THE PROFIT-MAXIMISING FIRM AND GOVERNMENT INTERVENTION[1]

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1. Introduction

A recent paper by Kamien and Schwartz {1}considered the topic of a profit-maximising firm's behaviour when new entry into the market was uncertain but the risk of new entry reflected the current activities of the firm.[2] In fact, the risk or 'hazard rate' of attracting a new entrant into the market at a moment in time, given that no new entry had previously taken place, was held to be a function of current price. If the current price was the 'limit price' then the risk was zero; if the price was greater the risk was positive, and the higher the price the greater the risk. Thus the firm maximised the expected value of V(t), the discounted stream of profits, where

$$V(t) = \int_o^t \Pi_{1(\tau)} e^{-\delta\tau} d\tau + \int_t^\infty \Pi_{2(\tau)} e^{-\delta\tau} d\tau$$

and $\Pi_{1(\tau)}$, $\Pi_{2(\tau)}$ are the current-value flow of profits respectively before and after the occurrence of the new entry; δ is the discount rate and t is the moment in time that new entry takes place.

The expected value of V(t) is then:

$$\varepsilon(V(t)) = \int_o^\infty V(t)\ P(t)\ dt$$

where P(t) is the probability of the new entry occuring at time t and thus the conditional risk or hazard rate H(t) is $\frac{P(t)}{1-F(t)}$ where F(t) is the cumulative probability distribution:

$$F(t) = \int_o^t P(\tau)\ d\tau$$

The concept of a hazard rate and the technical treatment of this kind of model is taken from the mathematical theory of reliability.[3] As can be seen above uncertainty arises from the fact that the profit stream may switch from Π_1 to Π_2 at any time t. Furthermore the probability of this occurring at time t given that it has not occurred before is the hazard rate, and as this is dependant on the current activities of the firm, an optimal rate of hazard through time $\dot{H}^{o}(t)$ can be jointly determined in the maximisation of the expected discounted stream of profit.

In the present paper, the Kamien and Schwartz model is adapted to analyse the profit-maximising firm's behaviour in a situation where government intervention may occur as a response to the firm making 'too high' a level of profit.(4) I will assume that this intervention takes the form of either a take-over of control - directly by taking over management or indirectly by regulating the present management - or by nationalisation. In either case I will assume that the result as far as the shareholders are concerned would be to change the uncontrolled flow of profits to a constant income flow, which is either the controlled level of profits or the interest on a fixed amount of compensation. I will call this constant income flow R. For the present I will assume that it is known with certainty, but in a later part of the paper I will show that it can be considered as an expectation of a random variable with known probability distribution.

We will find it useful to assume that H is a strictly convex function of profit flow ($\Pi(t)$) minus R. In order to obtain reasonably explicit solutions at some stages, a stronger assumption, that H is homogeneous of degree $V > 1$ in $\Pi(t)-R$ for $\Pi(t)-R \geqslant 0$, will be made. We will assume that $\Pi(t)-R < 0$ is always non-optimal by stating that there exists some $\Pi(t)$ which is greater than or equal to R and that the hazard rate is zero for all $\Pi(t) \leqslant R$.

The formalisation of the H function under the stronger assumption implies that V can be thought of as an elasticity of hazard, and that this V is constant whatever the value of $\Pi(t)-R$. The larger is V the greater is the risk-response to a λ% increase in profits above R.

It will be assumed that maximum prpfit flow is a constant Π^m throughout the period.

2. The Model and its Solution

If the government intervenes at time t then the present value of its flow of profits is V(t), where

$$V(t) = \int_o^t \Pi(t)e^{-\delta\tau}d\tau + \int_t^\infty Re^{-\delta\tau}d\tau$$

$$= \int_o^t (\Pi(t)-R)e^{-\delta\tau}d\tau + \frac{R}{\delta}$$

That is V(t) is equal to the sum of the present value of a constant stream R plus the present value of the flow of profits in excess of R. The expected present value of the profit stream is $\varepsilon(V(t))$, where $\varepsilon(V(t))$

$$= \int_o^\infty \left[\int_o^t (\Pi(\tau)-R)e^{-\delta\tau}d\tau + \frac{R}{\delta}\right] P(t)\ dt$$

$$= \frac{R}{\delta} + \int_o^\infty (\Pi(\tau)-R)e^{-\delta\tau}d\tau - \int_o^\infty (\Pi(t)-R)e^{-\delta t}F(t)\ dt$$

by integration by parts.

$$= \frac{R}{\delta} + \int_o^\infty (\Pi(t)-R)e^{-\delta t}\ (1-F(t)\ dt$$

If the government intervention was by nationalisation and compensation, then $\frac{R}{\delta}$ would be the amount of compensation.(5) The other part of $\varepsilon(V(t))$ is the expected profit in excess of R that can be made before intervention takes place.

