The (Bargaining) Power of Incentives: Cooperatives, For-Profit Firms and the Cost of Procurement

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Abstract

In this paper, we show formally that cooperatives can possess an informational - and hence cost - advantage compared to For Profit Firms. This advantage is directly linked to the goal alignment between the cooperative and its members, and is influenced by the extent of income redistribution between members. Hence the standard practice of modeling the cooperative and the FPF as having identical cost structures appears to be theoretically unsound.

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1 Introduction

An important insight obtained from incentive theory is that privately held information is valuable to those that posses it, while it imposes a cost on those that do not. With access to full information about the agents' characteristics, for instance, a principal is able to entice the agents to exert the most efficient effort or production at a cost (net of the agents' next best alternative) exactly equal to that incurred by the agents. If agents possess private information about their characteristic, however, a principal cannot perfectly price discriminate.[1] Instead, the principal must incur an additional cost in the form of an informational rent paid to all but the less efficient agent to elicit the appropriate effort. This additional cost arises because of the so-called agency problem - the goals of the principal do not align with those of the agents and the principal must incur a cost to bring these goals more into alignment.

The observation that goal alignment influences costs in principal-agent relationships suggests that organizations in which the goals of the principal and the agents are more closely aligned might have different cost structures than those organizations where the principal's and the agents' goals are widely divergent. More specifically, when information is privately held, the informational costs - the costs associated with eliciting the desired behavior from the agents - may differ from one organizational structure to another. In such a situation, organizational form becomes endogenous, with the organizational form that generates the least total cost expected to emerge as the dominant form.

Cooperatives and for-profit firms (FPFs) are a well-documented example of organizations with different alignments between the goals of the principle and the agents. In the FPF, the goals of the principal (the FPF) and the agents (the farmers) are in conflict - an increase in profit for the FPF typically means a loss of profit for farmers. Cooperatives, however, have different goals - these have been expressed variously as service at cost and the maximization

[1] Even with perfect information, a principal may be unable to perfectly price discriminate because of institutional factors - e.g., a principal may be required legally to make a contract offered to one person available to everyone else that requests it. Faced with this situation, the principal has to use the contract terms to elicit information about an agent's characteristics and the discussion in the text is applicable.

of member welfare (with or without consideration of membership size) - in both of these interpretations, the goals of the cooperative and its members are closely aligned.[2] This closer alignment of objectives, combined with the argument sketched out above, suggests that cooperatives may face a lower cost structure than their FPF counterparts and might thus provide cooperatives with an organizational advantage in situations where privately held information is important.[3] Of course, since cooperatives may be under pressure from the membership to redistribute returns among the members, the degree of goal alignment is unlikely to be perfect.

The purpose of this paper is to show formally that cooperatives can possess an informational - and hence cost - advantage compared to FPFs and that this advantage is directly linked to the goal alignment between the cooperative and its members. This informational advantage is influenced by the extent to which cooperatives have a goal of redistributing income among its members. Put another way, the power of incentives is stronger in cooperatives precisely because they exert less bargaining power vis-à-vis their members than do FPFs.

The results of the paper point out that the standard practice of modeling the cooperative and the FPF as having identical cost structures is theoretically unsound. Instead, all else equal, cooperatives should be modeled as having lower costs. As a result, cooperatives can be expected to be successful as an organizational form when informational asymmetries are significant and/or when their organizational disadvantages are small. Of course, cooperatives face numerous and well documented problems (e.g., horizon problem, free rider problem) that are derived from the poorly defined property rights (see, for example, Bonin, Jones and Putterman, Hansmann, Cook, and Vitaliano). The results of this paper show that these problems may be offset by informational advantages that cooperatives possess. These advantages may help explain why producer cooperatives, for instance, appear to be at least

[2] Sexton (1984) provides an overview of the objectives of cooperatives. Bonin, Jones, and Putterman. Carlson. Sexton 1986. Vercammen, Fulton and Hyde.

[3] See Fulton (1995) for a somewhat similar argument framed in terms of property rights.

as robust as FPFs once they come into existence (Bonin, Jones and Putterman).

In addition to developing some new insights into cooperatives, the paper departs from most of contributions to the theoretical literature on incentives. A standard result in the incentive literature is that the most efficient agent will always be enticed to produce at the first best level. As this paper demonstrates, this conclusion hinges on the assumption that each agent's production/effort has a differential impact on the benefits obtained by the principal. When, as is reasonably the case for many market situations, each agent's effort/production has an equal impact on the benefits obtained by the principal, it can be shown that the most efficient agent will overproduce compared to the first best level. The resulting production/effort pattern across agents in turn affects the distribution of informational rents among the agents.

The context for the problem examined in this paper is a group of producers that sell their output/services to a processor, who in turns sells a processed product further downstream. The producers differ in their productivity of producing the output/service; while this productivity is privately held information, the processor does have knowledge of the underlying distribution of producer types. The analysis begins with an examination of the pricing decisions made by an FPF, first under conditions of perfect information and then under conditions of asymmetric information. The analysis then moves to an examination of the pricing decisions made by a cooperative that is assumed to provide the processing service at cost - i.e., all revenues net of the non-raw product costs are returned to the members. The cooperative case captures production cooperative (or labor-managed firms) and agricultural marketing cooperatives. After a comparison of the pricing decisions by the FPF and the cooperative, and the resulting output and welfare levels, the paper concludes with some observations on how the analysis can be extended to capture other informational problems (e.g., moral hazard) and to examine oligopolistic market structures.

