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OPTIMAL HEDGING BY FIRMS WITH MULTIPLE SOURCES OF RISKY INCOME

by

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INTRODUCTION

Consider a competitive risk averse firm producing certain commodity under price uncertainty. It is well-known (see, for example, Sandmo (1971), Eithier (1973) and Baron (1976)) that such price volatility results in a lower level of production. It was demonstrated in the literature, in various models, that the existence of a commodity forward/futures market has the following two consequences. The first, known as the 'Separation theorem', is a striking observation: When a forward (or futures) market exists, the optimum production of the firm does not depend upon the (subjective) distribution of the random price nor upon the firm's attitude toward risk. This property was shown by Danthine (1978), Holthausen (1979), Feder Just and Schmitz (1980), Marcus and Modest (1984) and others. The second consequence, known as the 'Full-hedging theorem', states that when the futures market is *unbiased* the firm sells forward all its output and, by this forward contracting, it avoids price risk altogether (see, for example, Holthausen (1979), Kawai and Zilcha (1986)).

Many studies on optimal hedging claim that the Separation property does not hold when we introduce other sources of risk into the firm's revenue function. For example, with output uncertainty the Separation property does not hold (see, for example Losq (1982)). In this paper we study the effects of futures markets on a competitive risk averse firm producing certain commodity under price risk. The firm is also engaged in other activities, which contribute another source of riskiness to its revenues, and these two risky sources of income are correlated. To make this feature of our model more concrete we take the firm to be an exporting firm facing uncertainty about the exchange rate. The firm also controls other risky assets correlated to the random exchange rate (i.e., to the random price). Our main findings are (a) the Separation theorem *holds* when the firm's total revenue includes income from other risky sources correlated to the uncertain price of this commodity; (b) the Full–hedging theorem *does not hold* in this case; hence, with *unbiased* futures markets the firm may underhedge or overhedge its foreign exchange proceeds depending on the correlation between the exchange rate and the firm's other risky sources of incomes.

This work is a further contribution to the study of the impact of futures markets on the production and hedging behavior of competitive firms. The generalization of the Separation property to the type of models and the result that full hedging may not be optimal although *unbiased* futures markets exist are important observations in this literature.

2. THE FIRM

Consider a competitive risk averse firm which produces and exports a certain commodity. At the time when the decision about production takes place, the price of the output is a random variable. More precisely, we shall assume that the price abroad P^* is known, but the exchange rate $\tilde{e}$ is random. Since the firm is interested in revenues denominated in *domestic* currency this introduces randomness into its income. The firm's revenues contain another source of uncertainty, for example, financial investments or assets that the firm owns. We shall aggregate these additional sources of uncertain incomes to a single random variable $\tilde{I}$ and assume that its distribution is known at the time $t = 0$ when the production decision takes place. The output is available for sale at date $t = 1$ and we assume that the joint cummulative distribution function of the random variables $(\tilde{I}, \tilde{e})$, for realizations at $t = 1$, is known to the firm.

The cost of production is given by $C(Q)$, where Q is the level of output. We make the usual assumptions about the cost function: $C'(Q) > 0$, $C''(Q) \leq 0$. This competitive firm is risk averse, hence its von–Neumann Morgenstern utility function is $U(\cdot)$ and it satisfies: $U' > 0$ and $U'' < 0$.

When the production decision takes place (i.e., $t = 0$) the firm has access to a currency futures market (or in the less specific case a commodity futures market), where it can sell (or buy) foreign currency (for delivery at date $t = 1$) at a competitive futures exchange rate e_f. Denote by Z the forward contracting in foreign exchange of this firm at date $t = 0$. Thus the choice of output level Q and the forward sale (or purchase) of foreign exchange Z are made simultaneously at date 0 in a way that maximizes the expected utility of the total income at $t = 1$. Namely,

$$\max_{Q,Z} E\, U(\tilde{w})$$

s.t.

