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## A STOCHASTIC MODEL FOR TACTICAL HERBICIDE DECISION MAKING

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A simple model for determination of optimal herbicide rates for a single weed in a crop is developed. The effect of parameter uncertainty or variability is analyzed. It is found that even risk neutral decision makers may respond to uncertainty about some parameters and that, contrary to findings in the literature, the effect of risk is not necessarily to increase the optimal herbicide rate. The inclusion of tactical information about climate and weed density is found to have an important effect on decision making.

Weed competition for light, moisture and nutrients causes major losses of production and profits throughout the world. For example, annual weed induced losses have been variously estimated as \$20,400 million or 9.7% of the potential world cereal harvest (Cramer 1967), \$4500 million in the United States (Candler 1979) and \$2500 million in Australia (Combella 1987). Despite the questionable accuracy (Vere and Auld 1982) and usefulness (Pannell 1987) of estimates such as these, they do indicate that potential savings from improvements in efficiency of weed control are considerable. It is somewhat surprising, then, that agricultural economists have published little on the economics of weed control.

Many farmers attempt to improve economic efficiency by applying herbicides at rates other than those specified on chemical labels. However, in most discussion and analysis of the economics of herbicide use, the issue of optimal application rates is ignored. Rather the emphasis tends to be on whether the weed density exceeds the threshold

required to warrant treatment with a pre-defined chemical dose (e.g. Marra and Carlson 1983; Auld et al. 1987). This study is not based on the economic threshold concept but rather allows for the identification of optimal herbicide rates for weed control. The decision framework includes parameter variability and risk aversion. The model is applied in a tactical decision framework in which up to date climatic and agronomic information is incorporated in decisions made during the growing season.

In the following section the basic model is presented. Then the responses to risk by risk neutral and risk averse decision makers are analyzed. Finally the stochastic model is applied to tactical herbicide decision making. The aim of the tactical analysis is to determine whether tactical information is likely to have a significant impact on optimal herbicide strategies for weed control in crops.

#### *The Model*

Lichtenberg and Zilberman (1986) have shown that errors can arise if economic models used to determine pesticide use are formulated without considering technical or biological knowledge about the chemical/pest/crop system. As in their paper, the yield response function has two components: the weed survival function which represents the effect of herbicide rate ( $H$ ) on weed density ( $W$ ) and the actual crop yield function representing the effect of weed density on crop yield ( $Y$ ).

#### *Weed survival*

Weed kill ( $K$ ) is a function of herbicide rate and is represented as a proportion of weed density without spraying ( $W_0$ ). Based on field trial results for a range of weeds and herbicides, an exponential function was chosen to represent weed kill. Thus weed survival after spraying is given by:

$$(1) \quad W = W_0 [1 - K(H)] = W_0 \text{Exp}(-cH)$$

where  $c$  = marginal proportion of weeds killed as  $H \rightarrow 0$  ( $c > 0$ ).

#### *Actual crop yield function*

Crop yield is a function of weed density after spraying. Damage ( $D$ )

is defined as the proportion of weed-free yield ( $Y_0$ ) lost due to weeds. Cousens (1985) tested a range of functional forms for the representation of crop damage by weeds and found that a hyperbola best represented the relationship. Thus actual crop yield is given by:

$$(2) \quad Y = Y_0[1 - D(W)] = Y_0 \left[ 1 - \frac{a}{1 + a/(bW)} \right]$$

where  $a$  = maximum proportion of yield lost ( $a > 0$ ); and

$b$  = marginal proportion yield lost as  $W \rightarrow 0$  ( $b > 0$ ). Note that  $a$  is normally less than one; that is, even at extremely high weed densities some crop yield is obtained.

#### The economic model

Profit ( $\pi$ ) is given by:

$$(3) \quad \pi = P_y Y - P_h H - F$$

where  $P_y$  = price of output;

$P_h$  = price of herbicide; and

$F$  = fixed costs.

First order condition for profit maximisation on herbicide rate:

$$(4) \quad \frac{\partial \pi}{\partial H} = P_y \frac{\partial Y}{\partial H} - P_h = 0$$

$$\text{but } \frac{\partial Y}{\partial H} = \frac{\partial Y}{\partial W} \frac{\partial W}{\partial H},$$

$$\frac{\partial Y}{\partial W} = -Y_0 \frac{\partial D}{\partial W} \quad \text{and}$$

$$\frac{\partial W}{\partial H} = -cW_0 \text{Exp}(-cH),$$

so the optimal herbicide rate ( $H^*$ ) is given by:

$$P_y (-Y_0) \frac{\partial D}{\partial W} (-c)W_0 \text{Exp}(-cH^*) - P_h = 0$$

Rearranging gives:

$$(5) \quad H^* = \frac{1}{c} \left[ \ln \left[ \frac{\partial D}{\partial W} \right] + \ln(cY_0W_0) - \ln \left[ \frac{P_h}{P_y} \right] \right]$$

This indicates that the optimal herbicide rate is higher for higher values of  $\frac{\partial D}{\partial W}$  (at the optimal solution),  $c$ ,  $Y_0$ ,  $W_0$  or  $P_y$  or lower values of  $P_h$ . The marginal yield loss ( $\frac{\partial D}{\partial W}$ ) at the optimal solution is quite a complex term depending on a number of parameters.

### *Responses of Risk Neutral Decision Makers to Stochastic Parameters*<sup>1</sup>

Various authors have noted that risk aversion is not a necessary condition for decision makers to respond to risk (e.g. Just 1975; Antle 1983; Taylor 1986). Even if a decision maker is risk neutral variability can affect their decision making if it affects their expected returns. The herbicide decision problem provides four examples of this phenomenon. If initial weed density ( $W_0$ ), weed competitiveness ( $b$ ), herbicide effectiveness ( $c$ ) or herbicide rate ( $H$ ) are uncertain, a risk neutral decision maker may adjust their preferred herbicide application rate.