2 The model

Consider a continuum of farmers of mass unity. Individual production level q costs $c(q, \theta)$, where parameter θ belongs to the set $\Theta = [\underline{\theta}, \bar{\theta}]$ with $\underline{\theta} > 0$ and represents the farmer's ability to transform inputs into production. For simplicity, it is assumed throughout the paper that $c(q, \theta) = \theta c(q) + k$ with $c' > 0$, $c'' > 0$, so that farmers with a low type are more efficient in producing a given output and $k > 0$ is a fixed cost. The linearity assumption with respect to θ allows for the Spence-Mirrlees condition (that is $\partial^2 c/\partial\theta\partial q$ is of constant sign) to be naturally satisfied, but the analysis can be easily extended to more general cost functions under the Spence-Mirrlees assumption. Farmers are distributed along the line segment according to ability density function $f(\theta)$ and cumulative function $F(\theta)$. It is assumed that $f(\theta) > 0$ for every θ. Moreover, the hazard ratio $\frac{F(\theta)}{f(\theta)}$ is assumed to be strictly increasing in θ (monotone hazard rate property).[4] Importantly, parameter θ is private information to the farmer while the empirical distribution function $F(\theta)$ is common knowledge to farmers and processors. Equivalently, the processor knows the type of each farmer but institutional constraints prevents it from perfectly discriminating among farmers according to their ability. Regardless of the reason, however, the processor must consider pricing schedules that ensures self-selection by the farmers.

3 Contracting with an investor-owned-firm

Assume that farmers can only sell their product to a processor that is a for-profit firm (FPF). This firm transforms the raw product into a final product sold on a downstream market. Applying the revelation principle, the firm proposes to the set of farmers a menu of contracts composed of a production level $q(\theta)$ and the associated transfer $t(\theta)$. The FPF wants to maximize its expected profit, that is its expected net revenue minus the expected transfer to be paid to farmers. The net revenue function $R(Q)$ depends on total production

[4]This regularity condition is made in order to prevent the incidence of "pooling" in the optimal contract resulting from the probability function, that is a contract in which the same allocation is selected for different values of θ.

$Q = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)dF(\theta)$ and contains all costs of processing the final good. The net revenue function is assumed to be strictly concave.

3.1 Complete information situation as a benchmark case

Before analysing the optimal procurement policy under incomplete information, consider the complete information situation where the FPF knows the farmer's type. The FPF's program is:

$$\max_{q(.),t(.)} \int_{\underline{\theta}}^{\overline{\theta}} \left[R(Q) - t(\theta) \right] dF(\theta)$$

$$\text{s.t.} \quad \pi(\theta) = t(\theta) - \theta c(q(\theta)) - k \geq 0 \tag{IR}$$

$$Q = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)dF(\theta) \tag{1}$$

Since leaving rents in the hands of farmers is costly for the FPF, it is easily seen that (IR) constraints are binding at the optimum ($\pi^{PI}(\theta) = 0$, $\forall \theta$, where PI stands for perfect information). Replacing $t(.)$ by its value in the objective function gives:

$$\max_{q(.)} \int_{\underline{\theta}}^{\overline{\theta}} \left[R(Q) - \theta c(q(\theta)) \right] dF(\theta) - k$$

$$\text{s.t. } Q = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)dF(\theta).$$

To solve this problem, it is useful to proceed in two steps. In the first step, the optimal production schedule $q(.)$ that minimizes the cost of procurement for a given total production level Q is found by considering the following problem:

$$\max_{q(.)} \int_{\underline{\theta}}^{\overline{\theta}} -\theta c(q(\theta))dF(\theta) - k$$

$$\text{s.t.} \quad \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)dF(\theta) = Q.$$

Denote ν as the Lagrange multiplier of the constraint. The Lagrangian is written as follows:

$$\mathcal{L} = \int_{\underline{\theta}}^{\overline{\theta}} -\theta c(q(\theta))dF(\theta) + \nu \left(\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)dF(\theta) - Q \right).$$

Maximizing over $q(.)$ gives:

$$\nu = \theta c'(q(\theta)) \tag{2}$$

Equation (2) gives a production schedule $q(\theta,\nu)$ which implicitly depends on ν, where ν can be interpreted as the marginal cost of individual production. Note that from this first order condition, the marginal costs of all farmers, regardless of their θ-type, are equalized at the optimum. The marginal value ν is linked to the fixed Q through the constraint:

$$\int_{\underline{\theta}}^{\overline{\theta}} q(\theta,\nu)dF(\theta) = Q \Rightarrow \nu \equiv \nu(Q). \tag{3}$$

It is easily seen that $\nu(Q)$ is increasing in Q. Indeed, differentiating (3) w.r.t. Q, gives:

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial q(\theta,\nu)}{\partial \nu}\nu'(Q)dF(\theta) = 1 \tag{4}$$

which yields $\nu'(Q) = 1/\left(\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial q(\theta,\nu)}{\partial \nu}dF(\theta)\right) > 0$, since differentiating (2) w.r.t. ν gives $\frac{\partial q(\theta,\nu)}{\partial \nu} = 1/\left(\theta c''(q(\theta,\nu))\right) > 0$.

Hence, the minimum cost of procurement $C(Q)$ of a total quantity Q can be written as follows:

$$C(Q) = \int_{\underline{\theta}}^{\overline{\theta}} t(\theta)dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta c(q(\theta,\nu(Q)) + k\right] dF(\theta)$$

which is strictly increasing and convex.[5]

The second step of the resolution involves determining the optimal total quantity Q^{PI} by maximizing the revenue net of the procurement cost $(R(Q) - C(Q))$. The result of this maximization gives:

$$R'(Q^{PI}) = C'(Q^{PI}) = \nu(Q^{PI}).$$

Overall, the result is that the production level assigned to a type-θ farmer is the first best level which equalizes the marginal net revenue with the marginal cost of production:

$$R'(Q^{PI}) = \theta c'(q^{PI}(\theta)) \tag{5}$$

with $Q^{PI} = \int_{\underline{\theta}}^{\overline{\theta}} q^{PI}(\theta)dF(\theta)$. For simplicity, we implicitly assume that the total surplus of production is positive, that is $\int_{\underline{\theta}}^{\overline{\theta}} \left[R(Q^{PI}) - \theta c(q^{PI}(\theta)) - k\right] dF(\theta) \geq 0$.

[5] Indeed, we have $C'(Q) = \int_{\underline{\theta}}^{\overline{\theta}} \theta c'(q(\theta,\nu(Q))\frac{\partial q(\theta,\nu)}{\partial \nu}\nu'(Q)dF(\theta) = \nu(Q) > 0$ (using (4) and (2)) and $C''(Q) = \nu'(Q) > 0$.

