ELASTICITIES OF DERIVED DEMAND FOR HOGS

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The purposes of this paper are to illustrate the usefulness of a rather simple theoretical model in suggesting variables that enter in a statistical derived demand function and to provide estimates of price and income elasticities of demand for hogs at the farm level.[1]

Brandow discussed the distinction between alternative derived demand formulations, depending upon alternative assumptions about final demand for the product and supply functions for factors of production.[2] Briefly reviewing, and using Friedman's terms of reference, a "summation of demand curves of individual firms" has quantity of the factor demanded as a function only of product prices, price of the factor of interest, and prices of other productive factors.[3] This function is obtained by summing up individual firm derived demands without altering the assumption underlying the demand for the factor on the part of the individual firm--that of competition in both product and factor markets. However, recognizing that the price of the product is not constant along the derived demand curve for the industry taken as a whole, the assumptions are altered to allow product price to vary within the requirement that equilibrium is maintained in the product market. The alternate function, called by Friedman the "derived

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1 Perhaps the order of purposes should be reversed because of the paucity of farm level demand studies. But, in view of the fact that we are more interested in the contributions of economic theory to solutions of statistical problems, the order of purposes will serve us properly at this time. For a complete mathematical exposition of the model development, see Appendix A.

2 G. E. Brandow, "Demand for Factors and Supply of Output in a Perfectly Competitive Industry", Journal of Farm Economics XLIV,(1962), pp. 895-899. See also Milton Friedman, Price Theory, A Provisional Text, Aldine Publishing Company, Illinois (1962), pp. 172-183.

3 Friedman, Ibid., p. 181.

demand curve for the industry", tallies more closely with Marshall's derived demand; with Marshall's assumptions of fixed technological coefficients of production not being necessary in the specification.[4] This latter curve has quantity of the factor demanded as a function of its price, the prices of other factors and any exogenous variables that may affect the product demand curve.[5]

A simple specification consistent with the above remarks was estimated. The first construction had the per capita demand for hogs as a linear-in-logarithm function of the price of hogs and per capita disposable income. It was possible to arrive at the first specification by assuming:

(1) A logarithmic retail demand function for pork with per capita consumption of pork a function of the price of pork and per capita disposable income.

(2) A Cobb-Douglas production function for slaughterers with output (slaughter) a function of one input-hogs.

(3) Competition in hog slaughtering with profit maximization the objective of each firm.

Inclusion of population as a variable by deflating the dependent variable and income is not fully justified. Population enters the derived demand function because of its presence in the product (retail) demand function. However, depending on specification of the retail demand-supply structure and the values of such parameters as retail elasticities of supply and demand, the coefficient of population in the derived demand relationship may be other than one.

<u>Results of the First Specification</u>

The results of least squares estimation of the parameters were—

$$(1) \quad \hat{H}_t^D = 4.928 - \underset{(.125)}{.290}\, P_{Ht} - \underset{(.314)}{1.168}\, I_t \quad ^{6}$$

$$t = 1947, \ldots, 1964.$$

$$R^2 = .564 \quad \text{Durbin-Watson Statistic} = 1.180$$

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[4] Friedman, <u>Ibid</u>.

[5] Assuming factor prices fixed to the industry.

[6] The symbol H_t^D represents the log of the index of per capita hog slaughter in the U.S., P_{Ht} is the log of the deflated farm price of hogs, and I_t is the log of the deflated U.S. per capita disposable income. Data are shown in Appendix B.

Criteria for judging the fit are:

(a) The R^2 is low.

(b) The Durbin-Watson Statistic of 1.180 indicates that we cannot determine if there is auto-correlation in the calculated residuals.

(c) The income elasticity is negative with a coefficient over twice the standard error.

The poor empirical results were not too surprising, given the simplicity of the specification.

Respecification

Although the following Marshallian diagram adds nothing to the theory of derived demand as discussed previously, it is a convenient device for separating variables that shift the curve into two categories: (1) those that shift the retail (final) product demand curve, and (2) those that affect the derived demand curve through changes in supply conditions of other factors.