First consider weed density for which there are two sources of stochasticity: (i) uncertainty about the representativeness of the sample areas in which weeds were counted and (ii) spatial variability in weed density. As a result of the convexity of the actual yield function (given by (2)) a mean preserving spread in the probability distribution for weed density increases the expected yield. This is illustrated in Figure 1. A crop containing weeds at uniform density  $W_2$  would produce an expected yield of  $\bar{Y}$ . A similar crop with a mixture of areas with weed densities  $W_1$  and  $W_3$  such that the average weed density for the crop was  $W_2$  would produce a higher expected yield of  $\tilde{Y}$ . The change is similar in nature to a reduction in weed competitiveness ( $b$ ). It reduces the marginal yield loss per weed and it can be shown (Pannell 1988) that it reduces the optimal herbicide rate.

Variability in weed competitiveness may result from differences between individual weeds (e.g. a staggered germination of weed seeds produces weeds with a range of sizes and competitive abilities) or differences in environmental conditions over space or time. The result is again an increase in expected yield so that even risk neutral decision makers can respond to the uncertainty. As with uncertain weed density, the response is likely to be a reduction in the optimal herbicide rate [see Pannell (1988)].

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<sup>1</sup> Proofs of results in this and the next section are not presented here but can be found in Pannell (1988).

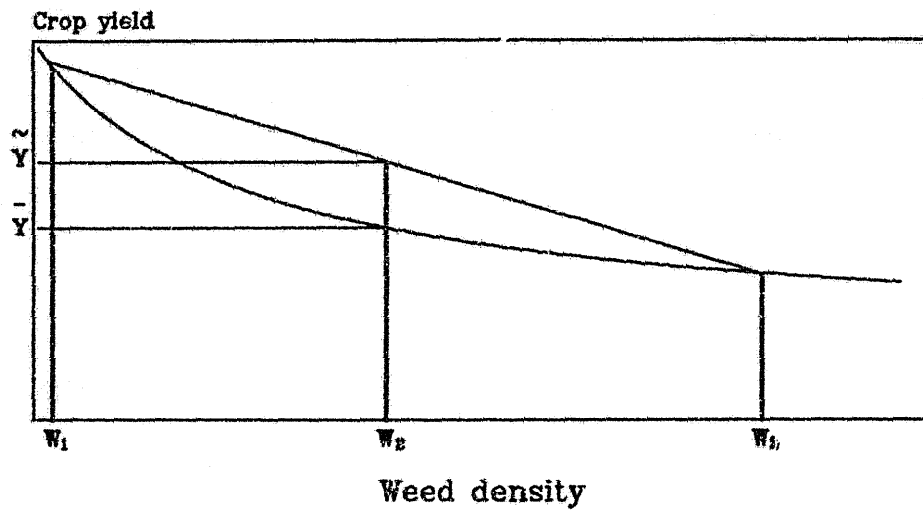


FIGURE 1 - Effect of variable weed density on expected crop yield.

These first two examples refer to uncertainty about the crop damage function. There are also two analogous examples for the kill function; uncertainty about the actual level of herbicide administered to a particular weed ( $H$ ) or about the proportion of weeds killed at a given herbicide dose (reflected in uncertainty about the value of  $c$ ) can both produce a response by risk neutral decision makers.

Uncertainty about herbicide rate is distinguished from that for other parameters in that herbicide rate is under the control of the decision maker. The uncertainty may arise if, for example, the herbicide is not applied with an even spatial distribution. This may apply to any agricultural input, but is particularly likely to be a problem with some of the modern herbicides which are applied at rates of a few grams of active ingredient per hectare.

Uncertainty about weed kill appears to loom large for many farmers and to be put forward as justification for applying higher herbicide rates to reduce the probability of many weeds surviving. This attitude may be rational even if the farmer is not risk averse. As before, the effect of uncertainty under risk neutrality is due to the convexity of a function: this time, the weed survival function (1). Uncertainty about  $c$  or  $H$  increases expected weed survival and the direct response to this is often to increase the optimal herbicide rate (Pannell 1988). However these sources of uncertainty also result in an uncertain weed density which, as has been shown above, reduces the optimal herbicide rate. The net result depends on the balance of forces.

#### *Responses of Risk Averse Decision Makers to Stochastic Parameters*

Apart from the above described effects on a risk neutral decision maker, variability or uncertainty will have additional effects on behaviour if the decision maker is risk averse. In this section a selection of parameters are analyzed individually for implications of their variability under risk aversion.

Feder (1979) and Robison and Barry (1987) have shown that uncertainty about the pest density increases the treatment rate which is optimal

under risk aversion. Under the risk aversion, the higher the treatment rate. The findings are true for uncertainty about weed density (Pannell 1988). The results arise because under weed or pest density uncertainty, increasing the level of control reduces the variance of income. Hence pesticides and herbicides are often described as "risk reducing inputs".

However the results described above were derived without considering the effect of uncertainty about weed or pest density on expected profit. In both the above publications (Feder 1979; Robison and Barry 1987), crop damage was approximated as a linear function of pest density. It is apparent from Figure 1 that in these circumstances uncertainty about pest density has no effect on expected yield and hence no effect on decision making under risk neutrality. But, as discussed in the previous section, if the actual yield (damage) function is convex (concave), as it is for weeds, uncertainty about pest density reduces the optimal treatment rate for risk neutral decision makers.

