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Use of Transition Probabilities Within a Regional Programming Model for Resource Allocation Among Subgroups and Through Time

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Abstract

The dynamic feedback problem in regional agricultural production response or planning models can be divided into two parts. One part is intertemporal dynamic feedback which involves the time-dating relationship between production response and price as a lagged phenomenon. The other part of the problem involves changing the aggregation of subgroups over time. This paper suggests including a first order Markov transition probability within the model to handle the problem of aggregate changes over time. Because the model is intertemporal, it also accounts for the interrelationship among time horizons in the addition to the interrelationship among subgroups in the model. Including progressive income tax, which affects production and consumption, normally would require integer programming to handle the nonconvexity. However, this is handled here by stepwise approximation and is uniquely connected to the dynamic system of aggregation in the model.

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Introduction.

The dynamic feedback problem in regional agricultural planning models can be separated into two parts. One is the intertemporal dynamic feedback which accounts for the time-lagged relationship between price and production response. The other dynamic problem is in aggregation [1, 2]. That is, the availability of resources for each subgroup changes from one period to the next and this in turn changes the coefficient or the weights of aggregation. This weight change has been handled outside the model by using Markov chains [3, 4]. Theoretically, they can be generated inside the model [5].

This paper proposes one possible way to handle the problem of dynamic aggregation inside the model by using transition probabilities of the first order Markov process. Given the initial availability of resources for each subgroup in the beginning period, the resources for each subgroup in the following period can be tranferred accourding to the transition probability matrix. By using an appropriate specification in the model, the number of farms in each subgroup is determined internally by the model. With the multi-period program, the proposed model not only takes into account the interrelationship between subgroups but also accounts for the changes through time covered in the study. Reaction to regional production response as well as regional resource constraints can easily be incorporated in the model. Progressive income tax, which needs integer programming to take care of the nonconvexity, is handled by stepwise approximation and is connected to the dynamic system of aggregation in the model.

A Multi-period Linear Program with Resource Transfer by Transition Probabilities.

General Assumptions.

For purposes of simplifications, we assume that when some proportion of land is transferred from one size group to another, the same proportion of labor and operating cash is also transferred, and this transfer occurs at the end of each annual period.[1] Based on these assumptions, the transfer of land, labor, and cash is accomplished by the same transition probability matrix. The transfer equation for land, labor, and cash can be derived in difference equation form as follows:

(1) $L_{T+k+1} = p' \cdot L_{T+k}$ for $k = 0, 1, \ldots, t$; or

(2) $L_{T+k+1} = p'^{k} \cdot L_T$ for $k \geq 1$.

where:

L_{T+k+1} is an rx1 column vector representing either total acreage of land, or total amount of operating cash, or total hours of family labor in each size group at the beginning of period $T + k + 1$,

p' is an rxr matrix of the transpose of the first order transition probability matrix.

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[1] This assumption can be relaxed in several ways: (1) when measures of past labor and capital among strata are available, separate transition probability matrices may be estimated and used to transfer these resources among strata from one period to the next as well as for land; (2) if we know the proportion of labor and/or operating cash which leaves agriculture from each stratum in each year, an adjustment coefficient such as 1-p can be used to transfer remaining resources within the stratum from one year to the next; or (3) with a pure market orientation to allow for competitive movement of resources, buying and selling activities for the resources, could be included in each stratum.

T is the starting time for the planning period,

L_T and L_{T+k} are rxl column vectors denoting either the total acreage of land, or the total amount of operation cash, or the total hours of family labor in each size group at the end of periods T and T + k respectively,

P'^k is the transpose of the kth power tranition probability matrix. The relationship of equation (1) reflecting the use of the first order transition probability matrix for year-to-year transfers, and the relationship in equation (2) uses either total acreage of land, or total amount of operating cash, or total hours of family labor in each size group in time T as the base for use of the kth power transition probability matrix for continuing transfer.

Specification of the Model.

