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Global Properties of the Logit, Translog and Almost Ideal Demand Systems

by

Jesus C. Dumagan
Timothy D. Mount

Department of Agricultural Economics
New York State College of Agriculture and Life Sciences
A Statutory College of the State University
Cornell University, Ithaca, New York, 14853-7801

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Abstract

Price elasticities for popular demand systems, e.g., Translog and AIDS, behave erratically when expenditure shares approach zero, and often violate strict quasi-concavity of the utility function, i.e., may have positive Hicksian own-price elasticities. A generalized linear logit model is shown to be well-behaved for all possible shares.

*Research Assistant and Professor, respectively, Department of Agricultural Economics, Cornell University, Ithaca, New York 14853-7801. This paper is part of a wider study on estimation applied to electricity and fuels in a complete demand system. They are grateful to Martha Czerwinski for expert computer assistance that has enhanced understanding of the subject of this paper. The research was supported by the New York State Department of Public Service.

1 Purpose and Overall Finding

Increased attention is being placed on long-run forecasting in applications such as evaluating the effects of global climate change. In these situations, it is important to use demand models that are data admissible (i.e., do not predict negative levels of consumption) and are consistent with economic theory (i.e., do not imply positive price responses). Unfortunately, the most widely used demand systems do not necessarily meet either of these criteria.

The Translog (Christensen, Jorgenson and Lau, 1975) and the "almost ideal demand system" (AIDS) (Deaton and Muellbauer, 1980) exhibit additivity, homogeneity and symmetry for any number of commodities, but they may predict negative shares (Lutton and LeBlanc, 1981) and their underlying (direct) utility functions are not necessarily strictly quasi-concave (Christensen and Caves, 1980).[1]

This paper, following Considine (1990), generalizes the linear logit model of budget shares for Marshallian demand (Tyrrell and Mount, 1982) and for Hicksian demand (or firm input demand) (Considine and Mount, 1984) so that it now has global symmetry of the Hicksian cross-price effects. As before, the new model predicts shares that never fall below zero or exceed one but always add up to one, i.e., it satisfies additivity. It also satisfies Marshallian zero-degree homogeneity in prices and income. For certain parameter values, it encompasses the characteristics of the Translog and AIDS models.

The main focus of this paper is on the parameter restrictions that are necessary and sufficient for strict quasi-concavity of the underlying utility function in each model. This is limited to the case of two goods to highlight the fact that, even in this simple case, the additivity, homogeneity and symmetry restrictions on parameters alone are not sufficient for the models to imply strict quasi-concavity. It is shown that there are necessary and sufficient conditions for strict quasi-concavity in terms only of the parameters of each model. However, these conditions have different implications on the price and income elasticities. In the case of the Translog, quasi-concavity implies that the goods cannot be Marshallian price-inelastic, cannot be gross complements and must have constant unitary income elasticities, i.e., the utility function is homothetic. While no such restrictions are implied for the AIDS Marshallian price and income elasticities, the AIDS Hicksian own-price elasticities could go to either plus or minus infinity when expenditure shares go to

[1] Christensen and Caves use the equivalent property of strict quasi-convexity of the indirect utility function.

zero.[2] This erratic behavior of the Hicksian elasticities applies to the Translog as well. In contrast, the necessary and sufficient conditions for strict quasi-concavity in the logit model do not restrict the Marshallian price and income elasticities and do not necessarily lead to erratic behavior of the Hicksian price elasticities.

2 Additivity, Homogeneity and Symmetry in a Linear Logit Model of Budget Shares

Let the prices and corresponding quantities at any time period t be given by p_{it} and x_{it}, $i = 1, 2, \ldots, n$. Given that income in the same period is I_t, then by definition of the budget constraint,

$$(1) \qquad w_{it} = \frac{p_{it}\, x_{it}}{I_t} \quad ; \quad 1 \geq w_{it} \geq 0 \quad ; \quad \sum_i w_{it} = 1.$$

In (1), w_{it} is the total expenditure or budget share of each commodity. In order to satisfy (1), define a logit specification of the budget share,

$$(2) \qquad w_{it} = \frac{e^{f_{it}}}{e^{f_{1t}} + \ldots + e^{f_{nt}}} = \frac{e^{f_{it}}}{\sum_j e^{f_{jt}}}$$

where f_{it} is a function of p_{it} and I. By defining the share, w_{nt}, of an arbitrarily chosen nth good in accordance with (2), it follows that

$$(3) \qquad \ln\left(\frac{w_{it}}{w_{nt}}\right) = f_{it} - f_{nt} \quad ; \quad i = 1, 2, \ldots, n-1.$$

Thus, the non-linear expenditure system evident from (2) can be estimated as a linear system in (3) by specifying f_{it} as a linear function shown later by (5).