Note also that differentiating (5) with respect to θ yields to:

$$\dot{q}^{PI}(\theta) = -\frac{c'(q^{PI}(\theta))}{\theta c''(q^{PI}(\theta))} < 0$$

which confirms that $q^{PI}(\theta)$ naturally decreases in θ.

3.2 Incomplete information

Under asymmetric information, the FPF's faces the following problem:

$$\max_{q(.),t(.)} \int_{\underline{\theta}}^{\overline{\theta}} [R(Q) - t(\theta)]\, dF(\theta)$$

$$\text{s.t.} \quad \pi(\theta) = t(\theta) - \theta c(q(\theta)) - k \geq 0 \tag{IR}$$

$$\pi(\theta) \geq t(\tilde{\theta}) - \theta c(q(\tilde{\theta})) - k \quad \forall \theta, \tilde{\theta} \tag{IC}$$

$$Q = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta) dF(\theta)$$

As above, the analysis is carried out in two steps. In the first step the optimal procurement cost $C(Q)$ of a fixed quantity Q is determined. Then, in the second step, the optimal Q is determined by maximizing the revenue net of the procurement cost $(R(Q) - C(Q))$.

The production schedule that allows total quantity Q to be produced at least cost can be obtained by solving the following problem:

$$\max_{q(.),t(.)} \int_{\underline{\theta}}^{\overline{\theta}} -t(\theta) dF(\theta)$$

$$\text{s.t.} \quad \pi(\theta) = t(\theta) - \theta c(q(\theta)) - k \geq 0 \tag{IR}$$

$$\pi(\theta) \geq t(\tilde{\theta}) - \theta c(q(\tilde{\theta})) - k \quad \forall \theta, \tilde{\theta} \tag{IC}$$

$$Q = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta) dF(\theta)$$

Usual arguments (see Guesnerie and Laffont (1984)) show that (IC) constraints can be reduced to:

$$\dot{\pi}(\theta) = -c(q(\theta)) < 0 \tag{6}$$

$$\dot{q}(\theta) \leq 0. \tag{7}$$

Equation (6) indicates that (IR) reduces to $\pi(\overline{\theta}) \geq 0$ because the rate of growth of rents is negative. Replacing $t(.)$ by its value given by (IR) and ignoring the monotonicity constraint (7) (it will be checked later), results in the following problem:

$$\max_{q(.),\pi(.)} -\int_{\underline{\theta}}^{\overline{\theta}} \{\pi(\theta) + \theta c(q(\theta)) + k\} \, dF(\theta)$$

$$\text{s.t.} \quad \pi(\overline{\theta}) \geq 0$$

$$\dot{\pi}(\theta) = -c(q(\theta))$$

$$Q = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta) dF(\theta)$$

Since it is clear that the individual rationality constraint binds at the optimum ($\pi(\overline{\theta}) = 0$), integrating (6) gives:

$$\pi(\theta) = \int_{\theta}^{\overline{\theta}} c(q(u)) du. \tag{8}$$

Replacing the above expression in the objective function and integrating by parts gives the following problem:

$$\max_{q(.)} -\int_{\underline{\theta}}^{\overline{\theta}} \left\{ \left[\theta + \frac{F(\theta)}{f(\theta)}\right] c(q(\theta)) \right\} dF(\theta) - k$$

$$\text{s.t.} \quad Q = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta) dF(\theta).$$

Denote η the Lagrange multiplier of the constraint in this problem. The Lagrangian can then be written:

$$\mathcal{L} = -\int_{\underline{\theta}}^{\overline{\theta}} \left\{ \left[\theta + \frac{F(\theta)}{f(\theta)}\right] c(q(\theta)) \right\} dF(\theta) + \eta \left(\int_{\underline{\theta}}^{\overline{\theta}} q(\theta) dF(\theta) - Q \right)$$

and maximizing over $q(.)$ gives:

$$\eta = \left[\theta + \frac{F(\theta)}{f(\theta)}\right] c'(q(\theta)) \tag{9}$$

which implies that $q = q(\theta, \eta)$. Compared to the complete information case, the marginal cost of individual production is augmented by the extra marginal cost due to incentive compatibility. Denote the total marginal cost as the *virtual marginal cost* of θ-type farmer. However, the presence of an added cost does not imply that individual production levels are distorted downward compared to the complete information case because the optimal value of the multiplier η differs from the one under complete information. In addition, note that the assumption on the distribution function $F(.)$ ensures that the production level decreases monotonically in θ so that the neglected monotonicity constraints (7) are satisfied.[6]

There is a connection between η and Q through the:

$$Q = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta, \eta) dF(\theta) \Rightarrow \eta = \eta(Q). \tag{10}$$

This connection means that the production level assigned to the most efficient farmer, which is given by $\eta(Q) = \underline{\theta} c'(q(\underline{\theta}))$, is distorted since the optimal Q under incomplete information will differ from the optimal Q under complete information.

Finally, the total procurement cost $C(Q)$ can be expressed as follows:

$$\begin{aligned} C(Q) &= \int_{\underline{\theta}}^{\overline{\theta}} t(\theta) dF(\theta) \\ &= \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \left[\theta + \frac{F(\theta)}{f(\theta)} \right] c(q(\theta, \eta(Q))) \right\} dF(\theta) + k \end{aligned}$$

with $C(Q)$ being strictly increasing and convex in Q.[7]

The second step in solving the FPF's problem consists of maximizing the (concave) objective function $R(Q) - C(Q)$ yields the optimal quantity Q^F under incomplete information (where F indicates the FPF). The main results on the comparison of incomplete and complete information cases are stated in the following proposition.

Proposition 1 *Consider a privately-owned processor. Under incomplete information,*

(i) *the total production level Q^F is lower than the total production level Q^{PI} under complete information,*

[6] From (9) and the convexity of $c(.)$, it is easily seen that the monotone hazard rate property is a sufficient condition for $q(.)$ to be strictly decreasing in θ.

[7] The proof is similar to the one under complete information. Indeed, $C'(Q) = \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \left[\theta + \frac{F(\theta)}{f(\theta)} \right] c'(q(\theta, \eta(Q))) \frac{\partial q(\theta, \eta(Q))}{\partial \eta} \eta'(Q) \right\} dF(\theta)$. Using (9) and differentiating (10) w.r.t. Q shows that $C'(Q) = \eta(Q)$ and hence $C''(Q) = \eta'(Q) > 0$.