The specification of the activities for land transfer are according to the transfer equation (1) and is given in Table 1. The model includes two strata and two annual periods. The program is formulated to maximize regional consumption over time and equity value (land and operating cash) at the end of the planning horizon, subject to the technical coefficients, aggregate resource constraints, and resource transfer limits. The model also is specified to allow land selling from the agricultural to the nonagricultural sectors of the region in each period, the amount of land sold is not known in advance. Therefore, continous land transfer among size groups in each period is impossible. Instead, a land accounting equality to net out the acreage of land actually existing in each size group at the end of each period after the possible activation of the land selling activities is used. Then the net acreage remaining in each size group is used as the base to multiply the first order transition probability matrix for year-to year land transfer. The activity (njLTR) is specified for this transfer.

The righthand side of the row for the land accounting equality for size group j in the first period (J1LE) is equal to the total acreage of cropland in that size group. Therefore, when land is sold, the land selling activity will absorb (add) one unit of land from this row. The number of times the activities 1jLTR are activated is equal to the total acres of land in size group j minus the total acres of land sold in size group j in that period. This net acreage of land is then distributed through activities 1jLTR to the proper size group in the following time period according to the transition probability matrix. From the second period on, whenever the activities njLTR allocate one unit of land to rows jnL in the next period, it also adds one unit of land in rows jnLE of that period. Thus, the righthand sides of jnLE from the second period on are equal to zero.

The specification of the activities for transfer of operating cash are given in Table 2. Since the amount of operating cash will be changed from one period to another due to the contribution of income from production in each period, and again this change is not known in advance. Therefore, transfer equation (1) will be used for year-to-year transfer, and activity njC3T1 is used for this transfer. The number of times activity njC3T1 is activated is equal to the total amount of net cash from the previous period in size j plus the total increase in cash in the current period minus the sum of the total outlays in production, minimum consumption, marginal consumption and taxes of this size group in the current period. This cash is then transferred through activities njC3T1 to the appropriate size group in the first subperiod of the following annual period as cash for size group j according to the first order transition probability matrix.

We assume that when farmers sell land, people originally in the farm sector will be accounted for there, but their labor could be used by obtaining off-farm employment. Based on this assumption, the only change in the supply of family labor among size groups from one period to another is controlled directly by the pattern of the transition probability matrix. Thus, the continous transfer of this resource through the three year planning period is possible by applying transfer equation (2) with first and second power transition probability matrices, and activities ljNTR on Table 3 are specified for this transfer. Since the righthand side of rows jlNE are equal to the total hours of family labor available in size j at subperiod m, and since entries in column ljNTR and rows jlNE are equal to one, the activity ljNTR will activate the number of times exactly equal to the number of hours of family labor specified in that righthand side. Therefore, activation of columns ljNTR multiplied by the first and second power transition probability matrices will generate the supply of family labor for each size group, respectively, in the second and third years. The specification will appear more explicitly if the entries in column ljNTR are moved to the righthand side and the sign of the entries is changed.

To allow farmers access to the outside capital market both for borrowing and investment opportunities, activities can be specified for capital borrowing and repayment, as well as activities for outside investment opportunities for the accumulated operating cash from production. Part time, half time, and full time hire in and hire out labor activities also could be specified in the model.

When farmers hire in (hire out) a unit of labor, it will add (subtract) one unit of labor to (from) the available family labor and subtract (add) the wage paid from (to) operating cash.

Equations for Controlling Cash Transfer, Consumption, Taxes, and Savings.

The entries of column njLSL and row jnCE in Table 1 is the selling price per acre of land. We assume the income generated from using the proceeds of land selling activities by farmers in either on-farm or off-farm investment are subject to income tax and influence the consumption of the period, but not the proceeds from the land selling activity itself. Therefore, this amount should be subtracted from the jnCE row and channelled directly through column njC3T1 in Table 2 to the next period as cash.