Thus risk averse decision makers facing a concave damage function are affected in two ways by pest density uncertainty. Its effect on expected yield tends to make them reduce the level of control, while its effect on yield variability prompts an increase in control. The net effect depends on which response is larger. A realistic numerical example shows that at moderate risk aversion levels the response due to expected profit dominates.<sup>2</sup> Since a risk neutral decision maker only shows the negative response, while a risk averter tempers their negative response with a smaller positive response, there arises the

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<sup>2</sup>For example consider the following parameter values:  $a = 0.6$ ,  $b = 0.01$ ,  $c = 3$ ,  $Y_0 = 1300$ ,  $P_y = 0.12$ ,  $P_h = 18$ ,  $F = 2.5$ . Then if  $W_0$  is deterministic with a value of 100,  $H^* = 1.04$ . But if  $W_0$  is stochastic and normally distributed with mean 100 and standard deviation 50 and the decision maker has a constant relative risk aversion utility function ( $U = k_1 + k_2 \Pi^{1-R}$ ) with  $R = 1.2$  and initial wealth of 100 then  $H^*$  is reduced to 1.03.



apparent paradox that risk neutral decision makers may respond more to risk than do risk averters.

Another important uncertain variable is the weed-free yield ( $Y_0$ ) which with the final weed density determines the actual yield. Yield uncertainty due to climatic variability is probably the major source of risk for dryland crop producers in Australia. Yield uncertainty does not affect herbicide decisions under risk neutrality but it may under risk aversion depending on the error structure assumed. Reasonable error structures, supported by empirical data, lead to a zero or negative but not positive effect of yield uncertainty on optimal herbicide rate. The negative response is greater for greater yield variance and for greater risk aversion (Pannell 1968).

Derivation of an analytical solution for the optimal response to uncertainty about the kill function under risk aversion is extremely difficult. In this study, resort has been made to a numerical approach to obtain desired qualitative results. In a numerical example using similar parameters to those shown in footnote 2, the net effect of weed kill uncertainty is an increase in the optimal herbicide rate. This includes effects due to both expected profit and risk aversion. However this result depends on the parameters used and on weed kill being the only source of uncertainty. As Table 1 shows, if weed-free yield is also uncertain, the introduction of uncertainty about weed kill may reduce the optimal rate. Given the system's complexity, it seems that analytical results for weed kill uncertainty alone are likely to be of little value even if derived easily.

TABLE 1

*Direction of change of optimal herbicide rate following introduction of weed kill uncertainty*

Other uncertain parameters	Direction of change
Nil	+
Weed-free yield ( $Y_0$ )	-
Initial weed density ( $W_0$ )	+
$Y_0$ and $W_0$	-

#### *Responses to Tactical Information*

Various authors have distinguished between strategic and tactical decision making (e.g. Trebeck and Hardaker 1972; Raiszadeh and Lingaraj 1986). In this study, tactical decisions are defined as those made in real time with use of up to date information. Any analysis using only average or expected values of parameters or distributions would be regarded as strategic, whereas a tactical analysis would generally include values which departed from long run expectations. Most weed control decisions are made after some climatic information for the relevant season has become available and some are made after actual weed densities have been observed. Consequently, a tactical analysis is often appropriate for weed control decision making. Results presented below are from calculations of optimal herbicide rates making tactical use of climatic and weed density information.

#### *Methods*

At the time of post-emergent herbicide application there are a number of variables about which expectations are likely to have changed since preliminary plans were made. If a parameter or variable is now expected to deviate from its long-run expected value, the application rate can be adjusted accordingly. In this study three types of tactical information are analyzed:

- (a) Weed density without treatment ( $W_0$ ). Weeds can be counted in the field and their untreated density estimated. Here it is assumed that the density is uniform across the field.

- (b) Weed free yield ( $Y_0$ ). Rainfall prior to treatment will affect the probability distribution of  $Y_0$ . On the day of treatment  $Y_0$  is still an uncertain variable but the mean and variance of its probability distribution has probably changed.
- (c) Herbicide effectiveness ( $c$ ). The proportion of weeds killed by any given dose of herbicide can be markedly influenced by climatic and soil conditions at and just prior to application (Caseley 1987). The most important determinant of  $c$  is probably temperature.

Tactical analyses require some means of ascertaining the implications of the dynamically observed information for the expected level and, possibly, distribution of net returns. Historical records, experiment results and/or subjective judgments may be used. Each of these sources was used in developing the dynamic simulation model of wheat growth used in this study to calculate the effects of climate on wheat yield. The other two parameters ( $W_0$  and  $c$ ) are treated as deterministic and, once observed, known with certainty in the tactical analysis.

In evaluating the approach, tactical information for rainfall was obtained from historical records for Merredin, a town in Western Australia's eastern wheatbelt. Values for  $W_0$  were generated by a normal random number generator using a mean of 100 and variance of 50 weeds per square metre. Values of  $c$  were calculated from randomly generated temperatures. Parameters of the kill and damage functions were based on Coupons' (1986) wild oats model. An iterative procedure on a microcomputer spreadsheet was used to find expected utility maximising herbicide rates assuming a constant relative risk aversion utility function ( $U = k_1 + k_2 \Pi^{1-R}$ ).

### *Results and discussion*

Figure 2 shows optimal herbicide rates for a slightly risk averse decision maker ( $R_R = 0.5$ ) for a sample period of 35 years. The horizontal line is the optimal rate calculated using long-run distributions, i.e. without considering tactical information. There is

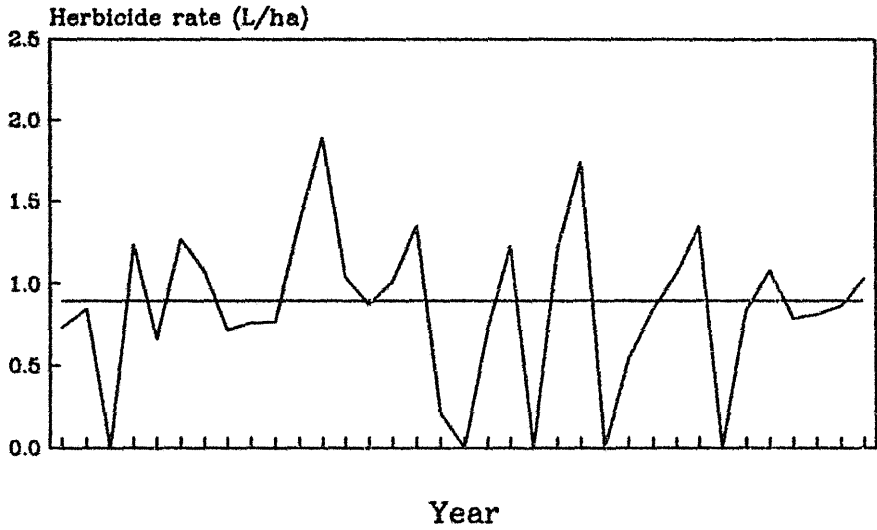


FIGURE 2 - Optimal herbicide rates given tactical information.

