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THE CONSUMER'S USE OF FUTURES

by

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## ABSTRACT

This paper shows that futures should be included in an efficient portfolio and that the degree of their inclusion depends critically on the evaluation of income in real terms. As planned consumption is balanced toward items traded in the futures market, holdings of futures are shown to increase.

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## I. Introduction

Although futures markets can provide an attractive investment opportunity for consumers in an inflationary era, traditional anti-inflation hedges, such as housing and other consumer durables, and the risk of dealing in futures may make futures a nonoptimal choice. This paper develops a mean-variance framework for analyzing the consumer's use of the futures markets. It differs from previous work by settling accounts in real terms and modeling the wealth and credit constraints close to the current institutional arrangements now prevailing for consumers, otherwise known as small investors.

The use of futures by consumers will have obvious consequences for the financial community and not-so-obvious consequences for farmers and storers. Should the rate of inflation be high enough to justify the use of futures by consumers, the financial industry will have a large incentive to develop institutions that make participation by this sector of the public easier. Since the current contract size is large compared to the consumption needs of any given consumer--and large even in real relation to a consumer's net wealth--the industry will need to develop highly liquid mutual funds of futures. As the theoretical results below show, these funds should be balanced with respect to the anticipated consumption bundle rather than being broadly representative of all the basic products traded on the exchanges. The implications for farmers and storers are not so immediate; they flow through an interpretation of futures markets as a place where "speculators" sell

insurance to storers and producers for a price. With the putative addition of consumers to the market and with consumers wanting a net long position at all times, the supply of insurance will be much greater when hedgers are net short, and the insurance premium transferred from storer and producer to speculator will be lower. During that part of the cycle when hedgers (storers and producers) tend to be net long, the addition of consumers to the demand side of the market will increase the insurance premium earned by speculators. Of course, these consequences depend upon the Keynes-Hicks-Cootner version of the pure theory of futures, a view with which these authors agree. In summary, inflation at the current or higher rates will provide financial incentives sufficient to lure consumers into the futures markets, provided that the industry develops mutual funds consistent with use by small investors. Should these new investors enter the market in large quantity, the cost of hedging in the futures markets will be driven down whenever they and the storers and producers are on opposite sides of the market and up, otherwise.

Section II develops the pure theory of consumption hedging in a mean-variance framework. In Section III, the model is applied to consumption hedging when the utility function of the consumers is Cobb-Douglas. The numerical example is limited to a few broad classes of assets but is sufficient to show the potential and the problems of consumer hedging. Conclusions are presented in Section IV.

## II. The Model

The choice of assets to minimize variance for a given mean income was the portfolio selection problem of Tobin [10] and Markowitz [5]. Although Markowitz originally suggested that the analysis be conducted in real terms, it was not until the inflation of the 1970s that this practice seemed empirically attractive. Chen and Boness [2] and Hagerman and Kim [4]

exemplified the modern trend toward real terms analysis. The first satisfactory handling of the price level was done by Grauer and Litzenberger [3]. They attacked the problem as one of capital asset pricing<sup>2</sup> instead of portfolio choice and were able to interpret the capital asset pricing equation in terms of an expected return, a covariance between the assets return and the price level, and a covariance between marginal utility and real return. In this section we produce a model of asset choice wholly in terms of the means and variances of prices of consumption goods and assets, exhibit the efficient portfolio, and discuss the first-order or capital asset pricing conditions.

An agent chooses a set of assets  $(z_1, \dots, z_m)$  to maximize his expected indirect utility  $V$  at prices  $p = (p_1, \dots, p_n)$  where the first  $m$  prices are those of both pure investment goods and consumption goods which can be held as assets and the last  $n - m$  prices are those of goods that can only be consumed. The agent's choice is constrained by his initial wealth  $W$  and the prices of the assets  $s_1, \dots, s_m$ . To write the wealth constraint and stochastic income  $y$  conveniently, define the  $n \times 1$  vectors  $z' = (z_1, \dots, z_m, 0, \dots, 0)$  and  $s' = (s_1, \dots, s_m, 0, \dots, 0)$ . With this notation,  $W = s'z$ ,  $y = p'z$ , and the choice problem is

$$\max_z EV(y, p) \quad (1)$$

subject to

$$y = p'z$$

and

$$W = s'z.$$



Since the portfolio choice problem without explicit consideration of prices of final goods is not generally solvable,  $V(y, p)$  must be suitably restricted to allow for analytic results in this more general case. A tractable restriction is a generalized mean-variance indirect utility function,<sup>3</sup>

$$EV(y, p) = g \{E y f(p), E [y f(p)]^2\} \quad (2)$$

where  $g, f \in C^2$ ,  $g_1 > 0$ ,  $g_2 < 0$ ,  $f_{p_i} < 0 \forall i$ , and  $f$  is convex and homogeneous of degree of minus one.

Particular interest is attached to an example often used in the financial literature,  $g = E y f - c E (y f)^2$ . This quadratic form will be examined in detail.