Engel aggregation or additivity is satisfied by the linear logit model from the fact that (2) strictly satisfies (1) for every set of predicted budget shares.

For homogeneity, consider first that by equating w_{it} in (1) to that in (2)

$$(4) \qquad \ln x_{it} = -\ln\left(\frac{p_{it}}{I_t}\right) + f_{it} - \ln \sum_j e^{f_{jt}}.$$

[2]The Hicksian own-price elasticity is derived in Section 3 from the Hicksian own-price effect of the Slutsky equation. This elasticity has the same sign as the own-price effect and must be negative; otherwise, the utility function is not strictly quasi-concave. In contrast, the Marshallian own-price elasticity need not be negative.

The Marshallian demand functions x_{it} in (4) are homogeneous of degree zero in prices and income given the following specification,

$$f_{it} = \alpha_{i0} + \sum_j \alpha_{ij}\,\theta_{ijt-1} \ln\left(\frac{p_{jt}}{I_t}\right) \tag{5}$$

where α_{i0} and α_{ij} are parameters. The variables are the logarithmns of the ratios of each commodity price to income, and θ_{ijt-1} is a lagged variable weight which will be shown later to determine global Hicksian symmetry.

By combining (4) and (5), it can be shown that the own-price, cross-price and income elasticities are

$$E_{iit} = \frac{\partial \ln x_{it}}{\partial \ln p_{it}} = -1 + \alpha_{ii}\,\theta_{iit-1} - \sum_j w_{jt}\,\alpha_{ji}\,\theta_{jit-1} \quad ; \tag{6}$$

$$E_{ikt} = \frac{\partial \ln x_{it}}{\partial \ln p_{kt}} = \alpha_{ik}\,\theta_{ikt-1} - \sum_j w_{jt}\,\alpha_{jk}\,\theta_{jkt-1} \quad ; \tag{7}$$

$$E_{iIt} = \frac{\partial \ln x_{it}}{\partial \ln I_t} = 1 - \sum_j \alpha_{ij}\,\theta_{ijt-1} + \sum_i \sum_j w_{jt}\,\alpha_{ji}\,\theta_{jit-1}. \tag{8}$$

Marshallian zero-degree homogeneity in prices and income implies that the sum of all price and income elasticities equals zero for each good, i.e.,

$$\sum_j E_{ijt} + E_{iIt} = 0\,. \tag{9}$$

It can be verified that (9) is satisfied by the elasticities of the logit model in (6) to (8).

Hicksian symmetry of the cross-price effects can be obtained by defining θ as the following function of lagged shares and by imposing symmetry on the α coefficients,

$$\theta_{ikt-1} = \frac{w_{kt-1}^{\gamma}}{w_{it-1}^{1-\gamma}} \quad ; \quad \alpha_{ik} = \alpha_{ki} \tag{10}$$

where γ is a parameter.

To show that the restrictions in (10) are sufficient for global Hicksian symmetry, consider the Slutsky equation

$$\frac{\partial x_{it}}{\partial p_{kt}} = \frac{\partial x_{it}^{h}}{\partial p_{kt}} - x_{kt}\,\frac{\partial x_{it}}{\partial I_t} \tag{11}$$

where x_{it}^h and x_{kt}^h are Hicksian demand functions whereas x_{it} and x_{kt} are Marshallian. Now, the Hicksian cross-price effect in (11) can be expressed in terms of Marshallian price and income elasticities and budget shares as

$$(12) \qquad \frac{\partial x_{it}^h}{\partial p_{kt}} = \frac{I_t}{p_{it}\, p_{kt}} \left(E_{ikt}\, w_{it} + w_{it}\, w_{kt}\, E_{iIt}\right) \qquad or$$

$$(13) \qquad \frac{\partial x_{kt}^h}{\partial p_{it}} = \frac{I_t}{p_{kt}\, p_{it}} \left(E_{kit}\, w_{kt} + w_{kt}\, w_{it}\, E_{kIt}\right).$$