(ii) *individual production levels $q^F(\theta)$ are upward or downward distorted depending on the type of farmers and are given by*

$$R'(Q^F) = \left[\theta + \frac{F(\theta)}{f(\theta)}\right] c'(q^F(\theta)) \qquad \text{with } Q^F = \int_{\underline{\theta}}^{\overline{\theta}} q^F(\theta) dF(\theta).$$

More precisely, there exists a unique interior threshold type $\tilde{\theta}$ which produces at his first best level ($q^F(\tilde{\theta}) = q^{PI}(\tilde{\theta})$). Any θ-type farmer more efficient than this threshold type over-produces ($q^F(\theta) > q^{PI}(\theta)\ \forall\theta < \tilde{\theta}$) whereas any θ-type farmer less efficient than this threshold type under-produces ($q^F(\theta) < q^{PI}(\theta)\ \forall\theta > \tilde{\theta}$).

(iii) *A strictly positive rent given by $\pi^F(\theta) = \int_\theta^{\overline{\theta}} c(q^F(u))du$ is left to any type of farmer except the least efficient one ($\pi^F(\overline{\theta}) = 0$).*

Proof: Part (i): the proof is by contradiction. Suppose that $Q^F \geq Q^{PI}$ or equivalently that $\eta(Q^F) \leq \nu(Q^{PI})$. Then because $\theta + \frac{F(\theta)}{f(\theta)} \geq \theta$, it would be the case that $q^F(\theta) \leq q^{PI}(\theta)$ $\forall\theta \in \Theta$. But this leads to $Q^F < Q^{PI}$ which contradicts the assumption. Hence, $Q^F < Q^{PI}$ necessarily.

Part (ii): From part (i), $\eta(Q^F) > \nu(Q^{PI})$. Then, when $\theta = \underline{\theta}$, individual virtual marginal costs are identical ($\theta c'(q)$) which implies that $q^F(\underline{\theta}) > q^{PI}(\underline{\theta})$. This remains true by continuity in a neighborhood of $\underline{\theta}$. Now, when $\theta = \overline{\theta}$, assume that $\eta(Q^F) \geq \left(\overline{\theta} + \frac{1}{f(\overline{\theta})}\right) c'(q^{PI}(\overline{\theta}))$. Then, it must be true that $q^F(\overline{\theta}) > q^{PI}(\overline{\theta})$. This results in over-production everywhere and hence to $Q^F > Q^{PI}$, which is impossible. Hence, it is necessarily the case that $q^F(\overline{\theta}) < q^{PI}(\overline{\theta})$, which by continuity of the monotonically decreasing production schedule yields a unique interior type $\tilde{\theta}$ such that $q^F(\tilde{\theta}) = q^{PI}(\tilde{\theta})$. Finally, $\tilde{\theta}$ is given by $q^F(\tilde{\theta}) = q^{PI}(\tilde{\theta})$ which implies from (2) and (9) that

$$\frac{\eta(Q^F)}{\tilde{\theta} + \frac{F(\tilde{\theta})}{f(\tilde{\theta})}} = \frac{\nu(Q^{PI})}{\tilde{\theta}}.$$

Part (iii): this comes from previous result (see (8)). ■

Thus, asymmetric information entails an upward shift in the procurement cost function compared to complete information. Indeed, as indicated in the proof, it is sufficient to notice that any incentive compatible and cost-minimizing production schedule $q(.)$ that results in total production Q would cost $k+\int_{\underline{\theta}}^{\overline{\theta}} \left\{ \left[\theta + \frac{F(\theta)}{f(\theta)}\right] c(q(\theta) \right\} dF(\theta)$ under asymmetric information whereas the corresponding cost would be $k + \int_{\underline{\theta}}^{\overline{\theta}} \{\theta c(q(\theta)\} \, dF(\theta)$ under complete information. As a consequence, the optimal quantity Q^F under incomplete information is lower than its counterpart Q^{PI} under complete information.

More interestingly, while asymmetric information induces a lower production level from an aggregate point of view, it also results in a reallocation of production from inefficient farmers to efficient farmers. Compared to the perfect information outcome, the most efficient farmers over-produce while the less efficient farmers under-produce. Individual production under asymmetric information only coincides with the first-best level for an intermediate-type farmer. Hence, we obtain a property that can be coined as "*efficiency somewhere in the middle*" rather than the "efficiency at the top" found in most adverse selection problems.

The "efficiency somewhere in the middle" result comes from the non-linearity of the total revenue function $R(Q)$ and can be contrasted with the standard result from the adverse selection literature where it is usually assumed that the principal's objective is linear with respect to aggregate production or that the non-linear revenue of individual production can be aggregated in a additive way to form total revenue (see Maskin and Riley (1984) in the context of price discrimination by a monopolist or Mussa and Rosen (1978) in the context of discrimination through quality). In the context of this model, the imposition of linearity would amount to assuming that $R(Q)$ is linear in Q or that the total revenue can be written as $R = \int_{\underline{\theta}}^{\overline{\theta}} r(q(\theta))dF(\theta)$ where $r(q)$ is the revenue of individual production. With this formulation, the usual result of efficiency at the top and under-production elsewhere would be obtained.

4 Contracting with a cooperative

A processor organized as a cooperative has a different objective compared to the FPF but nevertheless faces the same informational contraints. Assume that the cooperative manager who designs the contracts has to take into account equity considerations. A simple way to introduce this notion is to consider that the manager's objective is to maximize an increasing, concave function $\mathcal{W}(.)$ of profit levels ($\pi(\theta)$) obtained by farmers. The absolute degree of inequality aversion is thus given by $\sigma(.) = -\mathcal{W}''(.)/\mathcal{W}'(.)$. When $\sigma(.) = 0$, then the cooperative is not averse to inequality and the manager simply maximizes the average level of individual profit. When the cooperative is infinitely averse to inequality then the manager maximizes the lowest level of rent among members (Rawls criterion). However, the manager is constrained to exactly balance the budget, i.e., the net revenue from production must cover the total transfer to farmers.