When utility is quadratic in real income, the agent's problem can be reduced to the Lagrangian

$$\min_{\lambda} \max_z E f p' z - c E f^2 z' p p' z + \lambda (W - s' z) \quad (3)$$

with first-order conditions

$$E f p - 2 c E f p p' f z - \lambda s = 0 \quad (4a)$$

and

$$W = s' z. \quad (4b)$$

The solution is<sup>4</sup>

$$z^* = [2c (E f p p' f)]^{-1} (E f p - \lambda^* s) \quad (5a)$$

and

$$\lambda^* = \frac{s' (E f p p' f)^{-1} E f p - W 2c}{s' (E f p p' f)^{-1} s} \quad (5b)$$

Because there is no safe asset (in real terms), the solution does not have the portfolio separation property: different amounts of wealth or values for  $c$  lead to different portfolios (ratios of  $z_i/z_j$ ).<sup>5</sup> The solution, though calculable by numerical methods, is difficult to interpret—both because it involves fourth moments and because the underlying quadratic utility model does not necessarily have the property of gross substitutes—if the price of asset  $i$  increases, the resulting effect on  $z_j$  is not known [1].

An interpretation of the capital asset pricing conditions (the first-order conditions) in terms of expenditure on the bundle purchased at average prices is possible with a suitable approximation.<sup>6</sup> To get at this interpretation, write the first-order condition (4a)

$$E f(p) p_i - 2cE \{ [f(p) p' z] [f(p) p_i] \} - \lambda^* s_i = 0 \quad i = 1, n \quad (6)$$

and expand about  $p = E p$ . Let (real income)  $L(p) = f(p) p' z$  and (real return)  $M(p) = f(p) p_i$  so

$$E [f(p) p' z] [f(p) p_i] = E L(p) M(p)$$

and

$$E L(p) M(p) \approx E [L(\bar{p}) + L_p(\bar{p})' \tilde{p}] [M(\bar{p}) + M_p(\bar{p})' \tilde{p}]$$

where  $\tilde{p} = p - \bar{p}$ . Denote the variance-covariance matrix of  $p$  as  $\Omega = E \tilde{p} \tilde{p}'$ . Then

$$E [L(p) M(p)] \approx L(\bar{p}) M(\bar{p}) + \{ [(p' z) f_p' + f z'] \Omega [f_p p_i + f e_i] \} \Big|_{p=\bar{p}} \quad (7)$$

where  $e_i$  is the unit vector in the  $i$ th direction. Since  $x(p) = -y f_p / f$  (by Roy's identity [11]) and  $\bar{y} = \bar{p}' z = \bar{p}' x(\bar{p})$ , the stochastic term in braces in (7) can be rewritten as

$$f(\bar{p})^2 \left\{ \bar{p}_i \text{cov} \left[ (x - z)' \tilde{p}, \frac{\tilde{p}'x}{\tilde{p}'x} \right] - \text{cov} [(x - z)' \tilde{p}, \bar{p}_i] \right\} \quad (8)$$

where  $x = x(\bar{p})$  and  $L(\bar{p}) M(\bar{p}) = f(\bar{p})^2 \bar{p}'z\bar{p}_i$ . The mean of real income can

be similarly expanded:

$$E f(p) p_i \approx E [f + f_p' \tilde{p}] p_i = f \bar{p}_i - f \text{cov} \left( \frac{x'p}{x'p}, p_i \right) \Big|_{p=\bar{p}}. \quad (9)$$

Putting all this together, the first-order condition is:

$$\begin{aligned} \lambda^* = f \left[ \frac{\bar{p}_i}{s_i} - \text{cov} \left( \frac{\tilde{p}'x}{\tilde{p}'x}, \frac{\bar{p}_i}{s_i} \right) \right] - 2cf^2 \left\{ \frac{\bar{p}_i}{s_i} \text{cov} \left[ (x - z)' \tilde{p}, \frac{\tilde{p}'x}{\tilde{p}'x} \right] \right. \\ \left. - \text{cov} \left[ (x - z)' \tilde{p}, \frac{\bar{p}_i}{s_i} \right] + p'z \frac{\bar{p}_i}{s_i} \right\}. \end{aligned} \quad (10)$$

The term  $(x - z)$  is the vector of purchases of goods the agent would make at average prices after the state of nature has been revealed.

The first term in brackets, the real rate of return on asset  $i$ , is composed of the average real rate of return less the (real) covariance between the price level index  $(x'\tilde{p}/x'\bar{p})$  and return. Assets that pay off well when prices are high have a lower real rate of return than assets that pay off well when prices are low. The second-order terms are real mean income times asset return  $[\bar{p}'z (p_i/s_i)] f^2$ ; the covariance of the price index and excess income required to buy  $x(\bar{p})$ , all times average return; and covariance of the asset and excess required income. All other things equal, assets that pay off high when excess income is required are good.