Hicksian symmetry of the cross-price effects means that (12) equals (13) or that

$$(14) \qquad E_{ikt}\, w_{it} + w_{it}\, w_{kt}\, E_{iIt} = E_{kit}\, w_{kt} + w_{kt}\, w_{it}\, E_{kIt}\,.$$

Hicksian symmetry in this model holds for any set of predicted budget shares. For infinitesimal changes of shares, the time lag defined by the original data, $t-1$ may be replaced by an infinitesimal lag, $t-\delta$ where $\delta \to 0$. This means that the elasticities may be computed conditionally, using on the right-hand side the shares evaluated at time t. Thus, substituting (6) and (8) into the Hicksian symmetry condition in (14) gives

$$(15) \qquad w_{it} \sum_j \alpha_{jk} \left(w_{jt}\, w_{kt}\right)^{\gamma} + w_{kt} \sum_j \alpha_{ij} \left(w_{it}\, w_{jt}\right)^{\gamma} =$$

$$w_{it} \sum_j \alpha_{kj} \left(w_{kt}\, w_{jt}\right)^{\gamma} + w_{kt} \sum_j \alpha_{ji} \left(w_{jt}\, w_{it}\right)^{\gamma} \quad .$$

This result is true given the symmetry restriction, $\alpha_{ij} = \alpha_{ji}$, in (10). Global symmetry holds for every set of shares and for any value of γ, given any number of goods.

3 Global Convexity of the Indifference Curve

An important issue raised by Christensen and Caves (1980) is that standard systems which exhibit symmetry, such as the Translog, violate strict quasi-concavity of the utility function (or strict quasi-convexity of the indirect utility function) for much of the sample space, i.e., not just when shares are negative. In other words, Hicksian own-price effects become positive and illogical.

In a model with only two goods, if the utility function is smooth, increasing and strictly quasi-concave, then every indifference curve will be everywhere strictly convex to the origin (Chiang, 1984). To illustrate the implications for different models, the discussion now focuses on the two-good case in which strict or global convexity is defined to mean the above usual shape of the indifference curve. In this case, to demonstrate global convexity, it is only necessary to show for one good that the Hicksian own-price effect remains negative for all budget shares between zero and one.

Notice that zero-degree homogeneity in (9) and the Hicksian cross-price effect in (12) are true for any well-behaved demand system. It follows that in a two-good case, the own-price effect for good 1, can now be expressed as

$$(16) \qquad \frac{\partial x_1^h}{\partial p_1} = \frac{I}{p_1^2} w_1 [E_{11}(1 - w_1) - w_1 E_{12}]$$

where the time period subscript, t, is suppressed for simplicity. This yields the Hicksian own-price elasticity, E_{11}^h

$$(17) \qquad E_{11}^h = \frac{\partial x_1^h}{\partial p_1} \frac{p_1}{x_1^h} = \frac{x_1}{x_1^h} [E_{11}(1 - w_1) - w_1 E_{12}] \ .$$

Since Marshallian and Hicksian demand functions do intersect, (17) can be evaluated without loss of generality by letting $x_1 = x_1^h$ to give

$$(18) \qquad E_{11}^h = [E_{11}(1 - w_1) - w_1 E_{12}]$$

which shows that the Hicksian own-price elasticity is a function of the Marshallian own-price elasticity, E_{11} cross-price elasticity, E_{12} and budget share, w_1. It is necessary and sufficient for global convexity that (18) be negative for all w_1 between zero and one.

3.1 The Linear Logit Model With Two Goods

It follows from (6), (7) and (10), that

$$(19) \qquad E_{11} = \alpha_{11} w_1^{2\gamma-1}(1-w_1) - \alpha_{12} w_1^{\gamma}(1-w_1)^{\gamma} - 1 \ ;$$

$$(20) \qquad E_{12} = \alpha_{12} w_1^{\gamma-1}(1-w_1)^{\gamma+1} - \alpha_{22}(1-w_1)^{2\gamma} \ .$$

Substituting (19) and (20) into (18), gives the Hicksian own-price elasticity

$$(21) \qquad E_{11}^h = (1-w_1) J \quad ;$$

$$J = \alpha_{11} w_1^{2\gamma-1}(1-w_1) - 2\alpha_{12} w_1^{\gamma}(1-w_1)^{\gamma} + \alpha_{22} w_1 (1-w_1)^{2\gamma-1} - 1 .$$

Since $w_1 \geq 0$, E_{11}^h in (21) is negative if and only if J is negative.

