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Search for a Monetary Propagation Mechanism

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SEARCH FOR A MONETARY PROPAGATION MECHANISM

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Abstract

This paper examines a monetary propagation mechanism in an economy where exchanges in goods and labor markets involve costly search. It is shown that an increase in the money growth rate increases steady state employment and output when the money growth rate is low but reduces steady state employment and output when the money growth rate is already high. The model produces persistent, hump-shaped responses in employment and output to money growth shocks even when the shocks have no persistence. The model also generates desirable features on job vacancy, sales, inventory, and the velocity of money. All these features emerge here in an economy with perfectly flexible prices and wages.

Journal of Economic Literature Classification Numbers: E40, E30.

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1. INTRODUCTION

A striking monetary feature is that monetary aggregates significantly lead output over business cycles in post war U.S. data. As shown in Figure 1 and Table 1, output is positively correlated with monetary aggregates and the correlation with some lagged monetary aggregates is much higher than with contemporaneous aggregates. Vector autoregressive studies by Sims [41] have found similar phase shift and persistence in the correlation between money and output. Although the high correlation between money and output makes it difficult to infer about the direction of causality, a plausible interpretation for the phase shift and persistence of the correlation is that monetary shocks induce a persistent, hump-shaped output response and are an important cause of business fluctuations ([16]). General equilibrium monetary models have typically failed to generate such monetary propagation mechanism. After calibrating different versions of equilibrium monetary models, Cooley and Hansen [10] have concluded that “none of the models considered here can be interpreted as providing a strong theoretical argument for money growth shocks being the key impulses driving aggregate fluctuations. Monetarists must look elsewhere for a general equilibrium theory consistent with their interpretation of U.S. time series.”¹

Figure 1 and Table 1 here.

The present paper constructs an alternative equilibrium monetary model and explores its monetary propagation mechanism. Taking a step toward realism, the model assumes that exchanges in both labor and goods markets require costly search. Introducing search in the labor market allows for rich dynamics in employment. This seems necessary because previous monetary models, like their non-monetary counterparts (see [9]), feature quick employment responses to shocks. Since labor income has a large share in output, the corresponding output dynamics resemble too much of the exogenous shocks to capture sufficient internal propagation. Costly search in the labor market provides a necessary mechanism to induce persistent employment but, as described later, it alone is not sufficient for persistent monetary propagation.

¹The most frequently calibrated monetary model is the liquidity effect model (see [27, 17]), which is extended in [6] to endogenize broad monetary aggregates. There are also attempts to incorporate financial intermediation (see [18]) and nominal rigidities (see [8, 10, 31]).

Costly search in the goods market is the distinct feature of the model. It captures a search-inducing effect of money growth: By reducing the value of money, money growth increases buyers' surplus from trade and stimulates them to search more intensively. The higher buyers' search intensity increases firms' sales and induces them to hire more workers. Such a possible propagation mechanism that starts from the demand side of the goods market differs from the mechanism in the well-known liquidity effect model that starts from investment and the supply side of the goods market.

Money growth also has an inflation effect. Since firms' goods are sold for money, a higher money growth rate tends to reduce the shadow value of firms' sales, the surplus from hiring and employment. In the long run, the balance between the search-inducing effect and the inflation effect generates an inverted U-shaped relation between output and money growth. When the money growth rate increases from the level of time preference, steady state output increases. When the money growth rate is high enough, a further increase reduces steady state output. The critical money growth rate is lower than the calibrated level (1.2 percent quarterly) so that steady state output falls when the money growth rate increases from the calibrated level.

Along the transition path the search-inducing effect induces an additional dynamic effect, the inventory effect. That is, the rise in sales induced by buyers' high search intensity reduces inventory and the supply of goods in the next period. When the consumption smoothing motive is strong, buyers keep searching hard in the next period in order to maintain a smooth consumption profile. This persistently high search intensity keeps the firm's sales revenue high and induces firms to recruit more workers in subsequent periods.

The monetary propagation mechanism is summarized in Figure 2, where the top, middle and bottom chains depict the inflation effect, the search-inducing effect and the inventory effect, respectively. The inflation and search-inducing effects start the propagation of the monetary shock. When the shock is transitory, the inflation effect is weak so that the search-inducing effect dominates. Firms immediately increase job vacancies, which leads to rising employment and output. Then the inventory and search-inducing effects reinforce each other to keep the firm's

sale revenue persistently above the steady state and positively propagate the shock into output for a long time. The upswing of output ends when employment is above the steady state by so much that the force of the diminishing marginal product of labor finally reduces employment back to the steady state.

The reinforcing search-inducing and inventory effects produce persistent, hump-shaped output responses. Calibration shows that, even when the money growth shock has no persistence, output takes five quarters to peak and stays significantly above the steady state for another ten quarters. The propagation of a persistent monetary shock is similar, except that the inflation effect is stronger and so output may respond negatively to the shock in the initial transition.

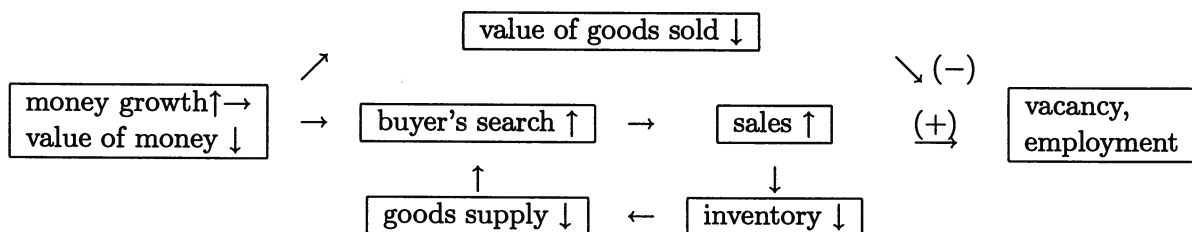


Figure 2 The monetary propagation mechanism

Both labor market search and goods market search are necessary for the persistent propagation. If the goods market were cleared in the Walrasian fashion, there would be no need for buyers to search nor for firms to maintain inventory. Consequently, the inventory effect would not exist and so the propagation would be short for a transitory monetary shock, despite the presence of search in the labor market. If, on the other hand, the labor market were cleared in the Walrasian fashion, there would be no need for maintaining vacancies, in which case employment and output would quickly peak after a transitory monetary shock, as in previous monetary models.

Of course, neither labor market search nor goods market search is entirely new in the literature. The modelling of labor market search follows recent developments [28, 2, 40, 39] to extend the equilibrium unemployment theory [29, 32] into an intertemporal framework. The modelling of goods market search is an attempt to extend the search monetary theory (e.g. [23, 24, 35, 36, 37, 42]) into a tractable intertemporal macroeconomic framework. The idea that inflation can stimulate buyers' search intensity has been formulated in [11, 12, 3, 26].

The remainder of this paper is organized as follows. Section 2 integrates search in the goods and labor markets into an intertemporal maximization framework. Section 3 defines equilibrium and examines how money growth affects the steady state. Section 4 calibrates the model and analyzes the monetary propagation mechanism. Section 5 highlights some features of the monetary propagation. Section 6 concludes the paper and the appendices provide necessary proofs.

2. THE ECONOMY

2.1. The Household and the Matches

There are a continuum of households with measure one, represented by points along a circle H . There are also a continuum of goods denoted by the same symbol H . A good h is storable (as inventory) only by its producers. To focus on the monetary propagation through employment, I abstract from the propagation through fixed investment by assuming that productive capital is fixed. Each household $h \in H$ is specialized in producing good h but wishes to consume a subset of goods that are different from its own product. Thus, exchanges are necessary for consumption. There is no Walrasian auctioneer so that agents must search for their desired goods. Given such an exchange process, it is likely that fiat money is valuable in facilitating exchanges, although explicitly establishing such a role for fiat money would require more detailed considerations of the exchange patterns (see [23, 24, 35, 36, 37, 42]). For the focus on monetary propagation, it suffices to assume that transactions require the use of money.

Since matching is random between producers and unemployed agents in the labor market, and between sellers and buyers in the goods market, agents face idiosyncratic risks in the matching outcomes. These risks induce distributions on buyers' money holdings, agents' employment status, firms' inventories and employment. Tracking the distributions is analytically intractable and numerically challenging. It is not clear whether such distributions have important consequences on aggregate variables. To focus on aggregate variables and to examine a general class of monetary shocks, I group different agents into large households, each consisting of a continuum of agents who share the same consumption and regard the household's utility as the common objective. Thus, idiosyncratic risks across agents generated by random matches are smoothed within each

household. This risk-smoothing modelling strategy resembles the one used in monetary models [27, 17] and in labor economics [34, 20].²

Table 2 Agents in a household

	entrepreneurs	unemployed agents	workers	buyers
set	A_p	A_u	A_{nt}	A_b
measure	a_p	u	$a_p n_t$	a_b

Table 3 Statistics of matches

	set of agents in suitable matches	rate of matches	quantities in each trade
entrepreneurs	A_p	$\mu(\hat{v}_t)$	\hat{W}_t units of real money balances,
unemployed agents	A_{ut}^*	$a_p \hat{v}_t \mu(\hat{v}_t)/u$	one unit of labor,
sellers	A_{pt}^*	$g_s(\hat{s}_t)$	\hat{q}_t units of goods,
buyers	A_{bt}^*	$g_b(\hat{s}_t)s_t$	\hat{m}_t units of money.

In each household, a group of members enjoy leisure, while others are active in markets and are classified into four groups: entrepreneurs, unemployed agents, workers, and buyers. An entrepreneur consists of two agents, a producer and a seller. The producer in household h hires workers to produce good h and the seller sells good h . An unemployed agent searches for jobs and becomes a worker when he finds a job. A worker inelastically supplies one unit of labor each period (see Section 6 for a discussion). A buyer searches to buy the household's desired good. Table 2 summarizes the notation for these active agents. Note that (a_p, u, a_b) are constant but n_t is made endogenous to allow for employment fluctuations. Let $B = a_b/a_p$ be the ratio of buyers to sellers in the goods market.

Table 3 lists the statistics of matches and quantities in each trade for a representative house-

²In the presence of a continuum of agents who face idiosyncratic risks, there are well-known measure theoretical problems with the law of large numbers. Since there are well-known solutions, I will simply assume that the law of large numbers holds for a continuum of independently and identically distributed variables.

At the macroeconomic level, the continuum of agents in each household is quantitatively indistinguishable from a perfectly divisible unit of time that an agent can allocate over different activities. Although the time interpretation is more natural, a formal implementation of such an interpretation is problematic. It is cumbersome to construct a matching technology that generates no aggregate uncertainty in matching probabilities throughout the trading period. Also, strategies are more difficult to detail because of the sequential nature inherited in the time interpretation.

Rios-Rull [33] reviews computable equilibrium models with heterogeneous agents. The difficulty of keeping track of buyers' money holdings in a search monetary model is illustrated by Diamond and Yellen [13], who managed to characterize the equilibrium under severe restrictions on trade that are not satisfied in the current model. In contrast, employing the risk-smoothing strategy allows for a straightforward characterization (see [37]). Similarly, partially adopting a risk-smoothing assumption allows Fisher and Hornstein [15] to analyze the inventory behavior in a framework that cannot be managed by the standard (s, S) inventory model.

hold. We explain the matching rates, leaving the quantities in trade for later discussion. In the goods market, a buyer chooses a search intensity s_t in each period t . Let \hat{s}_t be the search intensity per buyer, which is taken as given by individual agents. To focus on the effect of monetary policies on the buyer's search intensity, I assume that a seller's search intensity is fixed at a level normalized to one (see Section 6 for a discussion). The total number of matches in the goods market in each period is given by the following matching technology:³

$$g(\hat{s}) \equiv z_1(a_b \hat{s})^\alpha (a_p)^{1-\alpha}, \quad \alpha \in (0, 1).$$

Normalize $z \equiv z_1 B^{\alpha-1}$. Then the matching rate is $g_b(\hat{s}) \equiv z \hat{s}^{\alpha-1}$ for each unit of a buyer's search intensity and $g_s(\hat{s}) \equiv z B \hat{s}^\alpha$ for each seller, both depending only on buyers' aggregate search intensity. A buyer finds a desirable seller at a rate sg_b and a seller finds a desirable buyer at a rate g_s , as in Table 3. Under suitable restrictions on parameters, both rates are less than one and can be interpreted as probabilities. The measure of the set of buyers with suitable matches, A_b^* , is $sg_b a_b$ and the measure of the set of sellers with suitable matches, A_p^* , is $g_s a_p$. Note that $sg_b a_b = g_s a_p$ whenever $s = \hat{s}$.

In the labor market, each producer chooses the number of job vacancies, v , while each unemployed agent supplies one unit of search effort inelastically.⁴ Let \hat{v} be the number of vacancies per firm. The total number of matches between firms and unemployed agents is given by $(a_p \hat{v})^A u^{1-A}$, where $A \in (0, 1)$. The matching function is linearly homogeneous, in accordance with the finding in [4]. The number of matches per vacancy is:

$$\mu(\hat{v}) \equiv (a_p \hat{v}/u)^{A-1}. \quad (2.1)$$

Similarly, the number of matches per unemployed agent is $a_p \hat{v} \mu(\hat{v})/u$, as in Table 3.

³The current specification incorporates the possibility that a matched buyer does not like the seller's good. For example, one can let $1 - z_0$ be the probability of such an event in a match. Then the total number of desirable matches is $z_0 g$. This amounts to rescaling z_1 .

⁴The main results of this paper would be unchanged if unemployed agents' search effort is elastic, as long as it is less elastic than job vacancies. The latter assumption is supported by observations in [25].

2.2. The Household's Choice Problem

Consider a representative household h and use $-h$ to index any other household. Let j indicate an agent in household h and $-j$ the agent in household $-h$ with whom agent j is matched. At the beginning of each period t , the household evenly allocates the available money, M_t , to the buyers. Then the four groups of agents go to their own markets to exchange and are separated from each other until the end of the period. During the transaction period, a lump-sum monetary transfer τ_t is distributed to households through helicopter dropping (see Section 6 for a discussion on the timing of the transfer). At the end of the period, agents bring their trade receipts and residual balances back to their household. Workers' wage income and entrepreneurs' profits, both in terms of money, are added to the household's money balance for the next period's allocation.⁵

At the beginning of period t , the household chooses consumption for period t , c_t , the search intensity for each buyer, $s_t(j)$, the number of vacancies for each firm, $v_t(j)$, a new employment level for $t + 1$, $n_{t+1}(j)$, a new inventory level for $t + 1$, $i_{t+1}(j)$, and a new total money balance for $t + 1$, M_{t+1} . The quantities of trade in period t are as follows. Each buyer j in a suitable match with a seller $-j$ exchanges $\hat{m}_t(j)$ units of money for $\hat{q}_t(-j)$ units of goods in period t and each matched worker j receives a wage $\hat{W}_t(-j)$ in real money balances. The price of goods in the match between buyer j and seller $-j$ is $\hat{P}_t(j) = \hat{m}_t(j)/\hat{q}_t(-j)$ and the average price of goods is \hat{P}_t . The hat indicates that in making the decisions at t the household takes $\{\hat{m}_{t+r}, \hat{q}_{t+r}, \hat{W}_{t+r}\}_{r=0}^{\infty}$ as given (but see Section 3 for a discussion).