### III. Numerical Example

As an example of the import of conducting the analysis in real terms and including consumer durables and futures among the assets, consider the case of a consumer with a Cobb-Douglas indirect utility function with weights given by the consumer price index (CPI). The choice on consumer goods consists of the major CPI categories, and their prices are exactly the CPI components. Four

broad classes of assets are available: housing, represented by the price of new housing in San Francisco; stocks, represented by the Dow Jones 30 industrial average; futures, represented by the Dow Jones Commodities Futures index; and a nominal bond, represented by the mortgage rate. We call this an example precisely because we use indexes and limit the number of assets. The mortgage rate is the appropriate bond rate for a consumer because in the optimal choice the consumer holds a mortgage, which is equivalent to selling bonds.

The important credit constraint of an individual occurs in both the housing and the futures markets. Futures are settled daily and require only a small--in this case, 15 percent--deposit. Thus, holding a dollar in futures contracts ties up only 15 cents in wealth. It pays off the difference between the purchased price and the terminal price. Housing is also sold on credit. About 20 percent of the purchase price of a house is required in cash, and the rest is financed by selling a mortgage. Thus, 20 cents spent on housing requires the holding of 80 cents worth of mortgage and yields whatever the second period price of housing is less 80 cents times the interest rate on mortgages. These credit and institutional arrangements for futures and housing have been incorporated into the wealth constraint.

As explained in Section II, expected utility is taken as being the expected value of a quadratic in real income. The form of the quadratic is  $U = yf - c(yf)^2$ . For any choice of  $z$ , real income,  $yf$ , was computed for alternate months in 1978 and 1979 as was a value of  $U$ . The expected value of  $U$  was computed for each choice of assets  $z$ , and the optimal portfolio was  $z^*$ , the portfolio that maximized the expected value of  $U$ . This brute-force technique lacks any element of prediction in the returns of assets and amounts to the assumption that the returns follow a stationary time series. Economic analysis of the returns or formal time series analysis would be preferable, but this brute-force method suffices for an example.

The optimal allocation of \$100,000 among these assets, when  $c$  is zero or only expected value matters, is to buy a \$100,000 house, acquire an \$80,000 mortgage, and invest \$80,000 in the futures market. The result--that one should have been in futures in 1978-79--is easy to arrive at from a look at the data: futures increased at the astounding rate of 6 percent per month during that period. The result, if 1977 is included, still includes the purchase of a house; but futures are no longer as attractive so the optimal strategy is not to take a mortgage. When  $c$ , the parameter on the quadratic part of the utility function, is low--less than .006--the results are the same as if it were zero. However, when  $c$  is greater than .009 (that is, the investor is more risk averse), the entire portfolio is held in the form of stocks. There are interior solutions between .006 and .009; for instance, at .007 only \$50,000 worth of housing is purchased, \$10,000 is placed in futures, and \$80,000 is placed in stocks. These preliminary results show that at least some futures should be included in the portfolios of less risk-averse investors.

To see the importance of the consumption weights, the parameters of  $f$  in the indirect utility function, we computed the optimal portfolio choice with the weights on rent and food reversed. In the CPI, the weight of food is .17, and that on rent is .48. With the CPI weights, the optimal portfolio was to purchase \$100,000 of housing and place \$30,000 in futures and \$50,000 in stocks. With the weights reversed so that the consumer would spend close to half of his income on food, he would still buy a \$100,000 house; but now \$70,000 would be invested in futures and only \$10,000 in stocks. This shows that the consumption choice does influence the desirability of using the futures market and that it does so in the way one would naively anticipate; i.e., a higher weight on food which is well represented in the futures index results in more futures being purchased.

#### IV. Conclusions

With the high inflation rates of the late 1970s, the optimal portfolio choice of a mean-variance efficient consumer-investor is apt to include goods such as housing and futures that correlate well with anticipated consumption. Both the pure theory and the numerical example bear out this forecast of increasing futures use. The major bar to consumer use of futures is the need for appropriate mutual funds in futures. The need for these funds to be balanced in terms of anticipated consumption rather than as a broad market average is borne out in the numerical example. In that example the weights of housing and food were shifted, and the shift resulted in a change from stocks to futures. Much the same thing will happen in constructing a futures mutual fund. As the weights shift toward the consumption weights, the fund will become more desirable. Higher utilization of futures by consumers will decrease the price of the insurance contract storers that are said to purchase from spectators. This price decrease will make hedging a more desirable activity for the producers and storers of agricultural commodities.

## Footnotes

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<sup>2</sup>Mossin [6] and Rubinstein [7] provide excellent comprehensive summaries of the portfolio literature. Mossin's emphasis is on portfolio choice and stock market equilibrium while Rubinstein's is on the capital asset pricing model.

<sup>3</sup>Because variance  $(x) = E(x^2) - (Ex)^2$ , any function of mean and variance can also be written as a function of mean and  $E(x^2)$ .

<sup>4</sup>Providing  $E(fpp'f)$  is nonsingular.

<sup>5</sup>Rubinstein [8] shows that, in the absence of a safe asset--and putting everything in real terms assures there is no safe asset--all portfolios are a linear combination of a market portfolio and a portfolio uncorrelated with the market portfolio.

<sup>6</sup>In Sharpe's [9] model the interpretation of the first-order condition is sufficient for the interpretation of the whole model because the separation theorem assures that  $z^*$  is proportional to the stock of assets.

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