For the case where $\gamma = 1$, (21) yields

$$J = (\alpha_{11} + \alpha_{22} - 2\,\alpha_{12})\, w_1\, (1 - w_1) - 1 . \tag{22}$$

Observe that the maximum value of $w_1\,(1 - w_1)$ is 1/4 since w_1 is between zero and one. Therefore, the condition for global convexity is

$$(\alpha_{11} + \alpha_{22} - 2\,\alpha_{12}) < 4 \quad \Leftrightarrow \quad J < 0 \quad \Leftrightarrow \quad E_{11}^h < 0. \tag{23}$$

For $\gamma = 1$, (23) is the necessary and sufficient condition. Since the Marshallian elasticities in (19) and (20) become

$$E_{11} = (\alpha_{11} - \alpha_{12})\, w_1\, (1 - w_1) - 1 \quad ; \quad E_{12} = (\alpha_{12} - \alpha_{22})\,(1 - w_1)^2 ,$$

the conditions in (23) neither restrict E_{11} to be less than or greater than -1 nor restrict E_{12} to be positive or negative. That is, price-elastic or price-inelastic goods and gross substitutes or complements are compatible with the global convexity of the indifference curve in a logit model with $\gamma = 1$.[3] It is shown in the following sections that these desirable properties do not hold for the Translog and AIDS models.

3.2 The Translog Model

The budget shares of the Translog model (Christensen, Jorgenson and Lau, 1975) can be written as

$$w_i = \frac{\beta_{i0} + \sum_j \beta_{ij}\, ln\,(\frac{p_j}{I})}{\sum_i \beta_{i0} + \sum_i \sum_j \beta_{ij}\, ln\,(\frac{p_j}{I})} \quad ; \quad \beta_{ij} = \beta_{ji} \quad ; \quad \sum_i \beta_{i0} = -1 \; . \tag{24}$$

In the two-good case of (24), the own-price elasticity of good 1 and its cross-price elasticity with the price of good 2 are,

$$E_{11} = -1 + \left(\frac{1}{\Delta}\right)\left(\frac{\beta_{11}}{w_1} - \beta_{11} - \beta_{12}\right) \quad ; \tag{25}$$

$$E_{12} = \left(\frac{1}{\Delta}\right)\left(\frac{\beta_{12}}{w_1} - \beta_{12} - \beta_{22}\right) \quad ; \tag{26}$$

[3] In a two-good case, goods can only be Hicksian (net) substitutes but they could be either Marshallian (gross) substitutes or complements.

(27) $$\Delta = -1 + (\beta_{11} + \beta_{12})\, ln \left(\frac{p_1}{I}\right) + (\beta_{12} + \beta_{22})\, ln \left(\frac{p_2}{I}\right) \quad .$$

To determine the condition for convexity, substitute (25) and (26) into the expression for E^h_{11} in (18) to obtain

(28) $$E^h_{11} = w_1 - 1 + \frac{1}{\Delta}\{\beta_{11}\left(\frac{1}{w_1} + w_1 - 2\right) + 2\beta_{12}\,(w_1 - 1) + \beta_{22}\, w_1\}.$$

In evaluating (28), note that E^h_{11} assumes utility is held constant as p_1 changes, by definition of a Hicksian demand function. This means that I must be adjusted in the opposite direction to the change in p_1 in order to return to the original indifference curve. Given the price of the other good, p_2, it follows that Δ in (27) can switch signs as p_1 and I change, even with the same parameters, from the fact that (27) is a function of the logarithmn of the ratio of prices to income. For strictly positive prices and income, this ratio varies from less than to greater than one so that the logarithmn of this ratio changes from negative to positive, thus changing the sign of Δ and of E^h_{11} as well. To prevent this change in sign from happening as p_1 and I change, it is necessary that

