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A SEPARATE PEACE: A MATHEMATICAL MODEL FOR THE DISCUSSION OF TAXES FOR CONTROL OF THE ENVIRONMENT

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Abel and Cain are two college-age brothers residing in the same room. Abel likes to study in perfect silence. Cain prefers loud music to drown out white noise. Obviously, these desires conflict.

Both Cain and Abel wish to maximize individual learning. Each has the same k hours in which to study. For x_1 hours, quiet prevails. For the remaining x_2 hours, the stereo is on. Cain is larger, however, and has a propensity for violence. Therefore, the brothers operate at a level equal to Cain's optimum. Yet, Abel is tenacious and feels that if his own studying is impaired, he should argue and interrupt Cain's cogitation as well. Little learning occurs.

Negotiation, in its present form, seems to be ineffective in dealing with this crisis. Finals are approaching. Something must be done. At least, the brothers are in agreement in this. Cain and Abel, who are taking a course in Environmental Economics, take their problem to the professor. She will be glad to help.

The professor develops Cain's optimum study framework according to the precepts of learning theory. Cain's objective, she writes, is to maximize his learning,

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*This paper is intended as a serious treatment of the use of Pigovian taxes as a means of achieving societally optimal allocation of resources in the presence of externalities. The style was patterned after the work of H. A. Thomas, Jr. [1963], who provides a treatise on standards within an effective fictional story. There is, however, no other resemblance to his work.

$$L = \alpha_c \, x_1^{\beta_{1c}} \, x_2^{\beta_{2c}}$$

where $\alpha_c > 0$, $0 < \beta_{1c}, \beta_{2c} < 1$ and $\beta_{1c} < \beta_{2c}$. This suggests diminishing marginal learning to both x_1 and x_2. This can be placed in the constrained optimization context,

$$\phi = \alpha_c \, x_1^{\beta_{1c}} \, x_2^{\beta_{2c}} + \lambda(x_1 + x_2 - k),$$

where λ is the Lagrangian multiplier on the time constraint. The first order conditions then are:

$$(1) \quad \frac{\partial \phi}{\partial x_1} = \alpha_c \, \beta_{1c} \, x_1^{\beta_{1c}-1} \, x_2^{\beta_{2c}} + \lambda = 0$$

$$(2) \quad \frac{\partial \phi}{\partial x_2} = \alpha_c \, \beta_{2c} \, x_1^{\beta_{1c}} \, x_2^{\beta_{2c}-1} + \lambda = 0$$

$$(3) \quad \frac{\partial \phi}{\partial \lambda} = x_1 + x_2 = k$$

Division of (1) by (2) obtains:

$$(4) \quad \frac{\beta_{1c} \, x_2}{\beta_{2c} \, x_1} = 1$$

The use of equation (4) makes it possible to solve for the optimal amounts of time for Cain to study with silence and with noise, viz.,

$$(5) \quad x_1 = \frac{\beta_{1c} \, k}{\beta_{1c} + \beta_{2c}} \quad \text{and} \quad x_2 = \frac{\beta_{2c} \, k}{\beta_{1c} + \beta_{2c}}$$

The same equations can be developed for Abel. By definition of learning styles, it can be seen that for Cain $x_1 < \frac{1}{2}k$, $(\beta_{1c} < \beta_{2c})$ and for Abel, it is greater than $\frac{1}{2}k$, $(\beta_{1A} > \beta_{2A})$.

"To maximize the learning achieved by both, it is necessary to do more than simply sum the individual optima," the professor explains. This

is because each of them, in attempting to maximize his own learning, is interfering with the learning of the other. Now, to maximize aggregate learning, she formulates the equation:

$$\phi = \alpha_A x_1^{\beta_{1A}} x_2^{\beta_{2A}} + \alpha_c x_1^{\beta_{1c}} x_2^{\beta_{2c}} + \lambda(x_1 + x_2 - 2k),$$

and the necessary conditions become:

$$(6) \quad \frac{\partial \phi}{\partial x_1} = \alpha_A \beta_{1A} x_1^{\beta_{1A}-1} x_2^{\beta_{2A}} + \alpha_c \beta_{1c} x_1^{\beta_{1c}-1} x_2^{\beta_{2c}} + \lambda = 0$$

