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## Optimal Clearing Arrangements for Financial Trades

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# Optimal Clearing Arrangements for Financial Trades\*

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## Abstract

Clearinghouses support financial trades by keeping records of transactions and by providing liquidity through short-term credit that is periodically cleared by participants. We study efficient clearing arrangements for formal exchanges, where traders must clear with a clearinghouse, and for over-the-counter (OTC) markets, where trades can be cleared bilaterally. When clearing is costly, we show that it can be efficient to subsidize the clearing process for OTC transactions by charging a higher price for the clearing of transactions in exchanges. This necessitates a clearinghouse that operates across both markets. As a clearinghouse offers credit, intertemporal incentives are needed in order to ensure settlement. An increase in the costs of liquidity provision worsens the incentives to settle. Hence, when liquidity costs increase, concerns about default must lead to a tightening of liquidity provision.

*Keywords:* Clearing, OTC vs. Exchanges, Private Information, Liquidity Costs, Default

*JEL Classification:* G14, G23, E42

# 1 Introduction

Exchanges formalize and standardize trading. They also offer another crucial service: the *clearing* of financial transactions. Clearing is usually performed within a single clearinghouse associated with the exchange. The clearinghouse is the neuralgic centre of the exchange. It gathers information on trades, which it then uses to calculate the resulting net positions of participants. It adjusts the exposure of participants by granting short-term credit. Finally, it ensures that the resulting positions are properly settled. Given the information on hand, the clearinghouse can regulate and influence trading activities in order to limit the probability of participants defaulting. This can be accomplished in a variety of ways. For example, the clearinghouse can adjust the frequency of settlement, impose certain trading limits, or vary collateral requirements. As pointed out by Bernanke (1990), given the immense volume of transactions involved, designing the rules of operation for clearinghouses is important for efficient trading.

Somewhat surprisingly, centralized clearinghouses (CH) are not in operation in some important markets, such as over-the-counter (OTC) markets. Instead, trades in these markets are usually cleared bilaterally and on an ad-hoc basis. More recently, however, many have called for the creation of a CH on the grounds that it will achieve more transparency and improve the efficiency of trading by reducing operational and settlement risk.<sup>1</sup> As a response, some exchange clearinghouses plan to engage in the clearing of transactions in certain segments of OTC markets in the near future.<sup>2</sup>

The operation of a CH will necessarily differ across these markets. A CH operating in a centralized exchange inevitably becomes aware of all realized trades since they are automatically processed and cleared within the system. In this sense, trades in exchanges can be *monitored* by the CH. On the other hand, trades conducted OTC can be cleared

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<sup>1</sup>There is an on-going discussion on how to ensure proper risk management practices in derivatives markets through formal clearing arrangements (see, e.g., Geithner, 2006 or Counter-Party Risk Management Policy Group, 2005).

<sup>2</sup>See, for example, The Clearing Corporation (2008) for a short discussion of the clearinghouse introduction in the OTC market for credit default swaps. NYMEX is in the process of offering clearing services for some OTC derivatives markets.

and settled bilaterally. Therefore, trades in OTC markets are *non-monitored*: as traders can always choose to trade and settle bilaterally, they can misrepresent their true exposures by settling only a share of the trades through the CH. In other words, in an OTC market the CH will have to provide incentives for traders to clear their trade through the CH.

A CH aggregates transactions and, thus, transforms bilateral exposures into multilateral ones. This allows the CH to defer settlement and, hence, to economize on settlement costs. In this sense, a CH is subject to economies of scale in settling transactions. How much it can defer settlement depends, of course, on the participants' incentives to default on their outstanding positions. Thus, a key trade-off for designing an optimal clearing arrangement is between providing liquidity to participants in the form of credit at low cost, while at the same time ensuring proper incentives for carrying out and settling transactions. In contrast, when OTC trades clear bilaterally, costs are incurred *per trade* and immediately.

This raises the question of how a CH can support efficient trade in financial markets by economizing on settlement costs while, at the same time, providing incentives to settle the obligations arising from transactions. Can a CH operate in OTC markets and, if so, is it beneficial if it also operates in an exchange? Furthermore, how do the costs associated with clearing and settlement influence the amount of liquidity that is optimally granted by the CH?<sup>3</sup>

As traders have no choice but to settle through the CH if they trade in the exchange, we find that a CH operating in an exchange can use its monitoring power to extract surplus from transactions. For a CH to clear OTC transactions, however, it needs to offer a cost advantage that will be passed on to traders in the OTC market, as such traders have the option to bypass the CH and clear bilaterally.

If the cost advantage *per OTC transaction* is too small, a CH that only operates on the OTC market is not attainable. A joint CH for both markets, however, can extract surplus from its exchange clearing and use it to subsidize clearing costs for OTC transactions. By

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<sup>3</sup>The optimal design of a CH will also depend on a number of other factors such as the exact nature of the financial transactions cleared, heterogeneity among traders, or concerns about contagion if collateral is fungible across markets. We will abstract from these issues in what follows.

inducing OTC trades to clear within the CH, the CH increases cost-savings for each OTC transaction. Due to increasing returns, a CH also saves on overall settlement costs and, hence, increases overall welfare. In a broader context, our findings suggest that it is optimal to set interdependent clearing rules across markets whenever information on trades differs across these markets.

Conditions in the OTC market will determine whether it is feasible for a CH to operate across markets. We demonstrate that the larger the value of OTC transactions, the less viable a CH is in the OTC market. This result depends not only on the surplus generated by OTC trades, but also on the market power of sellers in the OTC markets. When sellers have more market power, buyers have to pay more for a given transaction. Thus, the CH faces a tighter default constraint when settlement is deferred and sellers' market power increases. If one thinks of dealers as OTC sellers with a relatively large market power, this implies that clearing OTC trades through a CH will be more difficult when dealers represent a large share of OTC trades.

How do settlement costs influence these results?<sup>4</sup> The risk of a settlement default increases with the cost of settling obligations. It is, thus, optimal for the CH to tighten credit in order to neutralize the increase in default risk – or, in other words, to tighten liquidity provision when the costs of providing such liquidity increase.<sup>5</sup> Hence, an increase in settlement costs results in a lower volume of transactions. More interestingly, it is optimal for the CH to decrease the size of transactions and, at the same time, to increase the frequency of settlement. Thus, while liquidity decreases, the overall costs of liquidity in the CH increase. The reason is intuitive. An increase in the fixed costs associated with settlement also increases the incentives to default. To balance this risk, the CH must ensure that participants have the incentive to settle. This is achieved by lowering the total credit available in the sys-

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<sup>4</sup>For evidence on strategic settlement failures related to the actual settlement costs, see, for example, Garbade and Fleming (2002).

<sup>5</sup>There is some recent literature studying liquidity (credit) provision in payment systems. Examples include Martin (2004), who studies the optimal pricing of intra-day credit in the presence of moral hazard, and Martin and McAndrews (2007), who analyze the use of a queueing mechanism for economizing in liquidity costs. Rather than restricting attention to a particular form of credit provision, we analyze the general interaction between credit and liquidity provision in a payment or settlement system.

tem.<sup>6</sup> To our knowledge, this result is not present in the existing literature, which does not consider endogenous incentive effects and simply trades off exogenous default costs against the opportunity costs of funding liquidity.<sup>7</sup>

To obtain these results, we build on the framework of a payment system developed by Koepl, Monnet and Temzelides (2007). Traders are randomly matched, and a transaction can occur if one trader has a preference for holding an asset (say, a contract), while the other trader is willing to sell (write) a contract. For concreteness, we call the two traders *buyer* and *seller*, respectively. Furthermore, we assume that there is an optimal transaction size and that there are immediate benefits and costs of a transaction for the buyer and the seller, respectively.

We model the CH as an authority that records transactions and provides credit. Writing a contract requires a payment from the buyer to the seller, and the CH assigns individual balances and specifies how these balances are updated given the participants' trading histories. Sellers are rewarded for writing a contract through balance increases, while buyers incur costs through balance decreases. These rewards and costs materialize when the resulting balances, which summarize the traders' credit positions, are settled with the CH.<sup>8</sup> Since settlement is costly, participants need an incentive to settle their outstanding balances. As a result, the CH might have to limit the overall volume of transactions or, equivalently, the total available supply of liquidity.

We further assume that a fraction of trades are monitored by the CH, while others are

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<sup>6</sup>This is reminiscent of results in the literature on real time gross versus net settlement systems. In a real time gross settlement system (RTGS), trades are settled at the time of the transaction. This imposes large liquidity requirements on banks. As shown by Kahn, McAndrews and Roberds (2003), such requirements can potentially create gridlocks in the system. Central banks remedy this problem by providing (costly) intra-day credit. Kahn and Roberds (2001) argue that the main costs of RTGS systems are associated with the cost of obtaining credit from the central bank. Net settlement, whereby obligations are netted and then settled, offers banks short-run credit automatically and, hence, introduces the potential for default. Kahn and Roberds (1998) show that this incentive should be seriously taken into account when considering the optimal payment system. Such a system must balance saving on liquidity costs against risking default when extending credit.

<sup>7</sup>See, for example, Berger et al. (1996). Fujiki et al. (1999) provide an early study of default in payment systems from an incentive perspective.

<sup>8</sup>One can think, for example, of a large-value system where all intra-day positions are settled in the overnight market using funds obtained through interbank lending.



not.<sup>9</sup> We identify the first with trades in an exchange and the second with OTC trades. Upon default, traders are excluded from clearing through the CH. Hence, they can no longer carry out trades in the exchange. However, they can still participate in OTC transactions which are then cleared bilaterally. Finally, we assume that there are differences in clearing costs. A CH incurs an aggregate fixed cost whenever it requires its participants to settle, whereas each OTC trade is settled at an individual cost.