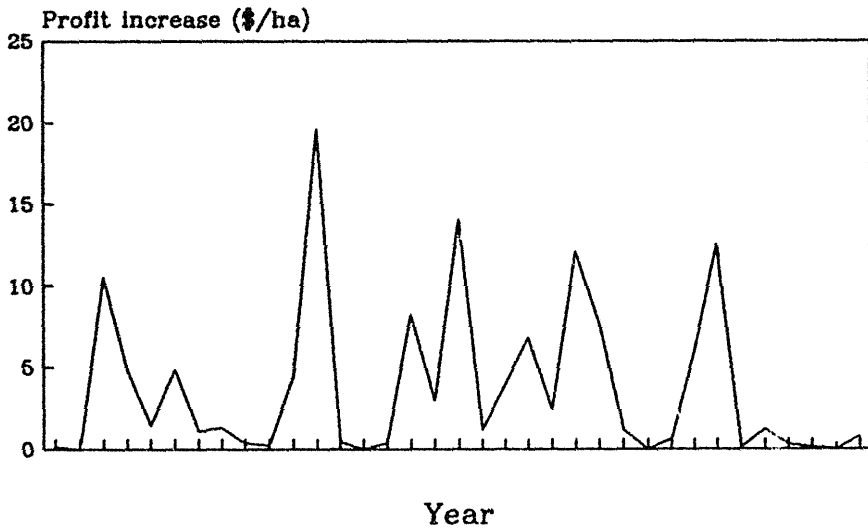


FIGURE 3 - Profit increase from tactical approach.

considerable between-year variation in the optimal tactical rate. There are several years in which no herbicide is justified, most due to poor yield expectations but one in this sample due to low weed density. The optimal rate is approximately as likely to be above the long-run optimum as below it.

Figure 3 shows the increase in expected profit from the tactical approach compared to the purely strategic approach. Annual improvements of up to \$20 per hectare are indicated, and the mean increase is \$4. Most of the larger increases are associated with years in which no herbicide was selected, but the greatest increase occurs in a year of high herbicide rate.

#### Conclusion

It has been shown that even risk neutral decision makers may respond to uncertainty about the crop/weed/herbicide system by adjusting herbicide rates. Despite the conventional wisdom that herbicides are risk reducing inputs, a number of circumstances have been identified in which risk reduces the optimal herbicide rate. The inclusion in decision making of tactical information about weed density and weather conditions has been shown to have a dramatic effect on selected treatment rates and a significant effect on expected returns from herbicide treatment.

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A STOCHASTIC MODEL FOR TACTICAL HERBICIDE DECISION MAKING:  
SUPPLEMENTARY NOTES

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Appendix A

PROOFS OF RESULTS FOR DETERMINISTIC HERBICIDE MODEL

A.1 The Basic Model

Effect of herbicide rate ( $H$ ) on weed density ( $W$ ):

$$W = W_0 [1 - K(H)] = W_0 \text{Exp}(-cH) \quad (\text{A.1})$$

where  $W_0$  = weed density before spraying;

$K(H)$  = proportion of weeds killed at herbicide rate  $H$ ; and

$c$  = marginal proportion of weeds killed as  $H \rightarrow 0$  ( $c > 0$ ).

Effect of weed density ( $W$ ) on crop yield ( $Y$ ):

$$Y = Y_0 [1 - D(W)] = Y_0 \left[ 1 - \frac{a}{1 + a/(bW)} \right] \quad (\text{A.2})$$

where  $Y_0$  = weed free crop yield;

$D(W)$  = proportion of yield lost due to weeds;

$a$  = maximum proportion of yield lost ( $a > 0$ ); and

$b$  = marginal proportion yield lost as  $W \rightarrow 0$  ( $b > 0$ ).

Profit ( $\pi$ ):

$$\pi = P_y Y - P_h H - F \quad (\text{A.3})$$

where  $P_y$  = price of output;

$P_h$  = price of herbicide; and

$F$  = fixed costs.

First order condition for profit maximisation on herbicide rate:

$$\frac{\partial \pi}{\partial H} = P_y \frac{\partial Y}{\partial H} - P_h = 0 \quad (\text{A.4})$$

$$\text{but } \frac{\partial Y}{\partial H} = \frac{\partial Y}{\partial W} \frac{\partial W}{\partial H},$$

$$\frac{\partial Y}{\partial W} = -Y_0 \frac{\partial D}{\partial W} \quad \text{and}$$

$$\frac{\partial W}{\partial H} = -cW_0 \text{Exp}(-cH),$$

so the optimal herbicide rate ( $H^*$ ) is given by:

$$P_y (-Y_0) \frac{\partial D}{\partial W} (-c)W_0 \text{Exp}(-cH^*) - P_h = 0$$

$$\text{Exp}(-cH^*) = \frac{P_h}{P_y} \frac{1}{\partial D / \partial W} \frac{1}{cY_0 W_0}$$

$$H^* = \frac{1}{c} \left[ \ln \left[ \frac{\partial D}{\partial W} \right] + \ln(cY_0 W_0) - \ln \left[ \frac{P_h}{P_y} \right] \right] \quad (\text{A.5})$$

