Risk Aversion, Uncertainty Aversion, and Variation Aversion in Applied Commodity Price Analysis

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Practitioner's Abstract: *Standard models of hedging behavior assume that either hedgers wish to minimize net price variation or they wish to balance variation versus profits. These models treat variation as risk and fail to distinguish between variation that is random and variation that is not random over time. Newer models of decision making differentiate between random and nonrandom variation somewhat, but they inadequately distinguish variation from risk. This paper reviews the distinctions among variation, uncertainty, and risk and calculates optimal hedge ratios for two models addressing the distinction. Empirical optimal hedge ratios typically decline toward zero when variation aversion is included in the models. These results may help explain why hedgers commonly hedge less than recommended by the standard models.*

Keywords: Generalized expected utility, Hedging, Recursive utility, Risk Aversion

Introduction

Not every source of variation involves uncertainty, and not every source of uncertainty involves risk. The distinctions among these three concepts have often been drawn, but their implications for risk management and price analysis have seldom been clear. This paper reviews the distinctions and explains their relevance to commodity price hedging models.

One way to account for the distinctions is known as recursive utility, which parameterizes aversion to temporal variation as distinct from risk and uncertainty. Recursive utility was developed by Epstein and Zin (1989, 1991) and Weil (1990) based on the work of Kreps and Porteus (1978, 1979). It has been used in agricultural contexts such as resource management (Knapp and Olson 1996) and farm finance (Lence 2000) and is also known as generalized expected utility. Its usefulness relies upon knowledge of the elasticity of intertemporal substitution between payments separated by time. Unfortunately, recursive utility inadequately distinguishes between risk and uncertainty. This paper explains the flaws in recursive utility and proposes an alternative.

After a thorough discussion of the issues involved, the paper turns to an empirical application to hedging. Expected utility, recursive utility, and alternative objective functions are considered using a simple discrete-time framework with a short horizon. Empirical examples demonstrate the differences among the approaches and highlight the distinctions among risk aversion, uncertainty aversion, and variation aversion with an eye toward practical applications.

This work is important because the distinctions among risk, variation, and uncertainty are so very important to the agricultural sector. Risk management education, in particular, must address these distinctions. The particular mathematical form of the objective function may not be of long-standing interest to industry participants, but the insights garnered from an intensive dissection of temporal decision making can be critically important for making good decision in the future. Therefore, the presentation of this work will focus on the distinctions among risk, variation, and uncertainty and on practical conclusions drawn from the empirical analysis.

Literature Review

Static choice under uncertainty is typically modeled using a the von Neumann-Morgenstern (1944) expected utility function or some variant (see Tuthill and Frechette (2002) for examples). A single coefficient of risk aversion often measures attitudes toward risk, which is measured by variance at a point in time. Intertemporal choice typically includes a discount factor to measure time preferences.

Intertemporal choice under uncertainty, however, is more complicated since decision makers consider not only risk and time preferences, but also the timing of events. Kreps and Porteus (1978, 1979) developed the foundation for representing the individual's utility when timing matters. Their preference functional is defined recursively by $U_t = V[y_t, E_t U_{t+1}]$, where V(.) is an aggregator function and y_t a control vector. Kreps and Porteus showed that the individual prefers the uncertainty to be resolved earlier rather than later (as most people do) if this utility function is convex in the second argument.

The aggregator function need not be linear, which means that U_t is not separable in time. Payoffs in different times are treated as if they were separate goods. Orange juice and housing are not additive, so why should income (or wealth) today be additive with income (or wealth) next June? Surely after discounting the two are substitutes, but cash flow budgeting requires a steady stream of income. It becomes more and more difficult to manage one's money as variation in payments over time becomes severe. Even when payments are deterministic and known in advance, there is an incentive to even-out the payments as much as possible and eliminate the variation. Therefore, substitution between time periods is imperfect.