(29) $$\beta_{11} + \beta_{12} = 0 \quad \textit{and} \quad \beta_{12} + \beta_{22} = 0 \quad .$$

Given (29), (27) and (28) yield

(30) $$E^h_{11} = w_1 - 1 - \frac{\beta_{11}}{w_1}$$

which shows that (29), while necessary, is not sufficient for (30) to be negative for all w_1 between zero and one. In this case, the necessary and sufficient conditions for global convexity of the indifference curve in the Translog model are

(31) $$\beta_{11} = \beta_{22} = -\beta_{12} \geq 0 \quad \Leftrightarrow \quad E^h_{11} < 0\,.$$

The sufficiency of (31) is obvious from (30). Necessity follows from the fact that if $\beta_{11} < 0$ then w_1 can always be made sufficiently close to zero such that (30) yields $E^h_{11} > 0$, which violates global convexity.

However, it follows from (31) that (25) and (26) yield

(32) $$E_{11} = -1 - \frac{\beta_{11}}{w_1} \leq -1 \;\; ; \;\; E_{12} = \frac{-\beta_{12}}{w_1} \geq 0 \;\; ; \;\; E_{1I} = -(E_{11} + E_{12}) = 1$$

where E_{1I} is the income elasticity. The results in (32) mean that global convexity implies the restrictions that good 1 cannot be price-inelastic, cannot be a gross complement and must have unitary income elasticity in the Translog model. It can be verified that these restrictions apply to good 2 as well. Thus, the implied underlying utility function must be homothetic in a globally well-behaved Translog model.

3.3 The AIDS Model

The budget shares of the "almost ideal demand system" (AIDS) (Deaton and Muellbauer, 1980) can be written as

$$(33)\qquad w_i = \lambda_{i0} + \sum_j \lambda_{ij}\, ln\, p_j + \tau_i\, ln\left(\frac{I}{P_s}\right) \;\; ; \;\; \sum_j \lambda_{ij} = 0 \;\; ; \;\; j = 1,2,\ldots,n$$

where P_s is a Stone price index (Phlips, 1983)

$$(34)\qquad P_s = \Pi_j\, p_j^{w_j} \quad .$$

Thus, (34) satisfies homogeneity. Moreover, it is required that

$$(35)\qquad \sum_i \lambda_{i0} = 1 \quad ; \quad \sum_i \lambda_{ij} = 0 \quad ; \quad \sum_i \tau_i = 0 \quad ; \quad \lambda_{ij} = \lambda_{ji}\,.$$

The first three restrictions in (35) are for additivity and the fourth is for symmetry.

For two goods, the AIDS model yields for good 1 the following own-price elasticity and cross-price elasticity (treating the shares in P_s as fixed)

$$(36)\qquad E_{11} = \frac{1}{w_1}\left(\lambda_{11} - \tau_1 w_1\right) - 1 \quad ;$$

$$(37)\qquad E_{12} = -\frac{1}{w_1}\left[\lambda_{11} + \tau_1\left(1 - w_1\right)\right] \quad .$$

To determine the convexity of the indifference curve, substitute (36) and (37) into the expression for the Hicksian own-price elasticity in (18) to obtain

$$(38)\qquad E_{11}^h = w_1 - 1 + \frac{\lambda_{11}}{w_1} \quad .$$

This yields the necessary and sufficient condition

$$(39)\qquad \lambda_{11} \le 0 \quad \Leftrightarrow \quad E_{11}^h < 0$$

for all w_1 between zero and one. The sufficiency of this condition is obvious from (38). At the same time, (39) is necessary because if $\lambda_{11} > 0$, then w_1 can always be made sufficiently close to zero such that (38) yields $E_{11}^h > 0$, which violates global convexity.

It follows that, since τ_1 could be of either sign, global convexity in the AIDS model does not impose restrictions on the Marshallian price elasticities in (36) and (37). However, since $\lambda_{11} = \lambda_{22}$, the Hicksian price elasticities must have the same realationships with the respective shares for both goods. Furthermore, the inverse function of w_1 in (38) implies that elasticities behave very erratically as w_1 approaches zero.

