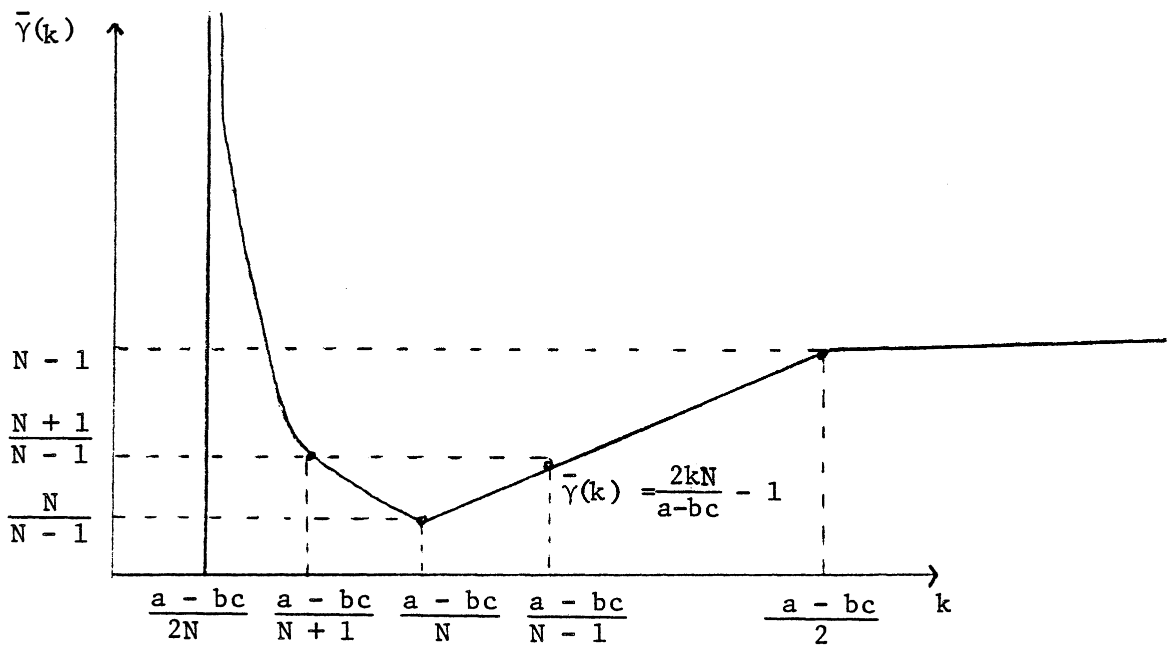


Figure 4.1

Turn now to the task of computing $\bar{\gamma}(\cdot)$ for the mixed strategy zone. If $\frac{a - bc}{N + 1} \leqslant k \leqslant \frac{a - bc}{N - 1}$, cheating occurs if

$$\left(\frac{a - bc}{2b}\right)k - \frac{(a - bc)^2}{4bN} > \gamma\left(\frac{(a - bc)^2}{4bN} - \frac{(a - (N - 1)k - bc)^2}{4b}\right)$$

Notice for the industry to be large enough to share the monopoly output, capacity k must satisfy $k \geqslant \frac{a - bc}{2N}$. But this holds whenever $N \geqslant 1$.

Now suppose $k = \frac{a - bc}{N + 1}$. Thus the critical γ can be found as the solution to the equation,

$$\frac{a - bc}{2b}\left(\frac{a - bc}{N + 1}\right) - \frac{(a - bc)^2}{4bN} = \gamma\left(\frac{(a - bc)^2}{4bN} - \frac{\left(a - \frac{N - 1}{N + 1}(a - bc) - bc\right)^2}{4b}\right),$$

or,

$$\frac{(a - bc)^2}{2b(N + 1)} - \frac{(a - bc)^2}{4bN} = \gamma\left(\frac{(a - bc)^2}{4bN} - \frac{(2a - 2bc)^2}{4b(N + 1)^2}\right),$$

or,

$$2(a - bc)^2N(N + 1) - (a - bc)^2(N + 1)^2 = \gamma[(a - bc)^2(N + 1)^2 - 4(a - bc)^2N]$$

i.e.,

$$\bar{\gamma}\left(\frac{a-bc}{N+1}\right) = \frac{2N(N + 1) - (N + 1)^2}{(N + 1)^2 - 4N} = \frac{N^2 - 1}{(N - 1)^2} = \frac{N + 1}{N - 1}.$$

If $k = \frac{a - bc}{N}$ then we must solve

$$\frac{(a - bc)^2}{4bN} = \gamma\left[\frac{(a - bc)^2}{4bN} - \frac{\left(a - \frac{N - 1}{N}(a - bc) - bc\right)^2}{4b}\right],$$

$$\frac{(a - bc)^2}{N} = \gamma\left[\frac{(a - bc)^2}{N} - \frac{(a - bc)^2}{N^2}\right]$$

i.e.,

$$\bar{\gamma}\left(\frac{a-bc}{N}\right) = \frac{1}{1 - \frac{1}{N}} = \frac{N}{N - 1}.$$

At this point we have computed $\bar{\gamma}(\cdot)$ for the values $k \geq (a - bc)/2$; $(a - bc)/(N - 1) \leq k \leq (a - bc)/2$; $(a - bc)/(N + 1) \leq k \leq (a - bc)/(N - 1)$. Turn now to the task of computing $\bar{\gamma}(\cdot)$ for $(a - bc)/2N \leq k \leq (a - bc)/(N + 1)$.

Routine (by now) computations yield

$$\bar{\gamma} = (N + 2)/(N - 2) \ , \quad k = (a - bc)/(N + 2)$$

$$\bar{\gamma} = (N + 1)/(N - 1) \ , \quad k = (a - bc)/(N + 1)$$

It is straightforward to demonstrate that $\bar{\gamma}(k)$ has the shape depicted in Figure 4.1 on the interval $((a - bc)/2N, (a - bc)/(N + 2)]$.

It is easy to see that $\bar{\gamma} = 0$ on $[0, \frac{a-bc}{N})$.