(1) $$\tilde{w} = \tilde{I} + \tilde{e}P^*Q - C(Q) + Z(e_f - \tilde{e})$$

where E is the expectation with respect to the joint probability distribution of $(\tilde{I}, \tilde{e})$. Due to the strict concavity of the maximand in Q and Z the unique optimal Q^*, Z^* are the solution of the first–order conditions:

(3) $$E\{[\tilde{e}P^* - C'(Q^*)]U'(\tilde{w})\} = 0$$

(4) $$E[(e_f - \tilde{e})U'(\tilde{w})] = 0.$$

We assume that the optimal output Q is positive for the given P^* and $\tilde{e}$. It is known (see Sandmo (1971) and Ethier (1973)) that the uncertainty in the exchange rate reduces the output, compared to the certainty equivalent case, if no risk–sharing markets exist. One of the striking results obtained when forward or futures markets are introduced is the 'Separation property' which claims that when such markets exist the optimal production level becomes independent of the probability distribution of the random price $\tilde{e}P^*$ and the attitude towards risk of the firm, i.e., the utility function U. We shall prove in the next section that this important result prevails when the firm owns other risky assets which are correlated to $\tilde{e}$.

3. THE SEPARATION PROPERTY

Let us examine the validity of the Separation property (see Danthine (1978) and Feder, Just and Schmitz (1980)) for our case.

PROPOSITION 1: Consider a competitive exporting firm facing exchange rate (or price) uncertainty and which possesses other risky sources of income. If currency futures markets exist, with futures exchange rate e_f, then the firm's production Q^* is determined by:

$$C'(Q^*) = e_f P^*. \tag{5}$$

Namely, it is independent of the (joint) probability distribution (the exchange rate and the other risky income) and the attitude towards risk of this firm.

Proposition 1 constitutes a meaningful extension of the Separation result. In particular the firm's optimum output, in the presence of futures markets, does not depend on the correlation between $\tilde{e}$ and $\tilde{I}$. This claim underscores the role of forward/futures markets in eliminating the adverse effects of price uncertainty on production, since the presence of other random sources of income which are correlated to the exchange rate seems to be natural.

PROOF of PROPOSITION 1: Using the first–order conditions (3) and (4) and substituting $E[\tilde{e}U'(\tilde{w})] = e_f EU'(\tilde{w})$, from equation (4), in equation (3) we obtain that $e_f P^* - C'(Q^*) = 0$ since $EU'(\tilde{w}) > 0$.

Even in the case where price uncertainty is the only source of uncertainty, it is known that the optimum hedging level Z^* does depend upon the distribution function of the price and on the utility function of the firm. In the next section we shall study the effects of the other sources of uncertainty, i.e., $\tilde{I}$, on the hedging behavior of the firm. In this case we shall see some major differences when $Cov(\tilde{e}, \tilde{I}) \neq 0$.

4. OPTIMAL HEDGING

Another result for competitive risk averse firms facing price uncertainty, implied by the existence of futures markets, is known as the 'full–hedging theorem.' It states that if futures markets are *unbiased* the firm hedges *all* its foreign proceeds, i.e., $Z^* = P^*Q^*$. When price is the only source of uncertainty using such futures contracting the firm avoids price risk altogether. The next proposition demonstrates that in our model the 'full–hedging theorem' holds if and only if $\tilde{e}$ and $\tilde{I}$ are uncorrelated. Surprisingly, we show later that full–hedging may occur when the futures market is *biased.*

We say that the currency futures market is *unbiased* if $e_f = \bar{e}$, where $\bar{e}$ is the *mean* of $\tilde{e}$. It exhibits *normal backwardation* (*contango*) if $e_f < \bar{e}$ ($e_f > \bar{e}$). We say that the firm *fully–hedges* if $Z^* = P^*Q^*$; namely, it sells forward all its foreign exchange proceeds. The firm *underhedges* if it sells forward only part of its foreign currency proceeds, i.e., $Z < P^*Q^*$. The firm *overhedges*, or *speculates*, if $Z^* > P^*Q^*$. Now we consider the case of *unbiased* futures market.

<u>PROPOSITION 2</u>: Assume that the currency futures market is *unbiased*, then

(a) The firm fully hedges if and only if $\text{Cov}(\tilde{e}, \tilde{I}) = 0$.