Subsidizing OTC trades is necessary since individual traders in these markets do not take into account the positive externality from clearing through the CH. Indeed, we demonstrate that, under certain conditions, it is optimal for the CH to operate across both OTC markets and exchanges. This is because a CH can extract surplus from clearing in the exchange in order to cross-subsidize the clearing of OTC transactions.

The paper proceeds as follows. Section 2 introduces our model and discusses trading and settlement in OTC transactions and in exchange trades. In Section 3 we analyze how information is used within a clearinghouse and demonstrate that it can be optimal for a clearinghouse to operate across OTC markets and the exchange. Section 4 studies how liquidity costs affect the optimal transaction volume. In Section 5 we further discuss clearinghouses and their costs, and we discuss some of the limitations of our analysis. The Appendix contains our proofs and introduces a continuous-time extension of the model used in Section 4.

## 2 Financial Trades and Settlement

### 2.1 The Environment

There is a continuum of infinitely lived traders. Time is denoted by  $t = 0, 1, 2, \dots$ . Traders discount the future according to a common discount factor  $\beta \in (0, 1)$ . We assume a periodic pattern of length  $n$ . Each *transaction stage*, consisting of a sequence of  $n$  bilateral trans-

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<sup>9</sup>For some early work on issues of private information and payments, see Aiyagari and Williamson (2000) and Temzelides and Williamson (2001).

actions, is followed by a *settlement stage*.<sup>10</sup> Discounting applies after each period, except between the last transaction stage and the settlement stage. We describe the transaction stage and the settlement stage in turns.

Traders are randomly and bilaterally matched in each round of the transaction stage. A trader can either sell or buy contracts to/from the trader he is matched with, each with probability  $\gamma$ . Thus, during each transaction stage, a trader is in a *trade meeting* with probability  $2\gamma$ , either as a seller or as a buyer. We assume that the *possibility of trading is private information* for the traders.

There are two types of trade meetings in each transaction stage. Trades can either take place in an *exchange*, with probability  $\alpha$ , or “*over-the-counter*” (OTC), with probability  $1 - \alpha$ . All contracts that are traded in the transaction stage are perfectly divisible and in zero net supply between the two counter-parties.

If a trader writes or sells  $q \geq 0$  contracts traded in the exchange, he incurs a one-time cost  $c(q)$  for holding this position. If a trader buys  $q \geq 0$  contracts in the exchange, he enjoys a one-time benefit equal to  $u(q)$ . Similarly, in OTC transactions, we assume a cost  $\sigma c(q)$  for the seller and a respective benefit  $\sigma u(q)$  for the buyer, where  $\sigma \geq 1$ . We assume that  $u(q)$  is increasing and concave, while  $c(q)$  is increasing and convex. Furthermore, we restrict attention to trades that are stationary and symmetric, and assume that there is a unique optimal transaction size that we denote by  $q^* > 0$ , reflecting the maximum surplus from trading. Since we concentrate on this quantity for most of the paper, we will simplify notation by letting  $u$  denote  $u(q^*)$  and  $c$  denote  $c(q^*)$ .

We will use this setup to study financial transactions. A buyer would like to take a long position in a contract, since he derives some benefits from it. For example, the buyer might need to purchase the contract in order to hedge, or he might wish to take a position in a customized derivative or futures contract. By writing the contract, the seller is willing to take the opposite short position. Creating this position, however, is costly. Writing an

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<sup>10</sup>Lagos and Wright (2005) introduced similar periodic trading patterns in monetary models. The continuum assumption precludes aggregate risk. Issues related to optimal market design in the presence of aggregate risk are of great interest, but beyond the scope of this paper.

option or futures contract implies no cap on the losses the issuer can incur. Hence, traders are unlikely to take any arbitrary high position in such contracts. Thus, there is an optimal size for the trade, with quantity  $q^*$  maximizing the joint surplus, irrespective of the type of trade. The assumption that  $\sigma \geq 1$  simply scales up the surplus from OTC trades. Still, while traders prefer OTC contracts, they are also more expensive to write.<sup>11</sup>

To analyze different clearing arrangements, we assume that information for the CH varies across transactions. First, recall that the possibility to trade is always private information for traders. When a trade takes place in the exchange (probability  $\alpha$ ), the CH can monitor the trade; i.e., it observes the traders' identity and their position with the CH, if any. Trades that take place in an exchange are always cleared through the CH.<sup>12</sup> In an exchange, clearing is *not a choice* for the trading parties, but part of the transaction according to the clearinghouse rules. When a trade is OTC (probability  $1 - \alpha$ ), the trade cannot be monitored by the CH. Such a transaction is only observable to the CH if the traders submit it for clearing. Hence, OTC traders have a choice over the clearing arrangement – clearing bilaterally or via a clearinghouse. We study these differences in more detail next.<sup>13</sup>

## 2.2 Clearinghouses

We begin this section by introducing the idea of a clearinghouse (CH). A CH records past transactions by assigning *balances* to traders. In addition, it specifies rules for how these balances are updated according to reported or observed transactions. We will refer to this set of rules as the *Payment System* associated with the CH.

The settlement of obligations (i.e., the actual payment) arising from financial transactions usually occurs at a later time than the transaction itself. In our model, settlement occurs periodically through trading a general asset during the settlement stage. Delivering  $\ell$  units

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<sup>11</sup>Here we take the stance that an OTC contract is better suited to the buyer's needs than a centralized contract. The analysis would go through if we were to assume  $\sigma < 1$  instead.

<sup>12</sup>This is reminiscent of anonymous computerized trading where settlement is automatically enacted after a trade.

<sup>13</sup>There are other differences between trading in OTC markets and trading in centralized markets (for example, in the way prices are determined, and in how trades are intermediated and executed). We abstract from these aspects in order to focus on the effects of differences in the information structure.

of a general asset implies a cost of  $-\ell$ , while obtaining  $\ell$  units gives a benefit  $\ell$ . Thus, trading this general financial asset is zero-sum and does not directly increase welfare. Nonetheless, actual settlement usually involves opportunity or funding costs that go beyond the direct cost of payments. In order to capture these costs in our model, we introduce an aggregate (average) resource cost  $\delta > 0$  whenever settlement occurs. This cost has to be paid in units of the general asset.

The settlement stage is not subject to informational frictions and is centralized; i.e., we model it as a direct transaction between traders and the CH. Balances are denominated in terms of the general asset, with  $d$  units of balances indicating a claim to receive  $d$  units of the general asset. Traders with low balances can increase their balances by delivering the asset, while those with high balances can reduce them by obtaining the general asset. Finally, we assume that the CH can permanently prevent traders who do not settle their balances from using its facility for settling trades in the future. This automatically excludes traders from all trades in the exchange.<sup>14</sup>

Formally, consider the problem of a trader during settlement at time  $t$ . Let  $V_t(d_t)$  denote his value function if he exits the last period in the transactions stage with a balance  $d_t \in \mathbb{R}$ . The problem of the trader is given by

$$V_t(d_t) = \max_{\ell_t, \hat{d}_t} -\ell + \beta E_t v_{t+1}(\hat{d}_t) \quad (1)$$

subject to

$$\hat{d}_t = d_t + \ell_t \quad (2)$$

$$\hat{d}_t \geq \min\{0, d_t\}, \quad (3)$$

where  $E_t v_{t+1}(\hat{d}_t)$  denotes the expected future value of a trader who exits the settlement stage with balance  $\hat{d}_t$ . The inequality constraint (3) rules out borrowing from the CH in order

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<sup>14</sup>This punishment might seem unrealistically harsh. Alternatively, we could assume that traders that default are allowed to participate only in certain trades, or that they are allowed to re-enter the system after a few periods of exclusion. In general, increasing the value of the outside option will increase the probability of default. Nonetheless, our results will remain qualitatively unchanged under any such specification of the outside option.

to consume the general asset. Notice that  $V_t$  is linear in  $d_t$ , which greatly simplifies our analysis.

During the transaction stage, traders are matched bilaterally in each period. Traders in the exchange who choose to trade are instructed to exchange a quantity  $q_t(d_t, d'_t)$  of the asset. Similarly, OTC traders can choose to identify themselves to the CH as being in a potential trade meeting. The potential seller is then instructed to sell  $\bar{q}_t(d_t, d'_t)$  of the asset to the potential buyer. Note that these quantities can depend on both traders' balances.<sup>15</sup>

Upon receiving the reports on OTC trades and the asset trading in the exchange, the CH makes an *adjustment*,  $X_t(d_t, d'_t) \in \mathbb{R}$  to each trader's balance. Recall that, in any given period, a trader can be a buyer, a seller, or he might not trade at all. In addition, each transaction is either monitored by the exchange, or is an OTC transaction submitted for central clearing. This results in six possible adjustments; i.e.,  $X_t \in \{L_t, K_t, B_t, \bar{L}_t, \bar{K}_t, \bar{B}_t\}$ . More precisely,  $L_t(K_t)$  is the adjustment for a buyer (seller) in the exchange, with the variables  $\bar{L}_t, \bar{K}_t$  defined for OTC transactions. The remaining adjustments refer to the case of a trader who reports not having any trade opportunity. We can then formally define the CH's rules.<sup>16</sup>

**Definition 1.** A Payment System *is an array*

$$S_t(d_t, d'_t) = \{L_t, K_t, B_t, q_t, \bar{L}_t, \bar{K}_t, \bar{B}_t, \bar{q}_t\}, \text{ for all } t.$$

We restrict attention to payment systems (PS) that are *incentive feasible*. We term a PS incentive feasible if (i) all traders have an incentive to participate in each transaction and do not default in the settlement stage, (ii) all OTC and exchange traders truthfully reveal their type, and (iii) the CH breaks even. The last requirement implies that, in each settlement

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<sup>15</sup>Throughout the paper, we apply a “mechanism design” approach to payments. Thus, when a buyer and a seller transact, it is *as if* the CH proposes both the quantity traded and the implicit price. Of course, we ensure that the prescribed trades are incentive compatible for the participating traders. In fact, we require that the realized trades are efficient. Our analysis, thus, abstracts from the implementation issue of how such trades would materialize in an actual market. We justify restricting attention to efficient trading by simply postulating that traders will find a way to explore all possible gains from their transactions.