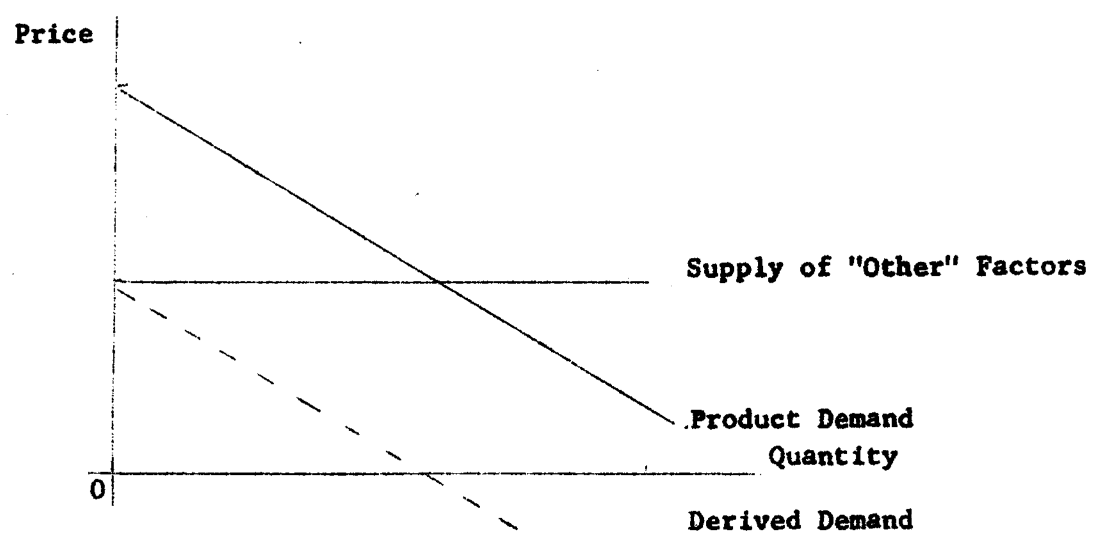

Figure 1. Marshall's Derived Demand

As shown in Figure 1, the supply of "other" factors is perfectly elastic. Therefore, the derived demand for the factor in question is simply the product demand projected downward - the vertical difference between product demand and the supply of "other" factors.

In the first specification, income and population - retail demand shifters - were included. However, the supply (price) of "other" factors was neglected.

Looking at variable inputs that enter into hog slaughtering, labor seemed an important factor. Therefore, wages per hour in the food and kindred products industry was considered as an additional variable. Since almost any wage data one can think of is highly correlated with per capita income over time, it seemed quite likely that multicollinearity would arise. Reflecting further, if the wage variable was an important determinant of demand for hogs over the period, the income variable in the first specification was forced to play two roles. While increasing income should shift the product demand and hence the derived demand upward, increases in wages should be associated with decreases in derived demand. Thus, although one would anticipate a high positive simple correlation between wages and income, the partial correlation of each variable with hog slaughter should have opposite signs.

Beef as a substitute in the product market also should play a role. If the quantity of beef is predetermined in time t, through, say, an inelastic supply, the quantity of beef is the correct choice of variable in the retail demand for pork and hence the derived demand for hogs. If supply is assumed to be perfectly elastic, the price of beef would be the better choice. As an alternative to specification and estimation of a simultaneous equation model, results obtained by using multiple regression including the quantity of beef are presented.

The empirical results of including these additional variables are presented below:[7]

$$(2)\quad \hat{H}_t^D = 3.367 - \underset{(.082)}{.275}\, P_{Ht} - \underset{(.271)}{1.070}\, W_t + \underset{(.432)}{.891}\, I_t - \underset{(.110)}{.230}\, B_t$$

$t = 1947, \ldots, 1964$

$R^2 = .868$ Durbin-Watson Statistic = 2.389

[7] New variables introduced are:

W_t = The log of deflated wages per hour in the food and kindred products industry.

B_t = The log of the index of per capita consumption of beef in the U.S.