(b) The firm underhedges if $\text{Cov}(\tilde{e}, \tilde{I}) < 0$.

(c) The firm overhedges if $\text{Cov}(\tilde{e}, \tilde{I}) > 0$.

This proposition provides some explanation why, at times, in the presence of unbiased futures markets, we do not observe full hedging. Consider, for example, a firm producing this commodity and owning assets with risky returns which have negative correlation to $\tilde{e}$, i.e., $\text{Cov}(\tilde{e}, \tilde{I}) < 0$. The revenues $\tilde{I}$ provide the firm in this case with partial insurance against low realizations of $\tilde{e}$. Thus, although $e_f = E\tilde{e}$ the firm sells forward only a fraction of its proceeds abroad. Similarly, when $\text{Cov}(\tilde{e}, \tilde{I}) > 0$ low realizations of the exchange rate at $t = 1$ increase the chance of lower values at $t = 1$ of $\tilde{I}$. Since the futures market, which satisfies $e_f = E\tilde{e}$, is a fair risk–sharing instrument it will be "used" as a vehicle for insuring this other risky income against low realizations, hence we observe overheding in the futures market in this case.

This observation regarding overhedging should be taken into account by the tax authorities. In the U.S., for example, one is considered as a speculator, for tax purposes, if he sells short in the forward market and he is not long in the commodity (or currency) itself. Using our result we can argue that selling forward more than one's output (or currency proceeds) may be considered as a hedging device in a multi–sources risky income case.

Another case where *overhedging* arises in the presence of unbiased currency futures markets was shown by Zilcha and Eldor (1991) in a multiperiod hedging problem. In this model the firm optimally hedges in a multiperiod optimization problem with random correlated exchange rates. It was shown that overhedging is optimal when there is a positive correlation between the random exchange rates (in dates 1 and 2 in that paper). However, the reasons for the overhedging, given the unbiasedness, in the two cases differ.

PROOF of PROPOSITION 2: We assume throughout this proof that $\tilde{I}$ and $\tilde{e}$ are not constants, i.e., each is a nondegenerate random variable. Let us rewrite equation (4) as follows:

$$(6) \qquad (e_f - \bar{e})EU'(\tilde{w}) - \mathrm{Cov}(\tilde{e}, U'(\tilde{w})) = 0.$$

Therefore, assuming unbiased futures market implies:

$$(7) \qquad \mathrm{Cov}(\tilde{e}, U'[\tilde{I} + \tilde{e}(P^*Q^* - Z^*) - C(Q^*) + e_f Z^*]) = 0.$$

The first conclusion from equation (7) is that if $\tilde{e}$ and $\tilde{I}$ are uncorrelated, then we must have $Z^* = P^*Q^*$. Now, when $\mathrm{Cov}(\tilde{e}, \tilde{I}) < 0$ if $P^*Q^* - Z^* \leq 0$ then due to the monotonicity of U'(i.e., a decreasing function) the covariance on the left–hand side of equation (7) must be positive. Therefore we must have in this case $Z^* < P^*Q^*$. When $\mathrm{Cov}(\tilde{e}, \tilde{I}) > 0$ if $P^*Q^* - Z^* \geq 0$ the covariance in equation (7) must be negative. Thus from (7) we derive that $Z^* > P^*Q^*$ in this case.

□

Let us consider now the cases of normal backwardation and contango. We shall demonstrate that the firm's optimal hedging behavior may deviate significantly from the cases discussed in the related literature. Namely, it has been shown in various models that under normal backwardation the firm's optimum forward contracting requires selling only *part* its future proceeds (or output in the case of commodity futures). However, in our case it is possible that the firm will optimally choose to *fully hedge* its foreign currency proceeds although the market is *biased.* The optimal hedging structure remains as in the nonrandom wealth case (see, for example, Feder Just and Schmitz (1980), Kawai and Zilcha (1986)) *only* under the conditions specified in the next proposition.