4 Comparison of Theoretical and Estimated Hicksian Own-Price Elasticities

The Hicksian own-price elasticities derived in the preceding section for the three models are illustrated in the following diagrams. To avoid repetition, any mention of "elasticity" in the following discussion is meant to refer to "Hicksian own-price elasticity", unless otherwise noted.

Notice that the Translog elasticity in (30) will generate the same locus as the AIDS elasticity in (38) by letting $-\beta_{11} = \lambda_{11}$. In Figure 1, two cases are illustrated. The first one is for $-\beta_{11} = \lambda_{11} > 0$ in which case the Translog and AIDS elasticities not only become positive but also increase to $+\infty$ as the expenditure share approaches zero, and consequently, violate global convexity. The other is for the reverse case when $-\beta_{11} = \lambda_{11} < 0$ in which the elasticities remain negative but decrease to $-\infty$ as the share approaches zero. While global convexity is not violated, this latter case illustrates that the Translog and AIDS elasticities change dramatically when the share, w_1, approaches zero. This Figure also illustrates the logit elasticities from (21) for the case where $\gamma = 1$ and the parameters are chosen to satisfy the necessary and sufficient condition for global convexity in (23). This case is illustrated by the solid line which shows that the logit elasticities not only remain negative (thus satisfying global convexity) but are also quite stable, ranging between -1 and zero as the share varies between zero and one.

Figure 2 illustrates a three-dimensional surface showing what happens to the logit elasticities when γ is below or above one, with the other parameters remaining the same as in Figure 1. The spikes in the surface appearing in the foreground correspond to cases where the logit elasticities violate global convexity. This possibility may be inferred from (21) in that when γ is less than $1/2$ then the exponent $2\gamma - 1$ is negative. Therefore, the influence of α_{11} and α_{22} will be accentuated as w_1 approaches zero. This behavior is similar to the characteristics of the Translog and AIDS models. Notice, however, that

