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Testing for Homogeneity and Habit Formation in a Flexible Demand Specification of U.S. Meat Consumption

by

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1. Introduction

Three common practical problems facing empirical analysts of demand relations are: (1) the choice of a functional form for econometric estimation, (2) the decision whether to deflate price and income data and the related question, "Are demand equations homogeneous of degree zero?", and (3) the representation of changing preferences.

The purpose of this paper is to test for homogeneity conditions and habit formation in a flexible demand specification. Box and Cox transformations are applied to four meat demand relations in order to allow for more flexible functional forms. Estimators of the demand parameters are obtained by using maximum likelihood techniques and tests of homogeneity and habit formation are based on the likelihood ratio procedure. The analyses utilize annual United States data on beef, pork, poultry, and fish for the years 1950 to 1975 and short run effects are emphasized. The motivation for the study is further indicated below.

Many studies have focused on the demand for meat. Fuller and Ladd Hayenga and Hacklander, and Tryfos and Tryphonopoulos used linear functional forms. The log form was used in Fox and Breimyer. More recently, Chang used the Box-Cox transformation in a dynamic model to investigate aggregate demand for meat in the United States. The linear and logarithmic forms are special cases of this more general functional form.

The above studies make no attempt to test restrictions implied by consumer theory. In contrast, recently, Christensen and Manser applied a translog utility system to meat demand and estimated demand parameters and tested theoretical restrictions. However, most econometric analyses of demand in agricultural economics do not use explicit utility function formulations but use arbitrary reduced forms (one recent exception is the work by Green, Hassan, and Johnson). This study adopts the latter approach due to increased ease of estimation and the ability to incorporate greater complexity in the dynamic formulation. These relations can be made locally (or in some cases globally) consistent with utility maximization by imposing restrictions on parameters (Court, Byron). However, homogeneity can be imposed globally as well as locally within a demand equation for many log-linear demand equations (it is noted here that Court assumed, but did not test for, homogeneity; however, Byron and others have tested for homogeneity (and usually rejected it) in more aggregated log-linear static systems). Because all demand specifications (whether a system or single equation) are amenable to homogeneity tests and because of the general applicability of the deflation issue, this study focuses on homogeneity restrictions.

The plan of this paper is as follows: Section 2 contains a description of the demand model including the introduction of habit effects. Section 3 includes a discussion of the estimation procedures employed. Data sources and commodity classifications are presented in Section 4. Section 5 contains the parameter estimates, results of the likelihood ratio tests, and price and income elasticities estimates. Concluding comments are offered in Section 6.

2. Model

2.1 Functional Form

The transformation developed by Box and Cox and extended in Zarembka is of the form

$$(1) \qquad q_t^{(\lambda)} = (q_t^{\lambda} - 1)/\lambda,$$

where q_t is the t^{th} observation of a variable and λ is some real valued parameter. For a single equation demand specification and applying the Box-Cox transformation, the static model becomes

$$(2) \qquad q_{it}^{(\lambda)} = \beta_0 + \beta_1 P_{1t}^{(\lambda)} + \beta_2 P_{2t}^{(\lambda)} + \ . \ . \ . + \beta_n P_{nt}^{(\lambda)}$$

$$+ \beta_{n+1} Y_t^{(\lambda)} + U_t \qquad \begin{array}{l} i = 1, \ . \ . \ ., n \\ t = 1, \ . \ . \ ., T \end{array}$$

where q_{it} is the per capita quantity demanded of the i^{th} commodity in time period t, P_{jt} is the corresponding price of the j^{th} commodity in time period t, Y_t is the per capita disposable income in period t, and U_t is a random error. Equation (2) reduces to the linear form when λ equals one. This can be easily seen by inspection of the equation. As λ approaches zero, the model approaches the double log demand specification (see Box and Cox). Thus, the above functional form is a more general specification than the typical linear and double-log relations often employed by economists.[1/]

The elasticity of q_{it} with respect to one of the explanatory variables, say P_{jt}, can be shown to be

$$(3) \qquad \varepsilon_{q_i P_j} = \beta_j (P_{jt}/q_{it})^{\lambda}.$$

For the linear case, $\lambda = 1$, the elasticity approaches one as the explanatory variable (P_{jt}) increases (Chang, p. 356). For the double-log case, the elasticities are constant with respect to changes in the explanatory variable. The functional form in (2) allows the data to discriminate changes in the income or price elasticity with changing income or price levels. For example, if $\lambda < 0$ and the commodity is a superior good ($0 < \varepsilon_{qY}$), then ε_{qY} decreases as (Y_t/q_t) increases.

2.2 Homogeneity

Theoretically plausible demand systems, that is, ones derived from a utility maximization process, possess certain general properties such as the Slutsky symmetry, Engel aggregation, and homogeneity conditions. For reasons outlined above, the attention here is focused on homogeneity. A demand function is homogeneous of degree zero in all prices and income if and only if

$$(4) \qquad q_i\ (kP_1, \ldots, kP_n, kY) = q_i\ (P_1, \ldots, P_n, Y)$$

for all $k > 0$. In terms of elasticities, and by invoking Euler's theorem, a necessary and sufficient condition for homogeneity is that the sum of all direct and cross price elasticities and the income elasticity equals zero, that is,

$$\text{(5)} \qquad \sum_{j=1}^{n} \varepsilon_{ij} + \eta_{iY} = 0 \qquad\qquad i = 1, \ldots, n$$

where η_{iY} is the income elasticity associated with the ith commodity. For the Box-Cox functional form, this condition can be expressed as

$$\text{(6)} \qquad \sum_{j=1}^{n} \beta_j \, (P_j/q_i)^{\lambda} + \beta_{n+1} \, (Y/q_i)^{\lambda} = 0, \qquad i = 1 \ldots, n$$

or by multiplying equation (6) by q_i^{λ}, as

$$\text{(7)} \qquad \sum_{j=1}^{n} \beta_j \, P_j^{\lambda} + \beta_{n+1} \, Y^{\lambda} = 0, \qquad\qquad i = 1, \ldots, n.$$