<sup>16</sup>In principle, the payment system could also be a function of the distribution of balances (see Koepl, Monnet and Temzelides (2008)). This, however, is not important in our context.

stage,

$$\int_d (\widehat{d}_t - d_t) d\Psi_t + \delta = 0, \quad (4)$$

where  $\Psi_t$  is the distribution of balances across traders at  $t$ .<sup>17</sup>

To formulate incentive feasibility, we first describe the traders' value functions during the transaction stage. Recall that there are  $n$  bilateral rounds (one in each period) between settlement stages. To ease notation, we denote the current-period immediate return in period  $t$  by  $f(X_t)$ , where  $X_t \in \{L_t, K_t, B_t, \bar{L}_t, \bar{K}_t, \bar{B}_t\}$ . Thus,  $f(\bar{L}_t) = \sigma u(\bar{q}_t)$ ,  $f(\bar{K}_t) = -\sigma c(\bar{q}_t)$  and  $f(\bar{B}_t) = 0$ . Similarly, for monitored trades,  $f(L_t) = u(q_t)$ ,  $f(K_t) = -c(q_t)$  and  $f(B_t) = 0$ .

If the last settlement stage occurred in period  $t$ , the value function during each round  $s$ ,  $s = 1, \dots, n-1$ , of the current transaction stage is given by

$$E_{t+s-1}[v_{t+s}(d_{t+s-1})] = \int_{d'_{t+s}} E[f(X_{t+s}) + \beta E_{t+s}[v_{t+s+1}(d_{t+s-1} + X_{t+s})]] d\Psi_{t+s-1}, \quad (5)$$

where  $E$  denotes the expectation over the meeting the trader will be in during the current period. For the last period of the transaction stage,  $t+n$ , we have

$$E_{t+n-1}[v_{t+n}(d_{t+n-1})] = \int_{d'_{t+n}} E[f(X_{t+n}) + V(d_{t+n-1} + X_{t+n})] d\Psi_{t+n-1}. \quad (6)$$

Traders may decide to leave the CH at any point. They can choose to default and take the outside option of trading only OTC in the future. The value of this outside option,  $V_0$ , is endogenous and will be derived later. For the settlement stage, we have the following no default constraint:

$$V(d_{t+n-1} + X_{t+n}) \geq \beta V_0. \quad (7)$$

Of course, traders also need an incentive to participate in clearing through the CH during

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<sup>17</sup>This immediately implies that in each settlement stage the net production of the general asset covers the settlement costs; i.e.,  $\int_{d_t} \ell_t d\Psi_t = -\delta$ .

the transaction stages. Thus, if the last settlement stage occurred at  $t$ , we require that

$$f(X_{t+s}) + \beta E_{t+s}[v_{t+s+1}(d_{t+s-1} + X_{t+s})] \geq g(X_{t+s}) + \beta V_0, \quad (8)$$

for all  $X_{t+s}$ , and all  $s = 1, \dots, n-1$ . Finally, for  $s = n$  we require that

$$f(X_{t+n}) + V(d_{t+n-1} + X_{t+n}) \geq g(X_{t+n}) + \beta V_0 \quad (9)$$

for all  $X_{t+n}$ . Note that  $g(X)$  is the current period return of the outside option and depends on the type of meeting a trader is in. For OTC transactions we have  $g(\bar{X}) \geq 0$ , reflecting the fact that traders can still trade OTC with bilateral clearing. Here, the CH takes the terms of such trades as given, which pins down the current period return of the outside option. On the other hand, traders in the exchange need to clear through the CH. If they choose not to trade, we simply have  $g(X) = 0$ .

Traders can also mis-report their transactions. Assuming that the last settlement stage occurred at time  $t$ , a PS is incentive compatible during each round  $s$ ,  $s = 1, \dots, n-1$ , of the current transactions stage if

$$f(X_{t+s}) + \beta E_{t+s}[v_{t+s+1}(d_{t+s-1} + X_{t+s})] \geq f(B_{t+s}) + g(X_{t+s}) + \beta E_{t+s}[v_{t+s+1}(d_{t+s-1} + B_{t+s})]. \quad (10)$$

For  $s = n$ , incentive compatibility requires that, for all  $X_{t+n}$ ,

$$f(X_{t+n}) + V(d_{t+n-1} + X_{t+n}) \geq f(B_{t+n}) + g(X_{t+n}) + V(d_{t+n-1} + B_{t+n}). \quad (11)$$

In words, these constraints require that reporting the true state and clearing through the CH gives at least as high an expected utility as trading OTC or not trading in the exchange. Notice that a trader can misrepresent a potential transaction by indicating to the CH that he did not have an opportunity to trade. The value of lying depends again on whether a trader faces a potential OTC transaction with bilateral clearing or not ( $g(\bar{X}) \geq 0$  vs.  $g(X) = 0$ ).

To conclude this section, it is useful to specify a particular type of PS that we will use extensively in what follows. A PS is *simple* if balance adjustments do not depend on the traders' current balances. Hence, a simple PS does not condition on a trader's history. A PS is *simple* and *repeated* (SRPS) if, in addition, it satisfies that

$$X_{t+s} = \frac{X_{t+n}}{\beta^{n-s}} \quad (12)$$

$$X_{t+kn} = X, \quad (13)$$

where  $X \in \{L, K, B, \bar{L}, \bar{K}, \bar{B}\}$ ,  $s = 1, \dots, n$ , and  $k \in \mathbb{N}$ . In the above expressions,  $t$  represents the date of settlement. In words, adjusting for discounting, a repeated PS imposes the same balance adjustments in each period of the transactions cycle. In what follows, we shall concentrate on SRPS and will simply refer to them as PS.

### 2.3 OTC Trades and Bilateral Settlement

We now determine the outside option when trading OTC. If presented with an opportunity to trade OTC, traders can choose to remain anonymous to the CH. Then the buyer and seller engage in Nash bargaining to determine a spot price and the size of the transaction. This trade is then immediately settled by producing the general asset on the spot.<sup>18</sup> Immediate settlement, however, imposes a fixed cost,  $\tau$  – without loss of generality imposed on buyers – where  $\tau \leq \sigma(u - c)$ .

Denoting the relative bargaining power of buyers by  $\eta \in [0, 1]$ , the traders agree on the quantity  $q$  and the price  $p$  so as to solve<sup>19</sup>

$$\max_{p,q} (\sigma u(q) - p - \tau)^\eta (p - \sigma c(q))^{1-\eta}. \quad (14)$$

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<sup>18</sup>Due to the random matching friction, traders cannot commit to an action toward each other in the future. Hence, if the OTC contract is delivered today, settlement must take place on the spot.

<sup>19</sup>The generalized bargaining problem includes a continuation value for both parties and the outside option of not trading at all today as a threat point. These are both given by  $f(\bar{B}_{t+s}) + \beta E_{t+s}[v_{t+s+1}(d_{t+s-1} + \bar{B}_{t+s})]$  and therefore cancel out.



Notice that linearity in the general good causes the surplus to be independent of the bargaining power; i.e., utility is transferable between the buyer and the seller. Hence, as long as the efficient quantity  $q^*$  is produced, the total surplus is shared proportionally to the bargaining power,  $\eta$ . The first order conditions reflect this and are given by

$$q = q^* \tag{15}$$

$$p = (1 - \eta)\sigma u + \eta\sigma c - (1 - \eta)\tau. \tag{16}$$

This leads to the current period pay-offs from OTC trades. These are given by  $g(\bar{L}_{t+s}) = \eta[\sigma(u - c) - \tau]$  for buyers and by  $g(\bar{K}_{t+s}) = (1 - \eta)[\sigma(u - c) - \tau]$  for sellers, with  $g(\bar{B}_{t+s}) = 0$ . Finally, the payoff in autarky,  $V_0$ , is defined as the expected payoff from trading only OTC forever,

$$(1 - \beta)V_0 = (1 - \alpha)\gamma[\sigma(u - c) - \tau], \tag{17}$$

and is independent of the bargaining power.<sup>20</sup>

### 3 Efficient Use of Information

In the previous section we described the trading environment and possible clearing arrangements. We will now study three different scenarios and characterize conditions under which a payment system can implement the first-best allocation, in which the efficient contract  $q^*$  is exchanged in *all* transactions. Formally, we concentrate on the following.

**Definition 2.** *A PS is optimal if it is incentive feasible, and if it leads to the efficient level of asset trading,  $q^*$ , in all transactions.*

We will first study exchange trades ( $\alpha = 1$ ) and characterize under what specifications of

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<sup>20</sup>It is useful to differentiate our setup from the one used by Duffie, Garleanu and Pedersen (2005 and 2007) to study OTC markets. Apart from minor modelling differences, we add settlement as a feature associated with every transaction, and we concentrate on efficient trading outcomes. Rather than focusing on a particular trading protocol, we use mechanism design in order to specify a PS that induces traders to carry out transactions efficiently. Thus, we abstract from frictions related to pricing and bargaining, which are the focus of Duffie, et al. (2005 and 2007).

preferences and costs an optimal payment system exists. We then study the same question for OTC trades ( $\alpha = 0$ ). Finally, we investigate the situation where a CH operates across both markets ( $\alpha \in (0, 1)$ ). In the Appendix we offer a brief discussion of the nature of the constraints a CH faces under a SRPS.