$$(7) \quad \frac{\partial \phi}{\partial x_2} = \alpha_A \beta_{2A} x_1^{\beta_{1A}} x_2^{\beta_{2A}-1} + \alpha_c \beta_{2c} x_1^{\beta_{1c}} x_2^{\beta_{2c}-1} + \lambda = 0$$

$$(8) \quad \frac{\partial \phi}{\partial \lambda} = x_1 + x_2 = 2k$$

When (6) is divided by (7), the marginal products are equated and one obtains:

$$(9) \quad \frac{\alpha_A \beta_{1A} x_1^{\beta_{1A}-1} x_2^{\beta_{2A}} + \alpha_c \beta_{1c} x_1^{\beta_{1c}-1} x_2^{\beta_{2c}}}{\alpha_A \beta_{2A} x_1^{\beta_{1A}} x_2^{\beta_{2A}-1} + \alpha_c \beta_{2c} x_1^{\beta_{1c}} x_2^{\beta_{2c}-1}} = 1$$

While the individual optima could be solved for the general case, the societal optimum can only be solved by numerical techniques.

Here, then, is the problem: By what system can the societal optimum be reached if it cannot first be readily determined. For standards to be effective, the optimal levels must be known. In their Environmental Economics class, the brothers are studying Pigovian taxes advocated by many economists including W. J. Baumol and P. A. Diamond (which puts Cain and Abel in pretty good company). The professor explains to refresh the boys' memories. The tax to be levied upon Cain is equal to the marginal damages

imposed upon Abel by Cain. If Cain is a profit maximizer, he will continue to reduce his noise level to avoid payment until the marginal cost to his learning of noise reduction exactly equals Abel's marginal damages. This level is the societal optimum; a maximum amount of learning will occur. All that is necessary is for Cain to be given Abel's damage function. He knows his own, and will make the trade-off accordingly.

"As an example," she continues, "let's assume: Cain initially holds 60 units of knowledge and Abel, 100; $\beta_{1A} = .9$, $\beta_{1c} = .4$, $\beta_{2A} = .05$, and $\beta_{2c} = .5$. Further assume that 8 hours are available for studying." Abel's damage function is the first derivative of the learning curve (L).

$$(10) \quad \frac{dL}{dx_1} = \frac{\alpha_A}{x_1(k-x_1)}\left[\beta_{1A}\, x_1^{\beta_{1A}}\,(k-x_1)^{\beta_{2A}+1} - \beta_{2A}\, x_1^{\beta_{1A}+1}\,(k-x_1)^{\beta_{2A}}\right]$$

Using this equation and its equivalent for Cain, it is possible to map out the damage functions and to determine the socially optimal noise level. Several values are provided in Table 1 below and graphed in Figure 1.