Thus the Box-Cox demand function is globally homogeneous, that is, equation (7) holds for all prices and income, if and only if the following conditions are satisfied

$$\text{(8)} \qquad \lambda = 0 \text{ and } \sum_{j=1}^{n} \beta_j + \beta_{n+1} = 0.$$

The Box-Cox functional form is locally homogeneous at specified prices and income, $\bar{P}_j$ and $\bar{Y}$, if and only if

$$\text{(9)} \qquad \sum_{j=1}^{n} \beta_j \, \bar{P}_j^{\lambda} + \beta_{n+1} \, \bar{Y}^{\lambda} = 0.$$

In Section 5, tests for both local and global homogeneity in a Box-Cox model are performed and the results analyzed.

2.3 Dynamic Representations

The demand relation in equation (2) does not allow for persistence or inertia in consumption patterns. In order to explicitly account for habit formation, three habit-version demand specifications are considered. However, unlike previous research, we will adopt the Box-Cox transformation for its added generality. The first extension of the static model in (2) assumes that endogenous tastes can be treated by adding a time trend to the original model. The resulting econometric model is

$$(10) \qquad q_{it}^{(\lambda)} = \beta_0 + \beta_1 P_{1t}^{(\lambda)} + \ldots + \beta_n P_{nt}^{(\lambda)} + \beta_{n+1} Y_t^{(\lambda)} + \beta_{n+2} t^{(\lambda)} + U_t \qquad \begin{array}{l} i = 1, \ldots, n \\ t = 1, \ldots, T, \end{array}$$

where t takes on the value of 1 in 1950, 2 in 1951, etc. (Pollak and Wales). The second habit specification assumes that adjustment of actual consumption to desired consumption is only partially achieved during any given time period due to habit effects. This model is equivalent to one developed by Houthakker and Taylor and can also be derived by allowing the intercept term to be random and functionally dependent on previous consumption levels. The latter interpretation is in the spirit of Pollak and Wales' treatment of habit formation in linear expenditure systems where tastes at time t are a function of the quantity consumed at t-1. The model, with the Box-Cox transformations, can be expressed as

$$(11) \qquad q_{it}^{(\lambda)} = \beta_0 + \beta_1 q_{it-1}^{(\lambda)} + \beta_2 P_{1t}^{(\lambda)} + \ldots + \beta_{n+1} P_{nt}^{(\lambda)} + \beta_{n+2} Y_t^{(\lambda)} + U_t \qquad \begin{array}{l} i = 1, \ldots, n \\ t = 1, \ldots, T, \end{array}$$

where q_{it-1} is the quantity purchased of the i^{th} commodity in the previous time period. The third habit formation version is the state adjustment model of Houthakker and Taylor and discussed, for example, in Phlips (pp. 164-169). In this model the quantity demanded of the i^{th} commodity is assumed to be a function of the "psychological stock of habits

(S_t)," prices, and income. By assuming that the time rate of change in the transformed stock of habits is equal to current transformed purchases minus losses due to depreciation and substituting for the transformed unobservable psychological stock variable, $S_t^{(\lambda)}$, the model can be written (see, e.g., Phlips, p. 168)

$$(12) \qquad q_{it}^{(\lambda)} = \beta_0 + \beta_1 q_{it-1}^{(\lambda)} + \beta_2 P_{1t}^{(\lambda)} + \ldots + \beta_{n+1} P_{nt}^{(\lambda)} + \beta_{n+2} P_{1t-1}^{(\lambda)} + \ldots + \beta_{2(n+1)} P_{nt-1}^{(\lambda)} + \beta_{2n+3} Y_t^{(\lambda)} + \beta_{2n+4} Y_{t-1}^{(\lambda)} + U_t$$

$$i = 1, \ldots, n \qquad t = 1, \ldots, T$$

where q_{it-1}, P_{jt-1}, and Y_{t-1} are lagged values of the quantities, prices, and income, respectively.

The elasticity formulas for (10) - (12) are the same as given in (3). However, consider the effects of habits on elasticities. For example, the change in the elasticity with respect to a change in habits (represented by time) for the own price elasticity in (10) is

$$(13) \qquad \frac{\partial \varepsilon_{q_i P_i}}{\partial t} = - \lambda \, \varepsilon_{q_i P_i} \, \beta_{n+2} \, t^{\lambda-1} \, q_i^{-\lambda}$$

where β_{n+2} is the coefficient of time. For normal goods, sgn $\partial \varepsilon_{q_i P_i} / \partial t =$ sgn $(\beta_{n+2}\lambda)$. Therefore habits may increase, decrease, or leave unchanged the elasticities.

For example, if $\lambda = 0$, then (13) is zero. Therefore, global homogeneity, (8), implies constant elasticity and thus necessarily changing habits do not alter elasticities. When $\lambda = 1$, from (13), an increase in habits make demands more inelastic when β_{n+2} is positive. Similar results may be obtained for (11) and (12).

Concluding this section, we emphasize that the homogeneity condition defined in (7) is applicable for the dynamic models (10) - (12) as well. It asserts that current demands are unaffected by an equal deflation or inflation in all current prices and income given habits.