FIGURE 1: HICKSIAN OWN–PRICE ELASTICITIES

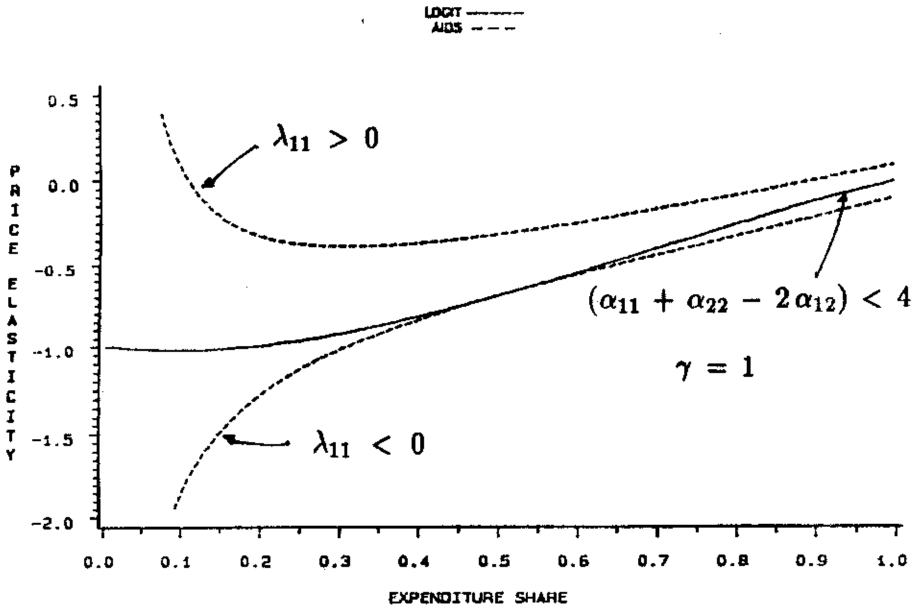


FIGURE 2: HICKSIAN OWN–PRICE ELASTICITIES LOGIT MODEL

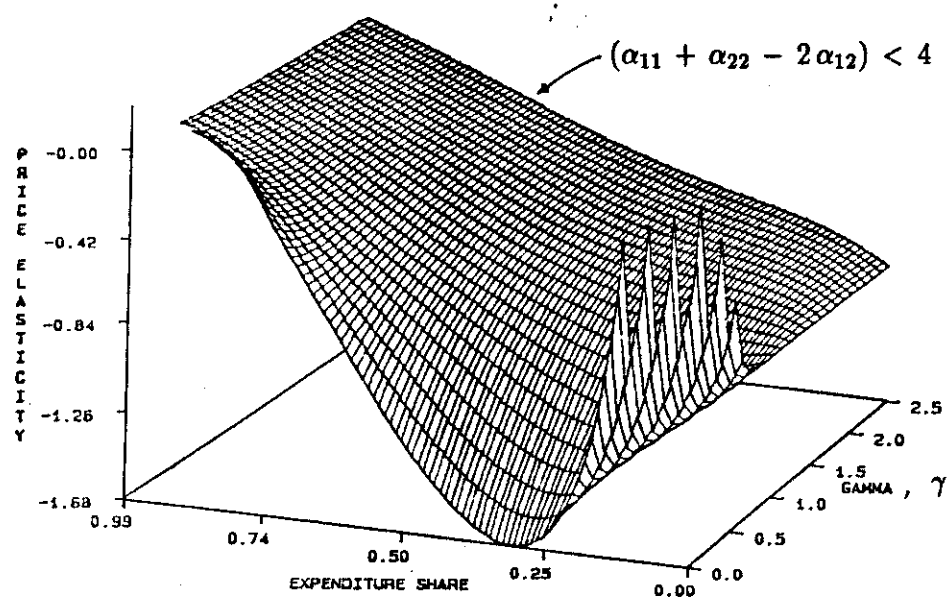
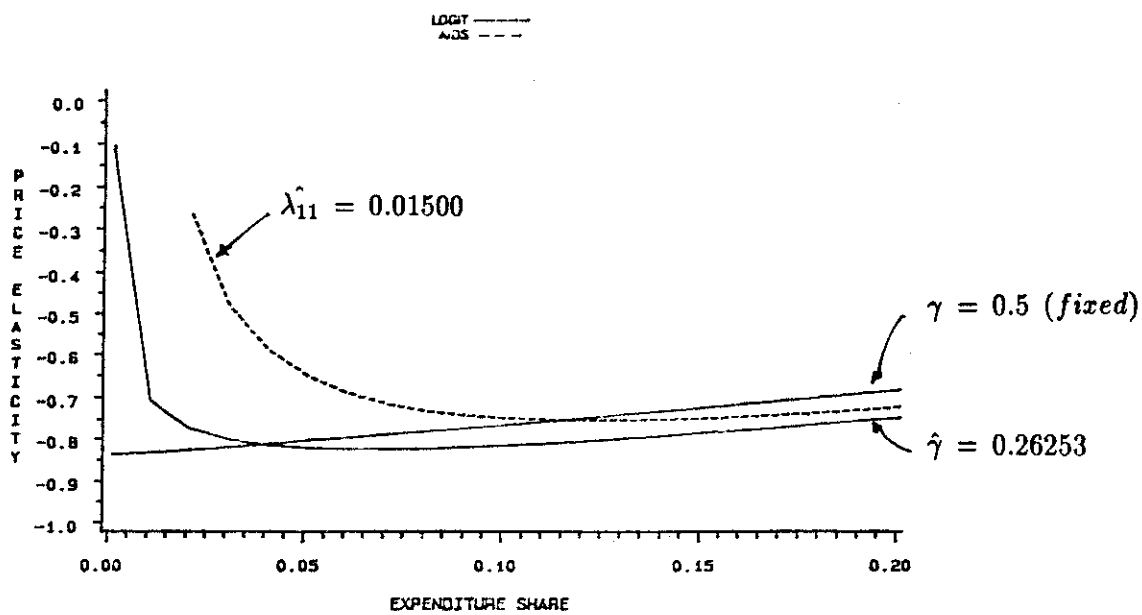


FIGURE 3: ESTIMATED HICKSIAN OWN–PRICE ELASTICITIES FOR ENERGY



when γ exceeds 1/2 the logit elasticities remain negative. Thus, the choice of γ is crucial to global convexity in the logit model.

As the preceding theoretical illustrations show, the Translog and AIDS models tend to behave erratically when shares are close to zero. Thus, for purposes of comparison, the AIDS and logit models were fitted to energy (electricity, natural gas and fuel oil combined) expenditure shares versus the shares of all other expenditures, using residential sector data for the state of New York from 1960 to 1987. This application seems appropriate since the data show that energy expenditure shares are small, ranging between 2.08% and 3.34% during the period examined.