Substitutability between periods is quantified using the elasticity of intertemporal substitution. The elasticity measures the straightness (lack of curvature) of the indifference curve representing the tradeoff between income in consecutive periods. It is the same concept as the elasticity of substitution from consumer theory and the elasticity of technical substitution in producer theory.

The conventional time-additive expected utility function implies the restriction that the elasticity of intertemporal substitution is equal to the reciprocal of the coefficient of relative risk aversion. That is, if agents are highly risk-averse, they must have low elasticity of intertemporal substitution. Intertemporal variation and static uncertainty are erroneously equated.

There is little evidence in favor of such a premise. Hall (1988) estimated a representative consumer's utility function, concentrating on the elasticity of intertemporal substitution. He determined empirically that its value can be very close to zero. His result implies that relative risk aversion can be nearly infinite!

It was clear from Hall's work that a new functional form for utility must be designed for modeling time preferences and uncertainty separately. Weil (1989, 1990) and Epstein and Zin (1989, 1991) developed an isoelastic utility function to fit the need. Their utility function is a non-expected utility function with a constant coefficient of relative risk aversion, and a constant but (seemingly) unrelated elasticity of intertemporal substitution. It is represented by

(1) $$U_t = [(1-\beta)y_t^{\rho} + \beta(E_t \tilde{U}_{t+1}^{\alpha})^{\rho/\alpha}]^{1/\rho}.$$

$\tilde{U}_{t+1}$ reflects random future utility and E_t is the conditional expectation given the information available to the agent at t. The parameter $\beta = 1 / (1+\delta)$ where δ is the rate of time preference, and ρ is equal to one minus the reciprocal of the elasticity of intertemporal substitution. Attitudes toward risk are modeled by the parameter α, which equals one minus the coefficient of relative risk aversion. Equation (1) is a specific example of a recursive utility function and has come to dominate the literature on the topic and to represent the entire class of recursive utility functions.

Epstein and Zin (1991) derived first-order conditions (Euler Equations) from (1) that can be written in terms of observable variables and estimate them using the Generalized Method of Moments (GMM). They use monthly U.S. data from 1959-86, which includes 4 different measures of consumption per capita, and returns for stocks and bonds. The empirical results show that the standard multi-period expected utility function is rejected; the elasticity of intertemporal substitution is less than 1, and the coefficient of risk aversion and around 1 for their data set.

Recursive utility has also been used in agricultural contexts, such as resource management and farm finance. Knapp and Olson (1996) used it to study rangeland and groundwater management under uncertainty. The optimal decision rule will be "rotated" under imperfect elasticity of intertemporal substitution, and this rotation of optimal decision rules smoothes the evolution of state and control variables over time. Lence (2000) applies recursive utility to U.S. aggregate annual farm data. His results show that the empirical performance of the recursive utility model is better than that of the expected utility model. He estimates the rate of time preference between 2.9% and 5.1%, and rejects the hypothesis that the elasticity of intertemporal substitution is less than one.

Barry, Robison, and Nartea (1996) relax time separability by allowing more general time patterns and developing explicit measures of changes in time attitudes. They introduce the concept of constant, increasing, or decreasing absolute time aversion, which is analogous to Arrow-Pratt risk attitudes. They address the same sort of issues that Kreps and Porteus (1978, 1979) discussed, but they do not use recursive utility.

Recursive is also widely used other areas, especially in solving consumption/ portfolio choice and asset-pricing problems. Kandel and Stambaugh (1991) set different value of risk aversion and intertemporal substitution, and determine the separate roles for these parameters in determining the mean and volatility of equity returns in an equilibrium risk pricing model. Increasing risk aversion raises the equity premium, while equity volatility decreases in the level of intertemporal substitution. Hung (1994) also uses the recursive utility model to determine the influence of preference parameters on the equity premium and risk-free rate. He claims that the equity premium puzzle can be resolved if non-expected utility is combined with asymmetric market fundamentals.