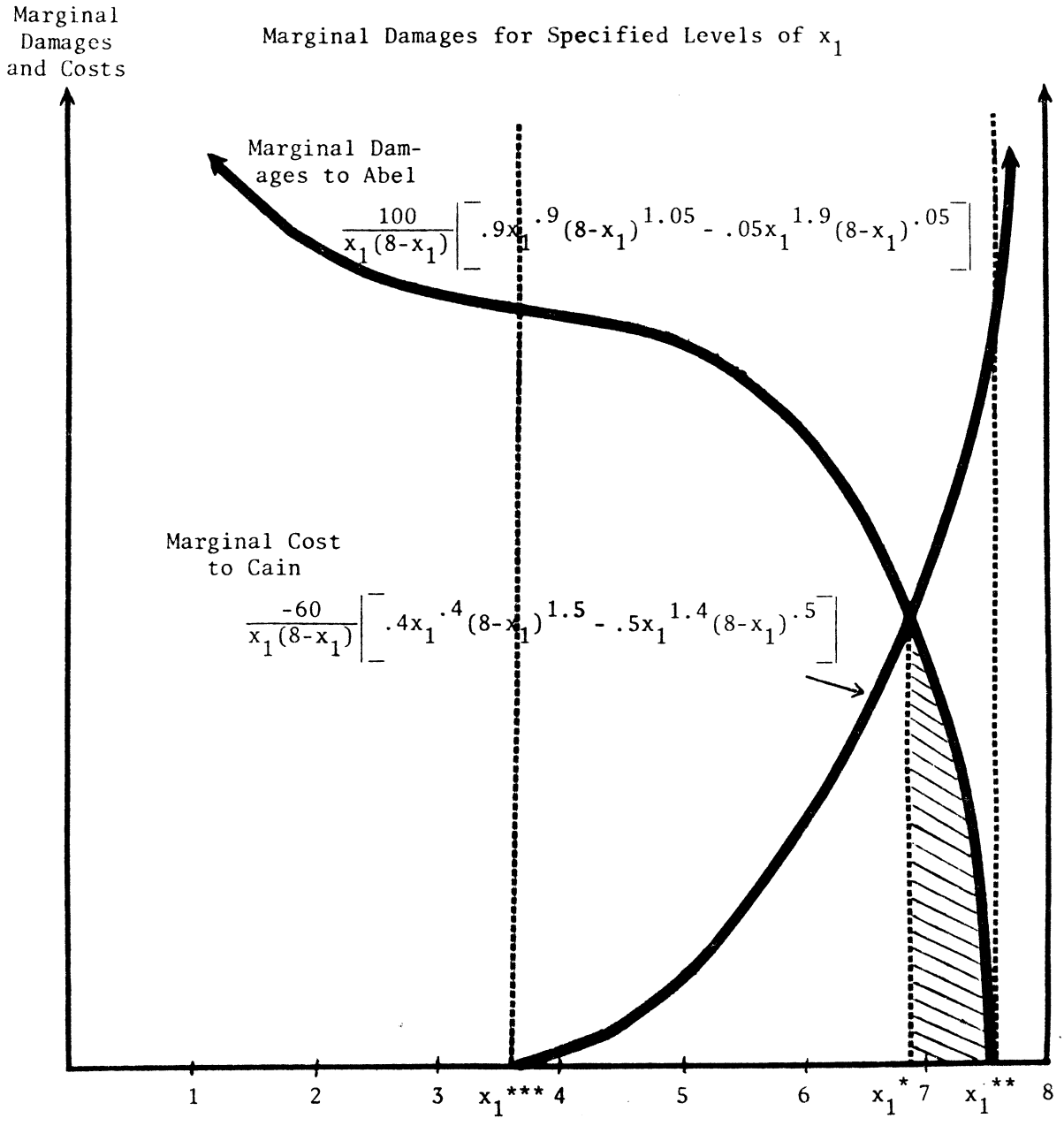
Table 1

Marginal Damages and Total Learning for Selected Levels of x_1

If x_1 Equals:	Abel's Marginal Damages	Cain's Marginal Cost	Total Learning
1	109.427	-52.158	268.964
2	90.141	-22.624	398.023
3	84.48	- 6.94	499.513
4	79.31	5.222	582.145
5	73.452	17.144	647.54
6	64.91	31.853	693.014
6.7	53.809	47.57	707.664
6.8	51.371	50.634	708.019
6.9	48.567	54.041	707.789
7	47.274	57.871	706.894
7.58	0	99.462	695.14

Figure 1

Marginal Damages for Specified Levels of x_1



$x_1 = 0$ $\qquad \frac{\beta_{1c}\, k}{\beta_{1c} + \beta_{2c}} \qquad \frac{\beta_{1A}\, k}{\beta_{1A} + \beta_{2A}}$

x_1^{*} = societal optimum

x_1^{**} = Abel's optimal level of x_1

x_1^{***} = Cain's optimal level of x_1

Here, we've determined that the social optimum is near 7. By calculating in smaller increments, the exactly optimal amount can be found.