At this point we have finished computing the values of $\bar{\gamma}$ depicted in Figure 4.1 The interesting economics contained in Figure 4.1 is the following: Tightening capacity (cet. paribus) encourages tacit collusion up to the point $k = (a - bc)/N$ then tightening capacity discourages tacit collusion!

This result is easy to explain. First look at Figure 4.1b below, holding N fixed.

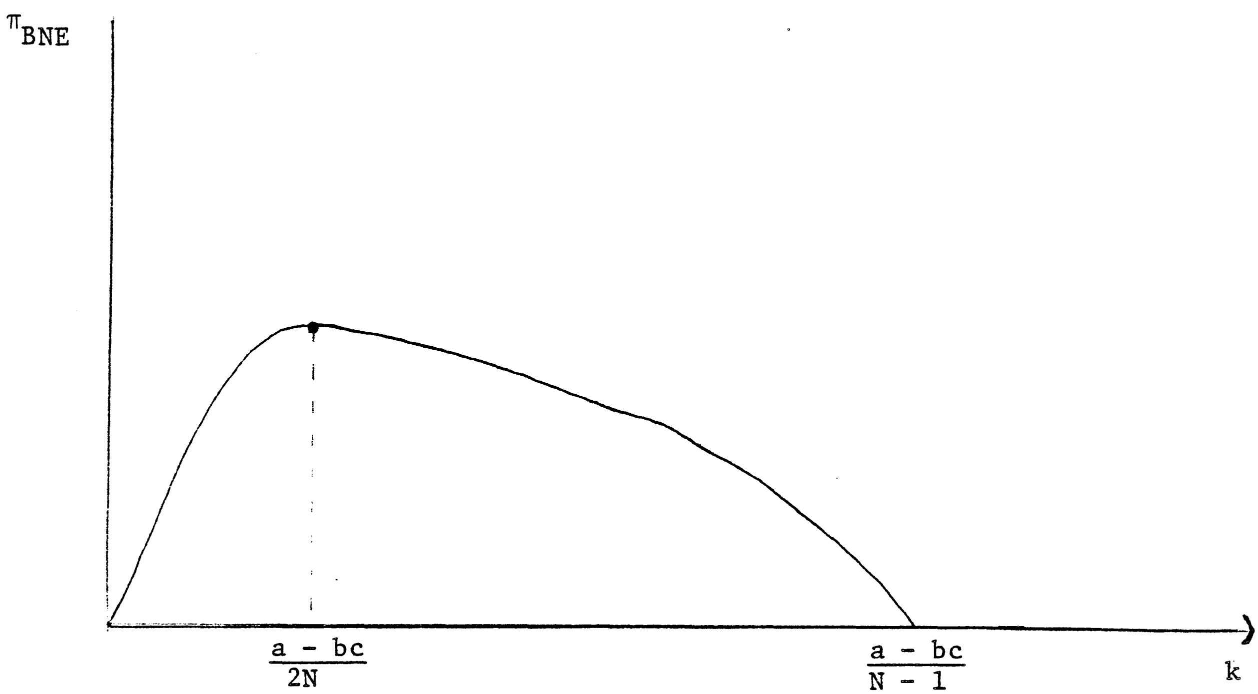


Figure 4.1b

Industry profits in static Bertrand-Nash Equilibrium, π_{BNE}, is plotted against capacity k. When capacity is small there is economic rent to capacity in competitive equilibrium. Parenthetically we remark that it is easy to see that a BNE is the same thing as a competitive equilibrium in the pure strategy zone. Hence competitive equilibrium profits will rise as a function of capacity for $k \in [0,(a - bc)/2N]$. Turn now to the zone: $k \in [(a - bc)/2N, (a - bc)/(N + 1)]$. Profits in this zone are computed following the proof of Lemma 1. The same Lemma shows that profits are zero for $k \in [(a - bc)/(N - 1),\infty)$. Turn now to the more novel mixed strategy zone: $k \in [(a - bc)/(N + 1), (a - bc)/(N - 1)]$.

Lemma 2 teaches us how to compute BNE profits for the mixed strategy zone. It is easy to see that the profitability curve has the shape depicted in Figure 4.1b.

The task of explaining the economics of Figure 4.1 is easy with the help of Figure 4.1b. For $k \in [(0,(a - bc)/2N]$ capacity is so scarce that an increase in capacity increases π_{BNE}. In this case the industry is already at the maximum possible profit given industrial capacity. The capacity constraint is so severe relative to the size of the market that the problem of optimum industrial structure does not even arise.

Suppose now that $k = (a - bc)/2N$. Let now a small increase in capacity Δk take place for each of the N firms. If a firm chisels he will receive net benefits of first order: $(P^m - c)\Delta k$. He will be punished forever after. But the punishment threat is of second-order in Δk since $k = (a - bc)/2N$ gives maximum profit to the industry regardless of industry structure. Hence a huge discount quantity, i.e., a tiny interest rate, is needed to deter chiseling for $k = (a - bc)/2N + \Delta k$, $\Delta k > 0$, Δk small. Thus there is an asymptote at $k = (a - bc)/2N$.

As k increases from the value $(a - bc)/2N$ the severity of the Bertrand competition unleashed by cheating begins to loom larger until it swamps the gain from chiseling and then the curve $\bar{\gamma}(k)$ begins to turn up again.

If capacity is greater than $(a - bc)/2$ then $\pi_{BNE} = 0$ but the maximum that a chiseler can get is the whole market since capacity no longer restrains the gains from chiseling. Hence

$$\bar{\gamma}(k) = N - 1, \quad k \geqslant (a - bc)/2 .$$

It is instructive to develop the pair of curves in Figures 4.1, 4.1b in N space holding k fixed. To this task we now turn. Notice that there is no <u>analytical</u> difference between holding N fixed while varying k and vice versa. In Figure 4.2 capacity k is held fixed but N is varied.