### 3.1 Simple Repeated Payment Systems

We denote the minimum balance adjustment in any given period by  $X_t^{\min} = \min\{L_t, K_t, B_t, \bar{L}_t, \bar{K}_t, \bar{B}_t\}$ . We also normalize the required starting balance after the settlement stage in period  $t$  to  $\hat{d}_t = 0$ . This simply implies that we start off the economy with a zero net position between the CH and all traders. The CH excludes from future clearing traders who do not reset their balances to zero in the settlement stage. This implies that the distribution of balances,  $\Psi_t$ , at the end of each settlement stage is degenerate.

Under a SRPS, the break-even condition for the CH (4) becomes

$$\sum_{s=1}^n [\alpha [\gamma (K_{t+s} + L_{t+s}) + (1 - 2\gamma)B_{t+s}] + (1 - \alpha) [\gamma (\bar{K}_{t+s} + \bar{L}_{t+s}) + (1 - 2\gamma)\bar{B}_{t+s}]] + \delta = 0,$$

or

$$\alpha [\gamma (K + L) + (1 - 2\gamma)B] + (1 - \alpha) [\gamma (\bar{K} + \bar{L}) + (1 - 2\gamma)\bar{B}] = -\delta(n), \quad (18)$$

where

$$\delta(n) = \delta \frac{\beta^n (1 - \beta)}{\beta (1 - \beta^n)}. \quad (19)$$

Such payment systems are convenient since the linearity of  $V$  implies that the incentive constraints for all  $s$ ,  $s = 1, \dots, n - 1$  are fulfilled whenever those for  $s = n$  hold. Similarly, all participation constraints are fulfilled as long as traders have no incentive to default in the settlement stage. One only needs to look at the worst adjustment among all traders, which is given by  $\sum_{s=1}^n X_{t+s}^{\min}$ . This is summarized in the following Lemma. We relegate the proofs to the Appendix.

**Lemma 3.** *All incentive constraints are fulfilled as long as*

$$\begin{aligned}
f(K_{t+n}) + K_{t+n} &\geq f(B_{t+n}) + B_{t+n} \\
f(L_{t+n}) + L_{t+n} &\geq f(B_{t+n}) + B_{t+n} \\
f(\bar{K}_{t+n}) - g(\bar{K}_{t+n}) + \bar{K}_{t+n} &\geq f(\bar{B}_{t+n}) + \bar{B}_{t+n} \\
f(\bar{L}_{t+n}) - g(\bar{L}_{t+n}) + \bar{L}_{t+n} &\geq f(\bar{B}_{t+n}) + \bar{B}_{t+n}.
\end{aligned}$$

*Furthermore, all participation constraints hold as long as traders with the worst possible balance adjustment have no incentive to default in the settlement stage and carry out the transaction in the last stage*

$$\begin{aligned}
V\left(\sum_{s=1}^n X_{t+s}^{\min}\right) &\geq \beta V_0 \\
f(X_{t+n}) + V\left(\sum_{s=1}^{n-1} X_{t+s}^{\min} + X_{t+n}\right) &\geq g(X_{t+n}) + \beta V_0,
\end{aligned}$$

*for all  $X_{t+n}$ .*

### 3.2 A Clearinghouse for the Exchange

We now consider the case where a CH only clears trades that take place in an exchange. This is equivalent to setting  $\alpha = 1$ . The CH needs to ensure that traders in the exchange report their trading opportunities truthfully. In other words, it needs to adjust settlement balances in order to elicit truthful revelation. The following result gives necessary and sufficient conditions for this to occur.

**Proposition 4.** *Suppose  $\alpha = 1$ . There exists an optimal PS if and only if*

$$\beta^n u - c \geq \frac{\delta(n)}{\gamma}. \tag{20}$$

The corresponding balance adjustments are given by

$$\begin{aligned} B_{t+n} &= L_{t+n} = -\delta(n) - \gamma c \\ K_{t+n} &= c + L_{t+n}. \end{aligned}$$

Notice that if  $n = 0$ , the condition reads  $u \geq c$ , while if  $n \rightarrow \infty$  the condition is violated. Therefore; given  $u > c$ , there exists  $\bar{n} \geq 1$  such that the condition holds for all  $n < \bar{n}$  and fails otherwise. In other words, if the settlement frequency is sufficiently high, there is an optimal PS.

It is instructive to rewrite our condition (20) in Proposition 4 as

$$\frac{\beta}{1-\beta} \gamma(u-c) - \frac{1}{1-\beta^n} \delta - \gamma c \frac{\beta(1-\beta^n)}{\beta^n(1-\beta)} \geq 0. \quad (21)$$

The first two terms in this expression give the total discounted net present value of the surplus from staying with the clearinghouse. This consists of the value of being able to transact in the future, net of the future settlement costs associated with trading. The last term gives the total outstanding balances for a trader that has consumed in every single transaction stage throughout the cycle. Hence, the difference between these terms is the net benefit from not defaulting for the trader that has the strongest incentive to default. If this net benefit is positive, all traders will transfer enough of the general asset to the CH to settle their trades.<sup>21</sup>

Finally, a comment is in order regarding the optimal payment system. One might have expected that the balance adjustment if no transaction takes place should be zero ( $B_{t+n} = 0$ ). This would imply that  $K_{t+n} = c$  and  $L_{t+n} = -(\delta(n)/\gamma) - c$ . While there are parameters for which such a payment system is incentive feasible, our findings assert that it is not the one that can implement the efficient transaction size  $q^*$  for the largest set of parameters. Having

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<sup>21</sup>Our approach does not offer a theory of which party in a transaction should cover the settlement costs. As long as an optimal PS exists, what matters for incentives are the *overall* balance adjustments (including costs). Hence, the PS can levy the costs arbitrarily on buyers, sellers, or non-trading members.

$B_{t+n} < 0$  can be interpreted as a fixed membership fee for participating in the CH.<sup>22</sup> Our model can be thought of as providing a possible justification for such fees.

### 3.3 A Clearinghouse for the OTC Market

We now turn to the case where there are only OTC trades ( $\alpha = 0$ ). The CH faces a more difficult problem in this case. The reason is that now traders have the option to clear bilaterally and on the spot at an immediate cost,  $\tau$ . Thus, since the outside option for traders has improved relative to exchange trades, the PS must ensure that traders are willing to clear through the CH rather than bilaterally. We again provide a characterization of when an optimal payment system through a CH exists.

**Proposition 5.** *Suppose that  $\alpha = 0$ . There exists an optimal PS if and only if*

$$\frac{1}{1 - \beta^n} \left[ \beta^n \tau - \frac{\delta(n)}{\gamma} \right] \geq p, \quad (22)$$

where  $p = (1 - \eta)(\sigma u - \tau) + \eta \sigma c$  is the price of OTC transactions. The optimal balance adjustments are given by

$$\begin{aligned} \bar{B}_{t+n} &= \bar{L}_{t+n} = -\delta(n) - \gamma p, \\ \bar{K}_{t+n} &= \bar{L}_{t+n} + p. \end{aligned}$$

For a CH to be able to operate, it must be the case that immediate bilateral settlement is sufficiently costly relative to deferred settlement. A necessary condition for this is given by

$$\frac{\beta}{1 - \beta} \gamma \tau > \delta \frac{1}{1 - \beta^n}. \quad (23)$$

This condition compares the costs from the two ways of settling transactions. The left-hand side expresses the expected net present value of settlement costs when all transactions are settled OTC. The right-hand is the equivalent expression for settling through the CH. Notice

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<sup>22</sup>See also Monnet and Roberds (2008).

that for OTC trades, the cost is incurred after every trade, while if clearing occurs through the CH, the cost is incurred independent of the number of trades. In a stark way, this captures economies of scale for the CH relative to bilateral clearing. For a CH to be able to clear OTC trades, it must offer a large enough cost advantage to traders.

### 3.4 A Joint Clearinghouse: Cross-Subsidizing Transactions

We now investigate the possibility of a clearinghouse that operates across both markets. To do so, we assume that  $\alpha \in (0, 1)$ , so that some transactions take place in an exchange, while others are OTC. When a CH operates across both markets, it can modify its clearing rules to affect the incentives to default, depending on where a transaction takes place. This opens up the possibility of cross-subsidizing different transactions in order to induce traders to clear trades within the CH. In particular, we demonstrate that, under certain conditions, a CH would not be viable if restricted to OTC trades but would become feasible if allowed to operate in both markets.

**Proposition 6.** *Suppose  $\alpha \in (0, 1)$ . There exists an optimal PS if and only if*

$$\alpha\gamma \left[ \frac{\beta^n}{1-\beta^n}(u-c) - c \right] + (1-\alpha)\gamma \left[ \frac{\beta^n}{1-\beta^n}\tau - p \right] \geq \frac{1}{1-\beta^n}\delta(n), \quad (24)$$

where  $p = (1-\eta)[\sigma u - \tau] + \eta\sigma c$  is the price for OTC transactions. The balance adjustments are given by

$$\begin{aligned} L_{t+n} &= \bar{L}_{t+n} = B_{t+n} = \bar{B}_{t+n} = -\delta(n) + \alpha\gamma c + (1-\alpha)\gamma p, \\ K_{t+n} &= L_{t+n} + c, \\ \bar{K}_{t+n} &= L_{t+n} + p. \end{aligned}$$

It is instructive to discuss condition (24) further. The left-hand side is a weighted average of the surplus that the CH can extract from traders in order to cover its settlement costs without inducing default. The weights reflect the relative size of exchange versus OTC

trades. Note that, in an exchange transaction, the CH can directly extract surplus from traders. For OTC transactions, however, the CH can only extract the cost-saving offered by deferred settlement through the CH (condition (22)).

