Numeraire Choice in Agricultural Supply Analysis

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Abstract

Should the choice of numeraire price for modeling profit functions be arbitrary, or is more careful study needed? Here, we examine the choice of numeraire using tests for models specification and out-of-sample predictive accuracy.

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In modeling producer's supply response behavior, the classic paper by Arrow *et al.* called into question the inherent restriction of the Cobb Douglas model that all elasticities factor substitution are equal to one. Researchers have since developed numerous flexible functions that allow substitution to be unrestricted (Berndt and Christensen; Diewert). Diewert introduced several flexible forms that are capable of performing comparative statics without imposing many prior restriction. These flexible functional forms (FFF) are created by second order numerical or second order differential approximations with (n+1)(n+2)/2 independent parameters. There exist several flexible functional forms, among them the normalized quadratic has been a popular one for economic analysis of profit functions (Lau; Eckstein; Diewert and Wales 1987 and 1988; Shumway and Alexander; Shumway and Gottret). The normalized quadratic function implies a quasi-homothetic technology and, except for numeraire netput, has strongly separable netput supplies. Some of the desirable characters of the normalized quadratic are that it is locally flexible, self dual (i.e., the production function has same functional form as the profit function), and its hessian is a matrix of constants. This latter property makes testing or imposition of properties which place restrictions on the elements of the Hessian (e.g. curvature restrictions) more straight forward, since the properties hold globally and do not have to be tested or imposed at each data point.

In a quadratic functional form, each output and input price is divided by one input or output price, referred to as the numeraire. This division of individual output and input prices by one of the price component maintains the linear homogeneity property implied by economic theory. When the profit function is normalized, all properties of the subsequently estimated function are conditional on linear homogeneity, and the homogeneity, since it has been imposed, cannot be tested.

The normalized restricted quadratic function can be shown as:

$$\Pi = b_o + CP + 0.5P^{/}DP \qquad (1)$$

where Π is a restricted profit divided by price of netput 1. P = [p_2 ... p_m, x_{m+1}...x_n] is the vector of prices of the variable netputs (P_2...P_m) divided by price of variable netput 1, and quantities of the fixed inputs and related exogenous variable (x_{m+1} ... x_n); and b, the vector C, and matrix D are parameters. Except for the numeraire equation, the first derivative output supply and input demand equation are linear in prices:

$$x_{it} = c_i + \sum_{j=2}^{m} d_{ij} p_{jt} + \sum_{j=m+1}^{n} d_{ij} x_{jt} \quad i = 2, \dots, m \qquad (2)$$

where t is time.

The numeraire (netput 1) equation is quadratic in normalized prices and other exogenous variables:

$$x_{1t} = b_o + \sum_{i=m+1}^{n} c_i x_{it} - 0.5 \sum_{i=2}^{m} \sum_{j=2}^{m} d_{ij} p_{it} p_{jt} + 0.5 \sum_{i=m+1}^{n} \sum_{j=m+1}^{n} d_{ij} x_{it} x_{jt} \qquad (3)$$

Unlike the translog functional form, there is no singularity in the covariance matrix of an estimated normalized quadratic. Therefore, the invariance of the estimates to choice of the deleted quadratic equation or numeraire cannot be guaranteed by maximum likelihood estimation. The numeraire equation can be, and often is, estimated as part of the system. However, changing the numeraire fundamentally changes the model specification by changing the right hand variables and meaning of the error term in all equations (Shumway and Gottret). In this paper, the performance of different input and output prices as numeraire will be evaluated based on each model's accuracy in out-of-sample forecasting of output supplies and input. Further, the choice of numeraire will also be examined by model specification tests of non-nested hypotheses, based on the P test (Davidson and MacKinnon).

Data

Annual data for all commercial agricultural outputs produced and inputs used in Iowa, for the period 1950-1988 were used to estimate the systems of supply and demand equations. Data for the years 1989-1993 were saved as out-of-sample data, and used in the assessment of forecasting accuracy.

The variable inputs considered are operating capital, fertilizer, pesticides, hired labor, and a miscellaneous inputs aggregate. The miscellaneous inputs aggregate consisted of all inputs not directly accounted for in the individual demand equations, plus numerous minor inputs which are not reported as separate categories. The output supply equations consisted of corn, soybeans, hogs, and cattle, with output aggregates being constructed for “other” crops, and “other” livestock. All aggregate price and implicit quantity indices were calculated using the tornqvist index. Because the relevant output prices are not known when most resources are committed to production, one year lagged prices were chosen to represent expected output price. These prices include both market price and value of government payments.