Looking at all the indicators, the above specification represents considerable improvement over the first specification. The signs of all coefficients are in agreement with what they should be. All coefficients are several times larger than their standard errors. The coefficient of determination is twice as large as it was in the first specification and respectable in terms of its absolute size. And the Durbin-Watson Statistic indicates model respecification did not worsen the serial correlation since again, the test is inconclusive.

The result that was most pleasing was the significance of the wage variable and its usefulness in establishing the significant coefficient for the income variable with the expected sign. Even though the simple correlation for the wage and income variables was +.98, the regression coefficients of these two variables were opposite in sign.

APPENDIX A

AN ECONOMIC MODEL OF DERIVED DEMAND

To facilitate a concrete exposition of a theoretical model for derived demand, it was assumed that the production function for the typical firm could be adequately represented as an exponential function.

$$(1)\quad Y = a_0 X_1^{a_1} X_2^{a_2};\ a_1 + a_2 < 1$$

The variable Y represents output (product) per unit time and X_1 and X_2 are variable inputs. It is assumed that certain factors are outside the firm's control for the time period in question and that their effect is contained in the constant term a_0.

Assuming that supply prices of the factors are fixed and known to the firm as P_{x_1} and P_{x_2}; that the demand price for the product is fixed and known as P_y and that the firm's objective is to maximize net revenue, the firm's demand functions for X_1 and X_2 will be of the form: [1]

$$(2)\quad X_1 = b_0 P_{x_1}^{b_1} P_{x_2}^{b_2} P_y^{b_3}$$

$$(3)\quad X_2 = c_0 P_{x_1}^{c_1} P_{x_2}^{c_2} P_y^{c_3}$$

The parameters in these functions are themselves functions of the parameters in the production function. For example, b_2, the price elasticity of demand for the factor X1 on the part of the firm, is related to the production function parameters as follows:

$$(4)\quad b_2 = \frac{-a_2}{1 - a_1 - a_2}$$

By substituting equations (2) and (3) into equation (1) we have the supply function for the firm.

$$(5)\quad Y = d_0 P_{x_1}^{d_1} P_{x_2}^{d_2} P_y^{d_3}$$

Where again, the d_i are functions of the b_i and c_i which are, in turn, functions of a_i.

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[1] These functions were obtained by differentiating the net revenue function with respect to X_1 and X_2, equating to zero, and solving simultaneously for X_1 and X_2.

Aggregation to the Industry

In textbook analysis of the market, the conventional device for dealing with aggregation is by assumption of homogeneity of individuals or firms. In economics we talk about an individual or firm, but do not define the individual or firm to represent anyone or any firm in particular. We describe autonomies so that when added together make a market or industry. Thus, economic theory can be viewed as an aggregate theory, recognizing that the "typical" firm or "typical" consumer represents the average of all firms or consumers. However, even when aggregation is viewed in these simplest terms problems of ceteris paribus arise.[2]

Summation of Firm Demand Curves Versus Industry Demand for the Factor

Assuming that the derived demand functions (2) and (3) are for the "average" or "typical" firm, summation of the functions does not change the basic form. Only the constant terms are affected. However, it is now unreasonable that a demand relationship between factor X_1 and its price can be established under the assumption that the price of the product remains constant. By definition of an industry, the totality of all firms producing Y faces a downward sloping demand curve for their product. Similarly, the industry as a whole may face an upward sloping supply curve for the other factor X_2.[3]

Assuming that the demand for the final product, pork, is an exponential function of the price of pork, the per capita consumption of beef (B), and per capita income (I); that the supply curve of "other variable factors" is completely elastic, and that aggregation of the summation of firm demand curves and the firm supply curve for pork only changes the constant terms, a partial model for the hog marketing system may be written as follows.[4]

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[2] See Friedman, op. cit., p. 181.

[3] It may be that an industry will face a more or less perfectly elastic supply curve for factors of production because the factor in question is used in other industries. However, the ceteris paribus conditions regarding product price is on an entirely different footing.

[4] The word "partial" is used because for the time being nothing is to be said about the supply of hogs; neither is the system closed with regard to the two additional variables entered in the retail demand equation, i.e., (I) and (B).