To set up the household's maximization problem, let $U(c)$ be the instantaneous utility function, which is strictly increasing and concave with $\lim_{c \rightarrow 0} cU'(c) = \infty$ and $\lim_{c \rightarrow \infty} cU'(c) = 0$. In addition, we impose the realistic restriction $RA \equiv -cU''/U' \geq 1$ (see [14]). Also, let $\Phi(s)$ be the disutility of a buyer's search intensity, φ the disutility of working (for one unit of time) and $K(v)$ the disutility of maintaining vacancies. The function Φ satisfies $\Phi' > 0$ and $\Phi'' > 0$ for $s > 0$, and $\Phi(0) = \Phi'(0) = 0$. The function K has similar properties.

⁵In a search model it may be desirable for a worker to be paid in terms of money rather than goods, because the worker may have no desire for the product produced by the firm he works for. Such an advantage of nominal repayments has been illustrated in a search monetary model (see [36]).

Taking the sequence $\{\hat{q}_t, \hat{m}_t, \hat{W}_t\}_{t \geq 0}$ and initial conditions (M_0, i_0, n_0) as given, the household chooses sequences $\{c_t, s_t, v_t\}_{t \geq 0}$ and $\{M_{t+1}, i_{t+1}, n_{t+1}\}_{t \geq 0}$ to solve the following problem:

$$(PH) \max \sum_{t=0}^{\infty} \beta^t \left[U(c_t) - \int_{A_{nt}} \varphi dj - \int_{A_b} \Phi(s_t(j)) dj - \int_{A_p} K(v_t(j)) dj \right]$$

subject to the following conditions for all $t \geq 0$:

$$c_t \leq \int_{A_{bt}^*} \chi_{mt}(j) \hat{q}_t(-j) dj, \quad (2.2)$$

$$\frac{M_{t+1}}{a_b} \geq \hat{m}_{t+1}(j), \quad \text{for all } j \in A_{bt+1}^* \quad (2.3)$$

$$i_{t+1}(j) + f(n_{t+1}(j)) \geq \hat{q}_{t+1}(j), \quad \text{for all } j \in A_{pt+1}^* \quad (2.4)$$

$$M_t + \tau_t - \int_{A_{bt}^*} \chi_{mt}(j) \hat{m}_t(j) dj + \int_{A_{nt}} \hat{P}_t \hat{W}_t(-j) dj + \int_{A_p} \pi_t(j) dj \geq M_{t+1}, \quad (2.5)$$

$$\int_{A_p} [(1 - \delta_n) n_t(j) + v_t(j) \mu_t(j) - n_{t+1}(j)] dj \geq 0, \quad (2.6)$$

$$(1 - \delta_i) \left[\int_{A_p} [i_t(j) + f(n_t(j))] dj - \int_{A_{pt}^*} \chi_{qt}(j) \hat{q}_t(j) dj \right] \geq \int_{A_p} i_{t+1}(j) dj. \quad (2.7)$$

where $\chi_m(j)$ and $\chi_q(j)$ are indicator functions defined by

$$\chi_{mt}(j) = \begin{cases} 1, & \text{if } \frac{M_t}{a_b} \geq \hat{m}_t(j) \\ 0, & \text{otherwise,} \end{cases} \quad \chi_{qt}(j) = \begin{cases} 1, & \text{if } i_t(j) + f(n_t(j)) \geq \hat{q}_t(j) \\ 0, & \text{otherwise.} \end{cases}$$

The three integrals in the maximand are the disutility of working, the disutility of buyers' search intensity and the disutility of maintaining vacancies, respectively. The constraint (2.2) states that the household cannot consume more than what the household's buyers obtain in period t . (Since there is no fixed investment, the household consumes all such goods.) The amount of consumption goods depends on whether the household buyers' money balance meets the required amount in exchange, indicated by the indicator function χ_m . To receive $\hat{q}_t(-j)$ units of goods, each buyer j must have at least $\hat{m}_t(j)$ units of money. Otherwise buyer j obtains no goods from the exchange and retains his money (as indicated by the appearance of the indicator $\chi_m(j)$ in (2.5)). The condition (2.3) specifies such a trading restriction for period $t + 1$ on the matched buyers. Since buyers obtain a positive surplus from the exchange if they have the

required amount of money, it is optimal for the household to choose such M that satisfies the constraint (2.3). Thus, $\chi_{mt}(j) = 1$ for all $t \geq 1$. For $t = 0$, however, both M_0 and \hat{m}_0 are exogenous to the household and so the trading restriction may fail. To simplify discussion, it is assumed that $M_0/a_b \geq \hat{m}_0(j)$ so that $\chi_{m0}(j) = 1$ for all $j \in A_{b0}^*$.

(2.4) is a similar trading restriction on sellers in period $t + 1$: To obtain a money balance $\hat{m}_{t+1}(-j)$, which appears in the expression of profit $\pi_{t+1}(j)$, the seller must have at least a quantity $\hat{q}_{t+1}(j)$ units of goods. Since there is a positive surplus for the seller if the restriction is satisfied, $\chi_{qt}(j) = 1$ is one for all $t \geq 1$ and we assume $\chi_{q0} = 1$. The amount of goods available to the seller in period $t + 1$ includes inventory at the beginning of the period, i_{t+1} , and output in the period, $f(n_{t+1})$. The production function f satisfies: $f' > 0$, $f'' < 0$, $f(0) = f'(\infty) = 0$, and $f'(0) = f(\infty) = \infty$.

The constraint (2.5) describes the motion of the household's money balance. On its left-hand side the first integral is the money balance paid by the household's buyers to other households' sellers; the second integral is the household's total wage income; and the third integral is the household's total profit from firms, where:

$$\pi_t(j) = \begin{cases} \chi_{qt}(j)\hat{m}_t(-j) - \hat{P}_t\hat{W}_t(j)n_t(j), & \text{if } j \in A_{pt}^*, \\ -\hat{P}_t\hat{W}_t(j)n_t(j), & \text{otherwise.} \end{cases}$$

Note that a firm pays the wage bill $\hat{P}_t\hat{W}_t(j)$ regardless of whether it has a suitable match in the goods market. Even when the firm does not succeed in selling its goods, the wage payment is made possible by resource sharing between firms within the household.⁶ The joint profit of the firms in a household is non-stochastic as a result of risk-sharing:

$$\Pi_t \equiv \int_{A_p} \pi_t(j) dj = \int_{A_{pt}^*} \chi_{qt}(j)\hat{m}_t(-j) dj - \hat{P}_t \int_{A_p} \hat{W}_t(j)n_t(j) dj.$$

As discussed before, the indicator function $\chi_{qt}(j)$ is set to one for all $t \geq 0$.

The condition (2.6) describes the law of motion of employment, where $v_t\mu_t$ is the number of

⁶Firms pay wages at the end of each period so that they do not have to enter the labor market holding cash. This assumption differs from that in [17], where firms must pay nominal wages before production. My assumption allows me to focus on search as the propagation mechanism by abstracting from the liquidity effect of money injection on firms.

newly hired workers per firm and $\delta_n \in (0, 1)$ is a constant job separation rate.⁷ (2.7) describes the law of motion of inventory, where $\delta_i \in (0, 1)$ is a constant depreciation rate of inventory (depreciation takes place at the end of each period). Note that firms produce goods regardless of whether it has a suitable match in the goods market. Unsold goods are added to the inventory.

Let Ω_{Mt} be the shadow price of money at the beginning of period $t + 1$, measured in terms of period- t utility (rather than period- $(t + 1)$ utility). Define Ω_{nt} and Ω_{it} similarly for employment and inventory at the beginning of period $t + 1$, respectively. Then Ω_{Mt} is the shadow price of (2.5), Ω_{nt} of (2.6) and Ω_{it} of (2.7). Also, let the shadow price be $\Lambda_{t+1}(j)$ for (2.3) and $\Omega_{qt+1}(j)$ for (2.4), both measured in terms of period- $(t + 1)$ utility. With $(\hat{q}, \hat{m}, \hat{P}, \hat{W})$ being taken as given, the choice of c is given by the equality form of (2.2) and the choices of $(M_{t+1}, i_{t+1}, n_{t+1}, s_t, v_t)$ are characterized by the following first-order conditions (the index j is suppressed):

$$\Omega_{Mt} = \beta[\Omega_{Mt+1} + g_{bt+1}s_{t+1}\Lambda_{t+1}], \quad (2.8)$$

$$\Omega_{it} = \beta[(1 - \delta_i)\Omega_{it+1} + g_{st+1}\Omega_{qt+1}], \quad (2.9)$$

$$\Omega_{nt} = \beta[(1 - \delta_n)\Omega_{nt+1} - \Omega_{Mt+1}\hat{P}_{t+1}\hat{W}_{t+1}] + f'(n_{t+1})\Omega_{it}, \quad (2.10)$$

$$\Phi'(s_t) = g_b(\hat{s}_t) \cdot [U'(c_t)\hat{q}_t - \Omega_{Mt}\hat{m}_t], \quad (2.11)$$

$$\Omega_{nt} = K'(v_t)/\mu(\hat{v}_t). \quad (2.12)$$

The slackness conditions associated with (2.3) and (2.4) are

$$\Lambda_t(j) \left[\frac{M_t}{a_b} - \hat{m}_t(j) \right] = 0, \quad \text{for all } j \in A_{bt}^* \quad (2.13)$$

$$\Omega_{qt}(j) [i_t(j) + f(n_t(j)) - \hat{q}_t(j)] = 0, \quad \text{for all } j \in A_{pt}^*. \quad (2.14)$$

The condition (2.8) equates the opportunity cost of acquiring money at time t , Ω_{Mt} , and the future benefit of using money. (2.11) equates the marginal disutility of search intensity and the expected gain from a higher search intensity. (2.12) equates the marginal disutility of vacancy,

⁷Since all firms in the household pool their employment and inventories, the rate δ_n should be interpreted as the net job separation of all firms. In this sense, the assumption of a constant job separation rate δ_n is not inconsistent with observed quits and layoffs at the level of individual firms.

$K'(v)$, and the expected benefit of an additional vacancy, $\mu\Omega_n$. (2.9) equates the discounted value of inventory including the capital gain, $\Omega_{it} - \beta(1 - \delta_i)\Omega_{it+1}$, and the cash flow $\beta g_{st+1}\Omega_{qt+1}$. (2.10) has a similar interpretation for the value of employment.

For future use, let us denote $\omega_t \equiv \hat{P}_t\Omega_{Mt}$ as the shadow value of the real money balance and similarly denote $\lambda_t \equiv \hat{P}_t\Lambda_t$. Also, denote $k(v) \equiv K'(v)/\mu(v)$. The properties of K and μ imply $k'(v) > 0$ for $v > 0$ and $k(0) = k'(0) = 0$.

2.3. Terms of Trade

We now describe the terms of trade in each match, denoted (q, m, W) , and the associated price $P = m/q$. To do so, we need to find the contribution of each match to the utilities of the households involved. Strictly speaking, such contribution is negligible because each agent is negligible in a household. As is standard, we can re-interpret each agent as an identity of a small measure, Δ , in the household, compute the terms of trade and then take the limit $\Delta \rightarrow 0$.

Consider first a match in the labor market at t between a producer from household h and an unemployed agent from household $-h$, each of size Δ . Add a bar to variables pertaining to household $-h$ and suppress the individual index. The firm and the worker negotiate the wage W_{t+1} to be paid to the worker at $t + 1$. The agent's expected wage income is $W_{t+1}\Delta$ in terms of real money balances which increases its household's utility by $\beta(\bar{\omega}_{t+1}W_{t+1} - \varphi)\Delta$. The producer will have Δ more workers at $t + 1$ and so the surplus is $[\Omega_{nt} - \beta(1 - \delta_n)\Omega_{nt+1}]\Delta$. Modifying (2.10) to compute the firm's cash flow generated from a Δ increase rather than a marginal increase in employment, we have:

$$[\Omega_{nt} - \beta(1 - \delta_n)\Omega_{nt+1}]\Delta = \Omega_{it}[f(n_{t+1} + \Delta) - f(n_{t+1})] - \beta\omega_{t+1}W_{t+1}\Delta.$$

The wage rate is assumed to maximize the weighted Nash product of the two agents' surpluses, where the bargaining weight of the firm is $\sigma \in (0, 1)$. Normalizing the surpluses by $\beta\Delta$, the wage rate solves

$$\max_{W_t} \left[\beta^{-1}\Omega_{it} \frac{f(n_{t+1} + \Delta) - f(n_{t+1})}{\Delta} - \omega_{t+1}W_{t+1} \right]^\sigma \cdot [\bar{\omega}_{t+1}W_{t+1} - \varphi]^{1-\sigma}.$$

Taking the limit $\Delta \rightarrow 0$ on the first-order condition, we have:

$$W_{t+1} = \sigma \cdot \frac{\varphi}{\bar{\omega}_{t+1}} + (1 - \sigma) \cdot \frac{\Omega_{it}}{\beta \omega_{t+1}} \cdot f'(n_{t+1}). \quad (2.15)$$

The wage rate is a weighted sum of the worker's opportunity cost, $\varphi/\bar{\omega}_{t+1}$, and the value of the marginal product of labor, $f'(n_{t+1})\Omega_{it}/(\beta\omega_{t+1})$. The firm's profitability from hiring is:

$$\beta^{-1}\Omega_{it} \cdot f'(n_{t+1}) - \omega_{t+1}W_{t+1} = \sigma \cdot \left[f'(n_{t+1})\beta^{-1}\Omega_{it} - \varphi\omega_{t+1}/\bar{\omega}_{t+1} \right]. \quad (2.16)$$

Now consider a match in the goods market at t between a seller from household h and a buyer from household $-h$. Again add a bar to variables pertaining to household $-h$. The trade at the terms $(q\Delta, \bar{m}\Delta)$ generates the following surpluses to the two agents' households:

$$\text{seller: } \Omega_{Mt}\bar{m}_t\Delta - [(1 - \delta_i)\Omega_{it} + \Omega_{qt}]q_t\Delta,$$

$$\text{buyer: } U(\bar{c}_t + q_t\Delta) - U(\bar{c}_t) - (\bar{\Omega}_{Mt} + \bar{\Lambda}_t)\bar{m}_t\Delta.$$

Since there is little empirical evidence on agents' bargaining weights in the goods market, equal weights are chosen here (but see Section 6). The terms of trade solve:

$$\max \left\{ \Omega_{Mt}\bar{m}_t - [(1 - \delta_i)\Omega_{it} + \Omega_{qt}]q_t \right\}^{1/2} \left[\frac{U(\bar{c}_t + q_t\Delta) - U(\bar{c}_t)}{\Delta} - (\bar{\Omega}_{Mt} + \bar{\Lambda}_t)\bar{m}_t \right]^{1/2}.$$

Denote $P_t = \bar{m}_t/q_t$. The first-order conditions lead to

$$U'(\bar{c}_t + q_t\Delta)\Omega_{Mt} = (\bar{\Omega}_{Mt} + \bar{\Lambda}_t)[(1 - \delta_i)\Omega_{it} + \Omega_{qt}],$$

$$2(\bar{\Omega}_{Mt} + \bar{\Lambda}_t)P_t = U'(\bar{c}_t + q_t\Delta) + \frac{U(\bar{c}_t + q_t\Delta) - U(\bar{c}_t)}{q_t\Delta}.$$

Taking the limit $\Delta \rightarrow 0$, we have:

$$P_t\Omega_{Mt} = \Omega_{qt} + (1 - \delta_i)\Omega_{it}, \quad (2.17)$$

$$U'(c_t) = P_t(\bar{\Omega}_{Mt} + \bar{\Lambda}_t). \quad (2.18)$$

The seller's surplus per sale equals Ω_q and the buyer's surplus per purchase equals $P\Lambda = \lambda$.