## A.2 Effect of Initial Weed Density on Optimal Herbicide Rate

From (A.5):

$$\frac{\partial H^*}{\partial W_0} = \frac{1}{c} \left[ \frac{1}{\partial D / \partial W} \frac{\partial^2 D}{\partial W \partial W_0} + \frac{1}{W_0} \right] \quad (\text{A.6})$$

$$\frac{\partial D}{\partial W} = \frac{a^2}{bW^2(1 + a/(bW))^2} > 0 \quad (\text{A.7})$$

$$\frac{\partial^2 D}{\partial W \partial W_0} = \frac{a^2}{b} \left[ \frac{-2}{W^3} \frac{\partial W}{\partial W_0} \frac{1}{(1 + a/(bW))^2} + \frac{(-2)}{W^2} \frac{1}{(1 + a/(bW))} \frac{-ab}{(bW)^2} \frac{\partial W}{\partial W_0} \right]$$

$$= \frac{a^2}{b} \frac{\partial W}{\partial W_0} \frac{(-2)}{W^4} \frac{1}{(1 + a/(bW))^2} (W - D/b)$$



$$= \frac{\partial W}{\partial W_0} \frac{(-2)}{W^2} \frac{\partial D}{\partial W} (W - D/b) \quad (\text{A.8})$$

Substituting (A.8) into (A.6):

$$\frac{\partial H^*}{\partial W_0} = \frac{1}{c} \left[ \frac{\partial W}{\partial W_0} \frac{(-2)}{W^2} (W - D/b) + \frac{1}{W_0} \right] \quad (\text{A.9})$$

which is greater than zero iff:

$$\frac{\partial W}{\partial W_0} \frac{2}{W^2} (W - D/b) < \frac{1}{W_0}$$

Substituting  $\text{Exp}(-cH)$  for  $\frac{\partial W}{\partial W_0}$  and multiplying by  $W_0$ :

$$W_0 \text{Exp}(-cH) \frac{2}{W^2} (W - D/b) < 1$$

Substituting  $W$  for  $W_0 \text{Exp}(-cH)$ :

$$2(1 - D/(bW)) < 1$$

Substituting  $bW/(1 + bW/a)$  for  $D$ :

$$W < a/b$$

which is true for any realistic problem. e.g. if  $a = 0.6$  and  $b = 0.01$  then  $\frac{\partial H^*}{\partial W_0} > 0$  if the weed density after spraying with the optimal herbicide rate is less than 60.

### A.3 Effect of Weed-Free Yield on Optimal Herbicide Rate

$$\frac{\partial H^*}{\partial Y_0} = \frac{1}{cY_0} > 0 \quad (\text{A.10})$$

$$\frac{\partial^2 H^*}{\partial Y_0^2} = \frac{-1}{cY_0^2} < 0 \quad (\text{A.11})$$

### A.4 Effect of Output Price on Optimal Herbicide Rate

$$\frac{\partial H^*}{\partial P_y} = \frac{1}{cP_y} > 0 \quad (\text{A.12})$$

$$\frac{\partial^2 H^*}{\partial P_y^2} = \frac{-1}{cP_y^2} < 0 \quad (\text{A.13})$$

#### A.5 Effect of Input Price on Optimal Herbicide Rate

$$\frac{\partial H^*}{\partial P_h} = \frac{-1}{cP_h} < 0 \quad (\text{A.14})$$

$$\frac{\partial^2 H^*}{\partial P_h^2} = \frac{1}{cP_h^2} > 0 \quad (\text{A.15})$$

#### A.6 Effect of Maximum Yield Suppression on Optimal Herbicide Rate

From (A.5):

$$\frac{\partial H^*}{\partial a} = \frac{1}{c} \frac{1}{\partial D / \partial W} \frac{\partial^2 D}{\partial W \partial a} \quad (\text{A.16})$$

$$\frac{\partial D}{\partial W} = \frac{a^2}{bW^2(1 + a/(bW))^2} > 0 \quad (\text{A.17})$$

$$\begin{aligned} \frac{\partial^2 D}{\partial W \partial a} &= \frac{1}{bW^2} \left[ 2a \frac{1}{(1 + a/(bW))^2} - \frac{2a^2}{bW} \frac{1}{(1 + a/(bW))^3} \right] \\ &= \frac{a^2}{bW^2(1 + a/(bW))^2} \frac{2}{a} \left[ 1 - \frac{a}{1 + a/(bW)} \frac{1}{bW} \right] \\ &= \frac{\partial D}{\partial W} \frac{2}{a} [1 - D/(bW)] \quad (\text{A.18}) \end{aligned}$$

Substituting (A.18) into (A.16):

$$\frac{\partial H^*}{\partial a} = \frac{2}{ac} [1 - D/(bW)] \quad (\text{A.19a})$$

Substituting  $bW/(1 + bW/a)$  for  $D$ :

$$\frac{\partial H^*}{\partial a} = \frac{2}{c} \left[ \frac{1}{a} - \frac{1}{a + bW} \right] > 0 \quad (\text{A.19b})$$

$$\frac{\partial^2 H^*}{\partial a^2} = \frac{2}{c} \left[ \frac{1}{(a + bW)^2} - \frac{1}{a^2} \right] < 0 \quad (\text{A.20})$$