3. Estimation Methods

With respect to estimating the above models, it is assumed that the error terms are normally and independently distributed with zero means and constant variances, σ^2, for a given λ. Box and Cox and Zarembka show that given the above stochastic specification, the concentrated log likelihood for fixed λ, is, except for a constant,

$$(14) \qquad L_{max}(\lambda) = - n/2 \ln \hat{\sigma}^2(\lambda) + (\lambda - 1) \sum_{i=1}^{n} \ln q_i$$

where $\hat{\sigma}^2(\lambda)$ may be considered an estimate of σ^2 obtained by regressing $q_t^{(\lambda)}$ on transformed prices and income.

There are two approaches one can take in estimation of (14). One can transform the data so that $q^{(\lambda)}$ is regressed on $P_j^{(\lambda)}$ and $Y^{(\lambda)}$ using OLS. Then a search is conducted by varying λ so as to maximize (14). Alternatively, the unconcentrated or concentrated likelihood function could be optimized by gradient methods that converge on the value of λ which maximizes (14). The former method is used here: the optimal value of λ is obtained by combining (14) and the OLS package such that the maximum likelihood estimate of λ is obtained to within three significant digits.[2/]

4. The Empirical Application

The equations given in (2), (10), (11) and (12), presume that all prices enter demand functions. Pragmatically, it is impossible to accommodate theory precisely. Researchers often follow two general approaches:

(1) Include the prices of close substitutes and complements directly and use a price index (e.g., Consumer Price Index, CPI) for all other prices either as a deflator or as a separate independent variable (Stone).

(2) Exclude all prices other than close complements and substitutes (Hassan and Johnson).

The former approach is taken here and was first used by Stone.[3/] Income per capita is deflated by the CPI leaving the homogeneity test[4/]

$$\sum_{j=1}^{n} \varepsilon_{ij} = 0 \qquad i = 1, \ldots, n.$$

The data used to obtain parameter estimates and to perform the above homogeneity tests are U.S. time series observations on beef, pork, poultry, and fish from 1950 to 1975. Variables used are per capita food consumption in retail weight equivalents (1970 base), per capita income deflated by the CPI, and implicit price indices (1970 = 100) for the commodity groups (Food Consumption, Prices, and Expenditures).

5. Empirical Results

Parameter estimates for the static demand equation and the three habit formation models for each of the four meat commodities are presented in Tables 1 through 4. Results are recorded for the linear, double-log and the functional form associated with the maximum likelihood value for λ. Though the estimates for the linear and double-log forms are MLE's given $\lambda = 1$ and $\lambda = 0$ respectively, only the estimates using the value of λ which maximizes (14) will be referred to as the maximum likelihood estimates.

With respect to goodness of fit measures, the adjusted R^2 values indicate high degrees of fit. In most cases, the adjusted R^2 values are in the 80 to 90 percent range. The Durbin-Watson values do not suggest rejection of the null hypothesis of zero autocorrelation. In general, for the case of lagged dependent variables, the Durbin h values are not suggestive of autocorrelation problems.[5/]

5.1 Income and Price Parameter Estimates

By observing Tables 1 through 4, all direct own price coefficients are negative for all commodities and functional forms with few exceptions. For the commodity, fish, in the double log form and for the static and state adjustment models, and for all of the time trend models, the own price coefficient is positive. However, in all such cases, the coefficients are not significantly different from zero using any commonly used significance levels. In addition, all the own price coefficients

TABLE 1

Maximum Likelihood Estimates of the Parameters for the Static Demand Equation, 1950-1975

Static model	Dep. var. q_{it}	Explanatory variables: Const.	Beef P.	Pork P.	Poultry P.	Fish P.	Income	λ	Statistics: lnL a/	$\bar{R}^2$ b/	D.W. c/
1. BEEF											
a. linear		48.880	-0.772	0.083	-0.054	0.379	0.851	1	-26.167	0.966	1.047
		(4.79) d/	(8.49)	(1.15)	(0.86)	(4.19)	(8.38)				
b. maximum likelihood		33.098	-0.787	0.094	-0.060	0.392	0.846	0.89	-26.159	0.966	1.057
		(4.82)	(8.62)	(1.26)	(0.94)	(4.18)	(8.19)				
c. double log		2.361	-0.891	0.195	-0.117	0.505	0.789	0	-27.420	0.967	1.236
		(4.65)	(9.14)	(2.06)	(1.60)	(3.88)	(6.17)				
2. PORK											
a. linear		71.671	0.190	-0.647	0.267	0.231	0.250	1	-17.963	0.912	1.510
		(9.63)	(2.86)	(12.30)	(5.85)	(3.49)	(3.38)				
b. maximum likelihood		47.878	0.199	-0.674	0.274	0.230	0.266	0.89	-17.825	0.913	1.498
		(9.53)	(2.97)	(12.40)	(5.93)	(3.36)	(3.52)				
c. double log		3.309	0.257	-0.859	0.318	0.195	0.377	0	-24.575	0.855	1.525
		(6.77)	(2.74)	(9.44)	(4.51)	(1.56)	(3.07)				
3. POULTRY											
a. linear		30.538	0.340	0.098	-0.467	-0.129	0.882	1	-25.201	0.990	1.326
		(3.11)	(3.88)	(1.40)	(7.76)	(1.48)	(9.02)				
b. maximum likelihood		1.287	0.319	0.325	-0.634	-0.112	0.684	-0.19	-15.386	0.995	1.460
		(6.18)	(5.30)	(5.43)	(13.84)	(1.33)	(8.41)				
c. double log		1.871	0.333	0.288	-0.609	-0.127	0.712	0	-15.872	0.995	1.376
		(5.86)	(5.44)	(4.85)	(13.24)	(1.56)	(8.86)				
4. FISH											
a. linear		66.839	0.244	-0.062	0.137	-0.072	0.120	1	-29.625	0.763	2.242
		(5.73)	(0.75)	(0.61)	(1.91)	(0.69)	(1.04)				
b. maximum likelihood		31315.6	0.211	-0.021	0.112	-0.076	0.115	2.51	-28.896	0.796	2.233
		(6.27)	(2.46)	(0.45)	(1.89)	(1.35)	(1.23)				
c. double log		3.160	0.196	-0.064	-0.125	0.006	0.061	0	-30.137	0.740	2.088
		(5.63)	(1.82)	(0.62)	(1.55)	(0.04)	(0.43)				

a/ Value of the log likelihood function ignoring the constant.

b/ Adjusted R^2.

c/ Durbin Watson value.

d/ Values in parentheses are t ratios.