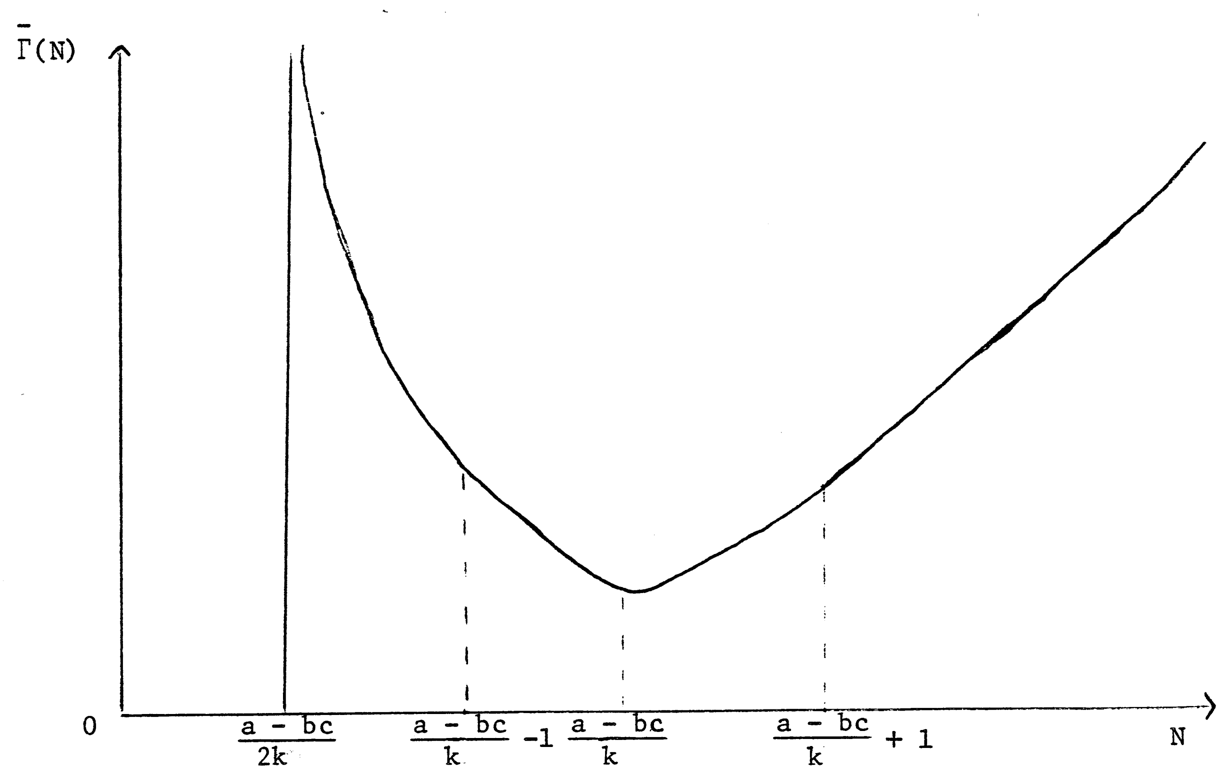


Figure 4.2

It is, by now, routine to verify that

$$\bar{\Gamma}[(a - bc)/k - 1] = 1 + 2k/(a - bc) = \bar{\Gamma}[(a - bc)/k + 1]$$

$$\bar{\Gamma}[(a - bc)/k] = (a - bc)/[(a - bc) - k] = \min_N \bar{\Gamma}(N)\cdot$$

The straightforward computations are the same as those displayed in Figure 4.1. The demonstration that $\bar{\Gamma}(N)$ has the shape depicted in Figure 4.2 follows straightforwardly from Figure 4.1. The shape may be explained by use of Figure 4.2b below in a manner analogous to Figure 4.1. Figure 4.2b is drawn below. Here ξ_{BNE} denotes BNE industry profit written as a a function of N holding k fixed.

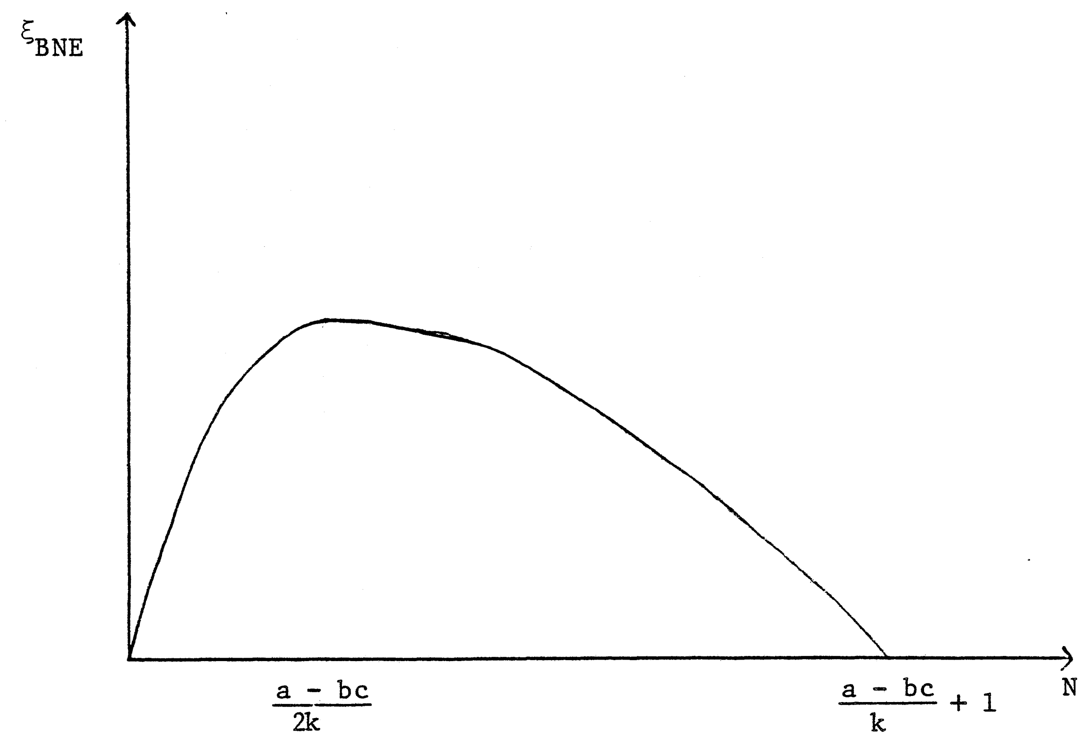


Figure 4.2b

What is the economics contained in Figure 4.2? We explain the shape from left to right. When N is small, BNE industry revenue is in the rising part of the industry revenue curve. Hence any discount factor is compatible with collusion in this case. At $N = (a - bc)/2k$ the point of maximum BNE industry revenue is reached and there is scope for output restraint to increase revenue to the collusion.