These marginal curves extend infinitely, but the only relevant portions are those between the individuals' optima since it is unreasonable that either Cain or Abel will reduce his own learning by demanding a greater amount of his preferred activity than is optimal. After Cain decreases his noise levels to the societally optimal level, learning will be stabilized, and Cain need only pay the amount of the shaded area. The effective tax (T), then, equals:

$$(11)\quad T = \int_{x_1^*}^{\frac{\beta_{1A}\, k}{\beta_{1A} + \beta_{2A}}} \frac{\alpha_A}{x_1^*(k-x_1^*)} \left| \beta_{1A}\, x_1^{*\beta_{1A}} (k-x_1^*)^{\beta_{2A}+1} - \beta_{2A}\, x_1^{*\beta_{1A}+1} (k-x_1^*)^{\beta_{2A}} \right| dx$$

"Good Lord!" says the professor who considers her opportunity costs as well as the likelihood that either Cain or the enforcement authority (Dad) would be willing or able to use equation (11) to set the tax. A linear approximation yields:

$$(12)\quad T = \frac{1}{2} \left| \frac{\alpha_A}{x_1^*(k-x_1^*)} \left(\beta_{1A}\, x_1^{*\beta_{1A}} (k-x_1^*)^{\beta_{2A}+1} - \beta_{2A}\, x_1^{*\beta_{1A}+1} (k-x_1^*)^{\beta_{2A}} \right) \left(k \frac{\beta_{1A}}{\beta_{1A} + \beta_{2A}} - x_1^* \right) \right|$$

"There," she says to Abel and Cain, "is your tax system. This should solve your problem." The professor is satisfied. Abel and Cain are pleased. They were getting tired from all those equations. The professor ushers them out of her office.

. . .

Abel and Cain leave the professor's office, their heads filled with equations. They have a tax system, but it seems so complicated. The brothers realize they need to know more. So, reading list in hand, they head for the library. To begin, they read through Fisher and Peterson's [1976] survey of the literature. There is so much to learn! How can so many knowledgeable economists arrive at such different conclusions? Abel and Cain decide that if they are to have any hope of dealing with the literature, they will have to split it up. Cain reads the material that is anti-tax, and Abel reads those authors that favor tax systems.

"Ronald H. Coase [1960] demonstrated that, if parties were able and willing to negotiate, state intervention was unnecessary to secure an optimum resource allocation. The same result would be achieved if the victim bribed the producer of a diseconomy to stop as would occur under a tax system. Later studies by Davis and Whinston [1962], Turvey [1963], Buchanan [1969], and Buchanan and Stubblebine [1962] concur with Coase's results. Additionally, they find the imposition of a tax to be so complicated, even in principle, that the a priori prescription of a tax is neither wise nor inexpensive."

"That's crazy, Cain," Abel interrupts. "For one thing, in this society, it is usually groups of people affected by pollution, not individuals.

How many times have you seen drivers on the highway bribe each other to get off the road? It just can't be done. But surely, additional drivers constrain movement and create diseconomies to other drivers. Coase also assumes that all gains and losses are commensurable in money terms, that all people have perfect knowledge of available alternatives to remove the externality, and that bargaining will take place on the margin. Without those unrealistic simplifications, he could never conclude that bargaining is viable. A major problem with benefit/cost analysis has always been our inability to measure environmental quality, utility and other goals in terms of money. Nobody has perfect knowledge of anything. And bargaining is rarely a marginal process. Rather, the bargaining limits are set by a discrete finite move away from the existing condition. Since, in most real world cases, externality producers neither inform their intended victims before the diseconomy exists nor compete for victims, bargaining will not take place as a marginal process, and the resulting solution is not likely to be optimal.

Coase also assumes all diseconomies to be by one producer upon another. Often, the victim of an externality will be a consumer who hasn't the access to capital markets necessary to bribe a corporation. Without this ability, the bargained amount of the diseconomy will differ according to incidence of liability through income effects. This point was made by Mishan [1967], Dolbear [1967], and Burrows [1970]. These economists describe the Coase argument as a special case that rarely could occur.

Kneese [1962] was probably the first person since Pigou's 1924 study to consider taxes or charges as a means of internalizing external

diseconomies in the environment. His analysis and later those of Kneese and Bowers [1968] and Baumol and Oates [1975] conclude that an effluent charge will achieve an environmental standard at the lowest possible economic and resource costs. In fact, Baumol [1972, p. 307] concludes that "... taken on its own grounds, the conclusions of the Pigovian tradition are, in fact, impeccable ... Pigovian taxes upon the generator of the externality are all that is required [for optimality to occur]."