TABLE 2

Maximum Likelihood Estimates of the Parameters for the "Time Trend" Demand Equation, 1950-1975

Model with time trend	Dep. var. q_{it}	Explanatory variables: Const.	Beef P.	Pork P.	Poultry P.	Fish P.	Income	Time	λ	Statistics: $\ln L$ a/	$\bar{R}^2$ b/	D.W. c/
1. BEEF												
a. linear		69.379	-0.803	-0.009	0.214	0.311	0.363	2.149	1	-22.729	0.973	1.412
		(5.53) d/	(9.69)	(0.13)	(1.71)	(3.61)	(1.63)	(2.40)				
b. maximum likelihood		9.485	-0.813	0.015	0.145	0.401	0.570	0.369	0.48	-21.002	0.978	1.659
		(6.41)	(10.44)	(0.18)	(1.57)	(4.40)	(4.63)	(3.13)				
c. double log		2.486	-0.842	0.072	0.047	0.490	0.657	0.063	0	-21.765	0.978	1.763
		(5.91)	(10.29)	(0.83)	(0.60)	(4.56)	(5.81)	(3.22)				
2. PORK												
a. linear		72.799	0.188	-0.652	0.282	0.227	0.224	0.118	1	-17.946	0.912	1.531
		(6.98)	(2.73)	(10.40)	(2.71)	(3.16)	(1.21)	(0.16)				
b. maximum likelihood		47.939	0.198	-0.675	0.276	0.230	0.264	0.008	0.89	-17.825	0.913	1.501
		(7.49)	(2.88)	(10.31)	(2.73)	(3.14)	(1.61)	(0.02)				
c. double log		3.308	0.257	-0.858	0.316	0.195	0.379	-0.001	0	-24.574	0.855	1.522
		(6.57)	(2.62)	(8.25)	(3.33)	(1.52)	(2.80)	(0.03)				
3. POULTRY												
a. linear		25.818	0.347	0.119	-0.529	-0.113	0.994	-0.495	1	-25.028	0.990	1.278
		(1.89)	(3.84)	(1.44)	(3.88)	(1.20)	(4.08)	(0.50)				
b. maximum likelihood		0.730	0.315	0.318	-0.611	-0.056	0.583	0.004	-0.57	-11.687	0.997	2.052
		(7.49)	(5.78)	(5.30)	(13.25)	(0.67)	(7.40)	(3.30)				
c. double log		1.898	0.344	0.261	-0.572	-0.130	0.683	0.014	0	-15.289	0.995	1.439
		(5.90)	(5.50)	(3.93)	(9.47)	(1.59)	(7.90)	(0.93)				
4. FISH												
a. linear		38.813	0.285	0.064	-0.229	0.022	0.787	-2.939	1	-24.362	0.842	1.689
		(2.91)	(3.23)	(0.80)	(1.72)	(0.24)	(3.31)	(3.08)				
b. maximum likelihood		17.763	0.232	0.087	-0.204	0.012	0.570	-1.097	0.70	-23.291	0.852	1.794
		(4.93)	(2.76)	(1.04)	(1.80)	(0.13)	(3.51)	(3.53)				
c. double log		3.021	0.142	0.072	-0.057	0.022	0.207	-0.070	0	-24.409	0.832	2.006
		(6.51)	(1.57)	(0.75)	(0.66)	(0.19)	(1.66)	(3.24)				

a/ Value of the log likelihood function ignorning the constant.

b/ Adjusted R^2.

c/ Durbin Watson value.

d/ Values in parentheses are t ratios.

TABLE 3

Maximum Likelihood Estimates of the Parameters for the Partial Adjustment Demand Model, 1950-1975