Campbell and Viceira (2001) bring up an interesting question about who should buy long-term bonds. They develop a model in which an infinite-lived individual with non-expected utility must choose consumption and portfolio weights in each period. The demand for long-term bonds is decomposed into "myopic" and "hedging" demand, and they conclude that when the level of risk averse increases, myopic demand decreases to zero and the demand for bonds is entirely for hedging purposes. They also suggest that inflation-indexed bonds are suitable for long-term conservative investors who seek a stable consumption path.

Koskievic (1999) uses this model to estimate the parameters of a consumption-leisure dynamic choice model using GMM. The empirical results indicate that the coefficient of relative risk aversion is very low (0.098) and the elasticity of intertemporal substitution is 3.17, which is significantly larger than Hall's (1988) estimate.

Weil (1989, 1990) and Epstein and Zin (1991) have successfully developed a recursive, isoelastic utility function based on Kreps and Porteus's (1978, 1979) insights that can be implemented empirically. The utility function is useful to analyze problems involving intertemporal choice under uncertainty. The advantage of this line of empirical work is that it distinguishes the elasticity of intertemporal substitution from the coefficient of relative risk aversion.

Still, there are some issues left unresolved. This line of research has done an excellent job of developing the elasticity of intertemporal substitution and identifying it as a variation-aversion parameter. It has not addressed the role of the so-called risk aversion parameter and its suitability to measure risk aversion over time. The parameter α has been described as one minus the coefficient of relative risk aversion, but the concept of risk across time periods has not been adequately defined. The next section starts from the simplest specification of an objective function and builds up an argument methodically to show that the literature has misinterpreted the role of α and that an alternative utility function is required to model intertemporal risk aversion properly.

Discussion

There are many different ways to specify objective functions for decision-making over time. A simple one is

$$(2) \qquad U_t = E_t \sum_{s=0}^{T} y_{t+s} .$$

Equation (2) restricts the objective function to have perfect dollar-for-dollar intertemporal substitution. The decision maker with this objective function is indifferent between income now and income later. A more reasonable specification would be

$$(3) \qquad U_t = E_t \sum_{s=0}^{T} (y_{t+s}\beta^s) .$$

Equation (3) is the Net Present Value rule with a constant discount rate, $\beta = (1+r)^{-1}$. Equation (3) improves over (2) because intertemporal substitution is no longer dollar-for-dollar. Income now is valued more highly than income later. However, the decision maker is now indifferent between income now and discounted income later. Some periods he may expect very high income, and some other periods he may expect very low income, but there is no way in (3) to account for his preferences against such intertemporal variation in y_t.

We can use a concave function V(.) to add this feature:

$$(4) \qquad U_t = E_t \sum_{s=0}^{T} [V(y_{t+s})\beta^s] \quad .$$

If V(.) is concave, then the decision maker is averse to variation in y_t over time and to uncertainty within a single period of time. He would prefer two average income years to one high income year and one low income year. He would also prefer an average income year to an equal chance of a high income year and a low income year. Equation (4) is called discounted expected utility because V(.) is like a von Neumann-Morgenstern expected utility function. The objective function can be rewritten to emphasize that it is the sum of discounted expected utilities into the future:

$$(5) \qquad U_t = \sum_{s=0}^{T} \{[E_t V(y_{t+s})]\beta^s\} .$$

If V(.) is concave, then the decision maker is averse to uncertainty in y_t at each time t. Unlike in (3), he would prefer to know the outcomes for certain than to wait for the uncertainty to be resolved.

The problem with (4) and (5) is that the function V(.) serves two roles – it embodies aversion to variation through time and to uncertainty at each time period. It seems unlikely that a decision maker will feel equally averse to both phenomena. Note also that V(.) is often said to embody risk aversion, but risk is harder to define in an intertemporal context. It is a different concept than variation aversion and uncertainty aversion.

