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HOTELLING RULE, ECONOMIC RESPONSES AND OIL PRICES

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In a recent article, Fishelson (1980) examined the effects of controlling energy supply to a small country that imports all its energy.[1] The controls that were examined were quotas and prices. They were studied for different production structures ranging from fixed proportions to variable elasticity of substitution (translog). The main disadvantage of that study was its static nature. All events occurred once and stayed forever. Furthermore, no changes took place in the production function. In the present study, some dynamics are introduced. However, we limit the examination to reactions to price increases. In the first section we modify the Hotelling model to explain the behavior of crude oil prices for the 1960-1970 period in which there were no changes in oil prices. In the second section, we hypothesize a Cobb-Douglas economy and allow for technological progress to offset the increase in oil prices. In the third section, we generalize the Cobb-Douglas function to a CES function and introduce explicitly the notion of energy saving technological progress.

The Determination of Energy Prices[2]

If we regard crude oil as an exhaustible resource with finite known reserves and zero extraction costs, the r percent rule of Hotelling (1931) applies.[3] This rule applies whether the industry that extracts crude oil is a competitive one or a monopoly. The only difference between the two is that for a monopoly the initial price would be higher and thus the first part of the path of prices would also be higher than that of a competitive industry.[4]

The r percent rule did not materialize in the 1960s although OPEC already existed. On the contrary, the real price of crude declined somewhat towards

1970 and started to increase from then on. The issue is how can we reconcile this price behavior with Hotelling's rule while still assuming rational behavior of the producers of crude?[5] The explanation we offer is that of simultaneous increases in reserves of oil, increases in the demand for oil, and changes in the costs of oil production. Each argument is introduced independently and then the effects are summed.

We assume that the world market demand for crude oil is of constant price elasticity,

$$(1.1) \qquad Q = BP^{-\alpha}.$$

Let the known reserves at t=0 be R_o. The cost of oil extraction is assumed at first to be zero. Thus, the solution to the problem

$$(1.2) \qquad \text{Max} \int_o^\infty P_t Q_t dt \qquad \text{s.t.} \int_o^\infty Q_t \leq R_o \qquad \text{and} \qquad \dot{R}_t = -Q_t$$

leads to an initial price

$$(1.3) \qquad P_o = (\alpha \cdot r \cdot \bar{R}_o/B)^{-1/\alpha}.$$

Hence, for any t in the future

$$(1.4) \qquad P_t = (\alpha \cdot r \cdot R_o/B)^{-1/\alpha} \cdot e^{rt}.$$

If reserves increase (ex post) at a rate of γ percent per year, i.e.,

$$(1.5) \qquad R_1 = R_o e^{\gamma}$$

then (view year 1 as the base year for the future path) the price for that year 1 (reevaluated, given the actual reserve level at year 1) is

$$(1.6) \qquad P_o' = (\alpha \cdot r \cdot R_o e^{\gamma}/B)^{-1/\alpha} \cdot e^{r}.$$

The ratio P_o'/P_o is $e^{-\gamma/\alpha+r} \lesseqgtr 1$, i.e. the price, instead of increasing, might decline. If demand is increasing over time such that

$$(1.7) \qquad Q_t = B'P_t^{-\alpha}$$

where $B' = B_o e^{\delta t}$, then,

$$(1.8) \qquad P_1' = (\alpha \cdot r \cdot R_o/B_o e^{\delta})^{-1/\alpha} \cdot e^r .$$

The ratio, P_1'/P_o is equal to $e^{r+\delta/\alpha}$, i.e., the price increases by more than r percent.

If both reserves and demand increase, the price at year 1 would be

$$(1.9) \qquad P_1'' = (\alpha \cdot r \cdot R_o e^{\gamma}/Be^{\delta})^{-1/\alpha} \cdot e^r = P_o \cdot e^{r+\delta/\alpha-\gamma/\alpha}.$$