Similarly, the nontaxable beginning amount of operating cash available for each size group in the first period is subtracted from the j1CE rows by specifying a negative righthand side for these rows in Tables 1 and 2 and the cash is thendistributed directly throught the 1jC3T1 activities to operating cash in the next period. For the second period, the righthand sides of jnCE rows are equal to zero. Instead of using the righthand sides, njC3T1 columns of the preceding period and jnCE rows of the next period are used to control the taxed cash flow of the preceding period from being taxed and influncing the consumption of the next period; that is, when activities njC3T1 transfer one unit of cash from the third subperiod of one annual period to the first subperiod of the next annual period, one unit of cash is subtracted from the njCE rows of the next period (Table 2). By using this specification, taxes and consumption are assumed to be functions of current income. Specification of the consumption response to past income and past capital gain is given by

Vandeputte and Baker [6].

The entries of columns njLTR and rows jnMC in Table 1 are equal to

-1/average farm size in group j

because when columns njLTR activate N times, these entries will become - N/ (average farm size in group j) which is the number of farms in size group j. Since the entries of column nJMC and row jnMC are equal to one, and the righthand side of jnMC row equals to zero, the activities jnMC will activate the number of times exactly equal to the number of farms. Assuming that the minimum consumption requirement is homogeneous among farms in each size group, the amount required for minimum consumption and taxes is subtracted from cash and contributes the present value of the consumption outlays to the objective function.

Similarly, the entries of columns njLTR and rows jnMTm are calculated as follows:

- M/ average farm size in group j

where M is the income range corresponding to a specific level of marginal propensity to consume.

In Table 1, we specified an activity for marginal consumption and taxes in addition to the minimum consumption and taxes. The difference of columns njMTm from columns njMC is that the unit of activities in njMC is the amount of minimum consumption and taxes while the unit of activities njMTm is one dollar. The entries of columns njMTm and row jnc are marginal propensity to consume and taxes while the entries of columns njMTm and rows jnCE are the marginal propensity to save. Therefore, when activities njMTm activate one time, one dollar of operating cash will be allocated to the proper place according to the marginal propensities for consumption, taxes, and savings. The activities of marginal consumption and taxes also transfer the present value of consumption outlays to the objective function.

Summary

Markov processes have been used in the literature to predict the distribution of various kinds of economic outcomes. This paper suggests use of appropriate Markov processes to handle the dynamic problem of resource allocation among strata and time periods inside aggregate programming model. The model could also incorporate sale and borrowing or investment opportunities to help predict distribution of farm size through land transfer and the income distribution among size groups through cash transfer.

Also, this paper shows for the first time a way to include specification of consumption, savings, and progressive income taxes in a macro model. This is accomplished by connecting entries of activities for consumption, savings, and progressive income taxes. Previously published linear programming models with specification for these items have been limited to micro analysis [6].

Use of this method is quite feasible for other types of problems such as distribution of enterprise size, tenancy, farm type, and even structure and performance of the market.

Table 1

Linear Programming Tableau for Land Transfer, Minimum Consumption, Marginal Consumption and Taxes