<u>Proposition 3</u>:

(a) If the currency futures exchange rate satisfies $e_f < E\tilde{e}$ and if $\mathrm{Cov}(\tilde{e}, \tilde{I}) < 0$ the firm will underhedge.

(b) If the currency futures exchange rate satisfies $e_f > E\tilde{e}$ and if $\mathrm{Cov}(\tilde{e}, \tilde{I}) > 0$ the firm will overhedge (or speculate).

Notice that in the normal backwardation case with constant $\tilde{I}$ the firm chooses $Z^* < P^*Q^*$, thus adding the condition $\mathrm{Cov}(\tilde{e}, \tilde{I}) < 0$ only intensifies this inclination due to risk aversion. Similarly, if $e_f > E\tilde{e}$ the firm overhedges when $\tilde{I}$ is constant, hence assuming that $\mathrm{Cov}(\tilde{e}, \tilde{I}) > 0$ only strengthens this property. However, observing the proof of Proposition 3, we conclude that in the normal backwardation case under $\mathrm{Cov}(\tilde{e}, \tilde{I}) > 0$ the firm may overhedge, fully hedge or underhedge. In the case where $e_f > E\tilde{e}$ if $\mathrm{Cov}(\tilde{e}, \tilde{I}) < 0$ then the firm's hedging behavior can be of either type.

<u>PROOF OF PROPOSITION 3</u>: Using equation (6) for the case where $e_f - \bar{e} < 0$ we obtain that:

$$\mathrm{Cov}(\tilde{e}, U'[\tilde{I} + \tilde{e}(P^*Q^* - Z^*) - C(Q^*) + e_f Z^*]) < 0. \tag{8}$$

By assumption $\mathrm{Cov}(\tilde{e}, \tilde{I}) < 0$ hence if $P^*Q^* - Z^* \leq 0$ the covariance on the left–hand side in (8) must be *positive* (since U' is a monotone decreasing function) which is a contradiction to (8). Thus $P^*Q^* - Z^*$ must be positive.

Consider now the case where $e_f - \bar{e} > 0$, then (6) implies that:

$$\mathrm{Cov}(\tilde{e}, U'[\tilde{I} + \tilde{e}(P^*Q^* - Z^*) - C(Q^*) + e_f Z^*]) > 0. \tag{9}$$

Given that $\mathrm{Cov}(\tilde{e}, \tilde{I}) > 0$ if $P^*Q^* - Z^* \geq 0$ we obtain, using the monotonicity of U', that the covariance on the LHS in (9) must be <u>negative</u> which is a contradiction. This proves part (b) of the proposition.

REFERENCES

Baron, D.P., "Flexible Exchange Rates, Forward Markets and the Level of Trade," *American Economic Review*, 1976, 66, 253–266.

Danthine, J.P., "Information, Futures Prices and Stabilizing Speculation," *Journal of Economic Theory*, 1978, 17, 79–98.

Ethier, W., "International Trade and the Forward Exchange Market," *American Economic Review*, 1973, 63, 494–503.

Feder, G., Just, R.E. and Schmitz, A., "Futures Markets and the Theory of the Firm Under Price Uncertainty," *Quarterly Journal of Economics*, 1980, 95, 317–328.

Holthausen, D.M., "Hedging and the Competitive Firm Under Price Uncertainty," *American Economic Review*, 1979, 69, 989–995.

Kawai, M. and Zilcha, I., "International Trade with Forward–Futures Markets Under Exchange Rate and Price Uncertainty," *Journal of International Economics*, 1986, 20, 83–98.

Losq, E., "Hedging with Price and Output Uncertainty," *Economics Letters*, 1982, 10, 65–70.

Marcus, A.J. and Modest, D.M., "Futures Markets and Production Decisions," *Journal of Political Economy*, 1984, 92(3), 409–424.

Sandmo, A., "On the Theory of the Competitive Firm Under Price Uncertainty," *American Economic Review*, 1971, 61, 65–73.

Zilcha, I. and Eldor, R., "Exporting Firm and Forward Markets: The Multiperiod Case," *Journal of International Money and Finance*, 1991, forthcoming.