4.1 Complete information

Once again, consider first the situation of perfect information. The program to be solved can be written as follows:

$$\max_{q(.),t(.)} \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{W}(\pi(\theta))dF(\theta)$$

$$\text{s.t.} \quad \pi(\theta) = t(\theta) - \theta c(q(\theta)) - k \geq 0 \tag{IR}$$

$$\int_{\underline{\theta}}^{\overline{\theta}} [R(Q) - t(\theta)]\, dF(\theta) = 0 \tag{BC}$$

$$Q = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)dF(\theta) \tag{11}$$

Intuitively, under complete information or equivalently perfect price discrimination, the processor organized as a cooperative should perform exactly as a privately-owned firm from the production point of view. This result is stated in the following proposition.

Proposition 2 *Under complete information, the processor organized as a cooperative offers contracts that induce perfect information individual production levels $q^{PI}(\theta)$ for any θ-type farmer and hence perfect information total production Q^{PI}. Moreover, whenever the absolute degree of inequality aversion σ is strictly positive, the level of rent is constant for any type of* *farmer and equal to average production surplus:*

$$\pi(\theta) = \pi^* = \int_{\underline{\theta}}^{\overline{\theta}} \left[R(Q^{PI}) - \theta c(q^{PI}(\theta)) - k\right] dF(\theta) \geq 0, \, \forall\theta \in \Theta.$$

Proof: See appendix A. ■

Hence, the FPF and the cooperative are equally efficient from a production viewpoint, since in both situations farmers produce at the perfect information level. This result does not depend on the downstream market structure, i.e., whether the Coop or the FPF are price takers or price makers. The result also does not depend on the degree of inequality aversion in the cooperative. Thus, under complete information, there is no conflict between incentives and income redistribution issues inside the cooperative.

However, while the two organizations are equally efficient, the distributional impacts are different. The FPF extracts all the surplus in the relationship, while the cooperative's manager transfers all the surplus to the farmers.

In the special case where there is no aversion to inequality ($\sigma = 0$) then the optimal rent level for any farmer remains undetermined. Indeed, any rent schedule that meets the budget constraint requirement, that is such that

$$\int_{\underline{\theta}}^{\overline{\theta}} \pi(\theta) dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \left[R(Q^{PI}) - \theta c(q^{PI}(\theta)) - k\right] dF(\theta),$$

is optimal. This result is easily understood, since the cooperative's manager has no specific preferences with respect to the distribution of incomes as soon as the budget constraint is met.

4.2 Incomplete information

With asymmetric information, the problem to be solved can be written as follows:

$$\max_{q(.),t(.)} \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{W}(\pi(\theta))dF(\theta)$$

$$\text{s.t.} \quad \pi(\theta) = t(\theta) - \theta c(q(\theta)) - k \geq 0 \tag{IR}$$

$$\pi(\theta) \geq t(\tilde{\theta}) - \theta c(q(\tilde{\theta})) - k \quad \forall \theta, \tilde{\theta} \tag{IC}$$

$$\int_{\underline{\theta}}^{\overline{\theta}} [R(Q) - t(\theta)]\, dF(\theta) = 0 \tag{BC}$$

$$Q = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)dF(\theta) \tag{12}$$

Recall first that usual arguments show that (IC) can be replaced by:

$$\begin{aligned} \dot{\pi}(\theta) &= -c(q(\theta)) < 0 \\ \dot{q}(\theta) &\leq 0, \end{aligned}$$

which indicates that individual rationality constraints reduce to $\pi(\overline{\theta}) \geq 0$. In addition, neglect for the moment the monotonicity constraints $\dot{q}(\theta) \leq 0$ (it will be checked later). Then, eliminating the transfer t in the budget constraint gives:

$$\max_{q(.),\pi(.)} \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{W}(\pi(\theta))dF(\theta)$$

$$\text{s.t.} \quad \pi(\overline{\theta}) \geq 0 \tag{IR}$$

$$\dot{\pi}(\theta) = -c(q(\theta)) \tag{13}$$

$$\int_{\underline{\theta}}^{\overline{\theta}} [\pi(\theta) + \theta c(q(\theta)) + k]\, dF(\theta) = R(Q) \tag{BC}$$

$$Q = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)dF(\theta) \tag{14}$$

The Lagrangean corresponding to this problem is:

$$\begin{aligned} \mathcal{L} \;=\; & \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{W}(\pi(\theta))dF(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} \lambda(\theta)\left[-c(q(\theta)) - \dot{\pi}(\theta)\right] d\theta \\ & + \mu \int_{\underline{\theta}}^{\overline{\theta}} [R(Q) - \pi(\theta) - \theta c(q(\theta)) - k]\, dF(\theta) + \nu \left(\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)dF(\theta) - Q \right) \end{aligned} \tag{15}$$

where $\lambda(\theta)$ is the co-state variable associated with π, μ is the multiplier associated with the budget constraint and ν is the multiplier associated with (14).

Integrate by parts the term containing $\dot{\pi}(\theta)$:

$$\begin{aligned}\int_{\underline{\theta}}^{\overline{\theta}} \lambda(\theta)\dot{\pi}(\theta)d\theta &= [\lambda(\theta)\pi(\theta)]_{\underline{\theta}}^{\overline{\theta}} - \int_{\underline{\theta}}^{\overline{\theta}} \dot{\lambda}(\theta)\pi(\theta)d\theta \\ &= \lambda(\overline{\theta})\pi(\overline{\theta}) - \int_{\underline{\theta}}^{\overline{\theta}} \dot{\lambda}(\theta)\pi(\theta)d\theta\end{aligned}$$

since $\lambda(\underline{\theta}) = 0$ ($\pi(\underline{\theta})$ is free). Substituting this expression in the Lagrangean (15) gives:

$$\mathcal{L} = \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{H} d\theta - \lambda(\overline{\theta})\pi(\overline{\theta})$$

where

$$\begin{aligned}\mathcal{H} &= \mathcal{W}(\pi(\theta))f(\theta) - \lambda(\theta)c(q(\theta)) + \dot{\lambda}(\theta)\pi(\theta) \\ &\quad + \mu\left[R(Q) - \pi(\theta) - \theta c(q(\theta)) - k\right] f(\theta) + \nu\left(q(\theta)f(\theta) - Q\right)\end{aligned}$$