		Period 1										Period 2										
		Size A					Size B					Size A					Size B					
		Pro-duce	Land		Consumption		Pro-duce	Land		Consumption		Pro-duce	Land		Consumption		Pro-duce	Land		Consumption		
			Sell	Transf.	Min.	Marg.		Sell	Transf.	Min.	Marg.		Sell	Transf.	Min.	Marg.		Sell	Transf.	Min.	Marg.	
Description	Code	1AP	1ALSL	1ALTR	1AMC	1AMTm	1BP	1BLSL	1BLTR	1BMC	1BMTm	2AP	2ALSL	2ALTR	2AMC	2AMTm	BP	2BLSL	2BLTR	2BMC	2BMTm	RHS
Objective Function	OBJ				a	a				a	a			a	a	a			a	a	a	=Max
Land, Size A	A1L	1	1																			=A1
Cash, Size A	A1C	(±)a	-a		a	a																≤A2
Min. consumption, Size A	A1MC			-a	1																	=0
Marg. consumption, Size A	A1MTm			-a		1																≤0
Cash, Accounting Equality	A1CE		a			a																=-A2
Land, Accounting Equality	A1LE		1	1																		≤A1
Land, Size B	B1L						1	1														=B
Cash, Size B	B1C						(±)a	-a		a	a											≤B2
Min. consumption, Size B	B1MC								-a	1												=0
Marg. consumption, Size B	B1MTm								-a		1											≤0
Cash, Accounting Equality	B1CE							a			a											=-B2
Land, Accounting Equality	B1LE							1	1													≤B1
Land, Size A	A2L			-Paa					-Pba			1	1									=0
Cash, Size A	A2C											(±)a	-a		a	a						≤0
Min. consumption, Size A	A2MC													-a	1							=0
Marg. consumption, Size A	A2MTm													-a		1						≤0
Cash, Accounting Equality	A2CE												a			a						=0
Land, Accounting Equality	A2LE			-Paa					-Pba				1	1								≤0
Land, Size B	B2L			-Pab					-Pbb								1	1				=0
Cash, Size B	B2C																(±)a	-a		a	a	≤0
Min. consumption, Size B	B2MC																		-a	1		=0
Marg. consumption, Size B	B2MTm																		-a		1	≤0
Cash, Accounting Equality	B2CE																	a			a	=0
Land, Accounting Equality	B2LE			-Pab					-Pbb									1	1			<0

Table 2
Linear Programming for Operating Cash Transfer

Description	Code	Period 1						Period 2						RHS
		1AC1T2	1AC2T3	1AC3T1	1BC1T2	1BC2T3	1BC2T3	2AC1T2	2AC2T3	2AC2TN	2BC1T2	2BC2T3	2BC3TN	
Objective Function	OBJ									1			1	=Max
Cash, Size A,														
Subperiod 1	A1C1	1												≤A2
Subperiod 2	A1C2	-1	1											≤0
Subperiod 3	A1C3		-1	1										≤0
Cash, Accounting Equality	A1CE			-1										=-A2
Cash, Size B,														
Subperiod 1	B1C1				1									≤B2
Subperiod 2	B1C2				-1	1								≤0
Subperiod 3	B1C3					-1	1							≤0
Cash, Accounting Equality	B1CE						-1							=-B2
Cash, Size A,														≤0
Subperiod 1	A2C1			-Paa			-Pba	1						≤0
Subperiod 2	A2C2							-1	1					≤0
Subperiod 3	A2C3								-1	1				=0
Cash, Accounting Equality	A2CE			Paa			Pba			-1				
Cash, Size B,														
Subperiod 1	B2C1			-Pab			-Pbb				1			≤0
Subperiod 2	B2C2										-1	1		≤0
Subperiod 3	B2C3											-1	1	≤0
Cash, Accounting Equality	B2CE			Pab			Pbb						-1	=0

Table 3
Linear Programming for Labor Transfer

Description	Code	Labor Transfer Size A 1ANTR	Labor Transfer Size B 1BNTR	RHS
Objective Function	OBJ			=Max
Labor, Size A, Subperiod 1	A1N1			≤A3
Subperiod 2	A1N2			≤A3
Subperiod 3	A1N3			≤A3
Labor, Accounting Equality	A1NE	1		≤A3
Labor, Size B, Subperiod 1	B1N1			≤B3
Subperiod 2	B1N2			≤B3
Subperiod 3	B1N3			≤B3
Labor, Accounting Equality	B1NE		1	≤B3
Labor, Size A, Subperiod 1	A2N1	-Paa	-Pba	≤0
Subperiod 2	A2N2	-Paa	-Pba	≤0
Subperiod 3	A2N3	-Paa	-Pba	≤0
Labor, Size B, Subperiod 1	B2N1	-Pab	-Pbb	≤0
Subperiod 2	B2N2	-Pab	-Pbb	≤0
Subperiod 3	B2N3	-Pab	-Pbb	≤0
Labor, Size A, Subperiod 1	A3N1	-[Paa^2 + PabPba]	-[PaaPab + PabPbb]	≤0
Subperiod 2	A3N2	-[Paa^2 + PabPba]	-[PaaPab + PabPbb]	≤0
Subperiod 3	A3N3	-[Paa^2 + PabPba]	-[PaaPab + PabPbb]	≤0
Labor, Size B, Subperiod 1	B3N1	-[PbaPaa + PbbPba]	-[PbaPab + Pbb^2]	≤0
Subperiod 2	B3N2	-[PbaPaa + PbbPba]	-[PbaPab + Pbb^2]	≤0
Subperiod 3	B3N3	-[PbaPaa + PbbPba]	-[PbaPab + Pbb^2]	≤0