The cumulative probability F(t) solves the differential equation :

$$H(\Pi(t)-R) = \frac{P(t)}{1-F(t)} = \frac{F'(t)}{1-F(t)}$$

where $F'(t)$ is $\frac{dF(t)}{dt}$

and $F(0) = 0$, and F is bounded from above by 1.

Therefore the maximisation of $\varepsilon(V(t))$ is identical to the maximisation of

$$V^* = \int_0^\infty (\Pi(t)-R)e^{-\delta t} (1-F(t))\, dt$$

subject to $F'(t) = H(\Pi(t)-R)\ (1-F(t)),\ F(0) = 0$

and $\Pi(t) \leqslant \Pi^m(t)$

This problem of variational calculus can be solved by setting up a Hamiltonian function and using Pontryagin's Maximum Principle.

The Hamiltonian ϕ is :

$$\phi = (\Pi(t)-R)\ (1-F(t))e^{-\delta t} + Z_1(t)\,(1-F(t))\ H(t)$$

write $Z_1(t) = -Z(t)\, e^{-\delta t}$

$$\phi = \left[\Pi(t)-R - Z(t)\ H(t) \right] (1-F(t))e^{-\delta t}$$

As H(t) is a strictly convex function of $\Pi(t)$-R, it is also of $\Pi(t)$.

Z(t) is the current cost of a higher probability of government intervention by period t, this cost being the lost future profit, and so $Z(t) \geqslant 0$.

Thus as H is convex in $\Pi(t)$-R, both $-H(1-F(t))$ and $\Pi(t)e^{-\delta t}\,(1-F(t))$ are concave in $\Pi(t)$ and F(t).

Thus the following conditions are necessary and sufficient for $\Pi^o(t)$, $Z^o(t)$ to be optimal in the problem, and to imply optimal F(t) and $H(\Pi(t)-R)$.[6]

The conditions are:-

(i) ϕ is a maximum with respect to $\Pi(t)$, subject to $\Pi(t) \leqslant \Pi^m(t)$

i.e. $1 - Z^o(t) \dfrac{dH}{d\Pi^o(t)} \geqslant 0$ and $\Pi^o(t) = \Pi^m(t)$

or $1 - Z^o(t) \dfrac{dH}{d\Pi(t)^o} = 0$ (1)

(ii) The co-state equations:

$$-(\Pi^{o}(t)-R - Z^{o}(t)\ H) = \dot{Z}^{o}(t) - \delta Z^{o}(t) \quad (2)$$

$$F'(t) = H(1-F(t)) \qquad F(0) = 0 \quad (3)$$

(iii) Transversality condition $Z_1(t),\ Z_1(t)\ F(t) \to 0$ as $t \to \infty$

Consider first the possibility that the profit stream can be maximised by maximising profits at each moment in time, i.e. $\Pi^{o}(t) = \Pi^{m}(t)$, all t. Then we have a non-strict inequality in equation (1) that is

$$1 - Z(t)^{o} \frac{dH}{d\Pi(t)} \geqslant 0$$

Also from equation (2)

$$Z(t)^{o} = \frac{\Pi(t)^{o} - R}{\delta + H} \quad (4)$$

as $\Pi(t)^{o}-R$ and $\therefore$ H are constants if $\Pi(t)^{o} = \Pi(t)^{m}$ = constant (by assumption). $Z(t)^{o}$ thus ensures the transversality conditions are satisfied. (4) is the only solution to equation (2) which does satisfy these conditions. Thus equation (1) becomes by substitution of $Z(t)^{o}$ from (4)

$$1 - \frac{dH}{d\Pi^{o}(t)} \left[\frac{\Pi(t)^{o}-R}{\delta + H}\right] \geqslant 0$$

but $\dfrac{dH}{d\Pi(t)^{o}} = \dfrac{dH}{d(\Pi^{o}(t)-R)}$

and if we assume the elasticity of hazard $\dfrac{dH}{d(\Pi-R)} \left[\dfrac{\Pi - R}{H}\right]$ to be a constant then (1) can be written as

$$1 - \frac{VH}{\delta + H} \geqslant 0$$

or $$H \leqslant \frac{\delta}{V-1}$$

Thus it will be optimal to maximise profits at time t if when profits are maximised the hazard rate is less than or equal to the rate of discount divided by the elasticity of hazard minus one.