Partial adj. model	Dep. var. q_{it}	Const.	Beef P.	Pork P.	Poultry P.	Fish P.	Income	qit-1	λ	$\ln L$ a/	$\bar{R}^2$ b/	Durbin h
		Explanatory variables								Statistics		
1. BEEF												
a. linear		45.386	-0.766	0.033	-0.017	0.410	0.810	0.057	1	-22.248	0.971	1.515
		(4.09) c/	(7.88)	(0.47)	(0.28)	(4.76)	(6.27)	(0.47)				
b. maximum likelihood		275.812	-0.679	-0.006	0.012	0.346	0.785	0.113	1.50	-21.998	0.970	1.515
		(3.66)	(7.39)	(0.10)	(0.20)	(4.84)	(6.20)	(0.91)				
c. double log		2.473	-0.927	0.149	-0.101	0.570	0.803	-0.037	0	-24.727	0.968	1.949
		(4.44)	(8.22)	(1.54)	(1.37)	(4.33)	(5.40)	(0.114)				
2. PORK												
a. linear		75.150	0.190	-0.689	0.285	0.243	0.254	-0.047	1	-17.082	0.913	1.465
		(7.31)	(2.75)	(11.45)	(5.58)	(3.52)	(3.33)	(0.53)				
b. maximum likelihood		43.663	0.202	-0.709	0.297	0.245	0.275	0.051	0.85	-16.813	0.915	1.492
		(7.29)	(2.911)	(11.63)	(5.73)	(3.37)	(3.52)	(0.58)				
c. double log		3.429	0.253	-0.893	0.335	0.224	0.372	-0.029	0	-23.398	0.857	1.536
		(5.18)	(2.58)	(8.80)	(4.35)	(1.70)	(2.94)	(0.25)				
3. POULTRY												
a. linear		29.457	0.286	0.085	-0.411	-0.108	0.723	0.163	1	-23,973	0.989	2.016
		(2.91)	(2.73)	(1.15)	(4.97)	(1.17)	(3.69)	(0.95)				
b. maximum likelihood		1.012	0.301	0.319	-0.656	-0.048	0.677	-0.036	-0.35	-12.559	0.996	0.757
		(6.78)	(4.90)	(5.24)	(10.42)	(0.56)	(6.03)	(0.37)				
c. double log		1.892	0.326	0.259	-0.600	-0.096	0.713	-0.010	0	-14.298	0.995	1.595
		(5.86)	(4.81)	(4.18)	(9.18)	(1.17)	(5.76)	(0.09)				
4. FISH												
a. linear		77.295	0.278	-0.034	0.131	-0.103	0.137	-0.147	1	-24.524	0.828	N.A. d/
		(3.70)	(2.91)	(0.37)	(1.47)	(1.05)	(1.27)	(0.53)				
b. maximum likelihood		139579.0	0.238	-0.029	0.124	-0.065	0.137	-0.307	2.79	-22.387	0.872	N.A.
		(4.34)	(3.40)	(0.64)	(1.80)	(1.40)	(1.71)	(1.11)				
c. double log		3.258	0.222	-0.001	0.105	-0.063	0.082	-0.045	0	-25.692	0.800	N.A.
		(3.41)	(2.20)	(0.005)	(1.07)	(0.45)	(0.63)	(0.16)				

a/ Value of the log likelihood function ignorning the constant.

b/ Adjusted R^2.

c/ Values in parentheses are t ratios.

d/ The Durbin h statistic cannot be computed because it entails taking the square root of a negative number.

TABLE 4

Maximum Likelihood Estimates of the Parameters for the State Adj. Demand Equation, 1950-1975

State adj. model	Dep. var. q_{it}	Const.	Beef P.	Pork P.	Poultry P.	Fish P.	Income	q_{i-1}	Beef P_{-1}	Pork P_{-1}	Poultry P_{-1}	Fish P_{-1}	Income_{-1}	λ	lnL[a]	$\bar{R}^2$[b]	Durbin h[c]
						Explanatory variables									Statistics		
1. BEEF																	
a. linear		-7.936	-0.693	0.035	0.019	0.165	0.354	0.726	0.636	-0.098	0.068	-0.203	0.082	1	-12.300	0.987	N.A.[d]
		(0.39)	(4.85)	(0.56)	(0.19)	(0.65)	(1.22)	(3.01)	(3.39)	(1.42)	(0.69)	(1.04)	(0.24)				
b. maximum likelihood		-28.880	-0.647	0.034	0.003	0.220	0.423	0.704	0.581	-0.103	0.080	-0.249	0.008	1.50	-11.793	0.987	N.A.
		(0.22)	(4.70)	(0.62)	(0.03)	(0.98)	(1.56)	(2.97)	(3.19)	(1.60)	(0.85)	(1.43)	(0.02)				
c. double log		-0.248	-0.789	0.064	0.027	0.064	0.253	0.655	0.666	-0.058	0.036	-0.114	0.253	0	-17.398	0.982	N.A.
		(0.22)	(4.41)	(0.65)	(0.22)	(0.17)	(0.66)	(2.34)	(2.91)	(0.61)	(0.28)	(0.39)	(0.58)				
2. PORK																	
a. linear		31.979	0.390	-0.578	0.136	-0.151	0.331	0.341	-0.117	0.322	0.051	0.130	-0.177	1	-9.382	0.953	N.A.
		(1.66)	(3.36)	(8.65)	(1.38)	(0.83)	(1.25)	(1.32)	(1.05)	(1.74)	(0.54)	(0.69)	(0.51)				
b. maximum likelihood		6.257	0.350	-0.703	0.164	0.063	0.345	0.335	-0.040	0.318	0.081	-0.156	-0.069	0.53	-7.317	0.960	N.A.
		(1.92)	(3.25)	(10.31)	(1.77)	(0.32)	(1.33)	(1.62)	(0.41)	(1.97)	(0.91)	(0.82)	(0.21)				
c. double log		1.301	0.300	-0.858	0.198	0.334	0.362	0.361	0.059	0.338	0.110	-0.540	0.057	0	-10.786	0.947	N.A.
		(1.58)	(2.39)	(9.68)	(1.83)	(1.30)	(1.12)	(1.76)	(0.52)	(1.93)	(1.05)	(2.19)	(0.14)				
3. POULTRY																	
a. linear		3.398	0.321	0.190	-0.580	-0.115	0.452	0.580	-0.051	-0.122	0.413	-0.132	0.028	1	-12.206	0.996	1.526
		(0.30)	(2.66)	(2.90)	(5.84)	(0.56)	(1.54)	(3.02)	(0.44)	(1.66)	(3.06)	(0.68)	(0.08)				
b. maximum likelihood		0.881	0.250	0.234.	-0.599	0.248	0.579	0.202	0.055	-0.066	0.187	-0.448	0.091	-0.16	-1.650	0.998	-4.63
		(2.59)	(3.28)	(4.48)	(9.77)	(1.50)	(2.93)	(1.16)	(0.64)	(0.91)	(1.46)	(2.89)	(0.37)				
c. double log		1.046	0.271	0.229	-0.596	0.191	0.599	0.272	0.034	-0.085	0.237	-0.415	0.039	0	-2.079	0.998	-3.194
		(2.23)	(3.49)	(4.41)	(9.50)	(1.16)	(3.00)	(1.57)	(0.39)	(1.22)	(1.89)	(2.68)	(0.15)				
4. FISH																	
a. linear		5.300	0.412	0.147	-0.267	-0.081	1.098	0.191	-0.025	-0.122	0.341	-0.127	-1.104	1	-15.643	0.915	N.A.
		(2.23)	(2.87)	(1.53)	(1.66)	(0.32)	(3.27)	(0.67)	(0.20)	(1.57)	(2.72)	(0.54)	(2.78)				
b. maximum likelihood		75035.4	0.424	0.068	-0.263	-0.314	0.600	0.167	-0.011	-0.057	0.310	0.200	-0.613	2.80	-12.504	0.942	N.A.
		(2.16)	(3.53)	(1.36)	(2.09)	(2.01)	(2.52)	(0.64)	(0.10)	(0.92)	(3.23)	(1.34)	(2.11)				
c. double log		2.660	0.303	0.159	-0.191	0.183	0.114	0.170	0.015	-0.139	0.278	-0.329	-1.170	0	-18.794	0.884	N.A.
		(2.38)	(1.89)	(1.20)	(1.07)	(0.52)	(2.69)	(0.57)	(0.10)	(1.44)	(1.95)	(1.03)	(2.41)				