The necessary conditions for an optimum are as follows:

$$\frac{\partial \mathcal{H}}{\partial q} = -\lambda(\theta)c'(q(\theta)) - \mu\theta c'(q(\theta))f(\theta) + \nu f(\theta) = 0 \tag{16}$$

$$\frac{\partial \mathcal{H}}{\partial \pi} = \mathcal{W}'(\pi(\theta))f(\theta) + \dot{\lambda}(\theta) - \mu = 0 \tag{17}$$

$$\frac{\partial \mathcal{L}}{\partial Q} = \mu R'(Q) - \nu = 0 \tag{18}$$

$$\frac{\partial \mathcal{L}}{\partial \pi(\overline{\theta})} = -\lambda(\overline{\theta}) \leq 0 \text{ with } \lambda(\overline{\theta})\pi(\overline{\theta}) = 0,\ \pi(\overline{\theta}) \geq 0 \text{ (slackness condition).} \tag{19}$$

From (17), the optimal value of the co-state variable $\lambda(\theta)$ can be obtained by integration:

$$\lambda(\theta) = \mu F(\theta) - \int_{\underline{\theta}}^{\theta} \mathcal{W}'(\pi(u))dF(u) \tag{20}$$

recalling that $\lambda(\underline{\theta}) = 0$ since $\pi(\underline{\theta})$ is free.

Rearranging (16) gives:

$$\psi = \frac{\nu}{\mu} = \left[\theta + \frac{\lambda(\theta)}{\mu f(\theta)}\right] c'(q(\theta))$$

and from (18):

$$R'(Q) = \psi = \frac{\nu}{\mu},$$

This last expression indicates that the marginal revenue of total production should be equated with the marginal value ν of individual production weighted by the shadow cost μ of the budget constraint. Overall, using (20), the optimal production schedule $q^C(.)$ (where C stands for the Coop) satisfies the following rule:

$$R'(Q^C) = \left[\theta + \frac{\phi(\theta)}{f(\theta)}\right] c'(q^C(\theta))$$

where

$$\phi(\theta) = \frac{\lambda(\theta)}{\mu} = F(\theta) - \frac{\int_{\underline{\theta}}^{\theta} \mathcal{W}'(\pi^C(u))dF(u)}{\mu}.$$

This indicates that the marginal revenue of total production should be equated with the virtual marginal cost of individual production, where the latter is the sum of the true marginal cost ($\theta c'(q(\theta))$) and the extra marginal cost due to incentive compatibility ($\frac{\phi(\theta)}{f(\theta)} c'(q(\theta))$).

Finally, integrating $\dot{\pi}(\theta)$, we get

$$\pi^C(\theta) = \pi^C(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} c(q^C(u))du.$$

Replacing this expression in the budget constraint gives:

$$\pi^C(\overline{\theta}) = \int_{\underline{\theta}}^{\overline{\theta}} \left[R(Q^C) - \int_{\theta}^{\overline{\theta}} c(q^C(u))du - \theta c(q^C(\theta)) - k\right] dF(\theta) \tag{21}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \left[R(Q^C) - \left(\theta + \frac{F(\theta)}{f(\theta)}\right) c(q^C(\theta)) - k\right] dF(\theta) \tag{22}$$

using integration by parts.

Therefore, we have two cases. From (19), either the participation constraint at $\overline{\theta}$ is binding or it is not, that is $\pi^C(\overline{\theta}) > 0$ or $\pi^C(\overline{\theta}) = 0$.

4.3 When the participation constraint is not binding

Assume that we have $\pi^C(\overline{\theta}) > 0$ and hence it is given by (21). Then $\lambda(\overline{\theta}) = 0$ and consequently from (20), we have that $\mu = \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{W}'(\pi^C(u))dF(u) > 0$.

First, we note that when the cooperative has no aversion to inequality ($\mathcal{W}'(.) = 1$), then $\mu = 1$ and any type of farmer produces at the first best level because $\phi(\theta) = 0$ for any

θ. Hence, there is no conflict between redistribution of wealth inside the cooperative and incentives in the absence of inequality aversion. This results does not remain true when the cooperative is strictly averse to inequality.

Indeed, the following proposition states that aversion to inequality entails a downward or upward distortion on production levels depending on the type, except for two intermediates values in Θ where the first-best production level is obtained. In addition, we show that asymmetric information leads the inequality averse cooperative to reduce globally production. But for this to be possible, the total level of rents left to farmers must be compatible with the budget constraint.

Proposition 3 *Assume that the optimal level of production is monotonically decreasing in θ. If leaving a strictly positive rent to any type of farmer is compatible with the budget constraint, then*

(i) *the total production level Q^C is lower than the total production level Q^{PI} under complete information,*

(ii) *individual production levels $q^C(\theta)$ are upward or downward distorted depending on the type of farmers and are given by*

$$R'(Q^C) = \left[\theta + \frac{\phi(\theta)}{f(\theta)}\right] c'(q^C(\theta)) \qquad \text{with } Q^C = \int_{\underline{\theta}}^{\overline{\theta}} q^C(\theta) dF(\theta).$$

More precisely, there exists two interior threshold types θ_1 and θ_2 with $\theta_1 < \theta_2$, which produce at their first best level ($q^C(\theta_i) = q^{PI}(\theta_i)$, $\forall i = 1, 2$). Any θ-type farmer more efficient than θ_1 or less efficient than θ_2 over-produces ($q^C(\theta) > q^{PI}(\theta)$ $\forall \theta < \theta_1$ or $\theta > \theta_2$) whereas any θ-type farmer less efficient than θ_1 but more efficient than θ_2 under-produces ($q^C(\theta) < q^{PI}(\theta)$ $\forall \theta_1 > \theta > \theta_2$).