(6) $Y = e_0 P_y^{e1} I^{e2} B^{e3}$ Demand for Pork

(7) $Y = d_0' P_{x1}^{d1} P_{x_2}^{d2} P_y^{d3}$ Supply of Pork

(8) $X_1 = b_0' P_{x1}^{b1} P_{x_2}^{b2} P_y^{b3}$ Summation of Firm Demand for Hogs

(9) $X_2 = c_0' P_{x1}^{c_1} P_{x_2}^{c_2} P_y^{c_3}$ Summation of Firm Demand for "Other Variable Factors"

(10) P_{x_2} = constant Supply of "Other Variable Factors"

In order to derive the industry demand for hogs (as opposed to the summation of firm demand for hogs), it is assumed that equilibrium is maintained in the final product market. That is, the price of the final product is determined as a function of income, the consumption of beef, the price of hogs, and the price of "other variable factors" through equating the demand and supply for pork. Substitution of the equilibrium determination of the price of pork (P_y) into the relationships (8) and (9) yield the form of the industry demand for factors of production - the derived demand functions. This "collapses" the five equation partial model into a three equation model and focuses attention upon the derived demand functions. The collapsed model is now of the form:

(11) $X_1 = f_0 P_{x_1}^{f_1} P_{x_2}^{f_2} I^{f_3} B^{f_4}$ Derived Demand for Hogs

(12) $X_2 = g_0 P_{x1}^{g_1} P_{x_2}^{g_2} I^{g_3} B^{g_4}$ Derived Demand for Hogs

(13) P_{x_2} = constant Supply of "Other Variable Factors"

The new parameters, f_i and g_i, are functions of all the parameters introduced up to now. A table is provided to summarize the relationships among parameters of the models.

The derivation presented here is about as simple as possible. For example, if one specified other than a perfectly elastic supply function for the "other variable factors", the price of "X_2" would be replaced in the derived demand function for hogs (equation 11) by a function obtained by equilibrating the demand and supply for "X_2".

The analysis, restricted as it is, still serves to point up the problems of *ceteris paribus* in constructing derived demand relationships. Anticipating statistical measurements of these relationships for the moment, it is clear that identification problems in the model specifying the summation curves (equations 6 through 10), are more ominous than for the collapsed model (equations 11, 12, and 13). That is, one would be more optimistic about estimating a statistical relationship stemming from the mathematical model for derived demand for hogs (equation 11) by single equation methods than one would be about similarly estimating the summation of firm demand relationship (equation 8).

Table 1. Relationships between various parameters involved in the derived demand relationships.

Equation where introduced	Parameter	Definition	Derivation
1)	a_1	Coefficient of the factor X_1 in the firm production function	——
	a_2	Coefficient of the factor X_2 in the firm production function	——
2) and 8)	b_1	Price elasticity of demand for hogs; individual firm demand and sum of firm demand	$\frac{a_2 - 1}{1 - a_1 - a_2}$
	b_2	Cross elasticity of the other factor(s) (X_2) affecting demand for X_1; individual firm demand and sum of firm demand	$\frac{-a_2}{1 - a_1 - a_2}$
	b_3	Elasticity parameter relating effect of a change in the price of the final product to the individual firm's demand for X_1 and sum of firms demand for X_1	$\frac{1}{1 - a_1 - a_2}$
3) and 9)	c_1	Cross elasticity of the other factor (X_1) affecting firm demand for X_2 and sum of firm demand for X_2	$\frac{-a_1}{1 - a_1 - a_2}$
	c_2	Own price elasticity of demand for X_2; individual firm and summation of firm demand	$\frac{a_1 - 1}{1 - a_1 - a_2}$
	c_3	Relates effect of a percent change in P_y to the percent change in the firm's and sum of firm demand for X_2	$\frac{1}{1 - a_1 - a_2}$
6)	e_1	Price elasticity of demand for pork	——
	e_2	Income elasticity for pork	——
	e_3	Cross elasticity of demand for beef	——

Table 1 (continued)