Why does $\bar{\Gamma}(N)$ fall on $((a - bc)/2k, (a - bc)/k)$ after jumping to infinity at $(a - bc)/2k$? This is due to the delicate tradeoff between one-period gains to chiseling and capitalized losses of unleashed Bertrand competition that emerges in this zone.

The reasoning is exactly the same as for Figure 4.1 since increasing N with capacity fixed is similar to increasing capacity with N fixed.

It is instructive to look at the closed form solution for the curve $\bar{\Gamma}(N)$. As pointed out above, the computations are very similar to those for the case of fixed N, variable capacity. We obtain,

$$\bar{\Gamma}(N) = 0 , \quad N \in [0, (a - bc)/2k) ;$$

$$\bar{\Gamma}(N) = (N + \ell)/N - \ell, \ \ell \equiv (a - bc)/k - N, \quad N \in ((a - bc)/2k, (a - bc)/k - 1]$$

$$\bar{\Gamma}(N) = (N^2 - (1 - \ell)^2)/[(N - 1 + \ell)^2 - \ell^2 N],$$

where $\ell = \ell(N)$ is defined by

$$k = (a - bc)/(N - 1 + \ell) , \quad N \in [(a - bc)/k - 1, (a - bc)/k + 1] ;$$

$$\bar{\Gamma}(N) = (2Nk - (a - bc))/(a - bc), \ N \in [(a - bc)/k + 1,\infty) .$$

Furthermore

$$\bar{\Gamma}((a - bc)/k - 1) = \bar{\Gamma}((a - bc)/k + 1) = (N + 1)/(N - 1)$$

$$\bar{\Gamma}((a - bc)/k) = N/(N - 1) .$$

5. Equilibrium Cartel Prices

In Section 4 we investigated the impact of changes on N,k on the critical discount factor as a function of N,k and found that it was not monotone in N,k. Unguided intuition would expect the critical discount factor to go up (i.e., the critical interest rate to go down) as N,k increase. Such a result would lead one to expect that the product price (holding the discount factor fixed) may not be monotone in N,k either. Indeed it is easy to see that price will not be monotone as a function of N for a value like Γ_o depicted in Figure 4.2. Moving N from left to right we see that price jumps from competitive to monopoly at $N = \frac{a-bc}{k} - 1$. Price drops to marginal cost c for $N > \frac{a-bc}{k} + 1$.

We wish to explore the non-monotonicity of price in a slightly different construct. For fixed discount factor γ and fixed capacity k we ask: What is the maximum profit that is consistent with cartel stability? In the region where total industry capacity does not exceed monopoly output the collusive price is equal to the BNE price and thus the question has a simple answer, namely that the collusion profit is consistent with cartel stability. For the region where total industry capacity exceeds monopoly output we already know that for a certain region of values of N, that we may denote by $[\underline{N},\overline{N}]$, the monopoly price may be sustained through trigger strategies. This region may be empty for very low values of γ, but if it is non-empty it is clear that for $N \epsilon [\underline{N},\overline{N}]$, the collusion profit is consistent with cartel stability. If total industry capacity (N) is larger than monopoly output (Q^m), but $N \notin [\underline{N},\overline{N}]$ we may also characterize the maximum profits which are consistent with cartel stability. Notice that if $N \epsilon [\underline{N},\overline{N}]$, and

$N k > Q^m$ since, by assumption $k < Q^m$ we have

$$(5.1) \qquad (p^m - c)\, Nk > (\gamma + 1)\, \Pi(p^m) - \Pi_{BNE}(N) \quad ,$$

where $\Pi_{BNE}(N)$ denotes the (expected) Bertrand-Nash equilibrium profits. Now suppose $p > p^m$. A defector may always change p^m, and thus sell his full capacity k. Since $\Pi(p) \leq \Pi(p^m)$, we have, if (5.1) holds,

$$(5.2) \qquad (p^m - c)\, N k > (\gamma + 1)\, \Pi(p) - \Pi_{BNE}(N) \quad .$$

This last equation expresses the fact that if p^m cannot be sustained, no price above the monopoly price is sustainable when trigger strategies are used. If profits are monotone on prices for prices below monopoly price--what is true in particular in our case of linear demand function and constant marginal costs up to capacity--we may transform our question on profits, when $Nk > Q^m$ and $N \notin [\underline{N}, \overline{N}]$, into one on prices--namely, what is the maximum price that may be sustained through trigger strategies of the type described in Section 3.

Thus for $N k > Q^m$ and $N \notin [\underline{N}, \overline{N}]$, a price is sustainable if $p \leq p^m$ and

$$(p - c)Nk < (\gamma + 1)\Pi(p) - \Pi_{BNE}(N) \quad .$$

Furthermore, the maximum sustainable price $\overline{p}(N)$ is defined by

$$\overline{p}(N) = \sup\{p \mid (5.3) \text{ holds}\} \quad .$$

Notice that we may now extend our definition of $\overline{p}(N)$ for all N, by letting $\overline{p}(N) = b^{-1}(a - Nk)$ if $Nk < Q^m$ and $\overline{p}(N) = p^m$ if $N \epsilon [\underline{N}, \overline{N}]$. We let $\overline{\Pi}(N)$ denote the associated profit.