The second possibility is that maximising profit at time t does not maximise the expected profit stream V^*. In this case equation (1) is a strict equality. It can easily be seen that the solution:

$$Z(t)^o = \frac{\Pi^o - R}{\delta + H}$$

$$H = \frac{\delta}{V - 1}$$

$$\Pi^o = \text{constant}$$

satisfies the necessary and sufficient conditions.

Whether or not the profit constraint is binding the solution is in terms of constant values of profit, hazard and Z(t). This is by no means surprising as there are no essentially dynamic features in the model. An optimal solution today would be the same tomorrow whatever we had done today providing the problem remained tomorrow, that is providing the government did not intervene today.

Because H is constant in both solutions, the cumulative probability F(t) can be easily found from equation (3) and theinitial condition F(0) = 0 to be:

$$F(t) = 1-e^{-H(t)}$$

where $H = H(\Pi(t)^m - R)$ if profit constraint is binding

$H = \frac{\delta}{V-1}$ if profit constraint is not binding

We have found that $H(\Pi(t)^m-R) \leq \frac{\delta}{V-1}$ if $\Pi(t)^m$ is optimal and so an alternative condition for $\Pi(t)^m$ to be optimal is that $F(t) = 1-e^{-H(\Pi(t)^m-R)t} \leq 1 - e^{-\frac{\delta}{V-1}t}$

that is that the probability of government intervention before period (t) if the firm maximises its profits should be less than or equal to $1-e^{-\frac{\delta}{V-1}t}$

The solution to the model implies that the hazard rate should be $\frac{\delta}{V-1}$ iff at maximum profits the hazard rate is greater than or equal to $\frac{\delta}{V-1}$. If at maximum profits the hazard rate is greater than $\frac{\delta}{V-1}$ then maximising profits does not maximise the expected value of the profit stream. Such a situation will be described as one of an 'effective' hazard. A situation where the profit constraint is binding will be described as one of non-effective hazard. In the latter situation small variations in the parameters of the model will produce no change in optimal policy. We can however assess the importance of small changes in the parameters of the model upon the optimal policy of the firm if hazard is effective.

The greater the elasticity of hazard (V) the lower is the optimal profit. The higher the rate of discount the greater will be the optimal hazard rate ceteris paribus as the benefits from chasing profits come now and the (opportunity) losses of incurring government intervention come later.

Altogether the optimal profit is found to depend on 3 factors - the rate of discount, the elasticity of hazard (having the role of a summary parameter of the hazard function) and the compensation or controlled profit resulting from government intervention. Thus the government has here two potential policy instruments for the regulation of profits. It has its readiness for responding to profit-making by involving itself in the regulation of firms (V), and it also has its typical compensation level $(\frac{R}{\delta})$ or controlled profits (R). A small threat of nationalisation with small compensation will have an equivalent effect, reasonably enough, to a larger threat with more compensation.

We have stated that Z(t) is the current (t period) value of the cost in terms of lost profit of higher F(t). The optimal value $Z(t)^o = \frac{\Pi(t)^o - R}{\delta + H}$ shows this to be true. $Z(t)^o$ is an infinite stream of the difference between the (constant) optimal profit and R, discounted at a rate equal to the sum of the 'pure' discount rate δ and the hazard rate H.

The expected optimal value of the profit stream is $\varepsilon(V(t))$

$$\varepsilon(V(t)) = \frac{R}{\delta} + \int_0^\infty (\Pi(t)^o - R) \quad e^{-(\delta + H)t} dt$$

$$= \frac{R}{\delta} + \frac{\Pi(t)^o - R}{\delta + H}$$

It is obviously sensible that the expected value of the profit stream should be the compensation $(\frac{R}{\delta})$ plus the (constant) profit flow in excess of R discounted at the rate δ + H. However, this result implies that

$$Z^o = \varepsilon(V(t)) - \frac{R}{\delta}$$

Again, the optimal Z is the difference between the maximum expected profit stream and compensation.