Equation where introduced	Parameter	Definition	Derivation
5) and 7)	d_1	Elasticity relating price of hogs to supply of pork	$a_1b_1 + a_2c_1$
	d_2	Elasticity relating price of "other factors" to supply of pork	$a_1b_2 + a_2c_1$
	d_3	Price elasticity of supply of pork	$a_1b_3 + a_2c_3$*
11)	f_1	Price elasticity of demand for hogs (industry demand)	$b_1 + b_3 \frac{d_1}{e_1-d_3}$ **
	f_2	Elasticity relating price of "other factors" to the derived demand for hogs	$b_2 + b_3 \frac{d_2}{e_1-d_3}$
	f_3	Income elasticity for hogs	$b_3 \frac{-e_2}{e_1-d_3}$
	f_4	Elasticity relating the price of beef to the derived demand for hogs	$b_3 \frac{-e_3}{e_1-d_3}$
12)	g_1	Relates effect of a percent change in price of hogs upon derived industry demand for unspecified factor	$c_1 + c_3 \frac{d_1}{e_1-d_3}$
	g_2	Own price elasticity of demand for unspecified factor	$c_2 + c_3 \frac{d_2}{e_1-d_3}$
	g_3	Income elasticity of derived demand for unspecified factor	$c_3 \frac{-e_2}{e_1-d_3}$
	g_4	Elasticity relating effect of "other" retail demand shifters on industry demand for unspecified factor (B)	$c_3 \frac{-e_3}{e_1-d_3}$

* Note that this is equivalent to the derivation in Z. Griliches, "The Demand for Inputs in Agriculture and a Derived Supply Elasticity", Journal of Farm Economics XLI, (1959) pp. 309-322.

** For a comparison of this demand curve with Marshall's conditions of derived demand, see William D. Diehl, "Analysis of Derived Demand for Hogs", Unpublished M.S. Thesis, North Carolina State University, 1962.

APPENDIX B

DATA

Per Capita Hog Slaughter (total live weight, including farm slaughter), Average Farm Price of Hogs, Wages per Hour in Food Products and Kindred Industries, Per Capita Disposable Income, and Per Capita Consumption of Beef and Veal.[a]

Year	Hog Slaughter[b] lbs. per cap.	Farm Price of Hogs[b] dollars per lb.	Wages-Food & Kindred Products Hourly earnings	Consumer Dispos. Inc.[d] dollars	Beef and Veal Cons.[b] lbs. per cap.
1947	127.3	.241	1.06	1180.2	80.4
1948	119.4	.231	1.15	1291.0	72.6
1949	121.6	.181	1.21	1271.5	72.8
1950	125.4	.180	1.26	1369.3	71.4
1951	133.6	.200	1.35	1473.8	62.7
1952	130.7	.178	1.44	1520.1	69.4
1953	109.0	.214	1.53	1581.7	87.1
1954	105.6	.216	1.59	1581.7	90.1
1955	116.2	.150	1.66	1660.3	91.4
1956	117.4	.144	1.76	1727.4	94.9
1957	107.0	.178	1.85	1782.1	93.4
1958	103.7	.196	1.94	1818.4	87.2
1959	117.8	.141	2.02	1912.2	87.1
1960	110.9	.153	2.11	1953.4	91.1
1961	107.2	.166	2.17	2002.0	93.4
1962	108.0	.163	2.24	2079.3	94.3
1963	110.6	.149	2.30	2144.5	99.1
1964	109.4	.148	2.37	2268.1	104.9

a / Prices and Income items were deflated by consumer price index (all items). See d / below.

b / Livestock and Meat Statistics, 1962, Bulletin No. 333, U.S. Dept. of Agric. Washington, D.C. July 1963, and Supplement for 1964 to Livestock and Meat Statistics, September 1965, U.S. Dept. of Agric., Washington, D.C.

c / U.S. Dept. of Labor, Bureau of Labor Statistics, Employment and Earnings Statistics for the United States, 1909-65, pp. 370-373, BLS Bulletin No. 1312-1.

d / U.S. Bureau of the Census, Statistical Abstract of the United States: 1960 and 1965 (81st and 86th editions) Washington, D.C., 1960 and 1965.