At this point the reader may notice that any price that satisfies (5.1) is sustainable whereas we are placing special emphasis on the one that yields maximum profits. Thus we are making an arbitrary selection among possible equilibria and at this point we have not much to offer as justification. Perhaps the best way to think about this selection is to require that players in this game actually play a cooperative game but that the agreements can only be enforced through the threats described above. Thus, if p is not sustainable an agreement to keep prices at p will not hold since defection pays off. On the other hand if p is sustainable such agreement will not be broken and thus players should agree on the best price that is sustainable.

Now suppose there is a fixed cost F of entry. Assume that from a large population of possible firms some will enter and some will not. Suppose further that after entry players will enjoy profits $\overline{\Pi}(N)$. Now, if we ignore the integer problem, an equilibrium will have to satisfy, in order to leave firms indifferent between entering and not entering,

$$\overline{\Pi}(N) = NF \quad . \tag{5.4}$$

Within our framework of linear demand functions we may show that (5.4) may be solved uniquely for $N(F)$. $N(F)$ thus describes the number of firms that will enter in equilibrium when fixed costs are given by F.

We wish to make three points in this section: First, $\overline{p}(N)$ is not monotone in N. Second, profits $\overline{\Pi}(N)$ is not monotone in N. Third, $\overline{p}(N(F))$ is not monotone on F. These facts form the basis of the welfare analysis that is done in the next section.

Look at Figures 5.1a, below. Figure 5.1a depicts the function $\overline{p}(N)$ for the case $c = 0$. (The case $c > 0$ is a minor modification of Figure 5.1a.) Notice that as N increases price $\overline{p}(N)$ falls from the choke price a/b to a price below the monopoly price a/2b then rises until the monopoly price is reached again. Maximum sustainable price remains flat until N reaches the value

$$N = \frac{\gamma + 1}{2} \frac{a}{k} .$$

Maximum sustainable price falls until $\overline{p}(N) = 0$ is reached.

Figure 5.1b depicts industry profits $\overline{\Pi}(N)$ which move in tandem with maximizing sustainable price. Obviously neither $\overline{p}(N)$ nor $\overline{\Pi}(N)$ possess monotonicity in N.

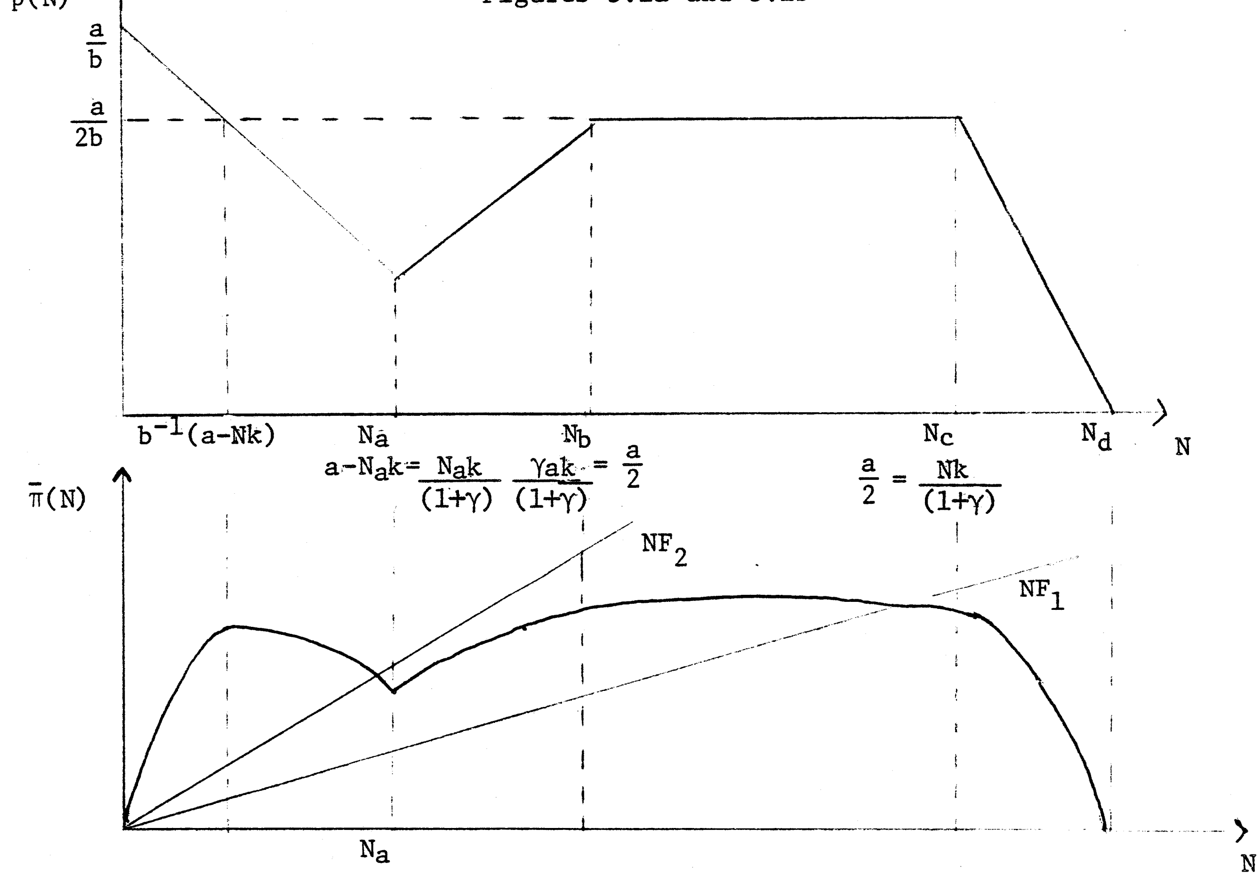
Now suppose it costs F to set up a plant and enter. Imagine that entry takes place so long as $\overline{\Pi}(N) > NF$.

Two equilibria are depicted in Figure 5.1b. The fixed costs $F_2 > F_1$ are chosen so that profits are higher and excess capacity is higher in the F_1 equilibrium. The economics may be explained by a parable.