In opposition, Dales [1968] advocates fixing the total amount of pollution to be permitted and to allow the price charged for pollution rights to adjust by means of auction. Theoretically, the solution will be identical to that of a tax system. However, the two approaches will yield the same result only if the regulatory agency is successful in selecting either the price or the quantity corresponding to the equilibrium level of pollution. Dales defends his position on the grounds that, although it is difficult to estimate how much pollution abatement is worth to a community, it is possible to make the political decision of how much abatement the community wants, irrespective of price.

Additionally, all these economists find tax systems to be equitable..."

"Now wait a minute!" Cain has had enough. "Many people will think it must be equitable if firms are forced to pay for damages their productions cause. Yet, if, for example, a smoke producing firm was constructed in an isolated area where no damages were expected to occur, it would be inequitable to require the firms to pay full damages occurring because people later moved into the area.

Those men make some pretty strong assumptions. They all assume the necessary information is available without cost. So much information is

needed; damage functions, cost functions, utility functions -- for each firm and individual involved. To assume away all those costs and call the system least-cost is presumptive. They also assume damage functions to be linear and additive. By doing that, they can make effluent taxes the same for each unit of effluent. This minimizes theoretical administrative costs and allows them to forget the system only works for two individuals. Kneese and Bower assume even more unrealistic characteristics of damage functions. They assume these functions to be vertical in their analysis. Thus the benefits of waste reduction are infinite up to some critical point whereupon there are no benefits. Even Kneese and Bower admit this to be false in reality but do not discredit results based on this assumption. Furthermore, they assume the initial charge to have been correctly specified. If it is not, there is no guarantee that an optimal result will occur. While Baumol and Oates do not use this assumption for most of their discussion, they instead assume the adjustment mechanism to be built in. That is, they assume an authority could instantaneously raise the tax without regard for the political process or to investments made by businesses to adjust production to the original tax.

Buchanan and Stubblebine found that if negotiation is possible, any tax not received as compensation for damages may prevent optimal resource allocation. This is because the amount that maximizes the difference between what one individual gains and the other loses may differ from the amount that maximizes the joint net gain. In the presence of negotiation, the computation of a tax is still more complicated if, as in our case, the external diseconomies are reciprocal. Davis and Whinston developed a series of bargaining games for this type of situation.

If tax rates are equal throughout an area, they will be inefficient because some producers whose discharge causes no damages will be paying too much and producing too little while others will pay too little and produce too much. An alternative set forth in Baumol and Oates is to create zones of tax rates. These are of two types. The first is to require high tax rates in highly polluted areas and low in pristine areas. If this scheme is used, polluting companies will choose to locate in low tax areas, resulting in an even distribution of pollution across zones. Since this is generally viewed as undesirable, the alternative system was developed. Under this system, low tax rates are set in zones where pollution levels are already high. This concentrates polluting activities in single areas, but affects income distribution. As posited in Baumol and Oates [pp. 204-205], rents in unprotected areas can be expected to increase as more polluters seek to locate there. As the poor tend to live in the more polluted areas to begin with, this type of policy tends to result in the poor living in yet less attractive areas and receiving less of a rent advantage relative to clean areas than they would in an absence of the program. Additional distributional effects involve declining production, rising prices, unemployment, loss of property rights, and potential profit decreases.

If we are committed to both pollution control and more equal distribution patterns, any policy would have to include governmental assistance to correct redistribution [Baumol and Oates, p. 211]. As many of the most drastic effects occur during transition, measures such as unemployment compensation, retraining programs and relocation assistance could be employed during this period.

Tax systems to control externalities are problematic to institute as well. Existing political institutions will not provide for easy administration of tax systems that traverse jurisdictions. These boundaries must be crossed as the effects of effluents within relevant land, air and water sheds do not conform to existing political boundaries. If a tax is charged for dumping toxic materials into a waterway, a producer may be motivated to burn the material instead, releasing toxins to the air.