a/ Value of the log likelihood function ignoring the constant.

b/ Adjusted R^2.

c/ The Durbin h statistic cannot be computed because it entails taking the square root of a negative number.

d/ Values in parenthesis are t ratios.

for the other commodities are significantly different from zero at the .01 level.

In general, the cross price effects are positive indicating gross substitution among commodities: the exceptions involve the commodity fish. The t values associated with cross price derivatives are usually smaller than those related to the own direct price coefficients.

All of the estimated income effects are positive, indicating superior commodities. In most cases, these coefficients are significantly different from zero at the .05 level. Also note that t values are usually higher for the MLE estimates.

5.2 Functional Forms

In Tables 1 through 4, the maximum likelihood estimates for the various models are presented. It is observed that the estimated λ's vary substantially across commodities and model specifications. They range in value from $\lambda = -.5$ to $\lambda = 2$. In order to determine if the linear and double log forms differ significantly from the functional form obtained by maximizing the likelihood function, likelihood ratio tests are performed.[6/]

The likelihood ratio test is based on

$$\tilde{\lambda} = \max_{\omega} L / \max_{r} L \tag{15}$$

where the numerator is the maximum value of the likelihood function, L, for the model with restrictions and the denominator is the maximum value of the likelihood function, L, for the model without restrictions. It can be shown that $-2 \ln \tilde{\lambda}$, under the null hypothesis, is distributed asymptotically, as chi-squared with the number of degrees of freedom equal to the number of restrictions to be tested (Theil, pp. 396-397).

The likelihood ratio test results presented in Table 5 indicate that linear demand functions cannot be rejected for beef regardless of the model specification. However, the double log form is rejected for both the partial and state adjustment models. For pork demand, the double log specification is rejected in every case while the linear formulation is rejected only for the state adjustment model. Almost the reverse is implied for poultry demands: the linear specification is rejected in every model while the double log is rejected only for the time trend model. The linear and double log specifications are rejected for fish demands in the partial and state adjustment models.

These results provide strong support for the position that more careful consideration needs to be given in the functional forms of demand relations for meat. The traditional linear and double log forms are frequently inadequate. These conclusions are based on the likelihood ratio test results given in Table 5.

5.3 Tests of Restrictions

Tests for global [refer to equation (8)] and local [refer to equation (9)] homogeneity conditions in the Box-Cox demand relations are provided in Table 6.

Global homogeneity is rejected for all models and all commodities with the exception of poultry in the state adjustment model. Thus, the

TABLE 5

Tests of Functional Forms Based on the Likelihood Ratio Procedure[a/]

	Models			
Commodity	Static	Time Trend	Partial Adjustment	State Adjustment
		values of $-2 \ln\lambda$		
1. Beef				
a. linear vs. MLE	0.016	3.454	0.500	1.014
b. double log vs. MLE	2.522	1.526	5.458*[b/]	11.210*
2. Pork				
a. linear vs. MLE	0.276	0.242	0.538	4.130*
b. double log vs. MLE	13.500*	13.498*	13.170*	6.938*
3. Poultry				
a. linear vs. MLE	19.630*	26.682*	22.830*	21.1112*
b. double log vs. MLE	0.972	7.204*	1.739	0.858
4. Fish				
a. linear vs. MLE	1.458	2.142	4.274*	6.278*
b. double log vs.	2.482	2.236	6.61*	12.580*

a/ The critical χ^2 value at the .05 significance level with 1 degree freedom is 3.84.

b/ * denotes significance at the 0.05 level.

hypothesis of no money illusion is rejected in nearly every case. To put these results in perspective, others using the Rotterdam and log-linear demand systems have also rejected the property of homogeneity using static aggregate systems (Barten, p. 46).

We also imposed local homogeneity conditions at the means of prices and income in a manner similar to Byron and Court who used a static log linear demand system. Tests for local homogeneity conditions are presented in Table 6. These results are somewhat in contrast to those obtained for global homogeneity. For example, local homogeneity is not rejected in any model for pork; however, local and global homogeneity is rejected for beef demand. In general, the frequency of rejection of local homogeneity is large but less than the frequency of rejection of global homogeneity.

In summary, the results of Table 6 indicate that homogeneity of meat demands is not a warranted maintained hypothesis. Therefore, our results suggest that a researcher should be cautious when choosing between regression methodologies using either deflated or nondeflated data.[7/] Because of the broad class of models considered and our choice of flexible functional forms, these results appear of general interest to researchers in demand analysis.