Doctors may practice in Los Angeles or Chicago. The fixed cost of practice in Los Angeles is less than Chicago because the burden of bearing Chicago weather is included in F. Doctors in Los Angeles and Chicago tacitly collude on price using trigger strategies but are unable to prevent movement into their field from other parts of the country. (The opportunity costs of hurdling the AMA's barriers to entry into US medical practice are included in F_1, F_2.) Each doctor has a daily service capacity k. Migration of doctors continues until N_2 are practicing in Chicago and N_1 are practicing in LA. But there is more excess capacity and prices are higher in LA than in Chicago. Furthermore the quantity of ervice provided is higher in Chicago than in LA. Also industry profits (much less profits per doctor) are higher in Chicago than in LA.

What makes the LA equilibrium "stable"? There is a large threat embodied in the amount of excess capacity that hangs over the LA medical community. Each doctor, in contemplating price competition, realizes that he may set off a gale of ruinous Bertrand competition. The threat of unleashed Bertrand competition does not loom so large in the Chicago case. Hence a higher price may be obtained in LA than in Chicago.

Figures 5.1a and 5.1b



Figures 5.1(a) and (b) may be used to illustrate an important insight concerning replication. Intuition and many economic models (Green [2]) suggest that when the market expands at the same rate as the number of firms then perfect competition will be achieved at the limit. The intuition is that the market share of each firm will diminish and hence, eventually, his gain from chiseling will exceed the capitalized losses of unleashed competition triggered by chiseling. Indeed equation (3.12) is independent of the size of the market.

But everything changes when capacity constraints appear. Look at (5.3). If N and the market demand at each price are doubled then inequality (5.3) remains unchanged. This trivial observation leads to an important economic insight. Replication does not change the viability of self-enforcing price fixing agreements. Hence the classical limit theorem that replication leads to perfect competition is false for our model.

The economics is simple. If the number of firms is doubled industry capacity doubles. However the net gain to each firm chiselling at p is still (p - c)k minus his share of monopoly profit. Monopoly profit is doubled but the number of firms to share it is also doubled. Hence net gain to each firm chiseling remains unchanged.
Each period the loss from chiselling is monopoly profits per firm minus BNE profits per firm. Doubling the market and doubling the number of firms leaves monopoly and BNE profits per firm unchanged. The <u>net</u> temptation to chisel at p remains unchanged.

The failure of the classical limit theorem on replication underscores the impact of capacity constraints on the predictions yielded by oligopoly models.

Turn now to welfare.

6. Welfare Analysis

Let us explore the welfare economics of government policies towards entry when tacit collusion is present. Return to Figure 5.1b. Notice that reducing fixed cost from F_2 to F_1 just leads to more entrants who create excess capacity. This excess capacity looms large as a threat that restrains chiseling on the tacit collusion. Hence industry prices and industry profits are increased by the reduction of fixed costs. Yet more resources are consumed by the increased number of entrants. We come to the bottom line conclusion on welfare: It never pays to subsidize entry. It may pay to tax entry.

In order to demonstrate the welfare conclusion we need some notation and a measure of welfare. Measure welfare by net surplus:

$$(6.1) \qquad \int_0^{Q(N)} D(\xi)d\xi - cQ(N) - NF \quad .$$

Here $Q(N)$ denotes total industry quantity produced by the N firms. Define $W(F,\tau)$ by

$$(6.2) \qquad W(F,\tau) \equiv \int_0^{Q(N(\tau))} D(\xi)d\xi - cQ(N(\tau)) - N(\tau)F$$

where $N(\tau)$ is the largest solution to the subsidized free entry equation

$$(6.3) \qquad \bar{\pi}(N) = NF(1 - \tau) \quad .$$

We remark parenthetically that the solution $N(\tau)$ to (6.3) is unique in the worked example below.

The subsidy τF is assumed to be financed by lump-sum non-distortionary taxes on the rest of the economy. The subsidy τF may be negative. Hence, if consumer surplus may be used, $W(F,\tau)$ is the correct measure of welfare generated by τ.

Our task is to prove the following theorem.

Theorem : Let

$$(6.4) \qquad D(Q) = b^{-1}[a - Q],\ Q(p) = a - bp \quad .$$

Then $W(F,\tau)$ is decreasing in τ for $\tau \geq 0$.

Proof: We prove this for the case $c = 0$. The case $c > 0$ is a straightforward generalization. Examine Figures 5.1a,b above where $\bar{p}(N)$, $\bar{\pi}(N)$ are calculated in closed form and are graphed. Notice immediately from the formula for $\bar{\pi}(N)$ that $N(\tau)$ is piecewise linear in τ.

The logic of the proof follows. The function $W(F,\tau)$ is piecewise differentiable and continuous. We shall show $W(F,\tau)$ decreases in τ by showing that the derivative of W with respect to τ is negative whenever it exists.

On $(0,N_a)$ the system is already at a competitive equilibrium. Hence one can only do harm by a small subsidy to entry. On (N_a,N_b) a small subsidy to entry causes price to rise. Hence consumer welfare falls. More resources are used up by entrants. Thus welfare falls. ON (N_b,N_c) a small subsidy to entry does not lower price yet uses up more resources on entry. Hence welfare falls. The interval (N_c,N_d) remains.