### A.7 Effect of Weed Competitiveness on Optimal Herbicide Rate

From (A.5):

$$\frac{\partial H^*}{\partial b} = \frac{1}{c} \frac{1}{\partial D / \partial W} \frac{\partial^2 D}{\partial W \partial b} \quad (\text{A.21})$$

$$\frac{\partial D}{\partial W} = \frac{a^2}{bW^2(1 + a/(bW))^2} > 0 \quad (\text{A.7})$$

$$\frac{\partial^2 D}{\partial W \partial b} = \frac{a^2}{W^2} \left[ \frac{-1}{b^2} \frac{1}{(1 + a/(bW))^2} + \frac{2aW}{b(bW)^2} \frac{1}{(1 + a/(bW))^3} \right]$$

$$= \frac{a^2}{bW^2(1 + a/(bW))^2} \frac{1}{b} \left[ -1 + \frac{a}{1 + a/(bW)} \frac{2abW}{(bW)^2} \right]$$

$$= \frac{\partial D}{\partial W} \frac{1}{b} \left[ -1 + \frac{a}{1 + a/(bW)} \frac{2abW}{(bW)^2} \right]$$

$$= \frac{\partial D}{\partial W} \frac{1}{b} \left[ \frac{2a^2}{bW + a} - 1 \right] \quad (\text{A.22})$$

Substituting (A.22) into (A.21):

$$\frac{\partial H^*}{\partial b} = \frac{1}{bc} \left[ \frac{2a^2}{bW + a} - 1 \right] \quad (\text{A.23})$$

which is greater than zero iff:

$$\frac{2a^2}{bW + a} > 1$$

i.e. iff:

$$W < a(2a - 1)/b$$

A necessary condition for  $\frac{\partial H^*}{\partial b} > 0$  is thus  $a > 0.5$ . Beyond this, no generalisations are possible. Using parameters reported by Cousens (1986) for wild oats ( $a = 0.6$ ;  $b = 0.01$ ) gives a cut off value for  $W$  of 20. That is,  $\frac{\partial H^*}{\partial b} > 0$  if the weed density after spraying with the

optimal herbicide rate is less than 20 plants per square metre. At current price relativities and expected yields, this will be true except for extremely high initial weed densities. Many weeds are less competitive with crops than wild oats are and so will have lower  $b$  values. If  $a$  is held constant, the cut off post-spraying weed density for these weeds will be even higher than  $20 \text{ m}^{-2}$ .

### A.8 Effect of Herbicide Effectiveness on Optimal Herbicide Rate

From (A.5):

$$\begin{aligned} \frac{\partial H^*}{\partial c} &= \frac{-1}{c^2} \left[ \ln \left( \frac{\partial D}{\partial W} \right) + \ln(cY_0W_0) - \ln \left( \frac{P_h}{P_y} \right) \right] + \frac{1}{c^2} \\ &= \frac{1}{c^2} \left[ 1 - \ln \left( \frac{\partial D}{\partial W} \right) - \ln(cY_0W_0) + \ln \left( \frac{P_h}{P_y} \right) \right] \end{aligned} \quad (\text{A.24})$$

Therefore  $\frac{\partial H^*}{\partial c} < 0$  iff:

$$1 + \ln \left( \frac{P_h}{P_y} \right) < \ln \left( \frac{\partial D}{\partial W} \right) + \ln(cY_0W_0)$$

i.e. iff:

$$e < W_0 c Y_0 \frac{\partial D}{\partial W} \frac{P_h}{P_y}$$

e.g. if  $a = 0.6$ ,  $b = 0.01$ ,  $c = 3$ ,  $P_y = 0.12$ ,  $P_h = 18$ ,  $W_0 = 100$  and  $Y_0 = 1200$  then  $H^* = 1.01$ ,  $W^* = 4.83$  and  $W_0 c Y_0 \frac{\partial D}{\partial W} \frac{P_h}{P_y} = 20.5$ , so  $\frac{\partial H^*}{\partial c} < 0$ .

## Appendix B

### PROOFS OF RESULTS FOR STOCHASTIC HERBICIDE MODEL

#### B.1 Risk neutral decision maker

##### B.1.1 Stochastic Pre-treatment Weed Density

Let  $W_0 = \bar{W}_0 + \varepsilon$  where  $\varepsilon \sim N(0, \sigma_w^2)$ . From (A.1), (A.2) and (A.3) the profit function is:

$$\pi = P_y Y_0 (1 - D(W_0(1-K))) - P_h H - F \quad (\text{B.1})$$

The damage function,  $D(W)$  can be approximated by a second order Taylor series approximation about  $\bar{W}_0$ , the expected value of  $W_0$ :

$$D(W) \approx \alpha + \beta W + \gamma W^2 \quad (\text{B.2})$$

where  $\alpha = D(\bar{W}_0) - \bar{W}_0 D'(\bar{W}_0) + \frac{1}{2} \bar{W}_0^2 D''(\bar{W}_0)$

$\beta = D'(\bar{W}_0) - \bar{W}_0 D''(\bar{W}_0) > 0$  since  $D'(W) > 0$  (see (A.7)) and

$$D''(W) = D'(W) \frac{a^2}{b} \left[ \frac{-2}{W} - \frac{a}{bW^2(1+a/(bW))} \right] < 0; \text{ and}$$

$$\gamma = \frac{1}{2} D''(\bar{W}_0) < 0.$$

Therefore:

$$\pi \approx P_y Y_0 \left[ 1 - \alpha - \beta W_0(1-K) - \gamma W_0^2(1-K)^2 \right] - P_h H - F \quad (\text{B.3})$$

The objective is to maximise expected profits.

$$E(\Pi) \approx P_y Y_0 \left[ 1 - \alpha - \beta \bar{W}_0(1-K) - \gamma (\bar{W}_0^2 + \sigma_w^2)(1-K)^2 \right] - P_h H - F \quad (\text{B.4})$$

The herbicide rate which maximises expected profit is given by the first order condition:

$$P_y Y_0 \left[ \beta \bar{W}_0 K' - \gamma (\bar{W}_0^2 + \sigma_w^2) (2)(1-K)(-K') \right] - P_h = 0$$

$$F = Y_0 K' \left[ \beta \bar{W}_0 + \gamma (\bar{W}_0^2 + \sigma_w^2) (2)(1-K) \right] - P_h / P_y = 0$$

$\frac{\partial H^*}{\partial \sigma_w^2} = - \frac{F_{\sigma_w^2}}{F_H^*}$  where subscripts denote partial derivatives. But

$F_H^* < 0$  for  $H^*$  a max. Therefore  $\frac{\partial H^*}{\partial \sigma_w^2} > 0$  as  $F_{\sigma_w^2} > 0$ . Now:

$$F_{\sigma_w^2} = 2Y_0 K' \gamma (1-K) < 0 \text{ since } \gamma < 0$$

so,  $\frac{\partial H^*}{\partial \sigma_w^2} < 0$ .