3. EQUILIBRIUM

3.1. Characterization

The following defines a symmetric search equilibrium:

Definition 3.1. *A symmetric search equilibrium is a sequence of households' choices $\{\Gamma_{ht}\}_{t \geq 0}$, $\Gamma_{ht} \equiv (c_t, s_t, v_t, M_{t+1}, i_{t+1}, n_{t+1})_h$, expected quantities in trade $\{\hat{X}_t\}_{t \geq 0}$, $\hat{X}_t \equiv (\hat{m}_t, \hat{q}_t, \hat{W}_t)$, and the terms of trade $\{X_t\}_{t \geq 0}$ such that*

- (i) *all these variables are identical across households and relevant individuals;*
- (ii) *given $\{\hat{X}_t\}_{t \geq 0}$ and (M_0, i_0, n_0) , $\{\Gamma_{ht}\}_{t \geq 0}$ solves (PH), with $(s, v) = (\hat{s}, \hat{v})$;*
- (iii) *X_t satisfies (2.15), (2.17), and (2.18);*
- (vi) *$\hat{X}_t = X_t$ for all $t \geq 0$.*

The above definition requires each household to take the sequence of \hat{X} as given when choosing Γ_h . With this definition, a symmetric equilibrium is determined as follows. First, for any given \hat{X} , the household's choices are a correspondence $\Gamma_h = G(\hat{X})$ (part (ii)). Second, the bargaining problem gives X as a correspondence of the particular household's and other households' choices, say, $X = g(\Gamma_h, \Gamma_{-h})$ (part (iii)). Finally, $\hat{X} = X$ and $\Gamma_{-h} = \Gamma_h$ so that (Γ, X) solve $\Gamma = G(g(\Gamma, \Gamma))$ and $X = g(G(X), G(X))$. Note that the restriction $\hat{X} = X$ is imposed in the final stage of solving the equilibrium but not in the household's problem.

An alternative definition would be such that requires each household to directly take into account of how its choices affect the terms of trade. With this definition, the bargaining problem is solved first, which generates $X = g(\Gamma_h, \Gamma_{-h})$. The household's problem is solved next, taking the bargaining outcome, $\hat{X} = X = g(\Gamma_h, \Gamma_{-h})$, as a constraint. The solution is a best response to other households' choices, say, $\Gamma_h = J(\Gamma_{-h})$. A symmetric equilibrium is a pair (Γ, X) that satisfies $\Gamma = J(\Gamma)$ and $X = g(\Gamma, \Gamma)$.

Given the description of the environment, the alternative definition seems more appropriate.⁸ The current definition is chosen here for two reasons. First, it is standard in the search theory of unemployment (see [32]), which assumes that firms have bargaining power in wage determination but take wages as given when choosing the capital stock and employment. Second, the alternative definition is more complicated algebraically, since the constraint $X = g(\Gamma_h, \Gamma_{-h})$ generates additional terms in the first-order conditions of the household's problem. Also, the first-order conditions typically involve the second-order derivatives of the utility and cost functions so that proper restrictions on the third-order derivatives of these functions are required to ensure optimality. In contrast, the first-order conditions under the current equilibrium definition involve only the first-order derivatives of these functions. It may be conjectured that this additional complexity does not lead to qualitatively different results.⁹

Given the symmetry requirement, I suppress the hat on aggregate variables and the bar on household specific variables. I further restrict attention to the equilibrium where $\lambda > 0$ and $\Omega_q > 0$. The restriction $\lambda > 0$ requires that a buyer prefer spending to hoarding his money (i.e., $U' > \omega$), and the restriction $\Omega_q > 0$ requires that a seller prefer selling to hoarding his product (i.e., $\omega > (1 - \delta_i)\Omega_i$). A positive nominal interest rate is sufficient for $\lambda > 0$ and a positive (but bounded) inventory is sufficient for $\Omega_q > 0$.¹⁰ These requirements will be verified around the steady state. Under these requirements, $q = i + f(n) = M/(a_b P)$ and so the price level is

⁸The two definitions differ conceptually in whether the household can commit to its choices. If the household can commit to its choices, each ex ante (before-match) choice Γ_h will induce an ex post (after-match) bargaining outcome $g(\Gamma_h, \Gamma_{-h})$. It is then appropriate to ask the household to take the bargaining outcome as a constraint. If the household cannot commit to its choices, the household might have an incentive to re-optimize after matches take place. Matched agents do not expect that an arbitrary choice Γ_h by the household will induce the bargaining outcome $X = g(\Gamma_h, \Gamma_{-h})$. If they did settle on X , the household would want to make a different choice, say Γ'_h . An equilibrium in this case requires that for any given terms of trade \hat{X} the household's choices leave no incentive for ex post re-optimization. Since our description of the environment precludes the opportunity of re-optimization after matches, the alternative equilibrium definition seems more appropriate.

⁹In a related model [38] I adopt the alternative equilibrium definition and show that money growth has effects similar to those generated under the current definition.

¹⁰To see that a positive nominal interest rate implies $\lambda > 0$, imagine that a household uses money to purchase a security at t that repays one unit of money at $t + 1$. Then the price of this security must be $\beta\Omega_{Mt+1}/\Omega_{Mt}$. The net nominal interest rate is $\Omega_{Mt}/(\beta\Omega_{Mt+1}) - 1$, which equals $g_{bt+1}s_{t+1}\lambda_{t+1}/\omega_{t+1}$ by (2.8). Thus, $\lambda > 0$ if and only if the nominal interest rate is positive.

$P_t = M_t/(a_b q_t)$. Define the gross rate of money growth between periods t and $t + 1$ by

$$\gamma_t \equiv M_{t+1}/M_t = (M_t + \tau_t)/M_t.$$

The gross inflation rate between periods t and $t + 1$ is $P_{t+1}/P_t = \gamma_t q_t/q_{t+1}$.

Equilibrium conditions can be expressed in terms of $(v, n, \Omega_i, \omega, q)$ by eliminating other variables $(i, \Omega_q, \lambda, c, \mu, \Omega_n, W, m, s)$. First, when $\Omega_q > 0$, $i = q - f(n)$ and Ω_q can be eliminated by (2.17); when $\lambda > 0$, $c_t = a_p B z s_t^\alpha q_t$ and λ can be eliminated by (2.18). Second, under symmetry, $\mu = \mu(v)$ and $\Omega_n = k(v) = K'(v)/\mu(v)$ (see (2.12)). Third, $m = Pq$ and W can be eliminated using (2.15). Finally, substituting the expression for g_b into (2.11) generates

$$s_t^{1-\alpha} \Phi'(s_t) = z[U'(a_p B z s_t^\alpha q_t) - \omega_t] q_t. \quad (3.1)$$

The search intensity can then be solved as a decreasing function of (ω, q) . Substituting these relationships into (2.6)–(2.10) generates the following dynamic system:

$$(D) \quad \begin{cases} k(v_t) = \beta(1 - \delta_n)k(v_{t+1}) + \sigma[\Omega_{it} f'(n_{t+1}) - \beta\varphi], \\ n_{t+1} = (1 - \delta_n)n_t + v_t \mu(v_t), \\ \Omega_{it} = \beta[(1 - \delta_i)\Omega_{it+1} + B z s_{t+1}^\alpha (\omega_{t+1} - (1 - \delta_i)\Omega_{it+1})], \\ \omega_t = \frac{\beta}{\gamma_t} \cdot \frac{q_{t+1}}{q_t} \{\omega_{t+1} + z s_{t+1}^\alpha [U'(c_{t+1}) - \omega_{t+1}]\}, \\ q_{t+1} = (1 - \delta_i)(1 - B z s_t^\alpha) q_t + f(n_{t+1}). \end{cases}$$

Among the five variables, (n, q) are predetermined and others are jump variables.

The equation (3.1) and the first equation of (D) are of particular importance. (3.1) characterizes buyers' equilibrium search intensity as a decreasing function of the shadow value of real money balances. That is, buyer's search intensity is higher if spending money for consumption generates higher marginal utility than retaining money. If the value of money were so high that agents were indifferent between spending money and retaining it (i.e., if $\omega \rightarrow U'$), there would be no incentive for buyers to search. The first equation in (D) characterizes the firm's equilibrium vacancy as an increasing function of the profitability from hiring, $\sigma[\Omega_{it} f'(n_{t+1})/\beta - \varphi]$. If workers were paid the value of the marginal product of labor, there would be no incentive for firms to hire

workers. Since the firm's profitability from hiring depends on the seller's future surplus per sale through the value of inventory (Ω_{it}), current vacancy respond to changes in future sales revenues.

3.2. Existence of the Steady State

The steady state, denoted with an asterisk, is given by the following equations:

$$\begin{cases} \Omega_i^* = \frac{\beta B z s^{*\alpha}}{1 - \beta(1 - \delta_i)(1 - B z s^{*\alpha})} \cdot \omega^*, \\ v^* \mu(v^*) = \delta_n n^*, \\ q^* = \frac{f(n^*)}{1 - (1 - \delta_i)(1 - B z s^{*\alpha})}, \end{cases} \quad (3.2)$$

$$\begin{cases} c^* = \frac{a_p B z s^{*\alpha} \cdot f(n^*)}{1 - (1 - \delta_i)(1 - B z s^{*\alpha})}, \\ s^* \Phi'(s^*) = \frac{c^*}{a_p B} [U'(c^*) - \omega^*], \\ z s^{*\alpha} = \frac{\gamma - \beta}{\beta} \cdot \frac{\omega^*}{U'(c^*) - \omega^*}, \\ k(v^*) \cdot \frac{1 - \beta(1 - \delta_n)}{\sigma \beta} = \frac{B z s^{*\alpha} \omega^*}{1 - \beta(1 - \delta_i)(1 - B z s^{*\alpha})} \cdot f'(n^*) - \varphi. \end{cases} \quad (3.3)$$

(3.2) gives (Ω_i^*, v^*, q^*) as functions of $(c^*, s^*, \omega^*, n^*)$. In particular, denote the solution for v^* as $v(n^*)$, which is an increasing function. Then the equations in (3.3) involve only $(c^*, s^*, \omega^*, n^*)$, which can be solved in two blocks. First, substituting the first equation of (3.3) into the second to solve for s^* as a function of (ω^*, n^*) , denoted $s(\omega^*, n^*)$. Under the assumption $RA \geq 1$, $s_\omega < 0$ and $s_n < 0$. Steady state consumption can then be expressed as $c^* = c(\omega^*, n^*)$, with $c_\omega < 0$ and $c_n > 0$. Also, from the second equation of (3.3), $c_\omega > c^*/(U' - \omega^* + c^* U'')$ so that $U'(c(\omega^*, n^*))/\omega^*$ is a decreasing function of ω^* .

Next, substituting $s(\omega^*, n^*)$ and $c(\omega^*, n^*)$ into the last two equations of (3.3) gives

$$z s^\alpha(\omega^*, n^*) \cdot \left(\frac{U'(c(\omega^*, n^*))}{\omega^*} - 1 \right) = \frac{\gamma}{\beta} - 1, \quad (3.4)$$

$$k(v(n^*)) \cdot \frac{1 - \beta(1 - \delta_n)}{\sigma \beta} = \frac{B z s^\alpha(\omega^*, n^*) \omega^* f'(n^*)}{1 - \beta(1 - \delta_i)(1 - B z s^\alpha(\omega^*, n^*))} - \varphi \equiv F(\omega^*, n^*). \quad (3.5)$$

(3.4) gives a relation between ω^* and n^* , denoted $n^* = n1(\omega^*)$, and (3.5) gives another, denoted $n^* = n2(\omega^*)$. The steady state value, ω^* , is a solution to $n1(\omega^*) = n2(\omega^*)$. To ensure $\lambda > 0$, the

solution must satisfy $U'(c^*) \geq \omega^* + \Delta$, where $\Delta > 0$ is an arbitrarily small number. That is, we require $n^* \leq N(\omega^*, \Delta)$ where $N(\omega, \Delta)$ is defined by:

$$U'(c(\omega, N(\omega, \Delta))) = \omega + \Delta. \quad (3.6)$$

The curves $N(\omega, \Delta)$, $n1(\omega)$ and $n2(\omega)$ are depicted in Figure 3.

Figure 3 here.

The following lemmas state the properties of these curves (see Appendix A for a proof).

Lemma 3.2. *For sufficiently small $\Delta > 0$, the function $N(\omega, \Delta)$ is well-defined and has the following properties: $N_\omega(\omega, \Delta) < 0$, $N(\infty, \Delta) = 0$, and $\lim_{\Delta \rightarrow 0} N(0, \Delta) = \infty$. The function $n1(\omega)$ satisfies $n1'(\omega) < 0$, $n1(0) = \infty$, and $n1(\infty) = 0$. Furthermore, the two curves $n1(\omega)$ and $N(\omega, \Delta)$ have a unique intersection at a level denoted $\omega1(\Delta)$ which satisfies $\lim_{\Delta \rightarrow 0} \omega1(\Delta) = 0$.*

Lemma 3.3. *$n2(0) = 0$, $n2(\infty) = 0$, and $n2'(\omega) < 0$ for sufficiently large ω . The two curves $n2(\omega)$ and $N(\omega, \Delta)$ have a unique intersection at a level denoted $\omega2(\Delta)$ which approaches infinity when Δ approaches zero.*

The function $n2(\omega)$ is not monotonic, because $F(\omega, n)$ may increase or decrease with ω . In spite of the non-monotonicity, existence of a steady state can be deduced from Lemmas 3.2 and 3.3. These lemmas imply that the curve $n1(\omega)$ crosses the curve $N(\omega, \Delta)$ from above and the curve $n2(\omega)$ crosses the curve $N(\omega, \Delta)$ from below. When $\Delta \rightarrow 0$, the curve $n1(\omega)$ crosses the curve $N(\omega, \Delta)$ before the curve $n2(\omega)$ does. Thus there exists at least one positive solution to the equation $n1(\omega) = n2(\omega)$, which lies below the curve $N(\omega, \Delta)$ (see Figure 3). The solution satisfies $\lambda > 0$, since $n^* < N(\omega^*, \Delta)$. The solution also satisfies $\Omega_i^* < \omega^*$ (i.e. $\Omega_q^* > 0$) (see the first equation of (3.2)). This result is summarized by the following proposition.