One could easily imagine a decision maker facing known temporal variation and no uncertainty. He would face no risk but would require a concave V(.) function. The decision maker may face no real risk and yet V(.) may rightly be concave to capture his preferences toward temporal variation. One could also imagine a decision maker facing uncertainty at each time period without any uncertainty on the whole, over the longer planning horizon. He may feel that he faces no risk at all because losses in one period are matched by gains in another. He would face no risk, yet a concave V(.) function may be appropriate if he is averse to period-by-period uncertainty. The three phenomena – *variation aversion*, period-by-period *uncertainty aversion*, and whole-horizon *risk aversion* – are different conceptually, and therefore (4) and (5) are oversimplifications that may fail to capture decision maker behavior in many situations.

Many authors have tried to capture these effects using an objective function based on "recursive utility," sometimes also known as "generalized expected utility." The terminology sometimes used for the objective function is a "dynamic utility aggregator functional" or some similar string of words. The recursive utility objective function based on a static power utility (CRRA) function can be written as

$$(6) \qquad U_t = [y_t^{\rho} + \beta(E_t U_{t+1}^{\alpha})^{\rho/\alpha}]^{1/\rho} \quad .$$

Utility is defined recursively with U_t depending on the expectation of future U_{t+1}. The expression can be written more simply as

$$(7) \qquad U_t = [y_t^{\rho} + \beta\chi(U_{t+1})^{\rho}]^{1/\rho},$$

where $\chi(U_{t+1})$ represents the certainty equivalent of future income. For the power utility function, $V(y_t) = y_t^{\alpha}$.

Equation (7) makes it clear that recursive utility does not depend on the power (CRRA) utility specification. Utility can be specified as a negative exponential (CARA) or in any other desired form. The discussion will continue using power utility because the literature has focused there exclusively to date, but the empirical application will use the negative exponential utility function to make the results more comparable to previous work on hedging, such as Frechette (2000, 2001).

From equation (7) it is clear that ρ measures the extent to which the decision maker abhors intertemporal variation, after discounting is applied. $(1-\rho)^{-1}$ is called the elasticity of intertemporal substitution. If $\rho = 1$, then intertemporal substitution is infinite, or perfect. If there is no uncertainty in this case, then the objective function reduces to equation (3). However, if $\rho = 1$ and there is uncertainty, then recursive utility does not reduce to equations (4) or (5). To see why more clearly, consider the two-period case.

In the two-period case, equation (6) becomes

$$(8) \qquad U_t = \{y_1^{\rho} + \beta[E_1[(y_2^{\rho\alpha})^{1/\rho}]]^{\rho/\alpha}\}^{1/\rho} = \{y_1^{\rho} + \beta[E_1(y_2^{\alpha})]^{\rho/\alpha}\}^{1/\rho}.$$

If y_2 were known with certainty, then

$$(9) \qquad U_t = (y_1^{\rho} + \beta y_2^{\rho})^{1/\rho}.$$

If $\rho = 1$, then

$$(10) \qquad U_t = y_1 + \beta y_2,$$

which is the same as equation (3). On the other hand, if $\rho = 1$ and y_2 is uncertain, then

(11) $$U_t = y_1 + \beta(E_1 y_2^{\alpha})^{1/\alpha} = y_1 + \beta\chi(y_2),$$

the sum of discounted certainty equivalents for the power utility function. A quick inspection reveals that (11) is not the same as (4) or (5). (11) is the sum of discounted certainty equivalents, while (4) and (5) are the sum of discounted expected utilities.

Recursive utility, in general, is the sum of discounted certainty equivalents with imperfect intertemporal substitution. There are three parameters – β, α, and ρ – which correspond to the time discount rate, the period-by-period uncertainty aversion, and the aversion to intertemporal variation. There are several other ways to write an intertemporal objective function with three parameters, and recursive utility is just the one most favored to-date.