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Appendix A

Derivation of Transfer Equations for Operating Cash and Labor.

Cash for individual size group j in Time T + k + 1 can be derived as follows:

$$(1)\quad C_{T+k+1,j} = \sum_{i=1}^{r} (C_{T+k,i} / L_{T+k,i}) \quad (L_{T+k,i} \cdot P_{ij})$$

$$= \sum_{i=1}^{r} \quad C_{T+k,i} \cdot P_{ij} \text{ for } k = 0, 1, \ldots\ldots\ldots, t$$

$$j = 1, 2, \ldots\ldots\ldots, r$$

$C_{T+k+1,j}$ is the amount of operating cash available in size group j at the beginning of year T + k + 1,

$C_{T+k,i}$ is the amount of cash for size group i at the end of year T + k,

$L_{T+k,i}$ is the total acreage of land in size group i at the end of year T + k,

P_{ij} is the transition probability for stratum i to j,

$C_{T+k,i}/L_{T+k,i}$ is the amount of cash per acre of cropland at the end of the year T + k, and

$L_{T+k,i} \cdot P_{ij}$ is the acreage of crop land in size group i at the end of year T + k, which will transfer to size group j at the beginning of year T + k + 1 according to the transition probability P_{ij}

In matrix form, (1) can be written as

$$(2)\quad C_{T+k+1} = P' \cdot C_{T+k} \text{ for } k = 0, 1, \ldots\ldots\ldots, t$$

or

(3) $C_{T+k+1} = p'^{k} \cdot C_T$ for $k \geqq 1$

Similarly, the transfer equation for labor can be derived as follows:

(4) $N_{T+k+1,\, j} \; \sum_{i=1}^{r} \; (N_{T+k,i} / L_{T+k,\, i})(L_{T+k,i} \cdot P_{ij})$

$$= \sum_{i=1}^{r} N_{T+k,\, i} \cdot P_{ij} \text{ for } k = 0, 1, \ldots\ldots\ldots, t$$

$$j = 1, 2, \ldots\ldots\ldots, r$$

where

$N_{T+k+1,\, j}$ is the total hours of family labor in size group j during subperiod m of period T + k + 1, and

$N_{T+1,\, i}$ is the total hours of family labor in size group i during subperiod m of period T + k.

In matrix for, (4) can be written as

(5) $N_{T+k+1} = P' \cdot N_{T+k}$ for $k = 0, 1, \ldots\ldots\ldots t$

(6) $N_{T+k+1} = p'^{1} \cdot N_T$ for $k \geqq 1$

Appendix B

Application

Farms in the four counties of Bureau, LaSalle, Marshall and Putnam in North Central Illinois were stratified into six groups based on the amount of cropland in farms. Size A includes farms with less that 260 acres of cropland. The acreage of size groups B, C, D, E, and F are, respectively, 260-339, 340-499 500-649, 650-799, and 800 acres or over. One representative farm was generated for each size group and an aggregate model was formulated to include six representative farms within the model.

The activities in each size group included production of corn, soybeans and oats, hog production, selling land, hire-in and hire-out seasonal labor, transfer activities for land, labor and cash, minimum consumption requirements, three finite ranges and one range with no upper bound for marginal consumption and taxes. Initial resource limitations were imposed for each size group and for the region as a whole. For each size group, these limitations included land, labor with three subperiods in each year, operating cash with three subperiods, minimum consumption requirements, three ranges for marginal consumptiona and taxes, and accounting equalities for land, labor, cash and corn production. The restrictions for the region as a whole included the amount of hog production, corn buying and selling formulated to force net exportation of corn for the region as a whole, and a limitation on the availability of hire-in labor for the region from outside the agricultural sector.