B.1.2 Stochastic Weed Competitiveness

Consider the actual yield function:

$$Y = Y_0[1 - D(W)] = Y_0 \left[ 1 - \frac{a}{1 + a/(bW)} \right] \quad (\text{A.2})$$

This can be approximated by a second order Taylor series approximation:

$$Y \approx Y_0 [\alpha + \beta(bW) + \gamma(bW)^2] \quad (\text{B.5})$$

where  $\alpha > 0$ ,  $\beta < 0$  and  $\gamma > 0$ .

Now let the parameter  $b$ , the main indicator of weed competitiveness, be a random variable  $b = \bar{b} + \xi$  where  $\xi \sim N(0, \sigma_b^2)$ . From (B.5) the expected yield is given by:

$$E(Y) \approx Y_0 (\alpha + \beta \bar{b}W + \gamma W^2 (\bar{b}^2 + \sigma_b^2)) \quad (\text{B.6})$$

Therefore,  $\frac{\partial E(Y)}{\partial \sigma_b^2} > 0$ . In other words, an increase in  $\sigma_b^2$  has similar

effects to a reduction in  $b$ : the extremities of the actual yield function are unchanged but for all weed densities greater than zero the expected yield is increased. It has already been shown in Appendix A that the effect of a reduction in  $b$  on  $H^*$  is ambiguous but is likely to be a reduction.

B.1.3 Stochastic Weed Kill

Consider the weed kill function:

$$W = W_0 \text{Exp}(-cH) \quad (\text{A.1})$$

This can be approximated by a second order Taylor series approximation:

$$W \approx W_0 (\alpha + \beta(cH) + \gamma(cH)^2) \quad (\text{B.7})$$

where  $\alpha > 0$ ,  $\beta < 0$  and  $\gamma > 0$ . Now let the parameter  $c$ , which encaptures the rate of weed kill, be a random variable  $c = \bar{c} + \psi$  where  $\psi \sim N(0, \sigma_c^2)$ . From (B.7) the expected weed density is given by:

$$E(W) \approx W_0 (\alpha + \beta \bar{c}H + \gamma H^2 (\bar{c}^2 + \sigma_c^2)) \quad (\text{B.8})$$

Therefore,  $\frac{\partial E(W)}{\partial \sigma_c^2} > 0$ . In other words, an increase in  $\sigma_c^2$  has similar

effects to a reduction in  $c$ : the extremities of the weed kill function are unchanged but for all herbicide rates greater than zero the expected weed density is increased. It has already been shown in Appendix A that the effect of a reduction in  $c$  on  $H^*$  is ambiguous but is likely to be an increase.

There is, however, a further impact of stochastic weed kill.

Uncertainty about  $c$  leads to uncertainty about  $W$  and from B.1.1

$\frac{\partial H^*}{\partial \sigma_w^2} < 0$ . Thus uncertainty about weed kill has two opposing effects

on  $H^*$ : (i) expected weed density is increased, increasing  $H^*$  and (ii) weed density is made uncertain, decreasing  $H^*$ . The net effect depends on the balance of forces; a range of numeric examples has shown that it can be in either direction.

#### B.1.4 Stochastic herbicide rate

The case of stochastic herbicide rate is very similar to the stochastic weed kill example above. Again there are two opposing responses with an ambiguous net effect.

Consider the weed kill function:

$$W = W_0 \text{Exp}(-cH) \quad (\text{A.1})$$

This can be approximated by a second order Taylor series approximation as shown in (B.7). Now let the variable  $H$ , the herbicide rate, be a random variable  $H = \bar{H} + \omega$  where  $\omega \sim N(0, \sigma_h^2)$ .

From (B.7) the expected weed density is given by:

$$E(W) \approx W_0(\alpha + \beta c \bar{H} + \gamma c^2(\bar{H}^2 + \sigma_h^2)) \quad (\text{B.9})$$

Therefore,  $\frac{\partial E(W)}{\partial \sigma_h^2} > 0$ . In other words, an increase in  $\sigma_h^2$  has similar

effects to a reduction in  $c$ . As before, the effect of a reduction in  $c$  on  $H^*$  is ambiguous but is likely to be an increase.

Again there is a further impact of stochastic weed kill. Uncertainty about  $c$  leads to uncertainty about  $W$  and from B.1.1  $\frac{\partial H^*}{\partial \sigma_w^2} < 0$ . Thus uncertainty about the herbicide rate actually applied has two opposing effects on  $H^*$  and the net effect can be in either direction.