Another related objective function is

(12) $$U_t = \{E_t[(\sum_{s=0}^{T} y_{t+s}^{\rho}\beta^s)^{\alpha/\rho}]\}^{1/\alpha}.$$

This function represents the certainty equivalent of the sum of the discounted values, with imperfect intertemporal substitution between values. It also accounts for three different kinds of behavior, but the difference is somewhat subtle. In (12), the discounted values are each raised to the power ρ and discounted, then summed. The sum is raised to the power $1/\rho$ to capture intertemporal substitution effects. This new value is uncertain, so it is raised to the power α and the expectation operator is applied. Finally the expectation is raised to the power $1/\alpha$ to capture whole-horizon risk aversion.

I make the distinction here between period-by-period uncertainty aversion and whole-horizon risk aversion because the treatment of risk and uncertainty is the distinguishing difference between the two objective functions. It can be shown that the two functions are the same if $\rho = \alpha$ or if there is no uncertainty, but optimization of the different functions will otherwise yield different results.

To make the distinction clearer, consider a situation where the decision maker faces a set of uncertain payoffs, but he is guaranteed a known net present value – he is just not sure when he will be paid. Recursive utility and the alternative (12) both capture the decision maker's variation-aversion through the parameter ρ, so focus on what happens when $\rho = 1$ (perfect intertemporal substitution). If there is no real risk involved over the whole planning horizon, then the decision maker is indifferent toward the timing of payments. (12) captures this aspect of the decision maker's behavior, but recursive utility does not. In recursive utility, the uncertainty on a period-by-period drags down the value of the objective function. Recursive utility is flawed in this way because it treats each period individually. Total recursive utility will be lower if the timing is unknown, but the total alternative utility from (12) will not be.

The core reason that recursive utility is flawed is that it does not recognize that future uncertain outcomes may be correlated. A decision maker may be indifferent among sets of

uncertain outcomes that all yield the same net present value, but he may still behave in a risk averse manner when net present value over the whole horizon is uncertain. These effects are not captured adequately in recursive utility because the parameter α is applied on a period-by-period basis and not over total net present value. In recursive utility, the parameter α drags along some of the decision maker's variation aversion that ought to be captured entirely by ρ.

The alternative utility function (12) does not suffer from this flaw. It handles variation aversion and whole-horizon risk aversion separately by applying the parameter α to net present value over the whole planning horizon. The two behaviors are conceptually distinct and ought properly to be distinguished in the objective function.

It is possible, then, to develop the following taxonomy: (i) *Variation aversion* is the desire to equalize the outcomes across time periods, for a given net present value. (ii) *Risk aversion* is the preference for the distribution of stochastic net present value to be as narrow as possible over to the whole planning horizon. (iii) *Uncertainty aversion* is the desire to know the timing of outcomes on a period-by-period basis in advance, holding variation and the distribution of net present value constant. Recursive utility addresses (i) and (iii), and the alternative specification (11) addresses (i) and (ii).

When these definitions are considered closely, (iii) is seen to be a preference for information over uncertainty. It is an important motivation that induces decision makers to demand better forecasts of market conditions so that they can "time" the markets. It measures the desire for decision makers to find ways to ride the market as it booms and bail out before it crashes.

Undoubtedly uncertainty aversion is important, but it is not risk aversion. It does not induce decision makers to hedge, and it does not make farmers buy crop insurance. Therefore, recursive utility may be appropriate for models of speculative behavior, but it is unlikely to make a substantial contribution to the risk management literature. The alternative specification (12) is the proper one to be used in studies of risk management, so next we shall turn to an empirical examination of its performance as a tool for managing risk in agricultural commodity markets.

Empirical Performance

Data

The data set is the same one used by Frechette (2000, 2001) and consists of (i) weekly corn cash prices collected by the Pennsylvania Department of Agriculture (PDA); and (ii) the nearby corn futures price in Chicago. Local cash prices were collected through surveys and phone calls for five regions: Southeastern, Central, South Central, Western, and the Lehigh Valley. Only the Southeastern region was used in this analysis. The prices were collected and reported by PDA on Monday mornings before the market opened and the futures price that corresponds most closely is the previous Friday's settlement price for the nearby futures contract. If the Chicago Board of Trade was closed due to a holiday, then the closest day was used, matching the information sets as closely as possible in each case. All prices are reported in cents per bushel, for the years 1997-1998.