Table 4 shows the initial availability of land, labor and operating cash for each size group and the transfers in the second and the third periods. Tables 5 and 6 give the organization and financial results from the optimal solution of the model.

Table 4. Resources Availability by Size of Farm

Size Group	A	B	C	D	E	F
Period 1						
Land (acres): Initial endowment	499,182	352,331	319,016	153,509	63,422	117,093
Sold	0	0	0	0	3,171	0
Total	499,182	352,331	319,016	153,509	60,251	117,093
Labor: (hour/subperiod)	5,460,772	1,644,384	1,210,892	455,216	131,224	148,420
Cash: (dollar)	37,743,542	15,540,067	14,931,540	8,563,165	3,109,737	4,083,353
Period 2						
Land: Transfer from period 1	491,994	342,995	321,822	177,094	58,243	109,234
	A A 491,994	A B 6,589	A C 599	C D 4,179	C E 5,934	C F 3,605
		B B 331,015	B C 21,316	D D 151,897	D E 1,228	D F 384
		C B 5,391	C C 299,907	E D 21,018	E E 51,081	E F 9,170
						F F 96,075
Sold	0	0	0	0	3,171	0
Total	491,994	342,995	321,822	177,094	55,072	109,234
Labor: Transfer from period 1	5,382,137	1,637,445	1,244,398	492,940	137,417	156,572
	A A 5,382,137	A B 72,082	A C 6,553	C D 15,863	C E 22,523	C F 13,683
		B B 1,544,899	B C 99,485	D D 450,436	D E 3,642	D F 1,138
		C B 20,464	C C 1,138,360	F D 26,641	E E 111,252	E F 19,972
						F F 121,779
Cash: Transfer from period 1	37,200,036	15,350,451	15,022,607	9,401,816	2,982,667	4,013,827
	A A 37,200,036	A B 480,215	A C 45,292	C D 195,603	C E 277,727	C F 168,726
		B B 14,599,893	B C 940,174	D D 8,473,251	D E 68,505	D F 21,403
		C B 252,343	C C 14,037,141	F D 732,962	E E 2,636,435	E F 473,302
						F F 3,350,391
Period 3						
Land: Transfer from period 2	484,909	334,177	323,886	199,059	54,093	102,088
	A A 484,909	A B 6,494	A C 590	C D 4,216	C E 5,986	C F 3,637
		B B 322,244	B C 20,751	D D 175,235	D E 1,417	F G 443
		C B 5,439	C C 302,545	F D 19,608	E E 46,690	E F 8,382
						F F 89,626
Sold	0	0	0	0	0	0
Total	484,909	334,177	323,886	199,059	54,093	102,088
Labor: Transfer from period 1	5,304,649	1,630,445	1,275,336	483,852	143,467	164,594
	A A 5,304,649	A B 138,878	A C 16,928	B D 1,299	B E 1,850	B F 1,118
		B B 1,453,109	B C 186,999	C D 33,057	C E 40,395	C F 27,557
		C B 38,458	C C 1,071,409	D D 445,911	D E 6,691	D F 2,613
				E D 3,585	E E 94,318	E F 33,320
				F D 48,220	F E 213	F F 99,986
Cash: Transfer from period 2	36,664,355	15,166,670	15,096,095	10,220,375	2,883,339	3,919,412
	A A 36,664,355	A B 491,040	A C 44,640	C D 196,796	C E 279,420	C F 169,755
		B B 14,421,748	B C 928,702	D D 9,303,097	D E 75,214	D F 2,350
		C B 253,882	C C 14,122,753	F D 720,482	E E 2,528,705	E F 453,962
						F F 3,293,345