5.4 Model Selection

To determine which specification is appropriate, the static or one of the habit version relations, t and likelihood ratio tests may be used.[8/]

TABLE 6

Results of Local and Global Homogeneity Tests Based on the Likelihood Ratio Procedure[a]

	Models			
Commodity	Static	Time Trend	Partial Adjustment	State Adjustment
	values of $-2\ \ln\tilde{\lambda}$			
1. Beef				
a. local	26.429*[b]	22.068*	14.001*	5.082*
b. global	29.475*	22.130*	29.367*	20.973*
2. Pork				
a. local	0.216	0.125	0.180	1.758
b. global	17.489	16.677*	16.274*	6.953*
3. Poultry				
a. local	10.136*	2.542	9.909*	1.86
b. global	13.788*	15.434*	16.219*	2.022
4. Fish				
a. local	21.273*	8.264*	--[c]	0.749
b. global	21.553*	14.411*	15.874*	17.493*

a/ In every case the models are compared against the unrestricted maximum likelihood forms.

b/ The computed values are to be compared with the critical χ^2 values of $\chi^2_{.05}$ (1) = 3.84 for the local homogeneity tests and $\chi^2_{.05}$ (2) = 5.99 for the global homogeneity tests. * denotes significance at the .05 level.

c/ No global maximum of the likelihood function could be obtained for reasonable values of λ.

The model with the time trend was compared with the static version.[9/] By observing the t ratios of the time coefficients in Table 2, it can be seen that accounting for endogenous tastes by adding a time trend results in a significant difference (at the .05 level) relative to the static model for beef, poultry and fish.

When the partial adjustment model is compared to the static model, the only difference is that of an additional explanatory variable, q_{it-1}, in the partial adjustment specification. By inspection of the t ratios of this variable in Table 3, it can be seen that this model does not differ significantly from that of the static version.

When the state adjustment model is compared with the static model, the likelihood ratio method reveals that there does exist a significant difference for all commodities. The computed test values ($-2 \ln \tilde{\lambda}$) of 28.73, 21.02, 27.47, and 32.78 for beef, pork, poultry, and fish, respectively, are to be compared with the critical χ^2 value, $\chi^2_{.05}(6) = 12.6$. Consequently, persistence in consumption patterns are present in the demand for meats and it appears that they can be represented well by the state adjustment relation.

The state adjustment model was also tested against the partial adjustment model; for all commodities, the restrictions implied by the partial adjustment model were rejected.[10/]

5.5 Price and Income Elasticities

The price and income elasticities for the state adjustment model (the one which was selected by the likelihood ratio procedure) are presented in Table 7. As can be observed from the entries in the table, generally the elasticities are relatively small (inelastic). Note that though homogeneity was usually rejected, the calculated elasticities, under the unrestricted, local and global homogeneity restrictions, are very similar. The values were computed at the means; however, their behavior will generally differ considerably over time depending upon the value of λ computed by the maximum likelihood procedure. An exception is when $\lambda = 0$. In this case, elasticities are constant and invariant with respect to taste changes.

5.6 The Effects of Tastes on Elasticities

Clearly, the static model implies that tastes do not affect the elasticities. For the time trend model λ is positive for all commodities except poultry. Further, all of the estimated time coefficients (MLE) are positive except fish. Following (13), the passage of time implies that beef and pork demands are becoming more inelastic, while poultry and fish demands are becoming more elastic. Also, following calculations similar to (13), the results of Table 2 indicate that beef and pork demands are becoming more income inelastic over time while poultry and fish are becoming more income elastic.

Equation (11) can be viewed as a reduced form from the partial adjustment model. Given this interpretation, the adjustment coefficient is estimated by one minus the coefficient associated with lagged quantity. Also, this coefficient should lie in the positive unit interval and habit persistence is indicated by an adjustment coefficient deviating from one.

TABLE 7

Price and Income Elasticities for the State Adjustment Model
Under Unrestricted, Global and Local Homogeneity

Commodity	Beef P	Pork P	Poultry P	Fish P	Income
1. Beef					
a. unrestricted[a/]	-0.690	0.042	0.003	0.262	0.448
b. global	-1.039	0.047	0.060	0.933	0.393
c. local	-0.679	0.058	-0.011	0.630	0.607
2. Pork					
a. unrestricted	0.316	-0.794	0.174	0.070	0.370
b. global	0.293	-0.860	0.201	0.365	0.366
c. local	0.322	-0.814	0.193	0.300	0.383
3. Poultry					
a. unrestricted	0.249	0.231	-0.601	0.245	0.580
b. global	0.296	0.235	-0.606	0.075	0.584
c. local	0.289	0.237	-0.609	0.084	0.572
4. Fish					
a. unrestricted	0.500	0.106	-0.295	-0.457	0.701
b. global	0.430	0.242	-0.323	-0.349	1.156
c. local	0.468	0.115	-0.268	-0.314	0.810

a/ The unrestricted model refers to the Box-Cox functional forms without the homogeneity conditions imposed.

Examining Table 3, all estimated adjustment coefficients are nearly one and in no case do they differ statistically from unity. Hence, the partial adjustment model may provide little information about changing tastes since it is not significantly different from the static model.

For the state adjustment model, adapting Houthakker and Taylor's approach here, it can be shown that the marginal effects of habits on demands (or elasticities) can be determined from the reduced form model.[11/] In our case, it was not possible to apply the restrictions on reduced form parameters as implied by the structure and perform nested hypotheses tests. Hence, these marginal effects cannot be estimated uniquely from the reduced form. However, given the state adjustment rationalization of (8), the estimated marginal effects of habits on the quantities demanded are positive for all commodities with the exception of fish.[12/] This corresponds to the results derived from the time trend model. Further, the marginal effects of habits on own price elasticities is given by (13), where t is replaced by the stock of habits, S_t, and α is its associated coefficient in the structural state adjustment model. Hence, beef and pork demands become more price inelastic as the stock of habits increases.