This is important to understand when a joint CH is optimal. Suppose, for example, that the cost advantage offered by a CH is not high enough to make clearing of OTC trades feasible. A CH that clears trades in an exchange can then extract surplus to cross-subsidize the clearing of OTC transactions. This is ex-ante efficient (i.e., before traders learn where they have to transact), as it saves on overall clearing costs. Interestingly, subsidizing OTC trades is necessary since individual traders in these markets do not take into account the positive externality from clearing through the CH. This can be summarized as follows.<sup>23</sup>

**Proposition 7.** *An optimal PS needs to subsidize the clearing of OTC trades whenever*

$$\frac{1}{1 - \beta^n} \left[ \beta^n \tau - \frac{\delta(n)}{\gamma} \right] < p.$$

Finally, certain features of the OTC market predict whether it is possible for a CH to cross-subsidize transactions across markets. Prices in OTC transactions vary with the relative benefits generated by such transactions. However, prices also depend on whether certain traders have a large bargaining power. OTC transactions are often highly specialized and, hence, create a large surplus for some traders. In addition, dealers in such markets might have considerable market power when writing contracts. In our model, as OTC transactions become more valuable, the price in OTC transactions increases. Similarly, our model predicts that the larger the bargaining power of sellers, the larger the negotiated price as they can extract more of the surplus from the transaction. Under these circumstances, clearing OTC transactions becomes more difficult for the CH. Formally, it is straightforward to show the following.

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<sup>23</sup>Using the 2009 annual report by Goldman Sachs (GS), back-of-the-envelope calculations imply that settlement costs represent a non-negligible 8.5 percent of GS's profit margin in 2008. GS's profit margin in 2008 was 10 times larger than the minimum level necessary to avoid default. Of course, GS was probably one of the better performing member of the DTCC. Still, the fact that GS's profit margin is relatively large compared with the default profit margin leads us to conclude that there is potential for cross-subsidization.

**Corollary 8.** *Suppose  $\beta^n u - c \geq \frac{\delta(n)}{\gamma}$ . Given clearing costs  $(\tau, \delta)$ , an optimal PS with cross-subsidization exists if and only if  $\alpha \geq \underline{\alpha}(\sigma, \eta)$ . Furthermore, the critical value  $\underline{\alpha}$  increases with the surplus created in OTC transactions,  $\sigma$ , and with the bargaining power of sellers,  $(1 - \eta)$ .*

## 4 Liquidity and Incentives

Our model naturally captures the fact that infrequent settlement through a CH increases the potential for default. Of course, less frequent settlement economizes on settlement costs. Indeed, as pointed out earlier, this feature links the frequency of settlement to higher levels of liquidity. We now turn to the question of how to optimally balance such costs against the higher default exposure associated with less frequent settlement. To simplify the analysis, we now set  $\alpha = 0$ .

Conditional on achieving an efficient transaction size ( $q^*$ ), an optimal payment system should minimize the incurred costs from settlement. The CH must then choose the maximum length of the transaction stage that is compatible with optimality, given the costs,  $\delta$ , as expressed by condition (20). Such an  $n$  exists as long as  $\delta \leq \gamma(\beta u - c)$ . Note also that, if  $n$  is large enough, we have that  $\beta^n u < c$ . Thus, if settlement is sufficiently infrequent, the participation constraint of a trader that bought  $n$  times in a row will be violated. In other words, there exists a maximum  $n$  such that condition (20) is satisfied.

However, our analysis so far has not studied the issue how the length of the settlement cycle interacts with the optimal transaction size. We turn to this question next.

### 4.1 Settlement Frequency and Transaction Size

It is widely recognized that actual payment systems involve liquidity costs for participants who hold reserves in the system. Hence, clearinghouses often make provisions, such as offering short-term credit facilities, in order to economize on such costs. Offering credit, however, is not without its own costs since it might increase default risk in the system. In



this section, we use our model to study the trade-off between liquidity creation and default.

So far in our analysis, we termed a PS optimal if it decentralizes the efficient amount of asset exchange,  $q^*$ , in all transactions. In the presence of settlement costs, however, an optimal PS might need to explore the trade-off between reducing the size of transactions versus lengthening the transaction stage. We now turn to the more general problem of determining *jointly* the efficient settlement frequency *and* the efficient transaction size.

We assume that  $\delta \leq \gamma(\beta u - c)$ , and that settlement costs are small enough, so that it is optimal for settlement to occur eventually. Based on our earlier findings, since  $\alpha = 0$ , we assume that the cost  $\delta$  is shared equally across all participants, and that it is covered by production in the settlement stage. In the Appendix we use a continuous-time formulation of our model to set up the problem of jointly choosing settlement frequency and transaction size. In order for the constraint set to be convex, it is sufficient that the cost function  $c(q)$  is log-convex. Under this assumption, we derive the following.

**Proposition 9.** *Assume that  $c(q)$  is log-convex. Any optimal PS implies that  $q_t = \hat{q} < q^*$ , for all  $t$ . Furthermore, as  $\delta$  increases, the optimal transaction size,  $\hat{q}$ , as well as the optimal length of the settlement cycle,  $\hat{T}$ , decreases.*

The first part of the Proposition confirms our intuition. Given that  $q^*$  can be decentralized, we already established in the previous section that we must optimally reduce the *extensive margin* – i.e., the settlement frequency – as much as possible in order to economize on settlement costs. But it is also optimal to economize further on settlement costs by reducing the *intensive margin* – i.e., the transaction size – below its first-best level. This is equivalent to reducing market participants’ exposures prior to the settlement stage by imposing a tighter *cap* on the total number of contracts that can be exchanged. Notwithstanding, it is still optimal to have a fixed transaction size across trades and across time.

The second part of the Proposition is somewhat surprising. It asserts that as a response to an increase in settlement costs, an optimal PS must adjust *both*  $\hat{q}$  and  $\hat{T}$  in the same direction. In other words, it must reduce the volume of balance adjustments that need to be settled in two complementary ways: shorten the length of the transaction stage *and* reduce

the transaction size. The explanation for this is as follows. The binding constraint is the participation constraint of a trader that has made the most purchases during the transaction stage and, as a result, has to settle a large negative balance. An increase in  $\delta$  makes it more likely that this constraint will be violated. Hence, in order to avoid default, we must decrease the potential exposure of this trader by reducing his negative balance adjustments. This involves reducing both the quantity produced *as well as* the time between settlement periods.

## 5 Discussion

There is an on-going policy discussion on whether the introduction of clearing arrangements in OTC markets will lead to superior outcomes. In this paper we used mechanism design to analyze the optimal organization of clearing across markets. This approach abstracts from certain characteristics of the CH, such as its ownership and its governance structure. In reality CHs are often operated in conjunction with a particular market or trading engine, thus reflecting a basic natural monopoly arising from increasing returns within the market or trading environment. If a CH operates as a monopoly, it is commonly either user-owned – i.e., it maximizes the benefits of its users –, or it is regulated to recover costs. In some cases, there is limited competition between CHs where one provider is public while other providers might be private.<sup>24</sup> In such cases, the public system is usually required to recover its costs in order to ensure “fair” competition.<sup>25</sup> We believe that this evidence supports our basic modelling approach of studying a planner’s problem, where all efficiency gains are passed on to traders, but additional subsidization from outside the trading environment is not feasible. At the very least, our model can be considered as normative, in other words, as identifying the rules under which optimal payment systems should operate.

Settlement costs are often neglected when looking at financial trades. One can argue that

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<sup>24</sup>An example is the European clearing landscape where the ECB is currently developing an alternative clearing and settlement engine, *Target2S*, for cross-border trading. This system is expected to be in competition with existing private providers.

<sup>25</sup>For example, the (publicly operated) *Fedwire* is required to break even.

these include not only operational costs related to clearing and settling a transaction, but also, and perhaps more importantly, the opportunity costs of holding liquidity in order to settle. One should interpret our settlement costs in this broad sense. We do not take a stance in this paper on the relative size of costs between the two different modes of settlement ( $\tau$  vs.  $\delta$ ). Nonetheless, we capture their essential differences. One refers to a per transaction cost, while the other is a total (average) cost. This distinction offers a straightforward way of incorporating increasing returns in clearing through a CH.

On the surface, direct operational costs seem quantitatively small, being in the order of a few cents per transaction. However, these add up over a typical transaction cycle taking place over a few days, as the volume of transactions can be very large.<sup>26</sup> More importantly, there are considerable fixed costs and costs for auxiliary services (such as custody fees) that are associated with being a member of a clearinghouse.<sup>27</sup>

Moreover, as pointed out earlier, one should also consider costs that are related to the in-house costs of settlement – related to the operation of a back-office and, more significantly, to the necessity to either hold liquid reserves for settlement, or to obtain such liquidity through costly short-term borrowing (e.g., in overnight markets). All these considerations make settlement costs a non-negligible issue for the organization of clearing across markets.

Finally, in order to concentrate on the role of differential information on clearing trades, we have abstracted from some interesting features of payments. Using SRPS is an obvious limitation, but it greatly sharpens the analysis. Similarly, a CH can rely on other risk management techniques (such as collateral requirements) in order to control their default exposure. Another short-coming of our analysis is that traders are assumed to have deep pockets that allow them to settle their obligations. Instead, we could introduce a trade-off by letting traders with extreme positions default, while lengthening the settlement cycle and, hence, saving on settlement costs. Incorporating positive levels of equilibrium default is a difficult, but interesting, extension which we leave for future research.

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<sup>26</sup>Indirect evidence on their importance is the heated discussion in Europe over the last few years concerning the reduction of cross-border settlement costs.

<sup>27</sup>For an example of such fees see the DTCC fee guide (DTCC (2009)).

## 6 Appendix

### 6.1 Proof of Lemma 3

We first show that all other participation constraints hold, if the one for the worst possible adjustment up to transaction stage  $n$  holds. To facilitate the exposition, we concentrate on the case where  $\alpha = 1$ . The arguments for the case where  $\alpha < 1$  are analogous.

