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Abstract

Value-at-risk (VaR) is increasingly being applied to problems in agriculture, especially valuation of crop insurance and agricultural lending risk exposure. VaR conveys the probability that losses exceeding a threshold will likely occur within a specified timeframe. However, it does not provide the expected value of losses, should they happen. When determining risk exposure for budget analysis, this latter amount is of keen interest. Expected tail loss (ETL) methods are developed and compared with VaR.

Keywords: Risk, Value-at-Risk, Expected Tail Loss, crop insurance, loan loss

Introduction

Value-at-Risk (VaR) is a popular method of quantifying risk exposure in financial, energy, and commodity markets. Holton (2003) summarizes the chronological evolution of VaR including the widespread dissemination of JP Morgan's *RiskMetrics Technical Document* in 1995. In its most popular form, RiskMetrics is a single index measure of risk that answers the following question: "What dollar loss is such that only p% of future losses will be worse over some holding period." The release of the Riskmetrics technical manual provided industry professionals with three practical methods of calculating VaR's – parametric, historical simulation, and Monte Carlo simulation methods. The rising popularity of VaR at the time was likely due to its ease of communication and estimation.

As adoption of VaR methods spread, estimated models began to integrate portfolio, financial engineering, or time series analysis literature. Variants of VaR were developed including Creditmetrics, a popular method for evaluating credit risk in loan portfolios. Other analysts applied the methodology to a broadening array of problems, including budget analysis. Holton (p. 22) proceeds to capture these variants and define VaR as a category of market risk measures that support VaR metrics. Of particular interest in this study, are expected tail loss (ETL) VaR metrics, which are more appropriate for budgetary analysis than standard VaR. As will be demonstrated below, standard VaR metrics convey the probability that losses exceeding a threshold will likely occur within a specified timeframe. However, they do not provide users with the expected value of losses, should they happen. When determining risk exposure for budget analysis, it is the latter amount that is of keen interest.

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This study reviews recent misapplications of VaR for budget analysis. The first determines federal budget loss exposure from unexpected crop reinsurance obligations. The second uses VaR to quantify the unexpected loan losses for an agricultural lender. Limitations of these applications are then reviewed. Finally, the paper reviews the ETL metric and compares the merits of its use for budget analysis with VaR.

Applications of VaR for Budget Analysis

Hayes, Lence, and Mason (HLM, 2003) evaluate various hedging strategies for reducing the federal government's risk as a reinsurer of crop insurance. Their specific focus is on the government's budget exposure as *"procedures used to budget for new crop insurance programs, or for changes to existing crop insurance programs with uncertain outlays, are based on worst-case scenarios. Thus, from a budgetary perspective, the worst-case budgetary outcome could be reduced if this risk could be managed"* (p.127). HLM proceed to examine several hedging strategies using VaR and conclude that risk reduction is appreciable, nearing one-half billion dollars. However, correlation stability, market effects, and transaction costs may limit risk reduction opportunities.

Katchova and Barry (KB, 2003) and Zech and Pedersen (ZP, 2003) use VaR methods to develop credit risk models that meet internal and regulatory capital reporting requirements. Following implementation of the Basel II Capital Accord in 2006, financial institutions will be required to adopt new risk-based capital requirements. These requirements involve standards for bank, regulatory, and economic capital. The latter is defined as the financial resources necessary to cushion unexpected losses.

Both studies estimate unexpected loan losses using VaR procedures (Barry, 2001). More specifically, ZP calculate economic capital requirements as:

Economic Capital = VaR - Mean (expected loss) + Market Risk Capital +Operational Risk Capital

where as KB develop the concept of unexpected losses:

$$\text{Unexpected Losses} = \text{VaR}\ (1-\alpha) = Z_\alpha * SD * LGD$$

where VaR $(1-\alpha)$ is the level of loss which will be exceeded with α probability, SD is the standard deviation of default for the portfolio, and LGD is the loss given default. Both studies employ a variant of VaR and strive to estimate the amount of capital required to cover unexpected loan losses for budget planning and regulatory reporting.