Land, family labor, and service flows from capital stock were treated as fixed inputs. The fixed capital variable was an aggregate measure of depreciation of various capital assets, including machinery, equipment, trucks, automobiles, and service structures. Land was included as the number of acres in farms. Time was included as a proxy for disembodied technical change in all netput supply equation. Temperature and precipitation are critical in crop growing months therefore were included in the output equation. Precipitation was included as the total for the first three months of the growing season. Precipitation was used in the model as an *ex post* output influencing measure rather than as an *ex ante* decision variable. The weather data were from Tiegen and Singer, and were monthly averages weighted by harvested cropland. Input and output price and quantity data are from USDA sources, originally compiled by Evenson and his associates, and updated by McIntosh.

Estimation

The Systems of six output supply and five input demand equations were estimated as a system of linear-in-parameters equations. Error terms were assumed to be additive, independently and identically distributed with mean zero and a constant covariance matrix. The estimation was carried out using iterative seemingly unrelated regressions (SUR). Symmetry of cross partial derivatives and linear homogeneity of the profit function were maintained in the estimation procedure. The normalized restricted profit function (1) was not included in the system of equations estimated. The numeraire equation (3) was included in the estimation.   Since our goal was to test the appropriateness of the individual netputs as numeraire, 11 sets of equations were estimated, each with a different numeraire.

Model and Criteria of Accuracy of Forecasting Measurement

Thirty-eight observations for the period 1951-1988 were used in estimating parameters for each of 11 systems of equations (one system for each individual numeraire). The parameters from the iterative SUR estimation were then used to forecast one-step-ahead estimates of the quantities of supplies and demands. That is, the parameters estimated through time period *t* were used along with the independent variables through time period *t+1* to obtain estimates of the dependent variables for time period *t+1.* The data for time period *t+1* was then added to the model and new parameters were obtained. The new parameters were then used to forecast the independent variables for time period *t+2,* and so on.

Various measures have been proposed for assessing the predictive accuracy of forecasting models (Theil; Fair). Most of these measures are designed to evaluate *ex post* forecasts. We calculate the out of sample forecasting efficiency of the systems of equations based on the various numeraires, using a minimum value-share weighted mean absolute percent error (MAPE). The value-share weighted MAPE is calculated each for input and output equations, from a weighted average of the individual equation MAPE's using the value-share of the individual input or output as weights. The individual equation MAPE's are calculated using the standard formula.

Tests of Non-nested Hypotheses in Multivariate Models

The systems of supply and demand equations for each numeraire were compared using a non-nested testing procedure. In each case, the models differed only in the choice of numeraire, and thus the relative composition of the linear and quadratic equations (2) and (3). Thus, one system cannot be obtained by simply imposing linear restrictions to any other, and the hypothesis test for appropriate numeraire is non-nested. Davidson and MacKinnon's P_1 test (a multivariate generalization of the P test) were used to evaluate the different specifications against the non-nested alternatives. Consider two alternative models:

$$
\begin{aligned}
H_o\colon\ y_{it} &= f_{it}(X_t,\beta) + \varepsilon_{oit} \\
H_1\colon\ y_{it} &= g_{it}(Z_t,\gamma) + \varepsilon_{1it}
\end{aligned}
\tag{4}
$$

where *i* indexes the equations. The y_{it} are the *t*th observation of the *i*th dependent variable and the f_{it} and g_{it} are non-nested functions which depend on vectors of exogenous variables X_t and Z_t and unknown parameter vectors β and γ. For a given *t* the $\varepsilon_{\mathrm{jit}}$ (j=0 or 1) are assumed to be serially independent and multivariate normal with unknown covariance matrix Ω_{j}. In order to test the validity of $\mathrm{H_o}$ in the presence of the non-nested alternative $\mathrm{H_1}$, an artificial compound model is constructed using the maximum likelihood estimates of [illegible] [illegible], and [illegible]. For this example, the artificial regression for testing the validity of $\mathrm{H_o}$ would be:

$$
(y_{it} - \hat{f}_{it}) = \alpha\hat{\Omega}_o\hat{\Omega}_1^{-1}\,(\hat{g}_{it} - \hat{f}_{it}) + X_{it}\beta + u
\tag{5}
$$