Having normalized  $\widehat{d}_t = 0$  for all  $t$ , the participation constraint for an optimal payment system at  $t + n - 1$  gives

$$\begin{aligned}
& f(X_{t+n-1}) + \beta E_{t+n-1}[v_{t+n}(d_{t+n-2} + X_{t+n-1})] \\
& \geq f(X_{t+n-1}) + \beta E_{t+n-1}[v_{t+n}(\sum_{s=1}^{n-2} X_{t+s}^{\min} + X_{t+n-1})] \\
& = f(X_{t+n-1}) + \beta \left[ \gamma(u - c) + E_{t+n-1}[V(\sum_{s=1}^{n-2} X_{t+s}^{\min} + X_{t+n-1} + X_{t+n})] \right] \\
& = f(X_{t+n-1}) + \beta \left[ \frac{1}{1-\beta} \gamma(u - c) - \delta \frac{\beta^n}{1-\beta^n} + \left( \sum_{s=1}^{n-2} X_{t+s}^{\min} + X_{t+n-1} \right) + E[X_{t+n}] \right] \\
& = f(X_{t+n-1}) + \beta X_{t+n-1} + \frac{\beta}{1-\beta} \gamma(u - c) - \beta \delta \frac{\beta^n}{1-\beta^n} + \beta \left( \sum_{s=1}^{n-2} X_{t+s}^{\min} \right) + \beta E[X_{t+n}] \\
& = f(X_{t+n}) + X_{t+n} + \frac{\beta}{1-\beta} \gamma(u - c) - \beta \delta \frac{\beta^n}{1-\beta^n} + \left( \sum_{s=2}^{n-1} X_{t+s}^{\min} \right) - \beta \delta(n) \\
& \geq f(X_{t+n}) + X_{t+n} + \frac{\beta}{1-\beta} \gamma(u - c) + \left( \sum_{s=1}^{n-1} X_{t+s}^{\min} \right) - \beta \delta \frac{\beta^n}{1-\beta^n} - \beta \delta \frac{(1-\beta)\beta^n}{\beta(1-\beta^n)} \\
& = f(X_{t+n}) + V\left(\sum_{s=1}^{n-1} X_{t+s}^{\min} + X_{t+n}\right),
\end{aligned}$$

which is the participation constraint associated with adjustment  $X$  in the last transactions round. The last inequality follows since  $X^{\min} \leq 0$ , and since we require that the CH must break even; i.e.,  $E[X_{t+n}] = -\delta(n)$ . Hence, the participation constraints at  $t + n - 1$  hold, provided that they hold for  $t + n$ . By induction, it follows that they also hold for any  $t + s$ ,  $s = 1, \dots, n - 2$ .

Next, for the incentive constraints, we have that

$$f(X_{t+s}) + \beta E_{t+s}[v_{t+s+1}(d_{t+s-1} + X_{t+s})] \geq f(B_{t+s}) + g(X_{t+s}) + \beta E_{t+s}[v_{t+s+1}(d_{t+s-1} + B_{t+s})]. \quad (25)$$

Solving for the function  $v_{t+s+1}$  and using the linearity of  $V$ , we obtain

$$f(X_{t+s}) + \beta^{n-s} X_{t+s} \geq f(B_{t+s}) + g(X_{t+s}) + \beta^{n-s} B_{t+s}. \quad (26)$$

Finally, using the fact that  $g$  and  $f$  do not depend on time, we obtain

$$f(X_{t+n}) + X_{t+n} \geq f(B_{t+n}) + g(X_{t+n}) + B_{t+n}. \quad (27)$$

The result then follows if we set  $g(X) = 0$  for transactions in the exchange.

## 6.2 Proof of Proposition 4

Using the fact that  $f(B) = 0$  and  $V_0 = 0$  (since here there is no OTC market), we obtain the following constraints for an optimal payment system

$$-c + K_{t+n} \geq B_{t+n}, \quad (28)$$

$$u + L_{t+n} \geq B_{t+n}, \quad (29)$$

and

$$V\left(\sum_{s=1}^n X_{t+s}^{\min}\right) \geq 0, \quad (30)$$

$$f(X_{t+n}) + V\left(\sum_{s=1}^{n-1} X_{t+s}^{\min} + X_{t+n}\right) \geq 0, \quad (31)$$

for all  $X_{t+n} \in \{L_{t+n}, K_{t+n}, B_{t+n}\}$ . In addition, we have the following break-even condition for the CH:

$$\gamma(K_{t+n} + L_{t+n}) + (1 - 2\gamma) B_{t+n} = -\delta(n). \quad (32)$$

We need to solve for the PS that fulfils these equations for the largest set of parameters. This is a well-defined linear program under inequality constraints. Notice that the first incentive constraint must be binding at the solution. If not, the second binds and we would have  $L_{t+n} < B_{t+n} < K_{t+n}$ . But, we can then increase  $L_{t+n}$  and reduce the other adjustments so as to still satisfy all constraints. Hence, the first incentive constraint must bind and we have that  $K_{t+n} = B_{t+n} + c$ . This implies immediately that  $B_{t+n} = L_{t+n}$ .

Using these facts and the break even condition for the CH, we obtain

$$\gamma c + L_{t+n} = -\delta(n). \quad (33)$$

We can now use the default constraint in the settlement stage and the participation constraints to characterize the set of parameters for which an optimal payment system exists. Again, by the linearity of  $V$  and the fact that  $K_{t+n} = L_{t+n} + c$ , all constraints hold if and only if the trader with the worst possible adjustment does not have an incentive to default in the settlement stage, or, if

$$V\left(\sum_{s=1}^n X_{t+s}^{\min}\right) = \sum_{s=1}^n L_{t+s} + \frac{\beta}{1-\beta}\gamma(u-c) - \delta\frac{\beta^n}{1-\beta^n} \geq 0. \quad (34)$$

Using condition (33), we obtain

$$V\left(\sum_{s=1}^n X_{t+s}^{\min}\right) = L_{t+n}\frac{\beta(1-\beta^n)}{\beta^n(1-\beta)} + \frac{\beta}{1-\beta}\gamma(u-c) - \delta\frac{\beta^n}{1-\beta^n} \quad (35)$$

$$= -\gamma c\frac{1-\beta^n}{\beta^n} + \gamma(u-c) - \delta\frac{1-\beta}{\beta}\frac{1}{1-\beta^n}. \quad (36)$$

This completes the proof.

### 6.3 Proof of Proposition 5

Since  $\alpha = 0$ , the value of trading outside the CH is now given by only trading OTC, or,  $(1 - \beta)V_0 = \gamma(\sigma(u - c) - \tau)$ . Using the linearity of  $V(X)$ , the fact that  $f(\bar{B}) = 0$ , and the expressions for  $g$ , we obtain the following incentive constraints

$$-\sigma c - (1 - \eta)[\sigma(u - c) - \tau] + \bar{K}_{t+n} \geq \bar{B}_{t+n}, \quad (37)$$

$$\sigma u - \eta[\sigma(u - c) - \tau] + \bar{L}_{t+n} \geq \bar{B}_{t+n}. \quad (38)$$

For the default and participation constraints we have

$$V\left(\sum_{s=1}^n X_{t+s}^{\min}\right) \geq \beta V_0, \quad (39)$$

and

$$f(X_{t+n}) + V\left(\sum_{s=1}^{n-1} X_{t+s}^{\min} + X_{t+n}\right) \geq g(X_{t+n}) + \beta V_0, \quad (40)$$

for all  $X$ . In addition, we again have the following break-even condition for the CH:

$$\gamma(\bar{K}_{t+n} + \bar{L}_{t+n}) + (1 - 2\gamma)\bar{B}_{t+n} = -\delta(n). \quad (41)$$

We again want to solve for the payment system that fulfils these equations for the largest set of parameters. By a similar argument as in the previous Proposition, only the first incentive constraint binds. Hence,  $\bar{B}_{t+n} = \bar{L}_{t+n}$ , and

$$\bar{K}_{t+n} = \bar{B}_{t+n} + (1 - \eta)[\sigma u - \tau] + \eta\sigma c. \quad (42)$$

Using these results in the break-even condition for the CH, we obtain:

$$\bar{L}_{t+n} + \gamma[(1 - \eta)[\sigma u - \tau] + \eta\sigma c] = -\delta(n). \quad (43)$$

Again, it is straightforward to verify that the payment system is incentive feasible if and only if the trader with the worst possible balance adjustment has no incentive to default in the settlement stage. Rewriting condition (39) we obtain:

$$V\left(\sum_{s=1}^n X_{t+s}^{\min}\right) = \sum_{s=1}^n \bar{L}_{t+s} + \frac{\beta}{1-\beta} \gamma \sigma(u-c) - \delta \frac{\beta^n}{1-\beta^n} \geq \frac{\beta}{1-\beta} \gamma [\sigma(u-c) - \tau]. \quad (44)$$

Using the expression for  $\bar{L}_{t+n}$  and the fact that we are studying a SRPS, this expression can be simplified to obtain:

$$\bar{L}_{t+n} \frac{\beta(1-\beta^n)}{\beta^n(1-\beta)} + \frac{\beta}{1-\beta} \gamma \sigma(u-c) - \delta \frac{\beta^n}{1-\beta^n} \geq \frac{\beta}{1-\beta} \gamma [\sigma(u-c) - \tau] \quad (45)$$

$$\left( -[(1-\eta)(\sigma u - \tau) + \eta \sigma c] - \frac{\delta(n)}{\gamma} \right) \frac{1-\beta^n}{\beta^n} - \frac{\delta(n)}{\gamma} \geq -\tau, \quad (46)$$

which completes the proof.