We calculate $\frac{dW}{d\tau} \equiv W_\tau$ explicitly for τ such that $N(\tau) \in (N_c,N_d)$. The other calculations are straightforward. For τ such that $N(\tau) \in (N_c,N_d)$, $N(\tau)$ is given by the solution to

$$(6.5) \qquad b^{-1}[a - \frac{Nk}{\gamma + 1}]\frac{k}{\gamma + 1} = F(1 - \tau) \quad .$$

Using subscripts to denote partial derivatives we obtain

$$(6.6) \qquad W_\tau = [D(Q)Q_N - F]N_\tau \quad .$$

It is clear from the formula for N_τ that $N_\tau > 0$. Hence it is sufficient to show

$$(6.7) \qquad D(Q)Q_N - F < 0 \quad .$$

Now on (N_c, N_d)

$$(6.8) \qquad Q(N) \equiv a - b\,\bar{p}(N) = a - b[\frac{a}{b} - \frac{Nk}{b(\gamma + 1)}] = \frac{Nk}{\gamma + 1} \; .$$

Hence

$$(6.9) \qquad Q_N = \frac{k}{\gamma + 1} \quad .$$

Also

$$(6.10) \qquad D(Q) \equiv b^{-1}[a - Q] = b^{-1}[a - \frac{Nk}{\gamma + 1}] \quad .$$

Thus

$$(6.11) \qquad D(Q)Q_N - F = b^{-1}[a - Nk/(\gamma + 1)](k/(\gamma + 1)) - F \quad .$$

It is important to develop some economic intuition for (6.7) on the zone (N_c, N_d). The zone (N_c, N_d) is an excess capacity zone with prices that are higher than the full capacity price $p = D(Nk)$. Excess capacity and excessive prices are maintained by mutual strategic tyrants that rely on the terror of small BNE profits. If government creates a new entrant via subsidy the net benefit to subsidy is

$$(6.12) \qquad D(Q)Q_N - F = \bar{p}(N)Q_N - F$$

It costs society F to create a new entrant and the benefits are extra output times price. But on (N_c, N_d) extra output is substantially less than k since the new entrant, via tacitly colluding with his fellows, will possess substantial excess capacity. Hence society gets little benefit for expenditure F. This observation leads one to suspect that $\bar{p}(N)Q_N - F < 0$ on (N_c, N_d). It is easy to see this for $N \geq \hat{N}$, where $[\hat{N}, \infty)$ is the set of N such $p = o$ is a BNE in pure strategies.

The exact formulae are

$$(6.13) \qquad \bar{p}(N)Nk = (\gamma + 1)\bar{p}(N)Q(N) - \gamma\pi_{BNE}(N) = (\gamma + 1)\bar{p}(N)Q(N) \;, \qquad N \geq \hat{N} \;.$$

Hence for $N \geq \hat{N}$ we have

$$(6.14) \qquad Nk = (\gamma + 1)Q(N) \,,\; k = (\gamma + 1)Q_N ,\; \bar{p}(N)Q(N) = NF(1 - \tau) \quad .$$

Hence

$$(6.15) \qquad \bar{p}(N)Q_N - F = \bar{p}(N)k/(\gamma + 1) - F = \bar{p}(N)Q(N)/N - F = - \tau F < 0 \quad .$$

Notice that the above demonstration was independent of the linearity of the demand curve provided $N \geq \hat{N}$. We do not have a proof for general demand curves for $N < \hat{N}$. Turn now to $N \in (N_c, \hat{N})$. Now

$$(6.16) \qquad \bar{\pi}(N) = NF(1 - \tau)$$

implies, via (6.8) and (6.10),

$$(6.17) \qquad b^{-1}[a - Nk/(\gamma + 1)](Nk/(\gamma + 1)) = NF(1 - \tau) \quad .$$

It follows immediately, using (6.9) and (6.17) that

$$(6.18) \qquad D(Q)Q_N - F = - \tau F < 0 \quad .$$

This ends the proof.

The underlying economics behind the proof of the welfare theorem suggests that the welfare theorem may hold for more general demand curves than linear ones. Although we have no general theorems it may be worthwhile to explain why we believe the theorem may be quite robust.

The logic of the theorem is contained in the general shape of the curves depicted in Figures 5.1a,b. As N increases a competitive equilibrium zone is followed by a zone of rising prices and profits. Prices and profits rise to the monopoly level. Eventually prices and profits begin to fall. Prices and profits fall to zero as N continues to increase.

The economic force that determines the shapes of the graphs of prices and profits is the trade-off between the gains from chiseling and the opportunity cost of Bertrand ompetition. This leads us to examine Bertrand equilibrium quantities and Bertrand equilibrium profits. Look at Figure 6.1 below. Hold k fixed and vary N. Plot Nk on the horizontal axis. Assume c = 0. Plot pure profits at Nk and Bertrand equilibrium profits at Nk on the vertical axis. Under general conditions on D(.) there will be three zones $[0,\tilde{N}]$, $[\tilde{N},\hat{N}]$, $[\hat{N},\infty]$.

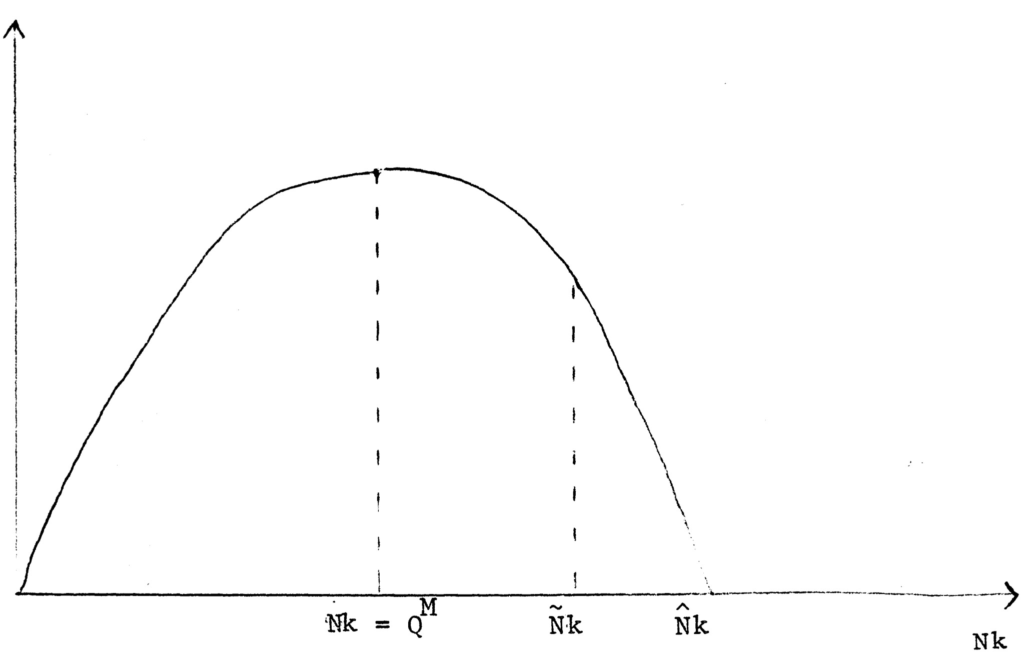


Figure 6.1

The zone $[0,\tilde{N}]$ is a pure strategy zone with $p = D(Nk)$ the equilibrium price if $\tilde{N}$ solves