Limitations of VaR for Budget Analysis

In addition to the limitations each of the authors note in their respective studies, two other shortcomings exist. First, standard VaR methods often yield biased estimates when loss functions are not normally distributed. In agricultural applications, this is especially important as many distributions are fat-tailed. Odening and Hinrichs (2003) review the pitfalls of traditional VaR models derived by historical simulation (a.k.a. HLM) and evaluate the merits of Extreme Value Theory (EVT) as an alternative.

More importantly, the VaR technique does not provide a direct answer for the fundamental question posed in each article. To estimate the federal government's risk exposure to crop reinsurance obligations, an ETL must be determined as suggested by Artzner, et al. VaR provides a loss value so that only 5% of potential losses will be worse. ETL provides an expected loss, given that a loss from the 5% tail actually occurs. It is this latter amount that is of actual budget consequence to the federal government. Basak and Shapiro (2001) demonstrate that when a large loss does occur, ETL methods result in lower losses than VaR derived measures.

These same limitations exist in the aforementioned credit risk models. LGD is simply an array of losses if borrowers were to default. At a confidence level p=99%, the VaR of a lender's given portfolio is the loss in market value that will be exceeded with probability (1-p). In this case, the loss exceeds the VaR with 1% probability. It is doubtful that capital standards based on VaR of a firm's loan portfolio are adequate.

As Barry argues, loan loss provisions should be designed to cover expected loan portfolio losses. Figure 1 depicts the likelihood of losses for an agricultural lender (distribution function). Moving from right to left, loan loss provisions set aside by lenders provide the first level of risk protection in the event a financial institution incurs losses from loan defaults.

In the event loan losses are greater than expected, equity capital as prescribed under the new Basel Accord capital standards, is intended to cover additional losses to the VaR, labeled as capital standards in Figure 1. Capital standards are unique to each institution and related to the bank, regulatory, and economic climate it faces.

However, financial institutions still have risk exposure to unexpected losses beyond VaR. Full coverage from credit risk only exists if capital is set-aside to cover the expected value of these losses as well. Financial managers and regulators would prudently desire a measure of this exposure to fully evaluate risk bearing capacity. ETL measures the expected value of losses beyond the VaR level, labeled as risk capital in Figure 1. For these reasons, VaR - based risk measures alone are inadequate for fully determining credit risk, especially in agriculture where large losses stemming from widespread weather and disease problems are likely to occur (Duffie and Singleton, 2003).