Procedures

The example hedger is a livestock farmer purchasing corn for feed, which results in an input cost hedge. The quantity of corn to be hedged is treated as predetermined by the number of animals in the herd/flock/etc. The ratio of corn to other ingredients in the feed are assumed to be fixed and do not vary with market conditions. These assumptions eliminate the need for modeling any additional sources of uncertainty.

Estimates of basis risk and expected basis depend on the structural forecasting model chosen by the hedger. There are many such models in use, such as naïve expectations, adaptive expectations, and rational expectations. The results depend on the model chosen, and yet there is no clear consensus in the literature to guide this choice. Fortunately, the results often are robust to any reasonable choice of forecasting method. Moschini and Hennesy (2000) consider this issue and conclude that a constant covariance matrix "may not be a bad approximation" and that "conditional variance does not do much better than unconditional variance" for use in estimating producers' responses to price risk. Each hedger has a unique perception of market structure, and no single model has come to dominate the literature.

To proceed, an adaptive expectations model is selected, as in Frechette (2000). To illustrate, note that adaptive expectations models can be written in autoregressive form as

(13) $E_t p_{t+1} = \alpha_0 + \alpha_1 p_t + \alpha_2 p_{t-1} + \ldots .$

In practice, (13) is truncated at a lag length sufficient to balance accuracy against degrees of freedom, and an error term is appended. If the error term satisfies standard assumptions, then Ordinary Least Squares can be used to get estimates of the α_i, which generate corresponding estimates of $E_t p_{t+1}$. The lag length is chosen by maximizing the Adjusted R-squared statistic and testing the standard OLS assumptions. The conditional covariance matrix is estimated by substituting expected local price minus expected futures price for expected basis. The conditional covariance matrix is assumed to be constant and to represent a bivariate normal distribution. These statistics represent actual results for the sample period, and therefore the results represent optimal *ex post* behavior in the sense that hedgers are assumed to have known the covariance matrix before the sample period began. Individual hedgers' expectations will depend on the sample period and available information.

A range of coefficients of absolute risk aversion was selected to span a range of possible farmer risk preferences, as in Frechette (2000, 2001). Reasonable values to span a range of risk preferences were chosen to be 2.00 for high risk aversion, 0.20 for moderate risk aversion, and 0.02 for low risk aversion. The elasticity of intertemporal risk aversion was allowed to vary indirectly by using a range of values for ρ from 0.1 through 1.0, which results in a range of elasticities from 1.11 through infinity (perfect substitution).

The hedging model is the same as found in Frechette (2000). The hedger faces futures price risk and basis risk and must pay a marginal transaction cost for hedging. He must balance the benefits or efficacy of hedging with the costs by maximizing his utility. In Frechette (2000)

each level of γ (CARA) yielded an optimal hedge ratio for each possible level of marginal transaction costs; however, simplistic intertemporal aggregation restricted the hedger's preferences toward intertemporal substitution to be infinitely elastic.

In the results to follow, this restriction is relaxed. Each combination of γ and ρ yields an optimal hedge ratio for each level of marginal transaction costs and each discount rate. Negative exponential utility is embedded within each specification, and the optimal hedge ratios under recursive utility are compared to those under the alternative utility function. Specifically, the objective functions are

(14) Recursive: $$\left(-\frac{b}{g}\log\{-E_t[-\exp(-gm_{t+1})]\}^r - (t \mid h \mid)^r \right)^{1/r}$$

and

(15) Alternative: $$-\log[-E_t\left(-\exp\{-g[b(m_{t+1})^r - (t \mid h \mid)^r]^{1/r}\}\right)]/g \ ,$$

where $m_{t+1} = -p_{t+1} + h(f_{t+1} - f_t)$ is the net gain in the spot and futures markets from procuring corn and hedging a fraction h of the amount to be procured. The other notation in (14) - (15) is defined as follows

β	discount factor, e.g. 1.000 or 0.985
h	hedge ratio
τ	marginal transaction cost of hedging
E_t	mathematical expectations operator
ρ	intertemporal substitution parameter
γ	coefficient of absolute risk aversion
|.|	absolute value operator

Expressions (14) - (15) are maximized over the control variable, h. Expectations over bivariate price risk are computed using trapezoidal integration in the Matlab computing language. Optimization proceeds using the simplex method. Further details of the optimization routine are available from the author.