$$(6.19) \qquad D'(Nk)k + D(Nk) = 0 \quad .$$

The reasoning is easy. Firm i gains nothing by lowering his price because he is already at capacity. He gains nothing by raising his price when

$$(6.20) \qquad \max_{q \le k} D[(N - 1)k + q]q \le D(Nk)k \quad .$$

But this holds when

$$(6.21) \qquad D'(Nk)k + D(Nk) \ge 0 \quad .$$

Hence $\tilde{N}$ defines the cutoff value of Nk where firm i gains nothing by raising his price.

The interval $[\hat{N},\infty]$ is the set of N such that $p = 0$ is a BNE in pure strategies. The set $[\tilde{N},\hat{N}]$ is a possible mixed strategy zone. In the linear case we demonstrated that it was a mixed strategy zone.

Let us summarize. We conjecture that all we need on D(.) to obtain the welfare theorem is something like the following: D(.) is regular enough so that profits are single peaked as a function of $Nk \equiv Q$ and there exist unique $\tilde{N},\hat{N}$ such that $D'(\tilde{N}k)k + D(\tilde{N}k) = 0$, $D^{-1}(0) + k = \hat{N}k$. The general case is left to future research.

7. Conclusions, Relevance, and Suggestions for Future Research

The result of this paper can be summarized thus: First price setting supergame equilibria may support higher industry price and lower industry quantity than quantity setting supergame equilibria -- contrary to the standard result for static games. Call such equilibria "self enforcing price (quantity) setting agreements." Second, the maximum price that can be supported by a self enforcing price setting agreement by N Bertrand cartelists each with capacity k is not monotonic in N and k. Unguided intuition would expect it to fall as N,k increase. Third, entry may decrease welfare if entrants become party to a self enforcing price fixing agreement with incumbants rather than competing. Furthermore the new entrants may strengthen the viability of a self enforcing price fixing agreement by increasing threat power by the extra excess capacity they bring to the market. Fourth and final is the finding that a subsidy for entry never increases welfare and it decreases welfare for "most" values of N,k.

These results are a bit counterintuitive. We offer them as results of an exploration of properties of price setting supergame models. It is time to address their relevance to the "real world" and to policy.

Many results generated by quantity setting game models are criticized by Bertrand type objections; "what happens if price is also a strategic variable?" But static price setting games behave badly unless products are differentiated. But our first set of results has shown that the predictions of price setting supergames do not differ wildly from those generated by quantity setting supergames. Recall this by looking at

Figure 3.1. The tacit collusion region for the price setting game (the quantity setting game) lies above B(N)(C(N)). Furthermore the second set of results on the values of N,k that result in tacit collusion do not fly in the face of common sense. This suggests that research on supergames with different strategy spaces may yield insights of use for industrial organization theory.

The results on entry and welfare seem strange. We think that the economic force that is being illustrated here is the following. One dimension of competition, viz. price, is easily observed. A price cutting rival can easily be observed and punished. But in many cases another dimension of competition e.g., new entry, cannot be stopped. If self enforcing price fixing agreements can be made between incumbants and new entrants then we have a typical "second best situation." Hence there is no reason to expect the admission of new entrants to improve welfare.

We certainly do not advocate the imposition of entry restrictions based upon the result of this paper! There are too many dimensions of reality left out. The results suggest that more realistic models where some dimensions of competition such as price may be readily observed but other dimensions of competition such as service quality are unobserved may help explain why some large groups such as professions seem to be able to support prices that appear to be collusive but yet entry appears excessive. Such models may be able to explain why socially excessive "service" competition breaks out in industries that appear to be ripe for socially beneficial price competition. This is a topic for future research.

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FOOTNOTES

[1]The calculations in the proof look arcane but they are actually quite intuitive when viewed properly.

Reason thusly. First, notice that on the support (P_e, P_N) of candidate mixed strategy equilibrium $\phi(\cdot)$ it must be the case that $\pi(P)$ is constant. If not, then firm one would gain by "stacking probability mass" onto the point P_m, where $\pi(P)$ is maximum. Hence firm one does not choose nondegenerate $\phi(\cdot)$ as best reply unless $\pi(P)$ is constant on the support (P_e, P_N) of $\phi(\cdot)$.

Second, given $P\varepsilon(P_e, P_N)$ calculate expected profit to firm one by noticing that firm one turns out to be the highest priced firm $\phi^{N-1}(p)$ percent of the time in which event it earns $[D(p) - (N-1)\mathbf{k}](P-c)$. Firm one is not the highest priced firm $1 - \phi^{N-1}(p)$ percent of the time in which event it earns $(P-c)k$. This is the content of equation (c). Now $\pi(P)$ is constant over (P_e, P_N). Therefore expected profit w.r.t. any distribution, in particular w.r.t. $\phi(\cdot)$, is just $\overline{\pi} \equiv \pi(P)$. Solve equation (c) to get the formula for $\underline{\phi}(\cdot)$ given in the statement of Lemma 2. At this point we still do not know $\overline{\pi}$, P_e, P_N.

Third, to find P_N, observe that firm one will choose price to maximize profits when the rest are charging slightly less than P_N. Profits to firms one in that case are, since the others are selling at capacity,

(i) $\pi(P) = (a - b\,P - (N-1)k)(P-c)$.

Choose P to maximize $\pi(P)$. This determines P_N. Plug P_N into equation (i) to find $\overline{\pi}$. Finally determine P_e by solving:

(ii) $\overline{\pi} = k(P-c)$.