Proposition 3.4. *For $\gamma > \beta$, there exists at least one steady state that satisfies $(\lambda, \Omega_q^*) > 0$.*

In principle, there can be more than one positive solution to (3.4) and (3.5). However, uniqueness is achieved with reasonable functional forms as in later numerical exercises. Thus, I will assume uniqueness of the steady state throughout this paper.

3.3. Steady State Effects of Money Growth

Let us focus on the effects of money growth on (ω^*, n^*) , which can be analyzed using Figure 3. It is easy to verify that an increase in the money growth rate shifts down the curve $n1(\omega)$ but has no effect on the curve $n2(\omega)$. Clearly, ω^* decreases, but n^* increases if and only if the intersection between the two curves is on the downward sloping side of $n2(\omega)$. Since a firm's marginal profitability from hiring decreases with employment (i.e. $F_n < 0$), $n2(\cdot)$ is downward sloping if and only if $F_\omega < 0$. This can be summarized in the following proposition:

Proposition 3.5. *The steady state value of real money balances decreases with the money growth rate. Steady state employment (and hence output) increases with the money growth rate if and only if, for given employment, the firm's profitability from hiring decreases with the value of real money balances.*

The negative effect of money growth on the value of real money balances is standard but the ambiguous effect of money growth on employment is not. To explain the ambiguity, note that changes in the money growth rate affect employment by affecting firm's profitability from hiring. A higher money growth rate generates two opposing effects on the profitability from hiring. The positive effect is a search-inducing effect: A higher money growth rate reduces the shadow value of money, widens the gap between spending and hoarding money, and encourages buyers to search more intensively. Since a higher search intensity generates faster sales, the firm's profitability from hiring increases. This can be seen from the positive dependence of the shadow price of inventory (Ω_i^*) on s^* for given ω^* . The negative effect is an inflation effect: Since firms' sales receipts are in terms of money, a higher inflation rate reduces the shadow value of sales, given the matching probabilities. This can be seen from the negative dependence of Ω_i^* on ω^* for given s^* . Since firms increase job vacancies only when the profitability from hiring increases, steady state employment increases with the money growth rate if and only if the search-inducing effect dominates the inflation effect. This requires the firm's sales revenue to be an overall decreasing function of ω , i.e., the function $F(\omega, n)$ to be a decreasing function of ω for given n .

Whether the search-inducing effect dominates the inflation effect depends on the money growth rate γ . When γ is sufficiently close to β , the curve $n1(\omega)$ in Figure 3 is very close to the curve $N(\omega, \Delta)$. The intersection between $n1(\omega)$ and $n2(\omega)$ is then on the downward sloping side of $n2(\omega)$, implying $dn^*/d\gamma > 0$. On the other hand, when γ is sufficiently large, the curve $n1(\omega)$ is very steep when ω is near zero but quickly gets close to the horizontal axis as ω increases. In this case ω^* is close to zero, implying $dn^*/d\gamma < 0$. The following corollary summarizes the above analysis (see Appendix A for a proof).

Corollary 3.6. *Steady state employment increases with the money growth rate γ when γ is sufficiently close to β but decreases with γ when γ is sufficiently large.*

The Friedman rule, which requires the deflation rate to be equal to the discount rate, does not maximize employment or output in the current model. It does not maximize welfare either, because employment and output approach zero when the money growth rate approaches the Friedman rule. Intuitively, the value of real money balances is so high under the Friedman rule that there is no incentive for buyers to search. Indeed, as γ approaches β , ω^* approaches the marginal utility of consumption so that the marginal gain from search approaches zero. As firms have almost no chance to sell their products successfully, employment and output approach zero. In this case, a marginal increase in the money growth rate generates a much larger search-inducing effect than an inflation effect. On the other hand, when the money growth rate is already high, a further increase reduces the value of real money balances and the firm's sales revenue close to zero so that the firm has little incentive to produce.

Since employment and output are asymptotically zero when either $\gamma \rightarrow \beta$ or $\gamma \rightarrow \infty$, the money growth rate that maximizes employment and output exceeds the Friedman rule and is finite. Similarly, since consumption is asymptotically zero when either $\gamma \rightarrow \beta$ or $\gamma \rightarrow \infty$, the money growth rate that maximizes welfare exceeds the Friedman rule and is finite. In principle, both the output-maximizing and welfare-maximizing net money growth rates can be positive.

4. MONETARY PROPAGATION

The last section has shown that a permanent increase in the money growth can increase steady state employment and output when the money growth rate is low. However, the analysis is not sufficient for drawing conclusions on how a general monetary shock is propagated. In particular, a transitory monetary shock can generate a quite different effect on output. It is possible that a transitory increase in the money growth rate can be positively propagated into output even though a permanent increase in the money growth rate reduces output. To analyze the propagation mechanism, it is necessary to examine the dynamics of the model. The dynamic system (D) does not admit analytical solutions so I proceed with calibration.

4.1. Parameter Values and Solution Method

First, let us interpret the length of a period as a quarter. The standard choice $\beta = 0.99$ gives an annual real interest rate of four per cent. The steady state (gross) money growth rate is $\gamma^* = 1.012$ that matches the quarterly average of inflation rate in postwar US data.

Next, the parameters in the labor market, $(\delta_n, A, \sigma, a_p, u)$, are determined. Set $A = 0.6$ to match the estimate on the matching function and $\delta_n = 0.06$ to match the quarterly transition rate from employment to unemployment (see [4, 30]). I choose $\sigma = 0.7$ to give workers 30% of the rent in wage bargaining (see [1] for more discussions). To identify (a_p, u) , let the labor participation rate, $a_p(1 + n^*) + u$, and the unemployment rate, $u/(u + a_p(1 + n^*))$, match the realistic values 0.7 and 0.06 respectively. Also, choose n^* to be 100. This value and the steady state relation, $v^*\mu(v^*) = \delta_n n^*$, imply an average unemployment duration, $u/(a_p v^* \mu) = 1.0745$, which is consistent with the finding in [25]. These three restrictions identify (a_p, u) and v^* .

Then I determine the parameters $(B, a_b, z, \alpha, \delta_i)$. Set the steady state inventory/output ratio (i^*/f) at 0.9 and the inventory investment/output ratio $(\delta_i i^*/f)$ at 0.0065 to match the quarterly average of these variables in postwar U.S. data. Also, choose the quarterly income velocity of money $(a_p f P/M)$ to be a realistic number, 1.0. Since the steady state values of these variables are endogenous, the procedure gives three restrictions that determine $(\delta_i, B, z s^\alpha)$. Set $\alpha = 0.8$

(see footnote 13 for a discussion). Let the shopping time of the population be 11.6% of the working time and the working time be 30% of agents' discretionary time to match the findings in [21, Tables 3 and 5].¹¹ Then $s^* = 0.116 \times 0.3a_p(1 + n^*)/a_b$. Since $zs^{*\alpha}$ is now known, z can be calculated.

The production function is assumed to be $f(n) = f_0 \cdot n^{e_f}$, where the constant f_0 is normalized to 1. The cost of vacancy is assumed to be quadratic, $K(v) = K_0v^2$, following the practice in the labor demand literature (see [19]). The disutility of the buyer's search intensity is assumed to be $\Phi(s) = \varphi \cdot (\varphi_0 s)^{1+1/\varepsilon_\Phi}$, where φ is the disutility of employment and φ_0 the efficiency units of a buyer's search intensity relative to a worker's time. To identify $(e_f, K_0, \varphi, \varphi_0, \varepsilon_\Phi)$, let the labor share of income be 0.64 (see [7]) and the hiring cost be 2% of the labor cost (see [19, pp285-287]). That is, $W^*n^*/f = 0.64$ and $K_0v^{*2}/(\omega^*W^*n^*) = 0.02$. (The vacancy cost is divided by ω in order to convert it into the same unit as the wage cost.) These two conditions and the last equation of (3.3) give e_f and the following restrictions on (φ, K_0, ω^*) :

$$\varphi = \frac{\omega^* f'}{\sigma} \left[\frac{0.64}{e_f} - \frac{(1 - \sigma)Bzs^{*\alpha}}{1 - (1 - \delta_i)(1 - Bzs^{*\alpha})} \right]; \quad K_0 = 0.02 \times 0.64\omega^*/v^{*2}.$$

The value of ω^* is calculated using the third equation of (3.3). To do so, let the utility function have an intertemporal elasticity $1/RA = 0.5$, which is in the admissible range estimated by [14]. I also re-interpret the model in order to make the consumption/output ratio realistic. This is necessary because the expenditure on fixed investment is a large fraction of GDP in reality but has been abstracted from so far. To incorporate such expenditure, let fixed investment be a constant fraction, FI_k , of aggregate sales. Since aggregate sales are exogenous to individual households, this amounts to subtracting a lump sum from each household's consumption and so the characterization of agents' choices in Section 2 remains valid. The equilibrium conditions in Section 3 continue to hold with the modification $c_t = (1 - FI_k)a_pBzs_t^\alpha q_t$. The constant FI_k can

¹¹The shopping time in [21, Table 5 there] varies widely and even does not add up to the total available time. To correct this error, I select the more reliable result obtained from time diary (experiment 3 therein). Assuming that people tend to over (or under) report shopping time in the same way as reporting the time spent on active leisure, I calculate the shopping time as a fraction of the time spent together on active leisure and social interaction. I then convert the shopping time to a fraction of work time by calculating the time of active leisure and social interaction as a fraction of work time, using Table 3 in [21].

be identified by setting steady state expenditure on fixed investment to 26.9% of output (see [7]). That is, $FI_k = 0.269 \cdot f / (Bzs^{\alpha}q^*)$. Once this is done, (c^*, ω^*) and hence (φ, K_0) are determined.

The value of φ_0 is found through the second equation of (3.3) once ε_{Φ} is identified. I set $\varepsilon_{\Phi} = 2$. Although this number is much larger than the labor supply elasticity documented by [22], it is reasonable to believe that shopping has a higher elasticity than market work.¹²

Table 4 Parameter values					
β	γ^*	RA	FI_k	B	z
0.99	1.012	2.0	0.2708	0.5263	0.118
δ_n	A	σ	u	a_p	e_f
0.06	0.6	0.7	0.0447	0.0069	0.6804
δ_i	φ	φ_0	ε_{Φ}	α	K_0
0.0072	10.603	0.4232	2.0	0.8	0.6615

Table 4 summarizes the identified parameters. It is worth noting that the chosen parameter values imply $F_{\omega} > 0$ so that a permanent increase in the money growth rate from the level $\gamma = 1.012$ reduces steady state employment and output. That is, the net money growth rate that maximizes steady state output is lower than 1.2 percent quarterly.

To complete the quantitative description of the model, let us assume that the equilibrium system is in the steady state at the beginning of period 1, where the money growth rate is γ^* . Then the path of future money growth rates is raised by $\{d \cdot \rho^{t-1}\}_{t \geq 1}$, where $\rho \in (0, 1]$ and $d > 0$ is a sufficiently small constant. That is, the money growth rates are changed to:

$$\gamma_t = \gamma^* + d \cdot \rho^{t-1}, \quad t \geq 1. \quad (4.1)$$

These changes can be alternatively interpreted as an unanticipated impulse of a magnitude d at $t = 1$ that persists through a known autoregressive process with a coefficient ρ . While agreeing on the restriction $\rho \in (0, 1)$, the literature seems to suggest a wide range of plausible values for the persistence of monetary shocks. I will examine three values: $\rho \in \{0, 0.4, 0.85\}$. The three cases are respectively referred to as an impulse shock, a transitory shock and a persistent shock.

¹²The result on propagation is robust to changes in the values of the search elasticity in the utility function, ε_{Φ} , and the elasticity in the matching function, α . I have experimented with $\varepsilon_{\Phi} = 0.4$, a value in the estimated range of the labor supply elasticity in [22], and with $\alpha = 0.5$, a value lower than the base value 0.8. In each experiment, the pattern of the propagation and the length of the delay of the output peak are the same as what is reported here while the magnitude of the peak output response falls by about 10%. When $(\varepsilon_{\Phi}, \alpha)$ are reduced to $(0.4, 0.5)$ simultaneously, the length of the delay of the output peak falls by one quarter.

I will compute the quantity $x_d(t)/x^*$ for each variable x in the set $(v, \Omega_i, \omega, q, n)$, where $x_d(t)$ is the derivative of $x(t)$ with respect to d . With $d = 0.01$, the quantity $x_d(t)/x^*$ is approximately the percentage change of the variable x from its steady state when γ increases by one percent (i.e., the net money growth rate increases from 1.2% to 2.2%). Since d is small, the system (D) can be approximated by the corresponding linearized system, which is obtained by differentiating (D) with respect to d and evaluating the derivatives at the steady state. Given the initial conditions $(q_{d1}, n_{d1}) = (0, 0)$, the system can be solved using standard techniques such as those in [5].

4.2. Propagation of the Monetary Shock

Let us first examine the propagation of an impulse increase in the money growth rate, i.e., the case $\rho = 0$. Figures 4a through 4d depict the percentage changes of selected variables in response to one percent increase in the gross rate of money growth at $t = 1$. The most prominent feature of the propagation is that employment (and hence output) follows a persistent, hump-shaped path, as depicted in Figure 4a. Employment takes five quarters to peak and then stays significantly above the steady state for ten more quarters. This persistent output response is in stark contrast with the exogenous monetary shock, which completely dies out in the second quarter (Figure 4b). The key element responsible for the persistent propagation is that buyers' search intensity persistently stays above the steady state (Figure 4b). Such persistently high search intensity is caused by the reinforcing search-inducing and inventory effects, as depicted in Figure 2 in the introduction.

Figures 4a through 4d.

Let us trace through the propagation process. The system responds in period 1 to the shock as follows. First, the increase in the money growth rate immediately reduces the value of real money balances in period 1, which increases the buyer's surplus per purchase, $U' - \omega$, and stimulates buyers to search more intensively (Figure 4b). The higher search intensity increases the probability of matches between sellers and buyers so that each seller experiences a larger quantity of sales in period 1, $Bzs_1^\alpha q_1$ (Figure 4c). Since output is predetermined, the increase in sales entirely reduces inventory in period 1. (Inventory investment per firm is defined as $i_{t+1} - (1 - \delta_i)i_t$, where

i_t is measured at the beginning of period t .)