In year t the price would thus be

$$(1.10) \qquad P_t'' = P_o e^{(r+\delta/\alpha-\gamma/\alpha)t}.$$

Between 1960 and 1971 reserves have doubled and demand has tripled while the price did not change. Hence, on the average for each year in that period we expect

$$(1.11) \qquad r + \delta/\alpha - \gamma/\alpha = 0.$$

If we let $r = 0.04$, $\alpha = .5$, $\gamma = 0.07$ and $\delta = 0.10$, the sum is 0.055, i.e., we would expect the price to rise by 5.5 percent per year. The fact that it did not might be due to the declining (and expected to decline) cost of extraction, C. Note that if $C > 0$, then the r percent rule implies that

$$(1.12) \qquad (P_t - C_t) = e^r(P_{t-1} - C_{t-1}) \quad \text{or,} \quad P_t = e^r P_{t-1} + (C_t - e^r C_{t-1}).$$

Hence,

$$(1.13) \qquad P_t/P_{t-1} \gtreqless e^r \quad \text{as} \quad C_t/C_{t-1} \lesseqgtr e^r.$$

Over the 1960-1971 period C_t/C_{t-1} of drilling was about $e^{.02}$ which is less than e^r, thus pushing P_{1971} towards P_{1960} (for additional analysis, see Pindyck, op. cit.).

The Cobb-Douglas Economy

Let the economy in which we are interested enjoy at first a constant price of oil. The firms there (or the central planner) assumed (ex post wrongly) that this stability would continue forever. The production function of this economy is of the Cobb-Douglas type, where the only two inputs are labor and energy. Output is denoted by Q. For simplicity, we assume a fixed labor force, $L = L_o$. Energy is denoted as E. Technological progress is neutral (by definition) at a rate λ.

$$(2.1) \qquad Q_t = Ae^{\lambda t} L_o^{\alpha} E_t^{1-\alpha}.$$

Let the price of Q be 1. Then the quantity of energy demanded at year t is,

$$(2.2) \qquad E_t = A^{1/\alpha} e^{\lambda t/\alpha} L_o (1 - \alpha)^{1/\alpha} P_t^{-1/\alpha}$$

where P denotes the price of energy. Output (the indirect production function) is

$$(2.3) \qquad Q_t = A^{1/\alpha} e^{\lambda t/\alpha} L_o (1 - \alpha)^{1/\alpha} P_t^{-\frac{(1-\alpha)}{\alpha}}.$$

As long as P_t was stable the economy grew at a rate of λ/α percent per year ($\lambda/\alpha > \lambda$ since $\alpha < 1$). Now consider the case where this small economy was suddenly faced with continuous increasing prices at a rate of ψ percent per year.[6] The elasticity of output with respect to oil prices is $\frac{-(1-\alpha)}{\alpha}$. Thus, the growth rate of the economy declined to $\lambda/\alpha - \psi \cdot \frac{1-\alpha}{\alpha}$. The only way the economy could return to its previous path of growth (offset the increasing price effect) is to increase its technological progress λ, (α is assumed to be unchanged) such that $\Delta\lambda = \psi(1 - \alpha)$. In other words, the neutral technological change should increase but by <u>less</u> than the increase in energy prices.[7] For example, if $\lambda = 0.03$, $\psi = 0.05$ and $\alpha = 0.90$ then $\Delta\lambda/\lambda = .20$ ($\Delta\lambda = .005$) which is not unrealistic. In order to get $\Delta\lambda > 0$, resources need to be diverted from production to the research and development of new technology of production. The explanation for this change of resource allocation is that research which is an indirect production process was less efficient than direct production before the increase in energy prices but became more efficient due to the increase of energy prices.

In Cobb-Douglas production function framework the above statement leads to an internal inconsistency when related to the analysis. It can be shown that for a Cobb-Douglas economy the profitability of investment in research and development has nothing to do with P nor with $\psi > 0$. Thus, if it is profitable to allocate resources to research and development when $\psi > 0$, it should also have been profitable when $\psi = 0$.[8] Thus, if research and development is observed when $\psi > 0$ while it was not observed before, and the production function did not change over time, then either the economy is not Cobb-Douglas, or it was not an optimizing economy.