This artificially nested model is estimated using generalized least squares. The ratio of the estimate of α to its estimated standard error provides the $\mathrm{P_1}$ test statistic which converges in distribution to *N*(0,1). It should be noted however, that the test is conditional on the truth of $\mathrm{H_o}$, not of $\mathrm{H_1}$. Thus, rejecting $\mathrm{H_o}$ does not make any implications regarding $\mathrm{H_1}$. If we desire to test the validity of $\mathrm{H_1}$, we must reverse the roles of the hypotheses and carry out the test again. It should also be noted that the tests are capable of rejecting or failing to reject both hypotheses at a given level of significance. Failure to reject a particular null hypothesis indicates that the data supports the null hypothesis in the presence of the specified alternative. Note also that the results are rarely transitive.

Due to the computational burden of conducting a non-nested test against 10 alternatives in a system of equations, the tests were preformed in sets of pair-wise comparisons. In this way, each numeraire choice was compared against all others, one at a time.

Results

The result of the analysis to choose among alternative numeraire are outlined in tables 1 and 2. Table 1 contains the P-test results from the pair-wise comparisons of the different numeraires. The test determines if the alternative model provides additional information or explanatory power beyond what is contained in the null hypothesis model. Ideally, we would look for a numeraire which for which we fail to reject H_o in all cases. Only one numeraire emerged un-rejected form all tests, and that was the Cattle aggregate. Three other potential numeraires were rejected by only one of the alternatives, those were the Other Inputs aggregate, Fertilizer, and Soybeans. Among these four potential numeraires, Cattle, the Other Inputs Aggregate, and Soybeans cause none of the others to be rejected; however, the Soybean numeraire is rejected in the presence of information contained in Fertilizer

The P-tests indicate that the price of cattle is the appropriate choice for numeraire given this data set and functional form. The P-tests indicated that, on the basis of average number of rejections, input and output prices were about the same, thus we cannot make a general conclusion regarding whether input or output prices are more appropriate. There also seemed to be no perceptible pattern linking the total value of an input or output to it’s relative success in the pair-wise P-tests. Thus our results do not support a heuristic of using the lowest value input or output as the numeraire.

To provide additional insight into the performance of the numeraire selection, the econometric supply-demand models are evaluated for out-of-sample predictive accuracy. The forecasting efficiency of the alternative numeraires is evaluated using a value-share-weighted mean absolute percent error measure; that is, for each equation, the absolute percent forecast errors are calculated, then averaged for input demands and output supplies, using each equation's share of the total value of demand or supply as a weight. These measures are shown in table 2. The top four numeraire equations, as indicated by the P-test results are then compared to find the one which achieves the lowest value-share weighted MAPE. Pair-wise t-test were conducted to determine if significant differences in forecasting accuracy existed. The pair-wise t-tests revealed no significance difference in forecasting accuracy between the numeraires in forecasting input demands. However, the model using Cattle prices as the numeraire achieved a statistically significant lower value-share weighted MAPE for output supplies when compared to the other three models at 10 percent level of significance. The model based on the Soybean price numeraire achieved a statistically significant lower value-share weighted MAPE for outputs when compared to Other Inputs and Fertilizer.

Summary and Conclusions

The P-tests results indicated a clear-cut choice for selecting the numeraire for our model. The results would have been even more convincing, perhaps, had the Cattle price numeraire model rejected all other potential models. Achieving somewhat ambiguous results from non-nested hypothesis tests is not surprising however, as previous research utilizing non-nested tests for model specification have failed to identify a single “best” model (Orazem and Miranowski, McIntosh and Shumway).

Among the top four models as indicated by the P-tests, no significant differences in forecasting ability for inputs was detected. In predicting outputs, the model based on the cattle price numeraire was shown to provide significantly better forecasts than the other three models, while the Soybean price numeraire models were superior to the remaining two models.

Of the 11 possible numeraires, the models based on cattle price provided the second best forecast of outputs as measured by the value-share-weighted MAPE’s (the model using the Other Outputs price as a numeraire achieved a slightly lower average MAPE of 0.0825). Of the 11 possible numeraires, the model using the price of other inputs as the numeraire achieved the overall lowest value-share weighted MAPE for inputs. It is also worth noting that on average, models using input prices as numeraire achieved greater accuracy in forecasting inputs than the models based on output prices. This result held likewise for the models using output prices as numeraires which were, on average, better predictors of outputs than were the models based on input prices.