The increase in the quantity of sales does not necessarily imply that firms have a higher sales revenue, $\omega_1 \cdot Bzs_1^\alpha q_1$, because the shadow value of each unit of goods sold is also reduced by the increase in the money growth rate. In fact, the decrease in the value of money initially dominates the rise in the quantity of sales so that sellers' surplus per sale falls in period 1 (Figure 4d). As a result, the sales revenue in period 1 falls. Despite the negative response of the sales revenue in period 1, firms immediately increase job vacancies (see Figure 4a). This is because a firm's hiring decision depends on future sales revenues. Figures 4c and 4d indicate that the firm's surplus per sale quickly increases above the steady state and so firms expect their sales revenue to increase quickly above the steady state. Thus, firms increase job vacancies in period 1.

Understanding why the firm's sales revenue increases above the steady state in period 2 is the key to understanding the monetary propagation mechanism. Loosely speaking, future sales revenue responds positively to the monetary shock because the buyers' search intensity stays persistently above the steady state while the fall in the value of money is temporary. Figure 4b shows that the search intensity stays significantly above the steady state even in the sixth quarter but the fall in the value of money lasts for only one period. What keeps the buyer's search intensity persistently above the steady state is the inventory effect of money growth induced by the search-inducing effect. That is, the immediate increase in sales in period 1 reduces inventory sharply so that the quantity of goods available for sales in period 2 falls (i.e., q_2 falls). Since the marginal utility of consumption is a decreasing function of q , households evaluate consumption more dearly in period 2. Since the consumption smoothing motive is strong ($RA = 2$), buyers keep searching hard for goods in order to maintain a smooth consumption profile, although the value of money has now returned to a level above the steady state.

The inventory effect and the search-inducing effect reinforce each other to produce a persistently positive effect on the firm's sales revenue and the profitability from hiring. Thus, job vacancy stays above the steady state and employment keeps rising until the fifth quarter, where the propagation reaches its peak. As employment is now above the steady state by a wide margin,

the marginal productivity of labor is significantly below the steady state in period 6 so that firms start to reduce hiring. Employment begins to fall back toward the steady state. The convergence is slow as the search-inducing effect and the inventory effect continue to work. Employment remains noticeably above the steady state even in the sixteenth quarter.

Figures 5a through 5b.

The propagation of more persistent money growth shocks is similar. Figures 5a and 5b depict the responses of vacancy and employment for the cases $\rho = 0.4$ and $\rho = 0.85$ respectively. The propagation in both cases is stronger than in the case $\rho = 0$, because a more persistent increase in the money growth rate generates larger search-inducing and inventory effects. The propagation in the case $\rho = 0.4$ is qualitatively the same as in the case $\rho = 0$; while the case $\rho = 0.85$ differs in that vacancy and employment respond negatively to the monetary shock in the first few periods of transition. The negative response occurs because a persistent increase in the money growth rate generates strong and persistent inflation that can dominate the search-inducing and inventory effects in the initial transition. However, as inflation dies down, the search-inducing and inventory effects overtake the prominent role in the propagation from the fourth quarter onward. Of course, if the shock is extremely but unrealistically persistent, say $\rho = 0.99$, job vacancies and employment will stay below the steady state during the entire transition.

5. DISCUSSIONS

In this section I discuss first the unique features and then the common features of the above monetary propagation mechanism in comparison with a liquidity effect model. To economize on space, let us illustrate these features only for the case of an impulse shock ($\rho = 0$).

Besides the persistent, hump-shaped output response, the monetary propagation mechanism described in the last section has four main features that differ from a liquidity effect model. First, the non-Walrasian features of the labor and goods markets are crucial for the monetary propagation. Were trade costless in the goods market, there would be no need for buyers to search or for firms to maintain inventory. In this case, neither the search-inducing effect nor

the inventory effect would arise. Consequently, output response would be quick, despite the existence of search in the labor market. Thus, simply adding labor market search into a liquidity effect model is unlikely to produce output response that is as persistent as in the current model. Similarly, if hiring could be done costlessly and instantaneously as in a conventional model, there would be no need for firms to maintain job vacancies. Like a conventional monetary model, this setup would imply quick responses in employment and output.

Figures 6a and 6b support the above argument. In Figure 6a, the inventory depreciation rate is increased from the calibrated value 0.0072 (Table 4) to 0.5. When inventory depreciates fast, the quantity of goods available for sales depends largely on new output and not so much on inventory (see the last equation of system (D)). In this case, the inventory effect is weak as increased buyers' search intensity in the first period of the shock has a small effect on the future supply of goods. Figure 6a shows that reducing the inventory effect reduces the lag of the output peak from five quarters in Figure 4a to three quarters. The magnitude of employment response is also less than one percent of that in Figure 4a.

In Figure 6b, the parameter K_0 in the vacancy cost function is reduced from the calibrated value 0.6615 in Table 4 to 0.05. When the vacancy cost is lower, firms can increase employment more quickly in response to shocks and so employment is less persistent. Figure 6b shows that the lag of the output peak decreases to three quarters from five quarters in Figure 4a. Compared with the case in Figure 4a, Figure 6b also shows a stronger immediate propagation, as firms respond to increased buyer's search intensity quickly with increased hiring.

The second interesting feature is that sales and inventory investment are more volatile than employment and output. As depicted in Figure 4c, sales increase quickly in response to the monetary shock; inventory falls quickly, serving as a buffer against the increase in the effective demand for goods. In contrast, employment and output respond smoothly. The relative volatility of sales to output seems a realistic feature that cannot be accounted by monetary models that rely heavily on the liquidity effect. In a typical liquidity effect model, the monetary shock immediately increases the loanable funds which in turn stimulate investment and output. As a result, there is

no significant difference between the timing of the peak responses of output and sales.

Third, job vacancies and unemployment follow a counter-clockwise trajectory that is documented by [25]. To see this, picture the vacancy-employment ratio, v/n , on the vertical axis against the unemployment-employment ratio, u/n , on the horizontal axis (not drawn). Since job vacancies increase immediately after the monetary shock while n is predetermined, there is an immediate discrete jump in the vacancy-employment ratio but no change in the unemployment-employment ratio. As the propagation continues, vacancy falls and employment rises so that v/n and u/n both fall. Since the two ratios eventually return to the steady state, the adjustment traces a counter-clockwise trajectory in the $(v/n, u/n)$ subspace.

Fourth, the velocity of money can persistently deviate from the steady state during the transition. To see this, note that the income velocity of money in period t is $(1 - (1 - \delta_i)(1 - Bzs_t^\alpha))/B$. As the monetary shock increases the search intensity and sales, money circulates faster during the transition. Such a deviation of the velocity of money from its steady state level persists, because the search intensity persistently deviates from the steady state.

Figure 7 here.

The current model also generates two results that are possessed by suitably enriched liquidity effect models. First, the real interest rate responds negatively to an increase in the money growth rate. To illustrate, define the real interest rate (rr_{t+1}) as the net return in consumption goods to one unit of consumption good lent in period t . Then the household's maximization problem implies $U'(c_t) = \beta(1 + rr_{t+1})U'(c_{t+1})$ and so

$$rr_{t+1} = \frac{1}{\beta} \left(\frac{c_{t+1}}{c_t} \right)^{RA} - 1. \quad (5.1)$$

Note that consumption responds to the monetary shock in the same way as the quantity of sales. Since sales jump up immediately after the shock and then decline, the consumption growth rate c_{t+1}/c_t dips immediately after the shock and then rises toward the steady state. That is, the real interest rate falls immediately after the shock and then rises. Figure 7 depicts such a response of the (gross) real interest rate to a transitory monetary shock, together with the responses of

the nominal interest rate and the rate of inflation. The negative response of the real interest rate implies that the nominal interest rate responds to the monetary shock only slightly, where the nominal interest rate is calculated as in footnote 11.¹³

The second result that the current model shares with a liquidity effect model such as [6] is that inflation peaks immediately after the increase in the money growth rate and then quickly falls below the steady state. This feature contrasts with the evidence that inflation peaks several quarters after the monetary shock (see [41]). However, the quick response of inflation is not an undesirable feature here. By allowing for perfectly flexible prices and wages, the model provides a robust hump-shaped output response that does not rely on nominal rigidities. Also, it is hard to imagine that prices do not react quickly to the shock in the money growth rate, given the assumption of perfect foresight and the absence of elements that delay inflation.

Whether nominal prices are sticky is unlikely to be important to our story, although the current analysis features perfectly flexible prices. What is important is that the shadow value of future real money balance immediately falls after an increase in the money growth rate. Since the shadow value of real money balances depends on both current and anticipated future price adjustments, it will immediately fall after an increase in the money growth rate, as long as agents anticipate inflation to rise *eventually*. Whether inflation rises immediately after the monetary shock or several periods later will not change the qualitative response of the shadow value of real money balances. Thus, one can imagine a modification of the current model where prices are sticky because goods are ordered several periods in advance of consumption. Even in this modified environment, an increase in the money growth rate will immediately reduce the shadow value of the real money balance. The buyers' search intensity will immediately rise, which again generates the reinforcing inventory effect and persistent, hump-shaped output response.

¹³The Fisher's relation between nominal and real interest rates does not hold here, because agents must search to exchange between money and goods. If one defines the nominal interest rate through the Fisher's relation while keeping (5.1) as the definition for the real interest rate, the response of the nominal interest rate will be larger than in Figure 7, but the relative magnitudes do not change.

6. CONCLUSION

This paper has examined a monetary propagation mechanism in an economy where exchanges in goods and labor markets require costly search. The monetary propagation is kickstarted by an increase in the effective demand for goods generated by a higher search intensity of buyers. The high demand increases firms' sales and induces firms to increase hiring. The increase in sales also reduces inventory and creates an inventory effect that further induces buyers to increase the search intensity. The search-inducing and inventory effects reinforce each other and create a powerful propagation mechanism. Even for an impulse increase in the money growth rate, employment and output take five quarters to peak and stay significantly above the steady state for ten more quarters. Since the current model passes the critical test of accounting for the persistent, hump-shaped response of output to monetary shocks, it is a valuable alternative to traditional monetary models.

Two features are important for the strong monetary propagation mechanism. One is the existence of surpluses in trades and the other is the predetermined nature of employment. The first feature is responsible for the search-inducing effect. It is by increasing the buyer's surplus from trade does an increase in the money growth rate stimulate buyers to search more intensively; it is by increasing the seller's surplus from trade does the higher search intensity encourage firms to hire more workers. The second feature is responsible for the inventory effect: Since employment (and hence output) is predetermined, the increase in sales reduces inventory and the supply of goods in the next period, further stimulating the buyer's search effort. The reinforcing search-inducing and inventory effects propagate the monetary shock.

The persistent monetary propagation mechanism is likely to be robust to a number of perturbations in the modelling details. First, the specific bargaining formulation adopted in Section 2 can be altered without affecting the results much, as long as the surplus from trade is monotonically shared by the two sides of the match. With monotonic sharing rules, the buyer's surplus from trade depends negatively on the value of real money balances and so the buyer's search intensity responds positively to money growth; the firm's profitability from hiring depends pos-

itively on the sales revenue and so a higher buyer's search intensity is transmitted into a higher profitability from hiring and more hiring.

Second, one can incorporate fixed investment into the model. Adding fixed investment adds another factor — the capital stock — which prolongs the real effect of a monetary shock. However, it also reduces (but does not completely eliminate) the inventory effect since new investment increases output and refills some of the reduced inventory after the monetary shock. When both effects are considered, it seems plausible that adding fixed investment would not change much the timing of the peak output response or the duration of monetary propagation.

Third, one can also allow the labor supply and the seller's search intensity to be elastic. A useful way to endogenize the labor supply is to endogenize the search intensity of unemployed agents. This extension will not change the results much since workers' search intensity is empirically much less elastic than job vacancy ([25]). An endogenized worker's search intensity may even increase the response of job vacancy to the monetary shock, as a higher search intensity by workers increases the rate at which vacancies are matched with workers. Similarly, endogenizing the sellers' search intensity in the goods market may also increase the buyer's search intensity and induce a stronger propagation.

Finally, the timing of the monetary transfer can be altered. In common with the liquidity effect models, the current analysis has assumed that the monetary transfer is distributed to households when agents have already gone to the goods and labor markets. With such timing, it takes one period for buyers to adjust their money holdings in response to the transfer. One can eliminate this delay of response by assuming that monetary transfers take place at the beginning of each period before agents go to the markets. A previous version of this paper has adopted this alternative timing and found that the dynamics were very similar to those in Figures 5a and 5b for transitory ($\rho = 0.4$) and persistent ($\rho = 0.85$) monetary shocks. In this case a pure impulse shock ($\rho = 0$) had a negligible effect on output because newly hired workers in any period t begin to produce in period $t + 1$, which makes firms' vacancy decision depend on future sales revenue rather than current revenue. A temporary increase in the money growth rate that reverses to its

original value next period has little effect on future sales revenue. If newly hired workers were assumed to begin production immediately, a pure impulse shock would generate a significant propagation as in Figure 4*a* even when the shock is realized before agents go to the markets.

APPENDIX

A. PROOFS FOR SECTION 3

A.1. Proof of Lemma 3.2

For the properties of $N(\omega, \Delta)$, rewrite (3.6) as $U'(c(\omega, n))/(\omega + \Delta) = 1$. Since the left-hand side of this equation is a decreasing function of (ω, n) when Δ is sufficiently small, the solution for n , denoted $N(\omega, \Delta)$, is a decreasing function of ω . The two properties $N(\infty, \Delta) = 0$ and $\lim_{\Delta \rightarrow 0} N(0, \Delta) = \infty$ can be shown in a similar way so only the proof for the latter is presented. To show $\lim_{\Delta \rightarrow 0} N(0, \Delta) = \infty$, note from the definition of $N(\omega, \Delta)$ that $c(0, N(0, \Delta)) = U'^{-1}(\Delta)$. The second equation of (3.3) implies:

$$s\Phi'(s) = \Delta U'^{-1}(\Delta)/(a_p B) \text{ for } s = s(0, N(0, \Delta)).$$

The assumed properties of U imply $U'^{-1}(0) = \infty$ and $\lim_{\Delta \rightarrow 0} \Delta U'^{-1}(\Delta) = 0$. Thus $s(0, N(0, \Delta))$ approaches 0 as $\Delta \rightarrow 0$. Expressing $f(n)$ in terms of (c, s) from the first equation of (3.3), one can easily show that $f(N(0, \Delta)) \rightarrow \infty$ as $\Delta \rightarrow 0$. This shows $\lim_{\Delta \rightarrow 0} N(0, \Delta) = \infty$.