The CES Economy

The economists in the constant elasticity of substitution, CES, production function economy have a harder life than their friends in the Cobb-Douglas economy since the former have to resort to linear approximations. Let output be a CES function of labor and energy,

$$(3.1) \qquad Q_t = e^{\lambda t}(\delta L_o^{-\rho} + (1 - \delta)E_t^{-\rho})^{-1/\rho}.$$

Also, let the demand for energy follow the optimality condition, that of value of marginal product equals price, $\partial Q/\partial E = P$. Hence,

$$(3.2) \qquad (Q/E)_t^{1/\sigma}(1 - \delta) \cdot e^{\lambda t(\sigma-1)/\sigma} = P_t$$

and

$$(3.3) \qquad E_t = Q_t(1 - \delta)^{\sigma} e^{\lambda t(\sigma-1)} P_t^{-\sigma}$$

where $\sigma = \frac{1}{1+\rho}$ is the constant elasticity of substitution between energy and labor. Substitution of (3.1) for Q in (3.3) leads to the specification of the demand for energy with respect to the exogenous variables.

$$(3.4) \qquad E_t = \frac{\delta^{-1/\rho}(1 - \delta)^{\sigma} \cdot e^{\lambda t\sigma} \cdot L_o}{(1 - e^{-\lambda t\sigma}(1 - \delta)^{\sigma})^{-1/\rho}} P_t^{-\sigma} = A_t P_t^{-\sigma}.$$

Note that for $P_t = P_o$ the rate of growth of energy demanded is not constant. However, with respect to its price the demand for energy is of a constant elasticity $-\sigma$.

Substituting (3.4) into (3.1) yields

$$(3.5) \qquad Q_t = e^{\lambda t}(\delta L_o^{-\rho} + (1 - \delta)(A_t P_t^{\sigma})^{-\rho})^{-1/\rho}.$$

From (3.4) it is clear that $\dot{A}/A$ declines over time and approaches $\lambda\sigma$ from above as t gets large, assuming $P_t = P_o$. Thus when energy prices are constant, output grows at a rate greater than $|\lambda|$. It is at least $\lambda(1 + \sigma)$. When the price of energy however increases, output growth rate declines by less than $\sigma\psi$ where ψ is the rate of growth of energy prices since the first term on the right in (3.5) is not affected by the energy price increase.

Hence, the lowest growth rate is $\lambda(1 + \sigma - \psi)$. The price of oil has to more than double each year in order to cause zero growth.

The interesting case for investigation is that for which $\rho > 0$ $(\sigma < 1)$. For $\rho < 0$ $(\sigma > 1)$ it might have paid to invest in R and D for increasing both the efficiency of labor and energy also before the energy price increase. This would be less likely in the case for which $\rho > 0$. To show this we use the extreme example of $\rho = \infty$ $(\sigma = 0)$. The production function is of fixed proportion (the first case dealt with in Fishelson (1980)). Denote rate of improved efficiency of labor by β and of improved efficiency of energy by γ. Accordingly, the inputs measured in efficiency units are

(3.6a) $$EL_t = e^{\int \beta_t dt} \cdot L_o = e^{\beta t} L_o \quad (\beta_t = \beta_{t'} = \beta)$$

(3.6b) $$EE_t = e^{\int \gamma_t dt} \cdot E_t = e^{\gamma t} E_t \quad (\gamma_t = \gamma_{t'} = \gamma)$$

The fixed proportion technology implies

(3.7) $$Q_t = \text{Min}\,(EL_t/a, \; EE_t/b).$$

Thus, unless (after reaching the proper ratio for full employment) the inputs measured in efficiency units grow at equal rates, units of one input will be wasted.[10] Furthermore, for $\sigma = 0$ and $L_t = L_o$ the quantity of energy that would be employed at year t is

(3.8) $$E_t = \begin{cases} L_o(b/a)e^{(\beta-\gamma)t} & \text{when } P_t = \bar{P} \leq e^{\gamma t/b} \\ 0 & \text{when } P_t = \bar{P} > e^{\gamma t}/b \end{cases}$$

Also for $\sigma = 0$ regardless whether $\beta \lessgtr \gamma$, the economy converges to a neutral technological progress path at a growth rate of β. Hence, even for $\sigma = 0$ and for sure for $\sigma > 0$, the economy should attempt to increase efficiency of its constrainted factor--labor. When the research production functions are

(3.9) $\beta = h^{L}(L_L)$ and $\gamma = h^{E}(L_E)$

then technological progress is not a free good. Labor has to be diverted from production to research if positive technological progress is desired. When $P = \bar{P}$, the diversion of labor from L to L_E becomes less profitable while the diversion of labor from L to L_L becomes more profitable as $\sigma \to 0$ (with regard to the latter there is risk of running into "Perpetuum Mobile" thus assuming internal equilibria constraints have to be imposed upon the properties of the production of improved technology, β and γ).