Table 5
Organization Results by Size of Farm

Size Group	A	B	C	D	E	
Period 1						
Number of farm	3,494	1,057	766	292	78	94
Corn production (acres)	499,182	352,331	319,016	153,509	44,780	0
Corn selling (bushels)	44,087,497	36,774,600	33,018,156	16,076,998	4,592,235	0
Soybean production (acres)	0	0	0	0	15,471	117,093
Hog raising (litter)	43,500	0	0	0	0	0
Labor hire in, subperiod 1 (hours)	0	0	0	0	0	41,271
Labor hire out, subperiod 1 (hours)	3,740,834	777,650	563,290	163,548	18,294	0
subperiod 2 (hours)	4,030,360	946,769	694,086	221,882	46,914	37,182
subperiod 3 (hours)	4,927,646	1,468,219	1,083,286	396,883	111,887	132,026
Period 2						
Number of farm	3,444	1,029	772	336	72	87
Corn production (acres)	491,994	342,996	321,822	177,094	55,072	109,234
Corn selling (bushels)	43,330,865	35,771,006	33,308,575	18,547,103	5,647,589	11,787,404
Soybean production (acres)	0	0	0	0	0	0
Hog raising (litter)	43,500	0	0	0	0	0
Labor hire in, subperiod 1 (hours)	0	0	0	0	0	29,125
Labor hire out, subperiod 1 (hours)	3,683,045	793,676	591,099	156,461	32,780	0
subperiod 2 (hours)	3,968,401	958,314	723,046	223,757	53,707	8,014
subperiod 3 (hours)	4,853,180	1,465,947	1,115,669	435,644	116,487	119,433
Period 3						
Number of farm	3,394		777	378	70	82
Corn production (acres)	484,909		323,886	199,058	0	102,087
Corn selling (bushels)	42,585,129		33,522,250	20,847,373	0	11,016,255
Soybean production (acres)	0		0	0	54,092	0
Hog raising (litter)	43,500		0	0	0	0
Labor hire in, subperiod 1 (hours)	0		0	0	0	8,886
Labor hire out, subperiod 1 (hours)	3,626,047		618,349	153,896	46,109	0
subperiod 2 (hours)	3,907,294		751,142	229,538	86,679	25,824
subperiod 3 (hours)	4,779,747		1,146,284	456,464	135,362	129,953

Table 6
Financial Results by Size of Farm

Size Group	A	B	C	D	E	F
Period 1						
Total income[3]	63,781,746	23,825,472	21,413,639	10,078,589	3,603,691	6,589,686
Tax payment[4]	11,238,142	4,494,582	4,501,982	2,390,281	983,428	2,041,405
Invome after taxes	52,443,604	19,330,890	16,911,657	7,688,308	2,620,263	4,548,281
Consumption outlays[5]	33,203,377	11,340,638	9,063,846	3,592,803	1,022,746	1,155,078
Period 2						
Total income	63,862,036	23,842,574	22,124,724	11,669,896	3,517,453	6,409,354
Tax payment	11,355,237	4,570,002	4,228,428	2,775,086	982,784	2,004,966
Income after taxes	52,506,799	19,272,572	17,896,296	8,894,810	2,534,669	4,404,388
Consumption outlays	33,016,595	11,172,452	9,717,876	4,140,146	883,652	1,073,818
Period 3						
Total income	63,944,092	23,724,551	22,776,943	13,223,375	3,589,040	6,226,040
Tax payment	11,427,350	4,600,547	4,859,006	2,555,569	1,017,355	1,964,837
Income after taxes	52,516,742	19,124,004	17,917,937	10,667,806	2,571,685	4,261,203
Consumption outlays	32,831,199	10,988,058	9,394,937	4,655,393	864,052	1,007,303

3. The number of times activities njMC activated times the amount for minimum consumption and taxes plus the sum of the times activities njMTm activated in the optimal solution.

4. The number of times activities njMC activated time the amount for taxes plus the sum of the times activities njMTm activated times their appropriate marginal propensities to taxes.

5. The activation of njMC times the amount for minimum consumption plus the sum of the product of the activation of njMTm and their appropriate marginal propensities to consume.