Let us now show the properties of $n1(\omega)$. The property $n1'(\omega) < 0$ is apparent from (3.4), whose left-hand side is a decreasing function of (ω, n) . To show $n1(0) = \infty$, let us first show $c(0, n1(0)) = \infty$. Suppose, to the contrary, that $c(0, n1(0))$ is bounded above by some constant c_0 . Then $U'(c(0, n1(0)))$ is strictly bounded away from zero. The second equation of (3.3) then implies that $\lim_{\omega \rightarrow 0} s(\omega, n1(\omega))$ is strictly bounded away from zero, since

$$\lim_{\omega \rightarrow 0} s\Phi'(s) = \lim_{\omega \rightarrow 0} cu'(c)/(a_p B) \text{ for } s = s(\omega, n1(\omega)) \text{ and } c = c(\omega, n1(\omega)).$$

In this case, (3.4) (which defines $n1(\omega)$) is violated, because its left-hand side approaches infinity as $\omega \rightarrow 0$. Thus, $c(0, n1(0)) = \infty$. Since the first equation of (3.3) implies

$$c(\omega, n1(\omega)) \leq \frac{a_p}{1 - \delta_i} f(n1(\omega)),$$

it is clear that $n1(0) = \infty$. Now that $n1(0) = \infty > N(0, \Delta)$, the curve $n1(\omega)$ must cross the curve $N(\omega, \Delta)$ from above if the two have a unique intersection as stated in the lemma and shown below. That is, $0 < n1(\omega) < N(\omega, \Delta)$ for $\omega > \omega_1(\Delta)$. Since $N(\infty, \Delta) = 0$, $n1(\infty) = 0$.

To show that $n1(\omega)$ and $N(\omega, \Delta)$ have a unique intersection, set $U'(c) = \omega + \Delta$ as the definition of $N(\omega, \Delta)$ requires. That is, $c = U'^{-1}(\omega + \Delta)$. The second equation of (3.3) gives

$$s = \left(\frac{\gamma - \beta}{\beta z} \cdot \frac{\omega}{\Delta} \right)^{1/\alpha}.$$

Substituting this function into the first equation of (3.3) gives:

$$\Delta U'^{-1}(\omega + \Delta) = a_p B \left(\frac{\gamma - \beta}{\beta z} \cdot \frac{\omega}{\Delta} \right)^{1/\alpha} \cdot \Phi' \left(\left(\frac{\gamma - \beta}{\beta z} \cdot \frac{\omega}{\Delta} \right)^{1/\alpha} \right). \quad (\text{A.1})$$

The left-hand side of (A.1) is a decreasing function of ω and the right-hand side is an increasing one. It is easy to verify the following properties:

$$LHS(\text{A.1})|_{\omega=0} > 0 = RHS(\text{A.1})|_{\omega=0},$$

$$LHS(\text{A.1})|_{\omega=\infty} = 0 < RHS(\text{A.1})|_{\omega=\infty}.$$

Thus, there exists a unique solution to (A.1), which is $\omega1(\Delta)$. For fixed ω , as $\Delta \rightarrow 0$, the left-hand side of (A.1) approaches 0 and the right-hand side approaches ∞ , in which case (A.1) is satisfied only when $\omega = 0$. That is, $\lim_{\Delta \rightarrow 0} \omega1(\Delta) = 0$. ■

A.2. Proof of Lemma 3.3

To show $n2(0) = 0$, note that

$$\frac{1 - \beta(1 - \delta_i)(1 - Bzs^\alpha)}{Bzs^\alpha} \geq \beta(1 - \delta_i) \text{ for all } s \in [0, \infty).$$

Then (3.5) (which defines $n2(\omega)$) implies

$$f'(n2(\omega)) \geq \frac{\beta(1 - \delta_i)}{\omega} [\varphi + k(v(n2(\omega))) \cdot \frac{1 - \beta(1 - \delta_n)}{\sigma\beta}] \geq \frac{\beta(1 - \delta_i)\varphi}{\omega}.$$

Thus, $f'(n2(0)) = \infty$ and so $n2(0) = 0$.

Similar to the feature $n1(\infty) = 0$ in Lemma 3.2, the feature $n2(\infty) = 0$ is a consequence of the properties of the intersection between the two curves $n2(\omega)$ and $N(\omega, \Delta)$ stated in Lemma 3.3. Also, since $n2(\omega) > 0$ for moderate ω , the feature $n2(\infty) = 0$ implies that $n2'(\omega) < 0$ for sufficiently large ω . To show the properties of the intersection between the two curves $n2(\omega)$ and $N(\omega, \Delta)$, set $\omega = U'(c) - \Delta$ as the definition of $N(\omega, \Delta)$ requires. Then the second equation of

(3.3) implies $s\Phi'(s) = \Delta c/(a_p B)$. For given (c, Δ) , denote the solution for s as $s(c, \Delta)$. Then $s_c > 0$ and $s(c, 0) = 0$. The first equation of (3.3) gives

$$n = n(c, \Delta) \equiv f^{-1} \left(\frac{c}{a_p B z s^\alpha} [1 - (1 - \delta_i)(1 - B z s^\alpha)] \right), \quad \text{where } s = s(c, \Delta).$$

Since $c/s^\alpha = (a_p B/\Delta) s^{1-\alpha} \Phi'(s)$, $n(c, \Delta)$ is an increasing function of c . Also, $n(c, 0) = \infty$. Substituting the functions $s(c, \Delta)$ and $n(c, \Delta)$ into (3.5) gives:

$$\frac{1}{f'(n(c, \Delta))} \left[\varphi + k(v(n(c, \Delta))) \cdot \frac{1 - \beta(1 - \delta_n)}{\sigma\beta} \right] = \frac{B z s^\alpha(c, \Delta) [U'(c) - \Delta]}{1 - \beta(1 - \delta_i)(1 - B z s^\alpha(c, \Delta))}. \quad (\text{A.2})$$

The left-hand side of (A.2) is an increasing function of c . Since $cU'(c)$ and $s^\alpha(c, \Delta)/c$ are decreasing functions of c and since $s^\alpha[U'(c) - \Delta] = (s^\alpha/c) \cdot c[U'(c) - \Delta]$, the right-hand side of (A.2) is a decreasing function of c . Also,

$$LHS(\text{A.2})|_{c=0} = 0 < \infty = RHS(\text{A.2})|_{c=0},$$

$$LHS(\text{A.2})|_{c=\infty} = \infty > 0 = RHS(\text{A.2})|_{c=\infty}.$$

Thus there is a unique solution for c to (A.2). Denote this solution by $c(\Delta)$. Then $\omega 2(\Delta) = U'(c(\Delta)) - \Delta$. Furthermore, since $\lim_{\Delta \rightarrow 0} s(c, \Delta) = 0$ and $\lim_{\Delta \rightarrow 0} n(c, \Delta) = \infty$, $LHS(\text{A.2}) \rightarrow \infty$ and $RHS(\text{A.2}) \rightarrow 0$ as $\Delta \rightarrow 0$. Thus $\lim_{\Delta \rightarrow 0} c(\Delta) = 0$ and so $\lim_{\Delta \rightarrow 0} \omega 2(\Delta) = \infty$. ■

A.3. Proof of Corollary 3.6

Since the proofs for the cases $\gamma \rightarrow \beta$ and $\gamma \rightarrow \infty$ are similar, I only prove the corollary for the case $\gamma \rightarrow \beta$ here. Fix Δ at a sufficiently small number so that the intersection between $n2(\omega)$ and $N(\omega, \Delta)$ satisfies $F_\omega < 0$ (this can be done because $n2(\omega)$ is downward sloping for sufficiently large Δ). Let us examine how the intersection between $n1(\omega)$ and $N(\omega, \Delta)$, given by (A.1), changes as γ is reduced toward β . The left-hand side of (A.1) is independent of γ . When $\gamma \rightarrow \beta$, the right-hand side of (A.1) approaches zero, in which case (A.1) gives $\omega 1(\Delta) \rightarrow \infty$. That is, there exists a level of γ_1 which is sufficiently close to β such that the intersection between $n1(\omega)$ and $N(\omega, \Delta)$ coincides with the intersection between $n2(\omega)$ and $N(\omega, \Delta)$. This is the solution to $n1(\omega) = n2(\omega)$ when $\gamma = \gamma_1$ and, by construction, satisfies $F_\omega < 0$. ■

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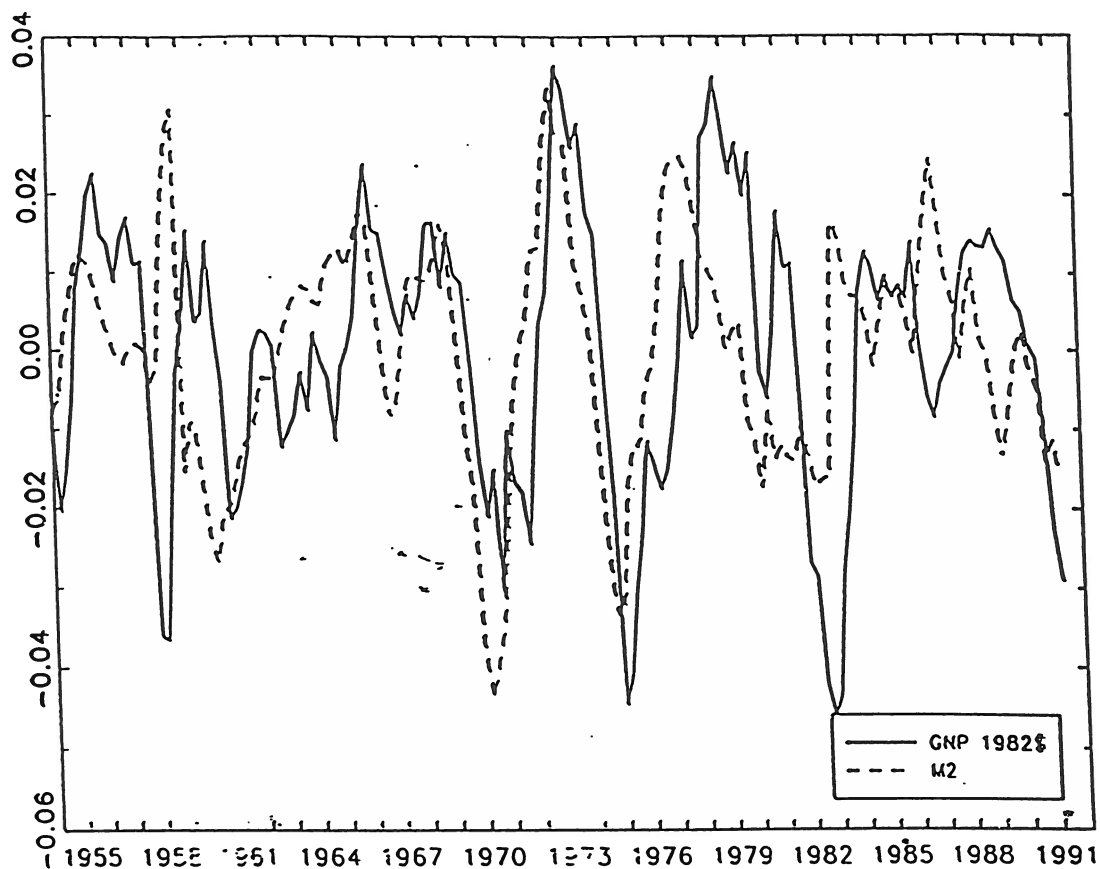


Figure 1 Detrended U.S. output and M2

Table 1 Correlation of output with monetary aggregates

Variables	Cross-correlation of output with:									
	$x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(1)$	$x(2)$	$x(3)$	$x(4)$
MB	.22	.33	.39	.42	.37	.30	.21	.15	.07	.03
M1	.16	.24	.33	.41	.39	.33	.21	.12	.05	.03
M2	.51	.58	.62	.62	.50	.33	.10	-.08	-.21	-.29

MB – monetary base; M1 – money supply 1; M2 – money supply 2.
Variables are logged and filtered through the H-P filter.