For $P_t = P_o e^{\gamma t}$ and $\sigma \to 0$, the diversion of labor from production to improving energy technology is more strongly justified. If initially $\gamma = 0$, then the minimal β needed to maintain the economy growth rate of γ equals ψ and its level increases as ρ diminishes (σ increases).

Using various approximations, the optimal investment in β and γ can be determined. We are not doing it here but just list the issues that should be considered.

1. Since labor is fixed at L_o, the diversion of labor to R and D implies losses of Q that equal the marginal product of labor.

2. If technological progress does not deteriorate, once positve β or γ are produced, they will last forever. The benefits from diverting labor to R and D is the present value of the flow of increases in Q due to the technological progress.

3. The increase in energy prices could not continue forever. Eventually a back stop technology would emerge (exogenous to the small country) that would produce energy at some constant price. At that price the economy will converge to a new path of growth obviously lower than the initial one, but over which the rate of growth would be the same as that before the price increases (Figure 1).[11]

Conclusions

Technological progress is economically justified to be biased (input saving) for the input whose quantity is limited by "nature" (zero population growth) or due to political actions (oil quotas). The need for technological progress becomes obvious when input prices increase (the historical labor-saving induced technological changes).[12] The CES model, despite its analytical inconvenience, proves useful for illustrating this equivalence. The intensity of research for improved technology is frequently related positively to the marginal productivity of labor in research and negatively to the productivity of labor in production. Furthermore, it is positively related to the rate of change of energy prices and to the elasticity of substitution between energy and labor in production.

The analysis and conclusions presented above is the first stage in the determination of the optimal allocation of resources between direct and indirect production activities where the objective function is the maximization of the social welfare function (e.g., $\int_0^\infty B(t)U(Q_t)dt$).[13] The specific element we introduce into this traditional maximization model is the continuously rising energy prices which accelerate the economic justification for investment in research for energy saving technologies. The model we have analyzed does not optimize. It is rather a satisfying model, i.e., it leads back to the path of growth that was experienced with constant energy prices. It should also show that research for improving energy and labor utilization were justified before the increase in energy prices. If they did, the discussion above is to be viewed in terms of intensifying it.

The drive towards increasing energy efficiency, i.e., to get more units of service from the same quantity of energy, started to be observed after energy shortages and higher energy prices were realized (e.g., more efficient air conditioners, gasoline efficient automobiles in terms of miles per gallon, more efficient space heat and water heating furnaces, and better housing insulation). Furthermore the increase in energy prices led to an increase of exploration activities and of the known reserves. The Hotelling r percent rule is still effective. Disregarding the shocks introduced in 1979 and 1980 because of the Iranian revolution and the Iran-Iraq war respectively, one finds that between 1975 and 1982 the real price of energy did not change while production costs have increased (the Alaska pipeline and more offshore and deeper drilling). Equation (1.11) explains this result. The same interest rate (.04) and demand elasticity (.5) with an increase of reserves at a rate of .02 and a decline of demand at a rate of .01 (world recession is another factor that lowered demand) should lead to a decline of oil prices at a rate of .02. The increase of the costs of production neutralized it to no change in price.

The interesting question is how long would the no-change-in-price hold? A slowdown of explorations and of energy saving technological progress, coupled with an upturn in the world economy, might lead to continuous increases in oil prices at rates between .10 and .15 per year. Whether such increases are sufficient to stimulate energy saving and explorations, or to slow economic growth, is questionable. At the margin, however, they would obviously occur. Hence, one can predict that in the future we are likely to observe cycling of oil prices at real rates between .05 to .15.