3. R as an expectation : the case of uncertain compensation

In this section I will show that the R used in the analysis so far can be considered as an expected value of a random variable with known probability distribution, providing we make two assumptions analogous to those made in the previous section. The first is that the probability density function of R, which will be called G(R), is independant of time and profits. The second assumption is that the hazard function is strictly convex in $\Pi(t) - \varepsilon(R)$, which might imply that the firm believes that its expectation of R is the same as the government's

The expected value of R is ε (R) where

$$\varepsilon(R) = \int_{R_1}^{R_2} R\, G(R)\, dR$$

Now for given government intervention at time t, the expected value of the profit stream is V(t)' where

$$V(t)' = \int_o^t \Pi(t)e^{-\delta\tau}\, d\tau + \int_{R_1}^{R_2} \int_t^{\infty} R\, e^{-\delta\tau}\, d\tau G(R)\, dR$$

Thus the expected value of the profit stream without the condition that government intervention takes place at time t is

$$\varepsilon(V(t)') = \int_o^{\infty} \left[\int_o^t \Pi(\tau)e^{-\delta\tau}\, d\tau + \int_{R_1}^{R_2} \int_t^{\infty} Re^{-\delta\tau} d\tau\, G(R)\, dR \right] P(t)\, dt$$

but $$\int_{R_1}^{R_2} \int_t^{\infty} Re^{-\delta\tau}\, d\tau\, G(R)\, dR = \int_t^{\infty} e^{-\delta\tau}\, d\tau \int_{R_1}^{R_2} R\, G(R)\, dR$$

$$= \varepsilon(R) \int_t^{\infty} e^{-\delta\tau}\, d\tau$$

$$= \int_t^{\infty} \varepsilon(R)e^{-\delta\tau}\, d\tau$$

$$\therefore \quad \varepsilon(V(t)') = \int_0^\infty \left[\int_0^t \Pi(\tau) e^{-\delta\tau} d\tau + \int_t^\infty \varepsilon(R) e^{-\delta\tau} d\tau \right] P(t)\, dt$$

$$= \frac{\varepsilon(R)}{\delta} + \int_0^\infty \int_0^t (\Pi\tau - \varepsilon(R)) e^{-\delta\tau} d\tau\, P(t)\, dt$$

The solution is thus $\Pi^o(t)$, $Z(t)^o$ as in the original model except that R is the expectation of the income flow associated with compensation or controlled profits.

4. Conclusion

The broad conclusions of the analysis undertaken in this paper is that it may not be optimal for a profit-stream maximising firm to maximise profits at a point in time if there is a possibility of incurring government intervention of a painful nature. However, it has also been found to be true that a sufficiently small hazard, or more correctly a sufficiently ineffective hazard function, may exist for continuous 'myopic' profit maximisation to be optimal. With the assumption of homogeneity of the hazard function this situation occurs if $H(\Pi^m - R) \leqslant \frac{\delta}{V - 1}$, when there is no accommodating behaviour on the part of the firm.

Due to the comparative static nature of the model, the solution found was one of constant profit flow, whether or not that flow was less than or equal to Π^m. The optimal level of profits depended on δ, the parameter(s) of the hazard function and R or ε(R). Furthermore the optimal path is independent of the initial value of F(0). That is, if the optimal policy is Π^o each day until intervention takes place, then if intervention does not take place on the first m days, then <u>no matter what</u> policy was followed on the first m days, Π^o is still optimal for day m + 1.

The expected length of independent existence of the firm (time before intervention) is the reciprical of the optimal hazard rate. As this is constant so is the expected time before intervention.

The model considered here is essentially comparative static, and efforts are being made to generalise it in several ways. Firstly the hazard function considered here states that only the current profit rate of the firm creates risk. It would be an improvement to give the government a memory of the profit record of the firm. Secondly, as the model stands there is an ambiguity in the result of an effective hazard. If $\Pi^o < \Pi^m$, then is this due to the firm selling on a different point on the demand curve, or is this due to cost inefficiency or profit-hiding?(7) As far as government regulating is concerned the distinction is of obvious importance.

Footnotes

(1) The author has benefited greatly from discussion with Richard Clarke, Graham Pyatt and participants of the Industrial Economics Workshop at the University of Warwick.

(2) Deterministic models rather than those involving uncertainty include Gaskin {2} and Pyatt {3}.

(3) See for instance Barlow and Proschan {4}.

(4) If a constant share-holding is assumed then profit may be considered as either the level of profit flow or the profit flow per share.

(5) This assumes that δ can be thought of as the appropriate rate of interest in that event.

(6) See Arrow and Kurz {5} page 49.

(7) The discretion available to management when profit-stream maximisation does not involve profit-maximisation does not seem to have had as much attention as it deserves.

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