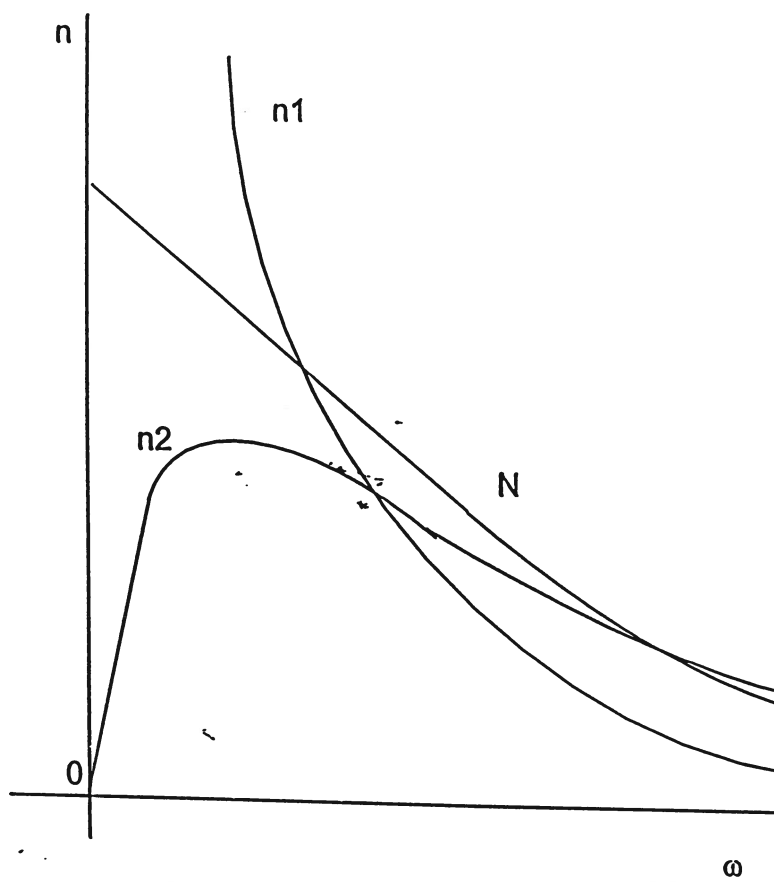


Figure 3 Existence of steady state

Figure 4a

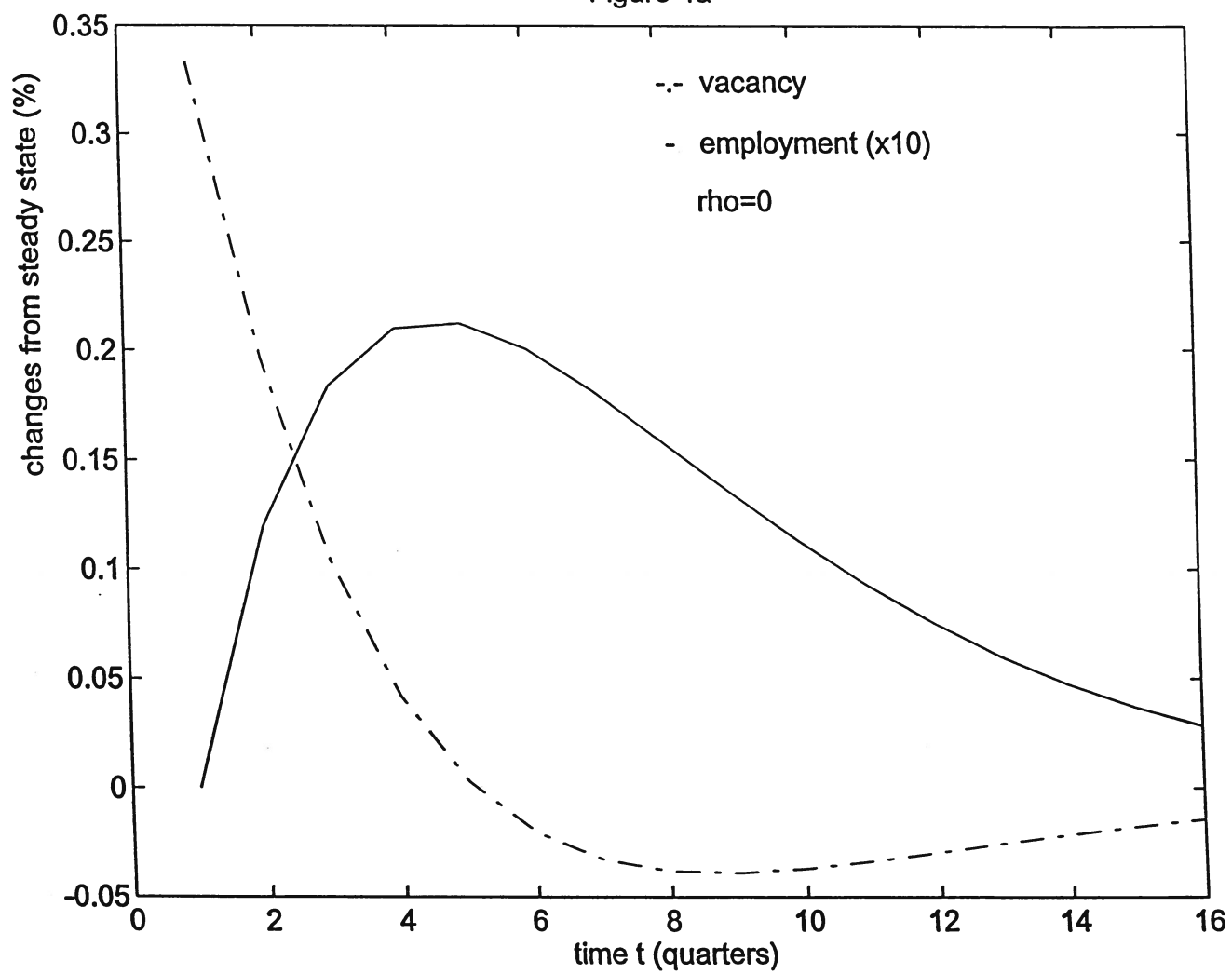


Figure 4b

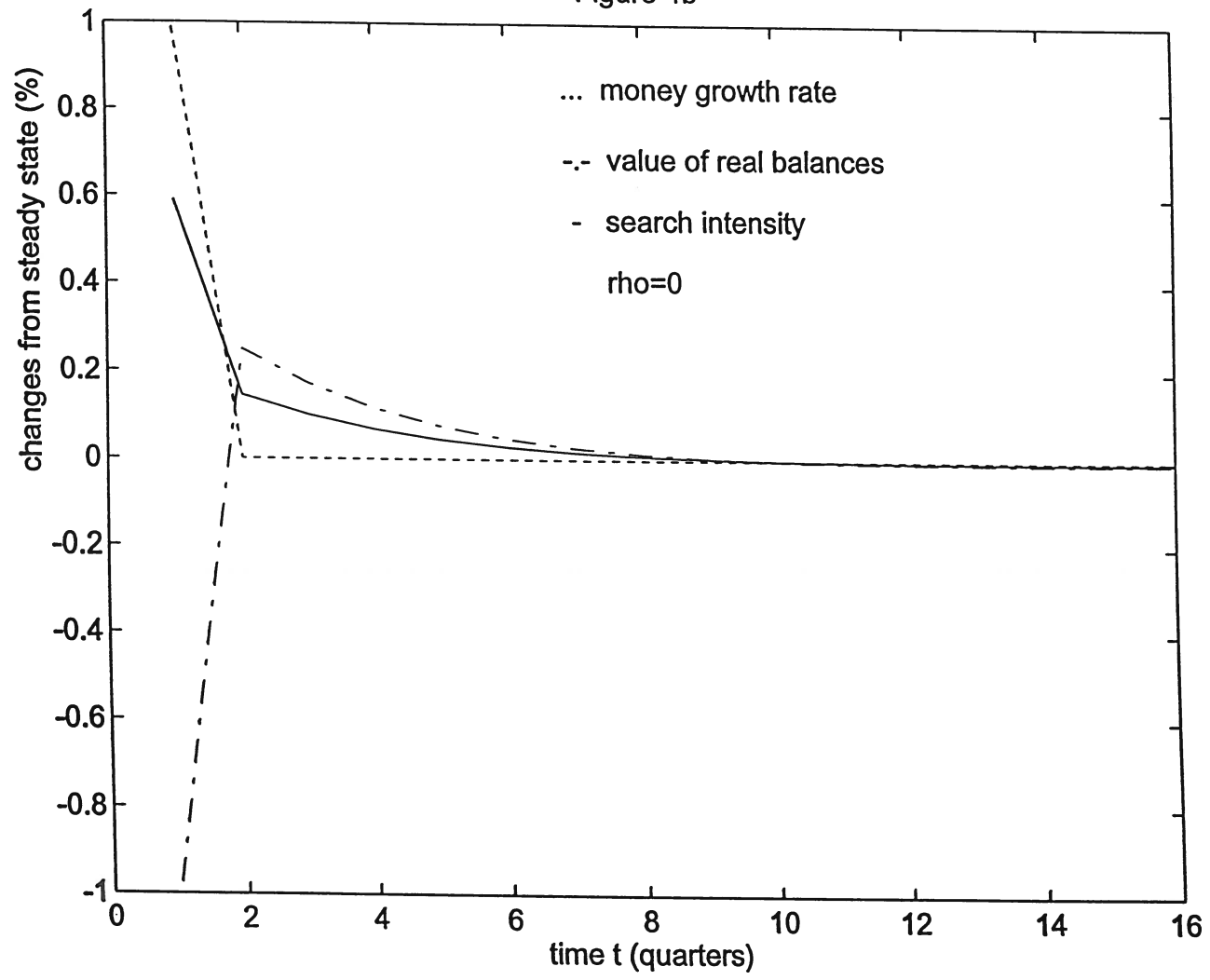


Figure 4c

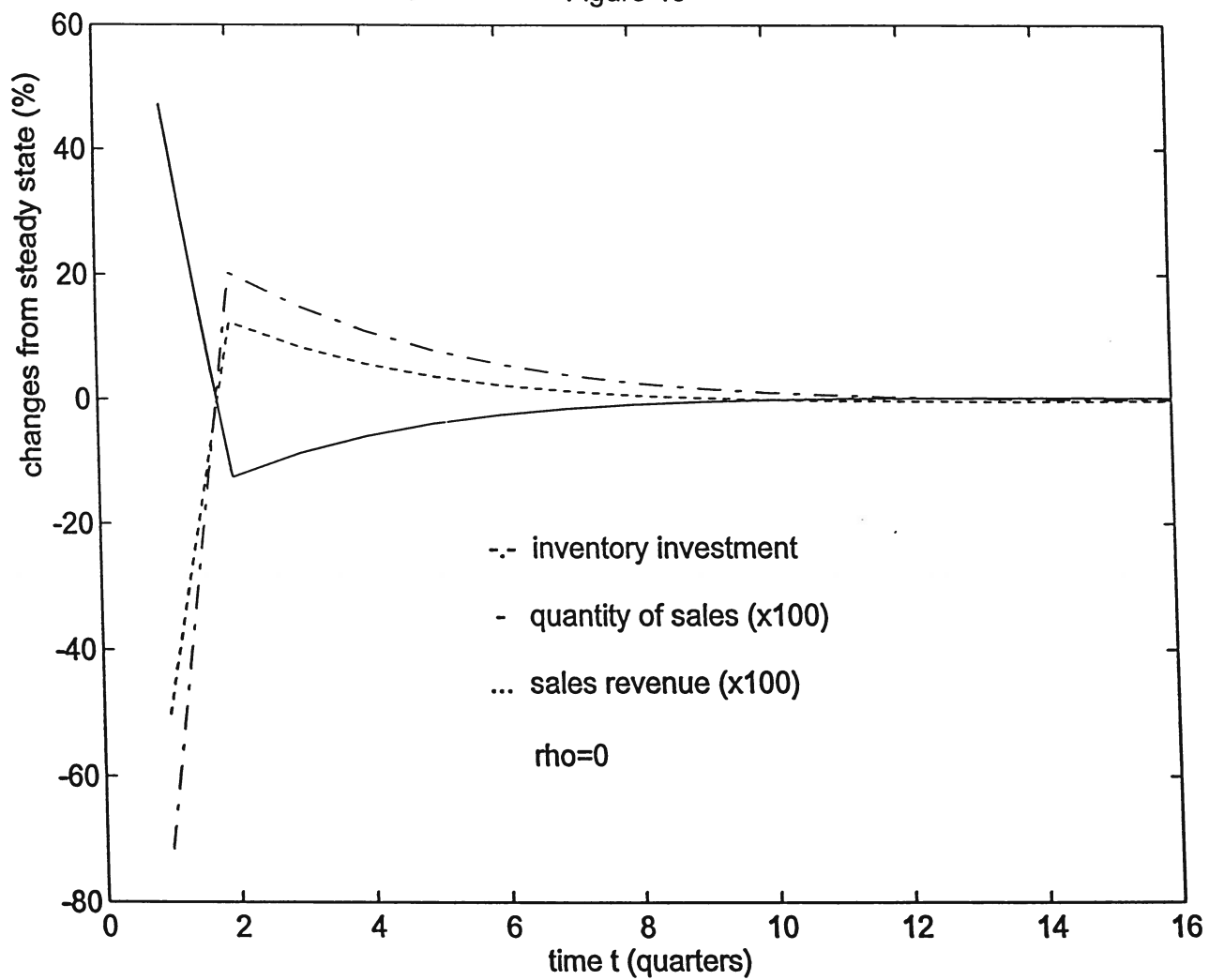


Figure 4d