## 6.4 Proof of Proposition 6

With  $\alpha \in (0, 1)$ , we now have two sets of incentive, participation and default constraints. Furthermore, the value of default changes to

$$(1-\beta) V_0 = \gamma(1-\alpha) [\sigma(u-c) - \tau],$$

reflecting the total size of the OTC market. It is also straightforward to verify from the incentive constraints that  $\bar{B}_{t+n} = B_{t+n}$ . Using this fact and that  $f(B_{t+n}) = 0$ , we have the following incentive constraints:

$$-\sigma c - (1-\eta)[\sigma(u-c) - \tau] + \bar{K}_{t+n} \geq B_{t+n}, \quad (47)$$

$$\sigma u - \eta[\sigma(u-c) - \tau] + \bar{L}_{t+n} \geq B_{t+n}, \quad (48)$$

$$-c + K_{t+n} \geq B_{t+n}, \quad (49)$$

$$u + L_{t+n} \geq B_{t+n}. \quad (50)$$



Finally, the break even condition for the CH is now given by

$$\gamma [\alpha (K_{t+n} + L_{t+n}) + (1 - \alpha) (\bar{K}_{t+n} + \bar{L}_{t+n})] + (1 - 2\gamma)B_{t+n} = -\delta(n). \quad (51)$$

Again, for the largest set of parameters for which an optimal PS exists, it must be the case that  $\bar{L}_{t+n} = L_{t+n} = B_{t+n} < K_{t+n}$ ,  $B_{t+n} < \bar{K}_{t+n}$  and

$$\bar{K}_{t+n} = L_{t+n} + (1 - \eta)[\sigma u - \tau] + \eta\sigma c \quad (52)$$

$$K_{t+n} = L_{t+n} + c. \quad (53)$$

These balance adjustments imply that we only have to consider the no-default constraint in the settlement stage for the worst possible adjustment, which is given by

$$\begin{aligned} V \left( \sum_{s=1}^n X_{t+s}^{\min} \right) &= \sum_{s=1}^n L_{t+s} + \frac{\beta}{1 - \beta} \gamma (\alpha + (1 - \alpha)\sigma) (u - c) - \delta \frac{\beta^n}{1 - \beta^n} \\ &\geq \frac{\beta}{1 - \beta} \gamma (1 - \alpha) [\sigma(u - c) - \tau]. \end{aligned} \quad (54)$$

From the break-even condition, we obtain

$$L_{t+n} = -\delta(n) + \alpha\gamma c + (1 - \alpha)\gamma [\eta\sigma c + (1 - \eta)(\sigma u - \tau)]. \quad (55)$$

Hence, our characterization becomes:

$$L_{t+n} \frac{\beta(1 - \beta^n)}{\beta^n(1 - \beta)} + \frac{\beta}{1 - \beta} \alpha\gamma(u - c) - \delta \frac{\beta^n}{1 - \beta^n} + \frac{\beta}{1 - \beta} \gamma(1 - \alpha)\tau \geq 0 \quad (56)$$

$$L_{t+n} + \frac{\beta^n}{1 - \beta^n} \alpha\gamma(u - c) - \delta(n) \frac{\beta^n}{1 - \beta^n} + \frac{\beta^n}{1 - \beta^n} \gamma(1 - \alpha)\tau \geq 0 \quad (57)$$

$$\begin{aligned} &\alpha\gamma \left[ \frac{\beta^n}{1 - \beta^n} (u - c) - c \right] \\ &+ (1 - \alpha)\gamma \left[ \frac{\beta^n}{1 - \beta^n} \tau - [\eta\sigma c + (1 - \eta)(\sigma u - \tau)] \right] \geq \frac{1}{1 - \beta^n} \delta(n), \end{aligned} \quad (58)$$

which completes the proof.

## 6.5 Proof of Proposition 9

In order to demonstrate Proposition 9, we find it convenient to use differential calculus. To this end, here we develop a continuous-time version of the model in the text. We assume that buying and selling opportunities follow a Poisson process with arrival rate  $\gamma$ . The (continuous) rate of time preference is now denoted by  $\rho$ . The fixed cost,  $\delta$ , is incurred whenever the transaction process stops and settlement occurs. This occurs after a deterministic interval of length  $T$ . As before, we denote balance adjustments by  $(K(t), L(t), B(t))$ . All other assumptions remain the same as in the text.

We let the random time before the next arrival of a trading opportunity be denoted by  $\tau$ . In that case,  $\tau$  has a distribution function given by

$$F(t) = \Pr(\tau \leq t) = 1 - \Pr(\tau > t) = 1 - e^{-\gamma t}. \quad (59)$$

Hence, the time until the next arrival of a trading opportunity is an exponentially distributed random variable with distribution function  $F(t) = 1 - e^{-\gamma t}$ .

Denote by  $V_0$  the expected future payoff for a trader at the end of the settlement stage. It is straightforward to show that an optimal PS involves a constant level of transactions. First, assume that there are no settlement costs. Since both consumption and production opportunities are independent, they arrive at rate  $\gamma$ , and they have the same continuation value, we have

$$\begin{aligned} V_0 &= \int_0^\infty e^{-\rho t} (u(q) - c(q) + V_0) d(1 - e^{-\gamma t}) \\ &= \frac{\gamma}{\gamma + \rho} (u(q) - c(q) + V_0), \end{aligned} \quad (60)$$

which yields

$$V_0 = \frac{\gamma}{\rho} (u(q) - c(q)). \quad (61)$$

This is analogous to the lifetime utility under a system that employs a threat of exclusion in the discrete-time version of the model presented in the text. In the absence of settlement

costs, equation (61) also gives the life-time expected payoff in a system that decentralizes transactions of size  $q$ .

When costly settlement occurs after each time length,  $T$ , it involves an aggregate (average) fixed cost  $\delta$ . Hence, the net present value of the settlement costs is given by

$$\sum_{n=1}^{\infty} e^{-n\rho T} \delta = \delta \frac{e^{-\rho T}}{1 - e^{-\rho T}}. \quad (62)$$

Thus, the continuous-time version of the value function,  $V_0$ , is given by

$$V_0 = \frac{\gamma}{\rho}(u(q) - c(q)) - \delta \frac{e^{-\rho T}}{1 - e^{-\rho T}}. \quad (63)$$

As before, we define the adjustments conditional on the traders' reports by

$$pK_t - c(q) = pL_t = pB_t, \quad (64)$$

for all  $t$ . Also, since the PS is repeated, we have that adjustments,  $X$ , satisfy

$$X_{nT+t} = X e^{\rho(T-t)}, \quad (65)$$

for all  $t \in [nT; (n+1)T]$ , where  $n$  is an integer. As in the discrete-time case, the above implies that all incentive constraints are fulfilled. In addition, it satisfies all PCs for the largest set of parameter values. Next, we derive the market clearing condition for the settlement stage. This is accomplished by approximating total balance adjustments in an interval of length  $T$ . First, note that the probability of having exactly  $n$  arrivals of trading opportunities in the interval  $[0, t]$  is given by

$$P[N_t = n] = e^{-\gamma t} \frac{(\gamma t)^n}{n!}. \quad (66)$$

For small  $\Delta$ , we then have that

$$P[N_\Delta = 1] \approx \gamma \Delta, \quad (67)$$

where  $P[N_\Delta > 1] = o(\Delta)$ . Next, define  $\Delta = \frac{T}{m}$ , where  $m \in [0, T]$  is an integer. The total adjustment for sellers over an interval of length  $T$  is then approximately given by

$$\begin{aligned}
& \gamma \Delta K_\Delta + \cdots + \gamma \Delta K_{(m-1)\Delta} + \gamma \Delta K_{m\Delta} \\
&= \gamma \Delta K \left[ e^{\rho(T-\Delta)} + \cdots + e^{\rho(T-(m-1)\Delta)} + e^{\rho(T-m\Delta)} \right] \\
&= \gamma \Delta K e^{\rho T} \left[ \frac{1 - (e^{-\rho\Delta})^m}{1 - e^{-\rho\Delta}} - 1 \right] \\
&= \gamma K \left[ \frac{\Delta e^{\rho(T-\Delta)} - \Delta}{1 - e^{-\rho\Delta}} \right]. \tag{68}
\end{aligned}$$

As  $\Delta \rightarrow 0$ , a trader will almost surely receive either none or one opportunity to trade during a time length  $\Delta$ . In that case, using L'Hôpital's rule,<sup>28</sup> the expected total adjustments for sellers are given by  $\frac{\gamma}{\rho} K (e^{\rho T} - 1)$ . The expected total balance adjustments for buyers are similarly determined and given by  $\frac{\gamma}{\rho} L (e^{\rho T} - 1)$ . Finally, expected balance adjustments for traders who have received no trading opportunities over this time interval can be determined as follows. For each interval of length  $\Delta$ , a measure  $2\gamma\Delta$  of traders are engaged in transactions ( $\gamma\Delta$  of them as buyers and  $\gamma\Delta$  as sellers). Therefore, the measure of traders who are not involved in any transactions over an interval of length  $\Delta$  is  $(1 - 2\gamma)\Delta$ . As a result, the aggregate balance adjustments for non-trading activities over the interval of length  $T$  are given by

$$\begin{aligned}
& (1 - 2\gamma) \Delta B_\Delta + \cdots + (1 - 2\gamma) \Delta B_{(m-1)\Delta} + (1 - 2\gamma) \Delta B_{m\Delta} \\
&= \frac{(1 - 2\gamma)}{\rho} B (e^{\rho T} - 1). \tag{69}
\end{aligned}$$

Market clearing during the settlement stage is then given by the following equation:

$$\frac{1}{\rho} (e^{\rho T} - 1) [\gamma p K + \gamma p L + (1 - 2\gamma)pB] = -\delta. \tag{70}$$

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<sup>28</sup>Both the numerator and the denominator in this expression go to zero as  $\Delta \rightarrow 0$ . In addition, we have  $\lim_{\Delta \rightarrow 0} \frac{f'(x)}{g'(x)} = \lim_{\Delta \rightarrow 0} \frac{-\Delta \rho e^{\rho(T-\Delta)} + e^{\rho(T-\Delta)} - 1}{\rho e^{\rho\Delta}} = \lim_{\Delta \rightarrow 0} -\Delta e^{\rho T} + \frac{e^{\rho T}}{\rho} - \frac{1}{\rho e^{-\rho\Delta}} = \frac{1}{\rho} (e^{\rho T} - 1)$ .