The estimation was carried out by SAS SYSNLIN, using the iterated Zellner's seemingly unrelated regression (ITSUR) option because of cross-equation constraints. The estimated parameters (with standard errors in parentheses) for the AIDS model are:

$$\hat{\lambda_{10}} = 0.10341\ (S.E. = 0.03341) \quad ; \quad \hat{\lambda_{11}} = 0.01500\ (S.E. = 0.00167)$$

$$\hat{\tau_1} = -0.01134\ (S.E. = 0.00385) \quad ; \quad R^2 = 0.7892$$

In the case of the logit model, two versions were fitted. One assumed a fixed value $\gamma = 0.5$, and the other version estimated γ along with the other parameters. The results are:

$$\hat{\alpha_{10}} = -2.54161\ (S.E. = 1.20199) \quad ; \quad \hat{\alpha_{11}} = 0.16331\ (S.E. = 0.13374)$$

$$\hat{\alpha_{12}} = 0.02524\ (S.E. = 0.00688) \quad ; \quad \hat{\alpha_{22}} = 0.16571\ (S.E. = 0.09203)$$

$$\gamma = 0.5\ (fixed) \quad ; \quad R^2 = 0.8722$$

and, when γ is estimated,

$$\hat{\alpha_{10}} = -2.43037\ (S.E. = 1.26663) \quad ; \quad \hat{\alpha_{11}} = 0.03373\ (S.E. = 0.19488)$$

$$\hat{\alpha_{12}} = 0.00027\ (S.E. = 0.02358) \quad ; \quad \hat{\alpha_{22}} = 0.02855\ (S.E. = 0.30344)$$

$$\hat{\gamma} = 0.26253\ (S.E. = 0.78252) \quad ; \quad R^2 = 0.8734\,.$$

It is interesting to note that when the hypothesis $\gamma = 0.5$ is tested, the t-test statistic gives $t = (0.26253 - 0.5)/0.78252 = -0.30347$. This indicates that γ is not statistically different from 0.5. This strengthens the case for $\gamma = 0.5$ where the estimated price parameters have smaller standard errors and R^2 is only slightly lower.

The plots of elasticities against expenditure shares using the estimated parameters are shown in Figure 3. Notice that the AIDS elasticities change much more than the logit elasticities over the sample range from 0.020 to 0.030. This shows that, although global convexity is violated for the estimated value of γ, the region of convexity for the logit is larger than that for AIDS. In the case of $\gamma = 0.5$, the logit elasticities remain negative, i.e., global convexity is satisfied, and are very similar to those of the other logit model with γ estimated over the data range.

5 Conclusion

This paper presented a linear logit model that has the same global properties as the Translog and the AIDS models with respect to additivity, homogeneity and symmetry of demand systems. The specific purpose was to examine the parameter restrictions in these models for global convexity to the origin of the indifference curve, in order to have logical price effects.

This paper finds that the restrictions for global convexity in the logit model permit goods to be Marshallian price-elastic or inelastic and to be gross substitutes or complements. Similarly, goods could be income-elastic or inelastic. Moreover, the Hicksian own-price elasticity is relatively stable and tends to remain negative as the expenditure shares vary between one and zero. In contrast, the parameter restrictions for global convexity in the Translog model restrict the goods not to be Marshallian price-inelastic, not to be gross complements and to have unitary income elasticities, i.e. the underlying utility function is homothetic. The restrictions in the case of AIDS allow the goods to be Marshallian price elastic or inelastic, as well as to be gross substitutes or complements, but the Hicksian own-price elasticity behaves erratically since it goes to $+\infty$ or $-\infty$ as shares approach zero. This erratic behavior applies to the Translog as well.

The overall conclusion is that the logit model covers realistic cases that would be ruled out by the Translog. It also encompasses the behavior of the AIDS model (when $\gamma < 0.5$). However, AIDS has the unappealing property that the elasticities always behave erratically when expenditure shares are small. In contrast, elasticities in the logit model, given appropriate parameter values, are stable for all expenditure shares. In this situation, the logit model will never give illogical predictions even if price and income levels differ substantially from the sample values.

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