6. Policy Implications and Conclusions

Theoretically, there is a direct linkage between demand relations which are homogenous and flexible. Our results indicate overwhelming evidence for rejection of homogeneity when a flexible functional form is used to model meat demand. These results, though at variance with the theory of individual choice,[13/] are consistent with the findings of other researchers who tested homogeneity in demand systems with more restrictive functional forms and representations of changing tastes (see, e.g, Barten). Therefore, the many varied ways of deflating data may lead to substantially different results.

The results reported here suggest that the linear and double log functional forms are inappropriate as maintained hypotheses. The best functional form is, however, sensitive to the manner in which changing tastes are represented and to the commodity studied. Yet, for pork demands, the often used double log form was rejected for every model considered. We conclude on the basis of likelihood ratio tests, that Box-Cox transformations may be a useful tool for analysts of meat demand since the double-log and linear functional forms were rejected in many of our cases.

Finally, the Box-Cox demand functions allow flexibility in the way habits affect demand elasticities. Suppose an increase in habits implies that the own price demand elasticity increases (becomes more inelastic), then the double log form would measure this phenomena incorrectly since elasticities do not change. Our results indicate that the impact of a future policy aimed at raising price may have a much lower effect on beef and pork demands (and larger impact on expenditures) than predicted from a double log model (*ceteris paribus*).

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FOOTNOTES

1/ By relaxing the restrictive assumption that all the λ's are the same for each variable the semi-log and log-inverse functional forms can be obtained. However, to reduce the computational burdens, only a single λ in each equation is considered.

2/ Using Cramer-Rao lower bound theory one can derive estimates of the asymptotic standard errors of parameters from the information matrix. However, the information matrix involves the expectations of nonlinear functions of random variables. A common procedure used for likelihood functions admitting sufficient statistics involves replacing the MLE's of β, λ, and σ^2 into the information matrix and removing the expectation sign (Goldfeld and Quandt). However, it can be shown that the Box-Cox density does not admit a sufficient statistic when λ is unknown. Hence, the conditional least squares approach, taken here, seems like a good pragmatic alternative to the maximization of the unconcentrated likelihood function, treating λ as an unknown parameter.

3/ All of the results reported in this paper were also obtained using the second approach. Often the results differed substantially; yet the major conclusions of this paper are unaltered. Based upon theoretical reasons and the qualitative results, Stone's approach appeared superior.

4/ Implicitly, this assumes that the CPI is homogeneous of degree one. The method used by Stone is attractive because it reduces collinearity as compared to the case where the CPI enters as a separate independent variable.

5/ Recall that the Durbin h statistic is tested as a normal deviate with large values leading to rejection of the null hypothesis of zero autocorrelation. In some cases it is impossible to compute the Durbin h statistic because it would entail taking the square root of a negative number.

6/ Equivalent tests could be run by using the confidence limits associated with the transformational parameter, λ. An approximate 100 $(1-\alpha)$ percent confidence region for λ can be found from (Box and Cox, p. 216)

$$L \max (\hat{\lambda}) - L \max (\lambda) < 1/2 \, \chi_1^2 (\alpha).$$

7/ If global homogeneity holds, then elasticity estimates should not be sensitive to the deflator used. However, if it does not hold then parameter estimates will depend upon the deflator used and regressions using nominal and deflated data will give different elasticity estimates. Hence, when homogeneity does not hold, a non-nested testing procedure must be used in order to find the "best" model. It should also be noted that local homogeneity is not a nested model of a globally homogeneous model.

8/ The likelihood ratio method can be used to run these tests because the static model is a restricted version of the habit formation models. The partial adjustment model is also a restricted version of the state adjustment model. Thus, the hypotheses to be tested are nested and this is essential for the LR procedure to be appropriate.

9/ The likelihood ratio test procedure is equivalent to using the asymptotic t values for the time variable when there is just one restriction as in this case.

10/ Note that the partial adjustment model is a nested version of the state adjustment model. The computed likelihood ratio values are 20.41, 18.99, 21.82, and 19.77, respectively for beef, pork, poultry, and fish. These values are to be compared with the critical value $\chi^2_{.05}(5) = 11.07$.

11/ The structural form of the state adjustment model for the Box-Cox case (ignoring prices) is

$$q_t^{(\lambda)} = \theta + \alpha\, S_t^{(\lambda)} + \gamma\, Y_t^{(\lambda)}$$

$$\frac{dS_t^{(\lambda)}}{dt} = q_t^{(\lambda)} - \delta\, S_t^{(\lambda)}.$$

The discrete approximation of the above model in its reduced form is

$$q_t^{(\lambda)} = K_0 + K_1\, q_{t-1}^{(\lambda)} + K_2\, Y_t^{(\lambda)}$$

where the structural habit coefficient of interest is (see Phlips, p. 168, equation 6.38)

$$\text{(F.1)}\quad \alpha = \frac{2(K_1 - 1)}{K_1 + 1} + \frac{K_2 + K_3}{K_2 - .5(K_2 + K_3)}.$$

Hence, $\partial q_t^{(\lambda)} / \partial S_t^{(\lambda)} = \alpha$ and $\partial q_t / \partial S_t = \alpha(S_t / q_t)^{\lambda-1}$.

12/ Using (F.1) in footnote 11, the estimates for α, the habit coefficient, are approximately $\hat{\alpha}$ = 1.7, 0.4, 1.1, -1.4, respectively for beef, pork, poultry and fish.

13/ Though received theory implies that micro demand functions are homogeneous of degree zero, a priori there is no reason for aggregate market demand functions to retain this property. Therefore, our results cannot be construed as a rejection of the micro theory.

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