Perhaps a better system would not only evaluate the effects of different disposal systems, but also not restrict itself to one form of corrective action. In deriving an environmental policy, it is necessary to consider:

1. Value judgments on the equity implications of a given system.
2. Whether negotiation can and will take place.
3. Whether compensation is to be given to damaged parties.
4. Relative ease or difficulty of estimating damage functions and thus the administrative costs of a system.
5. Costs of avoiding externalities through technological means.
6. Extent to which externalities are reciprocal and game theoretic concepts are applicable.

When there are so many points to consider, any a priori prescription is unlikely to be best. Further, it may well be that the most sensible scheme involves a combination of these approaches.

After considering these factors, in fact, Abel and Cain find the cost of avoiding externality to be the dominant factor. They chip in (with a subsidy from the central authority - Dad) and purchase a set of earphones for Cain.

Bibliography

Baumol, W. J., "On Taxation and the Control of Externalities", American Economic Review, LXII, (June 1972): pp. 307-22.

_____ and Oates, W. E., The Theory of Environmental Policy, Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1975.

Buchanan, J. M., "External Diseconomies, Corrective Taxes, and Market Structure", American Economic Review, LIX, (March 1969): pp. 174-77.

_____, The Demand and Supply of Public Goods, Chicago: Rand McNally, 1968.

_____ and Stubblebine, W. C., "Externality", Economica, XXIX, (November 1962): pp. 371-84.

Burrows, P., "On External Costs and the Visible Arm of the Law", Oxford Economic Papers, XXII, (March 1970): pp. 39-55.

Coase, R. H., "The Problem of Social Cost", Journal of Law and Economics, III, (October 1960): pp. 1-44.

Dales, J. H., "Land, Water, and Ownership", Canadian Journal of Economics, XXXIV, (November 1968).

Davis, O. A. and Whinston, A., "Eternalities, Welfare, and the Theory of Games", Journal of Political Economy, LXX, (June 1962): pp. 241-62.

Diamond, P. A., "Consumption, Externalities and Imperfect Corrective Pricing", Bell Journal of Economics and Management Science, IV, (Autumn 1973): pp. 526-38.

Dick, D. T., Pollution, Congestion and Nuisance, Lexington, Massachusetts: D. C. Heath and Company, 1974.

Dolbear, F. T., "On the Theory of Optimal Externality", American Economic Review, LVII, (March 1967): pp. 90-103.

Dorfman, R. and Dorfman, N. S., eds., Economics of the Environment: Selected Readings, New York: W. W. Norton and Company, Inc., 1972.

Fisher, A. C. and Peterson, F. M., "The Environment in Economics: A Survey", Journal of Economic Literature, XIV, (March 1976): pp. 1-33.

Kneese, A. V., Water Pollution: Economic Aspects and Research Needs, Washington, D.C.: Resources for the Future, 1962.

_____ and Bower, B. T., "Standards, Charges, and Equity", Managing Water Quality: Economics, Technology, Institutions, Baltimore: Johns Hopkins Press, 1968.

Mishan, E. J., "Pareto Optimality and the Law", Oxford Economic Papers, XIX, (November 1967): pp. 255-87.

Pigou, A. C., The Economics of Welfare, London: Macmillan Company, 1924.

Rausser, G. C., et al., "Learning, External Benefits, and Subsidies in Water Desalination", Water Resources Research, VIII, (December 1972): pp. 1385-99.

Senneca, J. J. and Taussig, M. K., Environmental Economics, Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1974.

Tietenberg, T. H., "Specific Taxes and Pollution Control", Quarterly Journal of Economics, LXXXVII, (November 1973): pp. 503-22.

Thomas, H. A., Jr., "The Animal Farm: A Mathematical Model for the Discussion of Social Standards for the Control of the Environment", Quarterly Journal of Economics, LXXVII, (February 1963).

Turvey, R., "On Divergencies Between Social Cost and Private Cost", Economica, XXX, (August 1963): pp. 309-13.

Wellisz, S., "On External Diseconomies and Government Assisted Invisible Hand", Economica, XXXI, (November 1964): pp. 345-62.