Using the above balance adjustments, one obtains

$$pB = -\delta\rho\frac{1}{e^{\rho T} - 1} - \gamma c(q). \quad (71)$$

The worst possible balance adjustment is assigned to traders that either never traded or never sold any contracts in the interval  $[0, T]$ . Following the above discussion, this adjustment is given by  $\frac{1}{\rho}(1 - e^{-\rho T})pB$ . This implies that the only PC that is potentially binding is given by

$$\frac{1}{\rho}(e^{\rho T} - 1)pB + \frac{\gamma}{\rho}(u(q) - c(q)) - \delta\frac{e^{-\rho T}}{1 - e^{-\rho T}} \geq 0. \quad (72)$$

This constraint is identical to the one in the discrete-time version, simply adjusting for the continuous time discount factor. Given these adjustments, an optimal PS chooses  $q$  and  $T$  in order to solve the following maximization problem:

$$\begin{aligned} & \max_{q, T} \frac{\gamma}{\rho}(u(q) - c(q)) - \delta\frac{e^{-\rho T}}{1 - e^{-\rho T}} & (73) \\ & \text{subject to} \\ & \frac{1}{\rho}(e^{\rho T} - 1)pB + \frac{\gamma}{\rho}(u(q) - c(q)) - \delta\frac{e^{-\rho T}}{1 - e^{-\rho T}} \geq 0, \\ & pB = -\delta\rho\frac{1}{e^{\rho T} - 1} - \gamma c(q). \end{aligned}$$

The objective function expresses the discounted lifetime utility of a representative participant. The second constraint summarizes the PC that is potentially binding, while the third constraint summarizes the IC and the market clearing conditions that must be satisfied in any incentive feasible outcome. The equality in the last equation follows from the fact that the PS works for the largest set of parameters if it makes all incentive constraints exactly bind. The constraint set can be rewritten as

$$\frac{\gamma}{\rho}[u(q) - e^{\rho T}c(q)] \geq \delta\frac{1}{1 - e^{-\rho T}}, \quad (74)$$

or

$$(1 - e^{-\rho T}) u(q) - (e^{\rho T} - 1) c(q) \geq \frac{\rho}{\gamma} \delta. \quad (75)$$

The objective function is strictly concave in  $(q, T)$ . In order to guarantee that the constraint set is convex, we need an additional assumption. Given any  $T(q)$ , the function on the left-hand side of the above inequality is concave in  $q(T)$ . However, the left-hand side is not necessarily jointly concave in  $(q, T)$  due to the second term, which is a product of two convex functions. A function is *log-convex* if its natural logarithm is convex. We have the following sufficient condition for the constraint set to be convex.<sup>29</sup>

**Lemma 10.** *Suppose that  $c(q)$  is log-convex. Then  $e^{\rho T} c(q)$  is a strictly convex function in  $(q, T)$ , and the constraint set is convex.*

*Proof.* Since  $c(q)$  is log-convex, we have that

$$\frac{\partial^2 \ln c(q)}{\partial q^2} = \frac{c(q)c''(q) - (c'(q))^2}{(c(q))^2} > 0. \quad (76)$$

The first term of the left-hand side in equation (74) is strictly concave in  $q$ , while the right-hand side is strictly convex in  $T$ . The remaining term has a Hessian given by

$$H(q, T) = \begin{pmatrix} \rho^2 e^{\rho T} c(q) & \rho e^{\rho T} c'(q) \\ \rho e^{\rho T} c'(q) & e^{\rho T} c''(q) \end{pmatrix}. \quad (77)$$

The first principal minor is positive, while the second principal minor is positive if and only if

$$c(q)c''(q) - (c'(q))^2 > 0. \quad (78)$$

Hence, as  $c(q)$  is log-convex,  $e^{\rho T} c(q)$  is convex. The result follows since the sum of two concave functions is concave.  $\square$

Taking first-order conditions with respect to  $q$  and  $T$ , we obtain the following character-

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<sup>29</sup>A weaker condition is given by  $-\frac{1}{e^{\rho T}} u''(q)c(q) \geq c'^2 - c''(q)c(q)$ .

ization of the solution:

$$\frac{u'(q) - c'(q)}{c'(q)} = \frac{\lambda}{1 + \lambda} (e^{\rho T} - 1) \quad (79)$$

$$\frac{\delta}{c(q)} \frac{\rho}{\gamma} \left( \frac{1}{e^{\rho T} - 1} \right) = \frac{\lambda}{1 + \lambda} (e^{\rho T} - 1), \quad (80)$$

where  $\lambda$  is the multiplier on the single constraint. This leads us to the following.

**Lemma 11.** *Let  $c(q)$  be log-convex. For any optimal PS with settlement, we have  $\hat{q} < q^*$ .*

*Proof.* Since  $\delta$ ,  $\gamma$ , and  $\rho$  are positive, and the optimal settlement length is finite ( $\hat{T} \in (0, \infty)$ ), we must have that  $\lambda > 0$ . Hence, equation (79) implies that  $u'(\hat{q}) - c'(\hat{q}) > 0$ . Since  $c$  is increasing and strictly convex, and  $u$  is increasing and strictly concave, this implies that  $\hat{q} < q^*$ .  $\square$

Eliminating  $\lambda$  from the first-order conditions (79) and (80), we obtain a single first-order condition

$$\frac{\gamma}{\rho} \frac{u'(q) - c'(q)}{c'(q)} (e^{\rho T} - 1) = \frac{\delta}{c(q)}. \quad (81)$$

This condition, together with the constraint (74), characterizes the solution  $(\hat{q}, \hat{T})$ . Solving these equations yields the optimal length of the transactions stage,  $\hat{T}$ , as a function of  $\rho$  and  $\hat{q}$ ; i.e.,

$$\frac{u(q) c'(q)}{u'(q) c(q)} = e^{\rho T}. \quad (82)$$

The optimal transaction size,  $\hat{q}$ , is given by

$$u(q) \left( 1 - \frac{c'(q)}{u'(q)} \right) + c(q) \left( 1 - \frac{u'(q)}{c'(q)} \right) = \delta \frac{\rho}{\gamma}. \quad (83)$$

A solution to the last equation exists by the Intermediate-Value-Theorem. Furthermore, any solution must lay in an interval  $[\underline{q}, q^*]$ , where  $\underline{q} > 0$ . The problem is that the left-hand side of equation (83) is non-monotonic. Hence, there will, in general, be more than one solution to this equation. The optimal solution, however, corresponds to the one closest to (and below)

$q^*$ . The next Proposition relies solely on the fact that at this solution,  $\hat{q}$ , the left-hand side of equation (83) is *locally* strictly decreasing.

**Lemma 12.** *Assume that  $c(q)$  is log-convex. As the settlement cost,  $\delta$ , increases, the optimal transaction size,  $\hat{q}$ , as well as the optimal length of the transactions stage,  $\hat{T}$ , decrease.*

*Proof.* We establish first that  $\hat{q}$  and  $\hat{T}$  move in the same direction; i.e., that  $\frac{d\hat{T}}{d\hat{q}} > 0$ . Differentiating the left-hand side of equation (82) with respect to  $q$ , we obtain

$$\frac{1}{(u'(q)c(q))^2} \left[ u(q)u'(q) (c(q)c''(q) - (c'(q))^2) + c(q)c'(q) ((u'(q))^2 - u(q)u''(q)) \right], \quad (84)$$

which is strictly positive, as  $u$  is strictly increasing and strictly concave, while  $c$  is log-convex.

Next, we show that  $\hat{q}$  is decreasing in  $\delta$ . Denote the left-hand side of equation (83) by  $\Gamma(q)$ . Differentiating  $\Gamma(q)$  with respect to  $q$  and collecting terms we obtain

$$\frac{\partial \Gamma}{\partial q} = c''(q) \left[ \frac{c(q)}{c'(q)} \frac{u'(q)}{c'(q)} - \frac{u(q)}{u'(q)} \right] + u''(q) \left[ \frac{c'(q)}{u'(q)} \frac{u(q)}{u'(q)} - \frac{c(q)}{c'(q)} \right]. \quad (85)$$

We can rewrite equation (83) as

$$\frac{\gamma}{\rho} \frac{u'(q) - c'(q)}{c'(q)} \left( \frac{u(q)}{u'(q)} \frac{c'(q)}{c(q)} - 1 \right) = \frac{\delta}{c(q)}. \quad (86)$$

Since  $u'(q) > c'(q)$ , for  $q < q^*$ , we obtain that  $\frac{u(q)}{u'(q)} > \frac{c(q)}{c'(q)}$ . Letting  $q \rightarrow q^*$ , this implies that

$$\frac{c(q)}{c'(q)} \frac{u'(q)}{c'(q)} - \frac{u(q)}{u'(q)} < 0, \quad (87)$$

and

$$\frac{c'(q)}{u'(q)} \frac{u(q)}{u'(q)} - \frac{c(q)}{c'(q)} > 0. \quad (88)$$

Hence,  $\frac{\partial \Gamma}{\partial q} < 0$ , or, equivalently, the left-hand side of equation (83) is strictly decreasing for  $q$  sufficiently close to  $q^*$ . Furthermore,  $\Gamma(q)$  converges to 0 as  $q \rightarrow q^*$ . Hence,  $\Gamma(q) > 0$  for  $q$  sufficiently close to  $q^*$  and, by the continuity of  $\Gamma(q)$ , there must exist a solution to equation



(83) for small enough  $\delta > 0$ . Finally, since  $\Gamma(q) \downarrow 0$  as  $q \rightarrow q^*$ , we must have that  $\Gamma'(\hat{q}) \leq 0$  (with  $\Gamma$  having possibly a local maximum at  $\hat{q}$ ). This completes the proof.  $\square$

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