The results of this study indicate that, for this data set and functional form, it does indeed make a difference which numeraire you choose. This is in contrast to previous studies which, in general, indicated that the choice of numeraire equation is arbitrary or, more commonly, offered no justification for the netput chosen (with the exception of Shumway and Gottret).

The results of this study further illustrate that the choice of a numeraire netput price is not trivial, and can have significant impacts on model specification, and the ability of the model to forecast output supply and input demand quantities. While conducting non-nested tests against a large number of alternative models is quite cumbersome, our results indicate a fairly consistent pattern between the forecasting ability of the models and their relative performance on the pair-wise non-nested tests. With the exception of the model using fertilizer price as a numeraire, the "best" five numeraires identified by the P-tests were also among the top five forecasters of either input demands or output supplies.

Table 2. Value-share weighted mean absolute percent errors for five one-step-ahead forecasts of input demands and output supplies.

		Value-share weighted MAPE's by equation			
	Year	Cattle	Other Inputs	Fertilizer	Soybean
Inputs	1	0.1413	0.1388	0.1406	0.1414
	2	0.1534	0.1516	0.1534	0.1538
	3	0.1759	0.1743	0.1756	0.1762
	4	0.1735	0.1717	0.1728	0.1735
	5	0.1670	0.1654	0.1664	0.1671
	mean	0.1622	0.1604	0.1618	0.1624
Outputs	1	0.0869	0.1232	0.1269	0.0899
	2	0.0659	0.0989	0.0986	0.0842
	3	0.0691	0.0851	0.0903	0.0720
	4	0.0887	0.1043	0.1083	0.0937
	5	0.1208	0.1413	0.1446	0.1192
	mean	0.0863	0.1105	0.1137	0.0918

Table 1. Result of pair-wise P-tests among alternative numeraire

Model 1	vs Model 2	H_o: Model 1 H_a: Model 2	H_o: Model 2 H_a: Model 1
		Asymptotic t-Statistics (p-values)	
Cattle	Other Livestock	0.66	-0.60
Hogs	Cattle	1.05	-0.48
	Other Livestock	-1.93	-1.49
Other Crops	Hogs	0.78	-0.09
	Cattle	-0.14	0.85
	Other Livestock	-1.61	1.17
Soybeans	Other crops	0.06	1.72*
	Hogs	0.85	-1.14
	Cattle	1.19	-0.46
	Other Livestock	0.33	-1.87*
Corn	Soybeans	-1.47	-0.28
	Other crops	-0.61	2.67*
	Hogs	0.10	0.78
	Cattle	0.35	1.02
	Other Livestock	-1.34	-0.99
Other Inputs	Corn	-1.40	2.90*
	Soybeans	1.51	-1.36
	Other Crops	1.13	-0.51
	Hogs	2.31*	-0.96
	Cattle	1.18	1.32
	Other Livestock	0.18	0.93
Pesticides	Other Inputs	0.35	-0.24
	Corn	-2.18*	2.24*
	Soybeans	-2.68*	-1.51
	Other Crops	1.85*	0.51
	Hogs	2.88*	2.52*

Table 1. Result of pair-wise P-tests among alternative numeraire (continued)

Pesticides (cont'd.)	Cattle	2.58*	1.59
	Other Livestock	1.07	2.93*
Labor	Pesticide	0.25	-0.41
	Other Inputs	-0.20	1.08
	Corn	-2.93*	-1.57
	Soybeans	-0.61	-0.24
	Other Crops	-0.09	1.05
	Hogs	-0.86	-1.86*
	Cattle	0.60	-0.46
	Other Livestock	0.79	2.46*
Operating Capital	Labor	-0.32	2.22*
	Pesticide	1.00	0.37
	Other Inputs	1.34	1.60
	Corn	-1.88*	1.63
	Soybeans	0.84	-1.62
	Other Crops	-0.90	-1.45
	Hogs	-0.13	-1.79*
	Cattle	1.24	1.39
	Other Livestock	2.02*	-3.23*
Fertilizer	Operating Capital	1.89*	-0.73
	Labor	-1.57	0.60
	Pesticide	0.13	-1.51
	Other Inputs	-1.23	-0.34
	Corn	-0.46	-0.71
	Soybeans	-0.88	-2.78*
	Other Crops	0.00	-0.84
	Hogs	-0.84	0.03
	Cattle	1.39	1.54
	Other Livestock	-1.29	-4.88*

* indicates significance at the 10% level, i.e., reject H_o that model 2 contains no new information.

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