Results

The results are shown in Tables I, II, and III. In Table I, Recursive utility hedge ratios are shown to vary little for high levels of risk aversion, regardless of the value of ρ. The highly risk averse hedger ($\gamma = 2.00$) faces an optimal hedge ratio of about 58% when $\rho = 0.1$ and 59% when $\rho = 1.0$. Alternative utility hedge ratios are always equal to recursive utility hedge ratios when $\rho = 1.0$ because both objective functions reduce to simple expected utility when $\rho = 1.0$. However, the two objective functions prescribe substantially different optimal hedge ratios for the risk averse hedger if ρ is less than 1.0. The alternative utility hedge ratio drops to 44% when $\rho = 0.5$ and then to zero for $\rho = 0.4$ or lower.

The moderately risk averse hedger ($\gamma = 0.20$) faces a different situation. When $\rho = 1.0$, the optimal (expected utility) hedge ratio is 55%. Both recursive utility and the alternative yield optimal hedge ratios then fall as r falls, with the alternative utility hedge ratios falling more dramatically than the recursive utility hedge ratios. Both hedge ratios become zero for low values of ρ. The effect is magnified further for low risk aversion hedgers ($\gamma = 0.02$). Both hedge ratios are 18% for $\rho = 1.0$, but for $\rho = 0.9$ or lower both are zero.

Experimentation with marginal transaction costs does not change the basic story. As shown in Table II, hedge ratios are lower when marginal transaction costs are higher, but they still differ between recursive utility and the alternative. Inelastic intertemporal substitution reduces hedge ratios and results in no hedging for low ρ – low γ combinations.

Experimentation with discount rates also results in little change to the pattern of hedge ratios. Increasing the discount rate from 0% to 10% per year ($\beta = 1$ to 0.9982 for weekly data) has very small effects on the optimal hedge ratios for both objective functions. As shown in Table III, the effects are significant only in the 3rd and 4th decimal places throughout the range of parameters considered in the experiments.

Conclusions

The main conclusions of this research are two-fold. The first conclusion is that optimal hedge ratios are lower when intertemporal substitution is inelastic, in some cases much lower. The alternative utility function yields hedge ratios that drop off considerably and fall all the way to zero for hedgers with low intertemporal substitution. The recursive utility function prescribes higher hedge ratios that the alternative does, but its optimal hedge ratios also fall quickly in some cases. Moderately risk averse and even highly risk averse hedgers may not hedge at all if they are averse to intertemporal variation.

The effect does not work the other way around. That is, hedgers who are nearly risk neutral will not hedge more than recommended by expected utility due to alterations of this sort in the objective function. Expected utility represents one extreme for both recursive utility and the alternative. Both reduce to expected utility when the elasticity of intertemporal substitution is infinite ($\rho = 1$).

The result is that hedgers who are averse to intertemporal variation may hedge considerably less than the minimum variance hedge ratio or expected utility hedge ratio that recommended by agricultural economists. Zero is an optimal hedge ratio for many people, according to the models discussed here. We must not be quick to conclude that hedgers with a zero hedge ratio are somehow "uneducated," "untrusting," or "fearful" with respect to price risk management. This research may help agricultural economists to understand hedgers' seeming paradoxical behavior in this ever-changing and complex field of choice under uncertainty.

The second main conclusion of this research is that recursive utility and the alternative suggested above differ markedly in their prescriptions for optimal hedge ratios. The two differ most when the elasticity of intertemporal substitution is low and coincide when it is perfect.