FOOTNOTES

1. Fishelson, G., "Control on Energy Supplies, the Damage to a Small Economy," Energy Economics, June 1980.
2. A recent study which is more detailed than the following section and which also discusses the role of uncertainty is Pindyck, R. S., "Models of Resource Markets and the Explanation of Resource Price Behavior," MIT-EL 7-9 062 WP, MIT Energy Laboratory, June 1979.
3. Hotelling. H., "The Economics of Exhaustible Resources," Journal of Political Economy 39 (1931):137-175. See also Fishelson, G.
4. Recall that $MR_t = MR_o \cdot e^{rt}$, while $MR_o = P_o(1 + 1/E)$ where E equals the own price elasticity of demand and $MR_t = P_t(1 + 1/E)$. See Herfindahl, O. C., "Depletion and Economic Theory," in M. Gaffney (ed.), Extractive Resources and Taxation, University of Wisconsin Press, 1967, pp. 63-90; and Herfindahl, O. C. and Kneese, Economic Theory of Natural Resources, Merrill (Columbia, Ohio: 1974) for a graphical exposition of this behavior.
5. For purposes of simplification we will not consider here the distribution of the responsibility of price (quantity) determination among the oil producing countries, OPEC and the oil companies.
6. Except for the 1973/74 price jumps, the real price of oil has been relatively stable. The reasons for this are analogous to the ones listed in the first section although the direction of change is different.
7. We talk about neutral technological change given the specific production function for which a biased change and a neutral change are identical.
8. In the simple economy that was postulated labor saving technological progress was needed since the fixity of labor implies that its price (equating demand and supply) was continuously increasing at a rate

of λ/α. The policy makers were not concerned, however, probably because this increase constituted returns to a domestic factor of production (did not <u>leak</u> out).

9. For empirical studies this implies identification difficulties not just for parameters but for function forms as well, e.g., is the originating production function C-D or CES.

10. If $L_t = L_o e^{nt}$, biased technological change at a rate of n is already called-for energy when E_t is constrained to equal E_o.

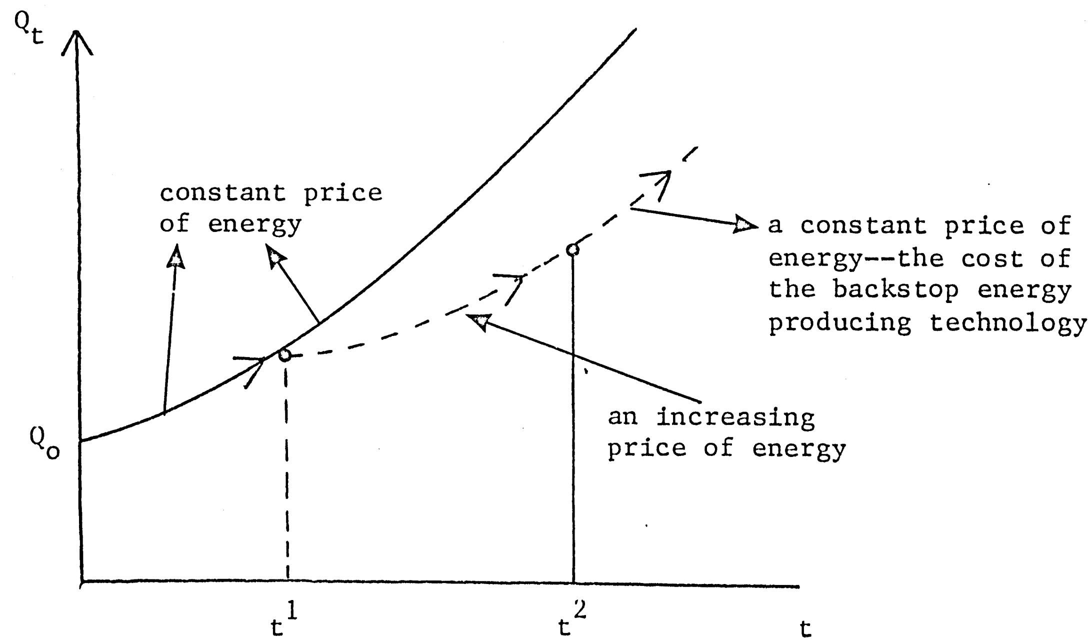
11. See Fishelson, G., "Dynamics in the Hotelling Model, A Note," Working Paper 35-82, Foerder Inst. Econ. Res., Tel Aviv University, Israel.

12. The fixed input case and the increasing input prices are actually the same. For the former, the relevant price is its shadow price.

13. The analogy for investment is perfect.

Figure 1

The Growth Path of the Economy



t^1 - energy price started to increase at a rate of r percent

t^2 - energy price reached he backstop technology