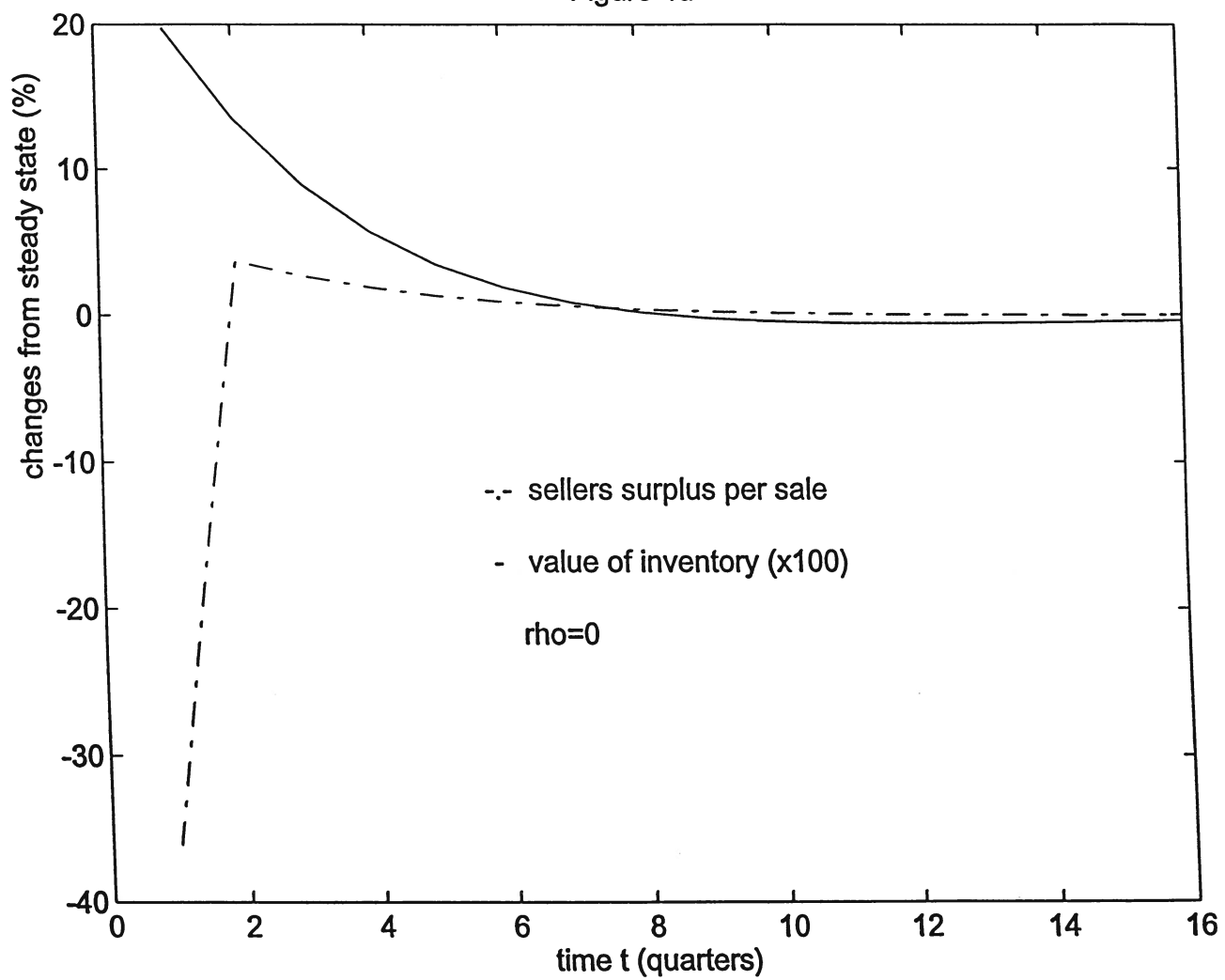


Figure 5a

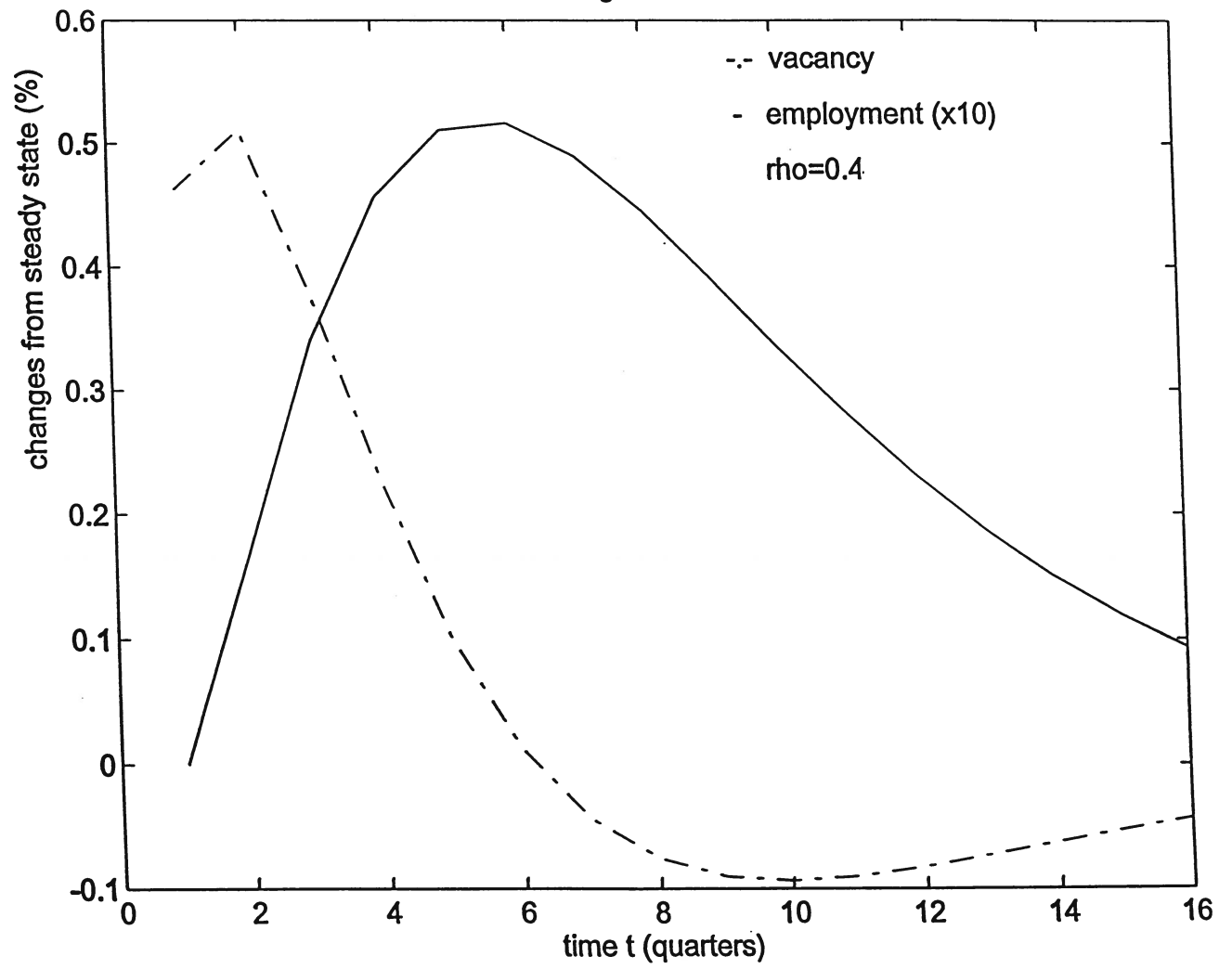
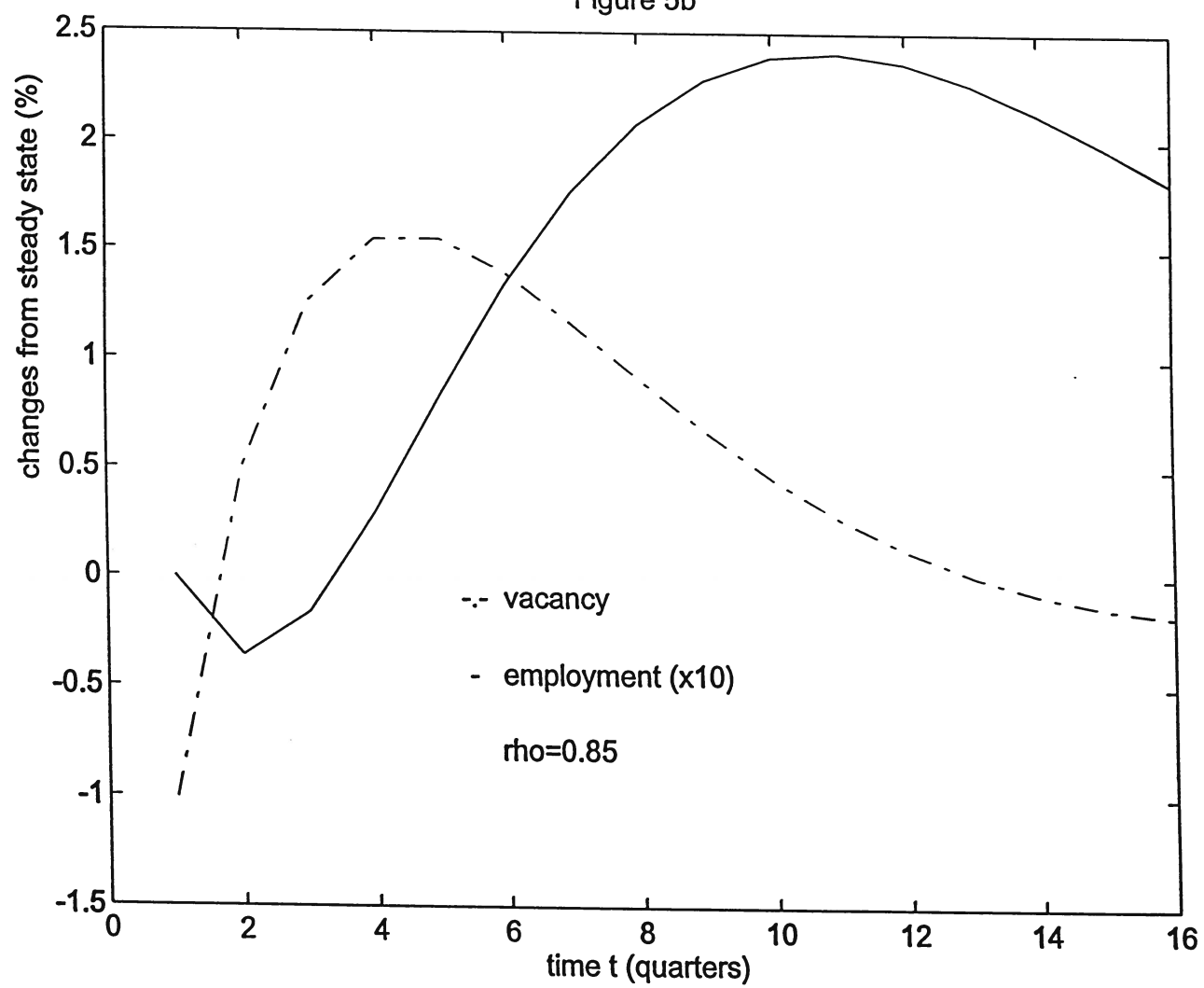


Figure 5b



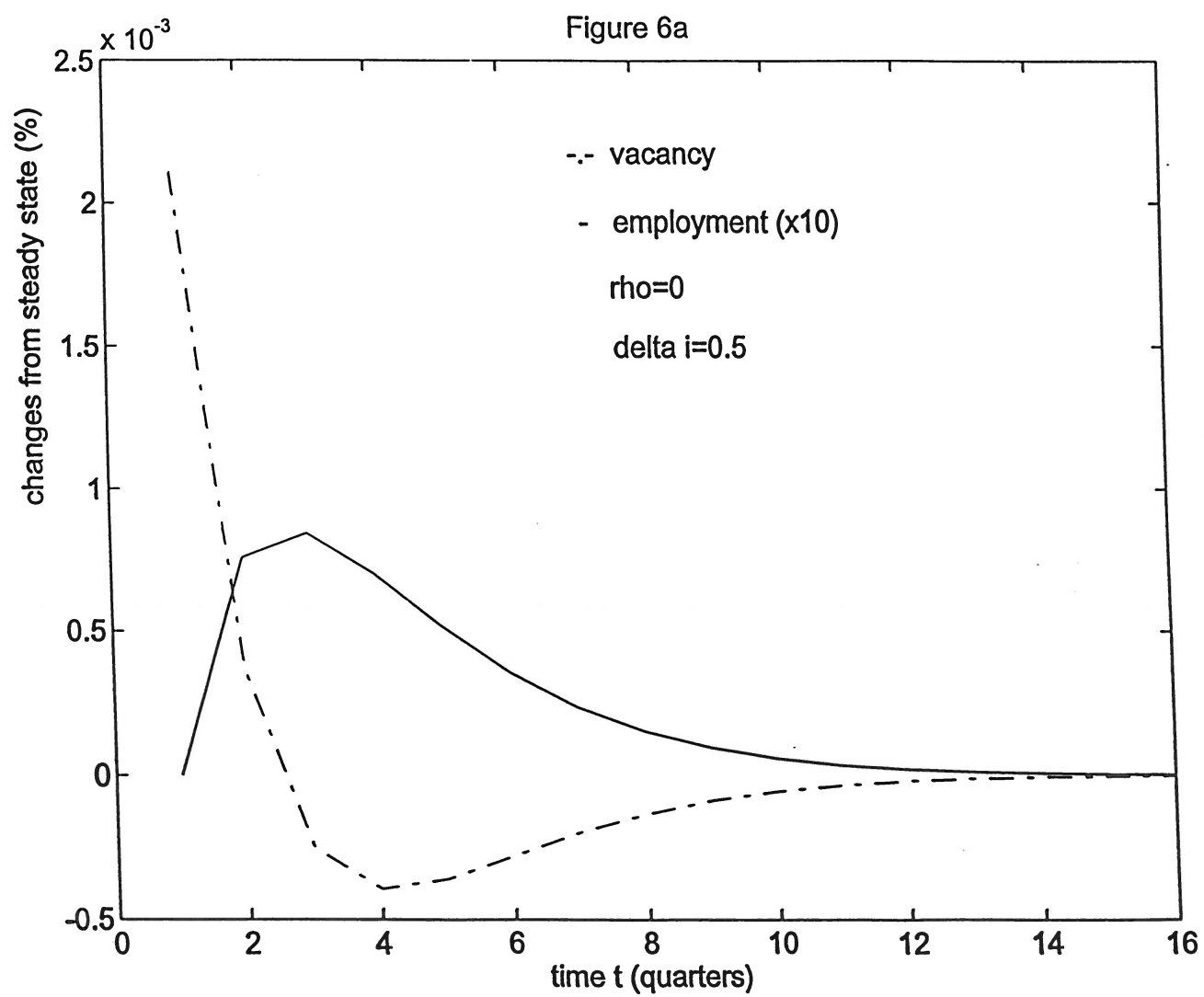


Figure 6b

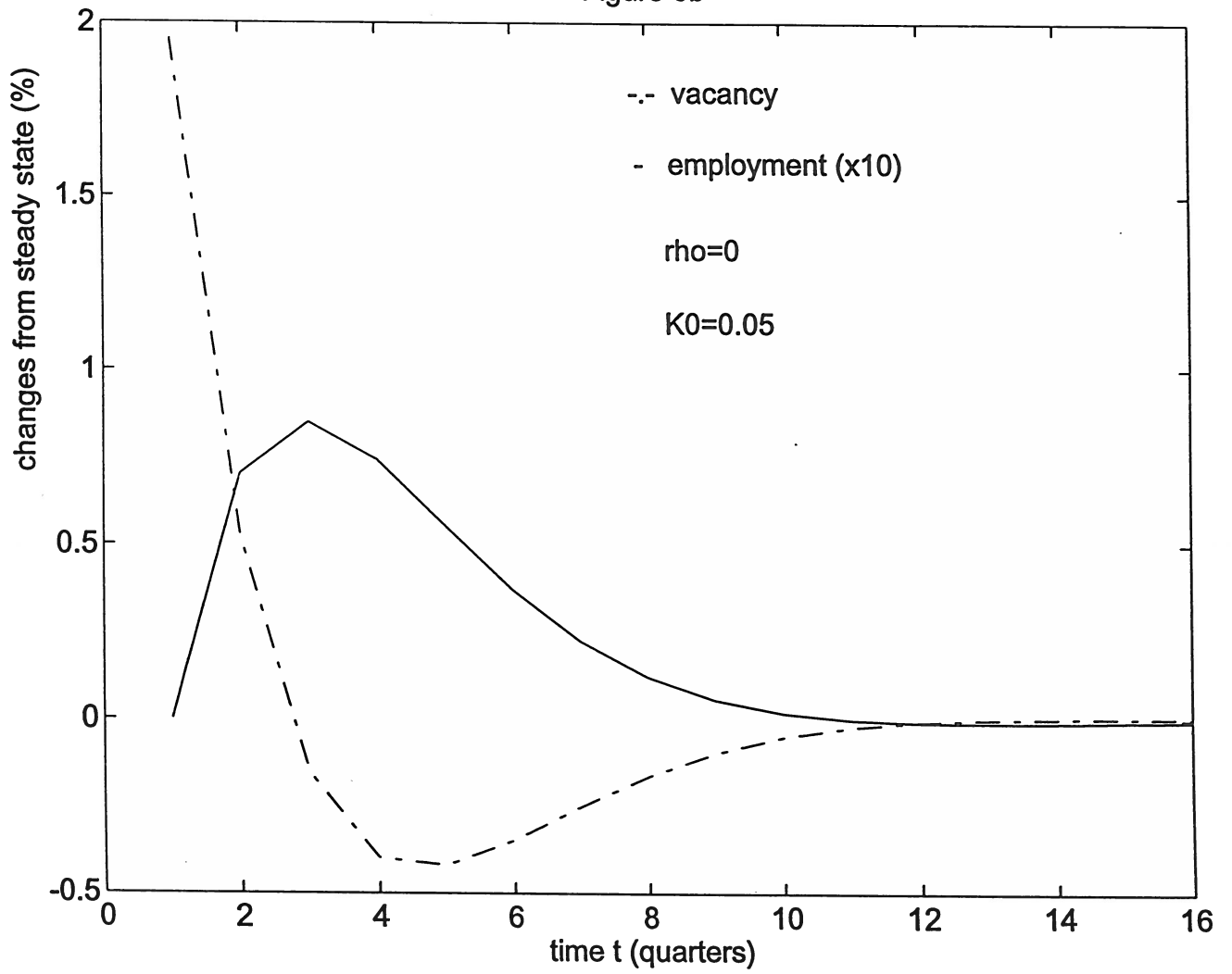


Figure 7