There is little difference between them when the level of risk aversion is low because optimal hedge ratios are already very low or zero.

The difference between the two objective functions implies that careful attention must be paid to the choice of intertemporal aggregator function. This paper makes the case in favor of the alternative utility function over recursive utility, but the subject is very much still open for debate and discussion. It is my desire that other researchers in the field of intertemporal choice under uncertainty will investigate the further properties of the two objective functions discussed here and that some additional insights may be gained from an ongoing discussion of the issues involved.

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Table I

Comparison of Optimal Hedge Ratios
With Different Objective Functions
Transaction Costs = 0.5 cents/bu
Discount Rate = 0%

ρ	Elasticity of Intertemporal Substitution	Coefficient of Absolute Risk Aversion (γ)					
		2.00		0.20		0.02	
		Recursive Utility	Alternative Utility	Recursive Utility	Alternative Utility	Recursive Utility	Alternative Utility
0.1	1.11	0.5779	0	0	0	0	0
0.2	1.25	0.5805	0	0	0	0	0
0.3	1.43	0.5826	0	0	0	0	0
0.4	1.67	0.5845	0	0	0	0	0
0.5	2.00	0.5860	0.4407	0.3164	0	0	0
0.6	2.50	0.5873	0.5252	0.4228	0	0	0
0.7	3.33	0.5884	0.5611	0.4783	0	0	0
0.8	5.00	0.5894	0.5782	0.5134	0.4184	0	0
0.9	10.00	0.5901	0.5868	0.5370	0.5111	0	0
1.0	Infinite	0.5908	0.5908	0.5535	0.5535	0.1802	0.1802

Table II

Comparison of Optimal Hedge Ratios
With Different Objective Functions
Transaction Costs = 1.0 cents/bu
Discount Rate = 0%

ρ	Elasticity of Intertemporal Substitution	Coefficient of Absolute Risk Aversion (γ)					
		2.00		0.20		0.02	
		Recursive Utility	Alternative Utility	Recursive Utility	Alternative Utility	Recursive Utility	Alternative Utility
0.1	1.11	0.5767	0	0	0	0	0
0.2	1.25	0.5782	0	0	0	0	0
0.3	1.43	0.5797	0	0	0	0	0
0.4	1.67	0.5811	0	0	0	0	0
0.5	2.00	0.5822	0.3172	0	0	0	0
0.6	2.50	0.5833	0.4859	0.3019	0	0	0
0.7	3.33	0.5843	0.5394	0.3955	0	0	0
0.8	5.00	0.5852	0.5657	0.4494	0.2554	0	0
0.9	10.00	0.5859	0.5794	0.4858	0.4359	0	0
1.0	Infinite	0.5867	0.5867	0.5120	0.5120	0	0

Table III

Comparison of Optimal Hedge Ratios
With Different Objective Functions
Transaction Costs = 0.5 cents/bu
Discount Rate = 10%/year

ρ	Elasticity of Intertemporal Substitution	Coefficient of Absolute Risk Aversion (γ)					
		2.00		0.20		0.02	
		Recursive Utility	Alternative Utility	Recursive Utility	Alternative Utility	Recursive Utility	Alternative Utility
0.1	1.11	0.5779	0	0	0	0	0
0.2	1.25	0.5805	0	0	0	0	0
0.3	1.43	0.5826	0	0	0	0	0
0.4	1.67	0.5845	0	0	0	0	0
0.5	2.00	0.5859	0.4397	0.3156	0	0	0
0.6	2.50	0.5873	0.5248	0.4224	0	0	0
0.7	3.33	0.5884	0.5609	0.4780	0	0	0
0.8	5.00	0.5894	0.5781	0.5133	0.4176	0	0
0.9	10.00	0.5901	0.5866	0.5369	0.5107	0	0
1.0	Infinite	0.5908	0.5908	0.5534	0.5533	0.1794	0.1787