(iii) *A strictly positive rent given by $\pi^C(\theta) = \pi^C(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} c(q^C(u))du$ is left to any type of farmer even for the least efficient one:*

$$\pi^C(\overline{\theta}) = \int_{\underline{\theta}}^{\overline{\theta}} \left[R(Q^C) - \left(\theta + \frac{F(\theta)}{f(\theta)}\right) c(q^C(\theta)) - k\right] dF(\theta) > 0.$$

Proof: Part (i): A preliminary result is needed to prove this. Let us first show that $\phi(\theta) = F(\theta) - \frac{\int_{\underline{\theta}}^{\theta} \mathcal{W}'(\pi^C(u))dF(u)}{\mu} \geq 0$ that is $\delta(\theta) \equiv \mu F(\theta) - \int_{\underline{\theta}}^{\theta} \mathcal{W}'(\pi^C(u))dF(u) \geq 0$. Indeed, $\delta(\underline{\theta}) = \delta(\overline{\theta}) = 0$. Moreover, $\delta'(\theta) = \left[\mu - \mathcal{W}'(\pi^C(\theta))\right] f(\theta)$. Note that $\mathcal{W}'(\pi^C(\theta))$ is strictly positive and also increasing in θ because $\mathcal{W}(.)$ is concave and $\pi^C(.)$ decreasing. Hence, we have three possible cases. First notice that $\mathcal{W}'(\pi^C(\overline{\theta}))$ cannot be lower than μ since otherwise $\delta(\theta)$ would be strictly increasing everywhere which is impossible. Second, $\mathcal{W}'(\pi^C(\underline{\theta}))$ cannot be higher than μ, otherwise $\delta(\theta)$ would be strictly decreasing everywhere which is also impossible. As a consequence, $\mathcal{W}'(\pi^C(\theta))$ necessarily intersects once the horizontal line μ and $\delta(\theta)$ is first increasing then decreasing. Overall, $\delta(\theta)$ is positive and consequently $\phi(\theta) \geq 0$.

Then the proof of part (i) is by contradiction. Suppose that $Q^C \geq Q^{PI}$ or equivalently that $R'(Q^C) \leq R'(Q^{PI})$. Then because $\theta + \frac{\phi(\theta)}{f(\theta)} \geq \theta$, it would be the case that $q^C(\theta) \leq q^{PI}(\theta)$ $\forall \theta \in \Theta$. But this would lead to $Q^C < Q^{PI}$ which contradicts the assumption. Hence, $Q^C < Q^{PI}$ necessarily.

Part (ii): From part (i), $R'(Q^C) > R'(Q^{PI})$. Then, when $\theta = \underline{\theta}$ or $\theta = \overline{\theta}$, individual virtual marginal costs are identical ($\theta c'(q)$) because $\phi(\underline{\theta}) = \phi(\overline{\theta}) = 0$. This implies that $q^C(\underline{\theta}) > q^{PI}(\underline{\theta})$ and $q^C(\overline{\theta}) > q^{PI}(\overline{\theta})$. Hence, by continuity of the monotonically decreasing production schedule, it must be true that the curve $q^C(\theta)$ intersects twice the curve $q^{PI}(\theta)$.

Part (iii): this comes from previous result. ■

Here again, asymmetric information entails an upward shift in the procurement cost function compared to complete information but only when the cooperative is averse to inequality. Consequently, the optimal quantity Q^C under incomplete information and aversion to inequality is lower than its counterpart Q^{PI} under complete information. We thus obtain a situation similar to the one when farmers contract with a FPF but for a different reason. Moreover, the conflict between redistribution and incentives yields to reallocate production from intermediately efficient farmers towards more efficient *and* less efficients farmers. Indeed, the most efficient farmers and the less efficient farmers overproduce while the other ones underproduce. We thus obtain a property that can be coined as "*efficiency somewhere* *twice in the middle*" which comes once again from the non linearity of $R(.)$.[8]

4.4 When the participation constraint is binding

Here, we have $\pi^C(\overline{\theta}) = 0$ which implies that $\lambda(\overline{\theta}) = \mu - \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{W}'(\pi^C(u))dF(u) > 0$. Consequently, we still have $\mu > 0$ and

$$\pi^C(\theta) = \int_{\theta}^{\overline{\theta}} c(q^C(u))du.$$

Hence, the budget constraint reduces to

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[R(Q^C) - \left(\theta + \frac{F(\theta)}{f(\theta)} \right) c(q^C(\theta)) - k \right] dF(\theta) = 0.$$

Moreover, we still have

$$R'(Q^C) = \left[\theta + \frac{\phi(\theta)}{f(\theta)} \right] c'(q^C(\theta)).$$

Proposition 4 *Assume that the optimal level of production is monotonically decreasing in θ. When the participation constraint is binding at $\overline{\theta}$, then*

(i) *the total production level Q^C is lower than the total production level Q^{PI} under complete information,*

(ii) *individual production levels $q^C(\theta)$ are upward or downward distorted depending on the type of farmers and are given by*

$$R'(Q^C) = \left[\theta + \frac{\phi(\theta)}{f(\theta)} \right] c'(q^C(\theta)) \qquad \text{with } Q^C = \int_{\underline{\theta}}^{\overline{\theta}} q^C(\theta)dF(\theta).$$

More precisely, there exists an upward distorsion for the most efficient farmer who overproduces compared to the first best.

Proof: Part (i): identical to part (i) in proposition 3.

Part (ii): From part (i), $R'(Q^C) > R'(Q^{PI})$. Then, when $\theta = \underline{\theta}$, individual virtual marginal costs are identical ($\theta c'(q)$) because $\phi(\underline{\theta}) = 0$. This implies that $q^C(\underline{\theta}) > q^{PI}(\underline{\theta})$.

[8] Note that under linearity of $R(.)$, we would have obtained first best levels of production for the upper and the lower bound of Θ and underproduction elsewhere.

Hence, by continuity of the monotonically decreasing production schedule, it must be true that the curve $q^C(\theta)$ intersects at least once the curve $q^{PI}(\theta)$. ■

Note that in the special case of no inequality aversion, we obtain that $\lambda(\overline{\theta}) = \mu - 1 > 0$. Hence, we have

$$R'(Q^C) = \left[\theta + \frac{F(\theta)}{f(\theta)}\frac{\mu - 1}{\mu}\right] c'(q^C(\theta)).$$

This indicates that when the participation constraint is binding, then even if there is no inequality aversion, the cooperative has to distort production schedules. This constrasts with the preceding case where leaving a strictly positive rent to any type of farmer is compatible with the budget constraint.