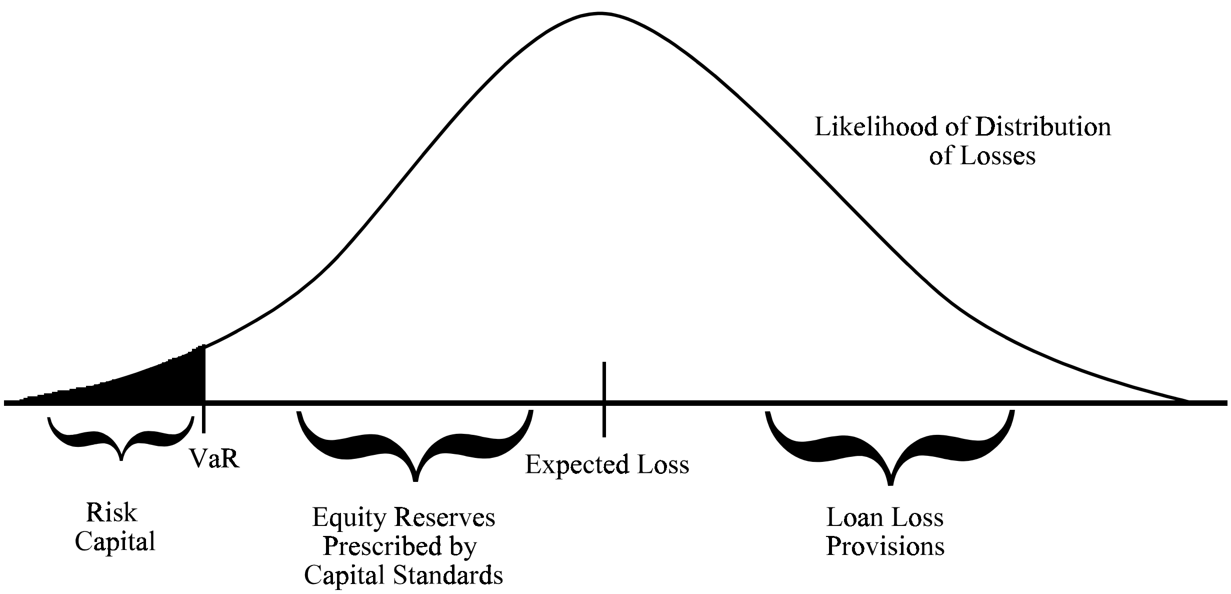

Figure 1. Determination of Credit Risk and Supporting Capital

The limitation of VaR is that it is not responsive to large losses beyond the threshold. Two different loan portfolios could have the same VaR, but have entirely different expected levels of loss. VaR calculations conceal the tail shape of distributions that do not conform to the normal distribution. In other words, two loss distributions that both have 1% VaRs could have quite different 0.1% or 0.01% VaRs depending on tail shapes. This divergence is of high interest to decision makers. Large VaR exceedences are much more likely to cause financial distress and capital shortfalls than are small exceedences. Thus, complete measures of risk capture both the magnitude and probability of losses occurring – the entire shape of the tail distribution of losses beyond VaR.

Expected Tail Loss (ETL) Methods

The ETL measure conveys the shape of the tail distribution with a single number that is derived by computing the average of tail losses multiplied by their probability of occurring. While ETL does not capture all the information about the tail shape, the key is that the shape of the tail beyond the VaR now impacts the index measure of risk being evaluated. ETL is defined in terms of log return as the expected value of losses exceeding VaR:

$$ETL_{t+1}^{p} - E_t\left[R_{t+1} \middle| R_{t+1} < -VaR_{t+1}^{p}\right]$$

where the negative signs in front of the expectation and the VaR are needed because the ETL and the VaR are defined as positive numbers. Log returns are used to facilitate compounding. (Compounded total returns are simply the sum of daily returns.) ETL is the expected value of tomorrow's return (R_{t+1}), conditional on it being worse than the VaR.

The expected value of a normal variable with zero mean return truncated at the VaR is

$$ETL_{t+1}^{p} = -E_t\left[R_{t+1} \middle| R_{t+1} < -VaR_{t+1}^{p}\right] = \sigma_{PF,t+1} \frac{\phi\left(-VaR_{t+1}^{p} / \sigma_{PF,t+1}\right)}{\Phi\left(-VaR_{t+1}^{p} / \sigma_{PF,t=1}\right)}$$

where ϕ (*) denotes the density function, Φ (*) denotes the cumulative distribution function of the standard normal distribution, and $\sigma_{PF,t+1}$ denotes portfolio variance. In the normal case

$$VaR_{t+1}^{p} = -\sigma_{PF,t+1} \Phi_p^{-1} \; .$$

Thus,

$$ETL_{t+1}^{p} = \sigma_{PF,t+1} \frac{\phi\left(\Phi_p^{-1}\right)}{p}$$

which has a structure very similar to the VaR measure. The ratio of the ETL to the VaR is

$$\frac{ETL_{t+1}^{p}}{VaR_{t+1}^{p}} = -\frac{\phi\left(\Phi_p^{-1}\right)}{p\Phi_p^{-1}} .$$

In the normal case, as the VaR coverage probability *p* gets close to zero, the ratio of the ETL to the VaR goes to 1.

In general, the ratio of ETL to VaR for a fat-tailed distribution will be higher than that of the normal. For the EVT distribution, when p goes to zero, the ETL to VaR ratio converges to

$$\frac{ETL_{t+1}^{p\approx 0}}{VaR_{t+1}^{p\approx 0}} \approx \frac{1}{1-\xi} \ .$$

Where ξ is the simple Hill estimator (Odening and Hinrichs, 2003). For fat-tailed distributions where $\xi > 0$, the fatter the tail, the larger the ratio of ETL to VaR. Thus, the ETL measure is more revealing than VaR about the magnitude of losses larger than the VaR.

Conclusion

Adoption of VaR metrics is becoming more widespread. Single index VaR statistics convey important risk information to decision makers. However, caution is urged with using VaR for budget analysis, especially when losses are not normally distributed. This article reviews conditions under which ETL measures yield more appropriate monetary loss measures for budget planning.

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