4.5 Comparison with the FPF

The following proposition states an important result which indicates that the cooperative is always more efficient than the FPF.

Proposition 5 *Assume that the optimal level of production is monotonically decreasing in θ. Whether the participation constraint of the least efficient farmer is binding or not, the processor organized as a cooperative is always more efficient than the For-Profit Firm from the production viewpoint ($Q^{PI} \geq Q^C \geq Q^F$).*

Proof: The difference between the incentive distortion when the processor is an FPF and the one when the processor is a cooperative can be written as follows:

$$\frac{F(\theta)}{f(\theta)} - \frac{F(\theta)}{f(\theta)}\left[1 - \frac{\int_{\underline{\theta}}^{\theta} \mathcal{W}'(\pi^C(u))dF(u)}{\mu F(\theta)}\right] = \frac{\int_{\underline{\theta}}^{\theta} \mathcal{W}'(\pi^C(u))dF(u)}{\mu f(\theta)} > 0.$$

This indicates that the marginal individual production cost is everywhere higher when the processor is a for-profit firm compared to the cooperative. Hence, it must be that the total quantity sold by a FPF is lower than the total quantity sold by a cooperative. ■

Our analysis has shown that both FPF and the cooperative distort production schedules in case of asymmetric information but for different reasons. While the FPF faces the classic rent extraction-efficiency trade-off, the inequality-averse cooperative faces an equity-efficiency trade-off. However, the aggregate distortion due to inequality aversion is always less important than the aggregate distortion implied by rent extraction.

Nevertheless, this does not mean that any type of farmer would produce more when contracting with a cooperative than when contracting with a FPF. Indeed, for the most efficient farmer ($\underline{\theta}$), it is easily seen that because the marginal cost is simply $\theta c'(q)$ in any case, then we have

$$q^F(\underline{\theta}) > q^C(\underline{\theta}) > q^{PI}(\underline{\theta})$$

and by continuity of production schedules, this remains true in a neighboorhood of $\underline{\theta}$. As a conclusion, a FPF would offer the most efficient farmers to produce more compared to a cooperative.

4.6 Monotonicity of production schedule

Finally we have to check whether the proposed production schedule are monotonically decreasing in θ in order to fulfill the second order conditions. [To be completed]

5 Conclusion

[To be completed]

Appendix

A Proof of Proposition 2

Once gain, we proceed in two steps. First, having eliminated the transfer t by replacing it by $\pi + \theta c(q) + k$, we look for the optimal production schedule $q(.)$ and optimal utility schedule $\pi(.)$ that maximizes the objective of the cooperative for a given total production level Q:

$$\max_{q(.),t(.)} \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{W}(\pi(\theta))dF(\theta)$$

$$\text{s.t.} \quad \pi(\theta) \geq 0 \tag{IR}$$

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[R(Q) - \pi(\theta) - \theta c(q(\theta)) - k\right] dF(\theta) = 0 \tag{BC}$$

$$Q = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)dF(\theta) \tag{23}$$

Denote μ the multiplier of the budget constraint (BC) and ν the multiplier of constraint (23). Omitting the (IR) constraints for the moment, the Lagrangean writes as follows:

$$\mathcal{L} = \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{W}(\pi(\theta))dF(\theta) + \mu\left(\int_{\underline{\theta}}^{\overline{\theta}} \left[R(Q) - \pi(\theta) - \theta c(q(\theta)) - k\right] dF(\theta)\right) + \nu\left(\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)dF(\theta) - Q\right)$$

Maximizing over the utility $\pi(\theta)$ yields to:

$$\left[\mathcal{W}'(\pi(\theta)) - \mu\right] f(\theta) = 0$$

which implies that the optimal level of utility is constant, which we denote π^* and hence we get $\mu = \mathcal{W}'(\pi^*)$. Now maximizing over $q()$ yields to:

$$\theta c'(q(\theta)) = \delta \tag{24}$$

where $\delta = \frac{\nu}{\mu}$. Notice that the marginal price of individual production is now weighted by the shadow cost of the budget constraint. This equation implicitely gives us a production schedule $q(\theta, \delta)$. Now, the link between δ and Q is established through constraint (23):

$$\int_{\underline{\theta}}^{\overline{\theta}} q(\theta, \delta)dF(\theta) = Q \Rightarrow \delta \equiv \delta(Q). \tag{25}$$

Finally, from the budget constraint, we obtain that π^*, which depends on Q, corresponds to the average surplus of production:

$$\pi^*(Q) = \int_{\underline{\theta}}^{\overline{\theta}} \left[R(Q) - \theta c(q(\theta, \delta(Q))) - k\right] dF(\theta).$$

It remains to maximize the objective of the cooperative with respect to total production Q:

$$\max_{Q} \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{W}(\pi^*(Q)) dF(\theta) = \mathcal{W}(\pi^*(Q))$$

which yields to the following first-order condition,

$$\pi^{*\prime}(Q) = 0$$

that is,

$$\begin{aligned} R'(Q) &= \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta c'(q(\theta, \delta(Q))) \frac{\partial q}{\partial \delta} \delta'(Q)\right] dF(\theta) \\ &= \delta(Q) \\ &= \theta c'(q(\theta)) \;\; \forall \theta. \end{aligned}$$

by using (24) and differentiating (25) w.r.t. Q. This indicates that at the optimum, farmers produce at the individual first-best level given by $R'(Q^{PI}) = \theta c'(q^{PI}(\theta))$ where $Q^{PI} = \int_{\underline{\theta}}^{\overline{\theta}} q^{PI}(\theta) dF(\theta)$. They also receive the lump sum transfer $\pi^* = \int_{\underline{\theta}}^{\overline{\theta}} \left[R(Q^{PI}) - \theta q^{PI}(\theta) - k\right] dF(\theta) \geq 0$ which satisfies the neglected individual rationality constraints.