## B.2 Risk averse decision maker

### B.2.1 Stochastic Pre-treatment Weed Density

Using a second order Taylor series approximation of the crop damage function gives the following approximation of profit:

$$\pi \approx P_y Y_0 [1 - \alpha - \beta W_0(1 - K) - \gamma W_0^2(1 - K)^2] - P_h H - F \quad (\text{B.10})$$

Let  $W_0 = \bar{W}_0 + \varepsilon$  where  $\varepsilon \sim N(0, \sigma_w^2)$ . Then:

$$E(\pi) \approx P_y Y_0 [1 - \alpha - \beta \bar{W}_0(1 - K) - \gamma(\bar{W}_0^2 + \sigma_w^2)(1 - K)^2] - P_h H - F \quad (\text{B.11})$$

Now let  $A = P_y Y_0 \beta(1 - K)W_0$

$$\text{and } B = P_y Y_0 \gamma(1 - K)^2 W_0^2$$

Then

$$\text{var}(\pi) = \text{var}(A) + \text{var}(B) + \text{covar}(A, B) \quad (\text{B.12})$$

$$\text{var}(A) = [P_y Y_0 \beta(1 - K)]^2 \sigma_w^2 \quad (\text{B.13})$$

$$\text{var}(B) = [P_y Y_0 \gamma(1 - K)^2] \text{var}(W_0^2) \quad (\text{B.14})$$

$$= [P_y Y_0 \gamma(1 - K)^2] (4\bar{W}_0^2 \sigma_w^2 + 2\sigma_w^4) \quad (\text{see proof below})$$

$$\text{covar}(A, B) = \rho [P_y Y_0 \beta(1 - K)] \sigma_w [P_y Y_0 \gamma(1 - K)^2] \text{stdev}(W_0^2) \quad (\text{B.15})$$



Proof that  $\text{var}(W_0^2) = 4\bar{W}^2\sigma_w^2 + 2\sigma_w^4$

$$W_0 = \bar{W}_0 + \epsilon$$

$$W_0^2 = \bar{W}_0^2 + 2\bar{W}_0\epsilon + \epsilon^2$$

$$E(W_0^2) = \bar{W}_0^2 + \sigma_w^2$$

$$\text{var}(W_0^2) = E(W_0^4) - E(W_0^2)^2$$

$E(W_0^2)^2$  can be easily derived; the challenge is  $E(W_0^4)$ .

$$W_0^4 = \bar{W}_0^4 + 4\bar{W}_0\epsilon + 6\bar{W}_0^2\epsilon^2 + 4\bar{W}_0\epsilon^3 + \epsilon^4$$

$$E(\bar{W}_0^4) = \bar{W}_0^4$$

$$E(4\bar{W}_0\epsilon) = 0$$

$$\begin{aligned} E(6\bar{W}_0^2\epsilon^2) &= 6\bar{W}_0^2E(\epsilon^2) \\ &= 6\bar{W}_0^2[E(\epsilon)^2 + \sigma_\epsilon^2] \\ &= 6\bar{W}_0^2\sigma_w^2 \end{aligned}$$

$$E(4\bar{W}_0\epsilon^3) = 4\bar{W}_0E(\epsilon^3)$$

= 0 ( $\epsilon$  is normally distributed and so has zero skewness)

$$E(\epsilon^4) = 3\sigma_w^4 \text{ (since for } \epsilon \text{ normal, kurtosis} = E[(\epsilon - \mu_\epsilon)^4]/\sigma_\epsilon^4 = 3 \text{ ;}$$

Hogg and Craig 1978)

$$\text{Thus } E(W_0^4) = \bar{W}_0^4 + 6\bar{W}_0^2\sigma_w^2 + 3\sigma_w^4$$

$$\text{but } E(W_0^2)^2 = \bar{W}_0^4 + 2\bar{W}_0^2\sigma_w^2 + \sigma_w^4$$

$$\text{so } \text{var}(W_0^2) = 4\bar{W}_0^2\sigma_w^2 + 2\sigma_w^4$$

Q. E. D.

From (B.13), (B.14) and (B.15) it can be shown that:

$$\frac{\partial \text{var}(A)}{\partial H} < 0$$

$$\frac{\partial \text{var}(A)}{\partial H} < 0$$

$$\frac{\partial \text{covar}(A, B)}{\partial H} < 0 \text{ (given } \rho > 0)$$

and therefore from (B.12)  $\frac{\partial \text{var}(\pi)}{\partial H} < 0$

which means that given a stochastic initial weed density, risk aversion increases the optimal herbicide rate and the greater the degree of risk aversion, the greater will be  $H^*$ . This can be shown as follows.

The certainty equivalent value ( $\pi_{CE}$ ) can be approximated as follows (Robison and Barry 1987):

$$\pi_{CE} = E(\pi) - \frac{\lambda}{2} \text{var}(\pi) \quad (\text{B.16})$$

Where  $\lambda$  is the absolute risk aversion coefficient. To find the certainty equivalent maximising herbicide rate set  $\frac{\partial \pi_{CE}}{\partial H}$  equal to zero:

$$\text{let } F = \frac{\partial \pi_{CE}}{\partial H} = \frac{\partial E(\pi)}{\partial H} - \frac{\lambda}{2} \frac{\partial \text{var}(\pi)}{\partial H} = 0 \quad (\text{B.17})$$

$\frac{\partial H^*}{\partial \lambda} = - \frac{F_\lambda}{F_H}$  where subscripts denote partial derivatives. But  $F_H < 0$  for  $H^*$  a max. Therefore  $\frac{\partial H^*}{\partial \lambda} > 0$  as  $F_\lambda > 0$ . Now:

$$F_\lambda = - \frac{1}{2} \frac{\partial \text{var}(\pi)}{\partial H} > 0$$

so  $\frac{\partial H^*}{\partial \lambda} > 0$ .

### B.2.2 Stochastic Weed-Free Yield

Consider again the profit function:

$$\pi = P_y Y_o [1 - D] - P_h H - F$$

Let  $Y_o = \bar{Y}_o + \tau$  where  $\tau \sim N(0, \sigma_\tau^2)$ . Then:

$$E(\pi) = P_y \bar{Y}_o [1 - D] - P_h H - F \quad (\text{B.18})$$

$$\sigma^2(\pi) = \sigma_\tau^2 P_y^2 (1 - D)^2 \quad (\text{B.19})$$

from (B.12)

$$\pi_{CE} = P_y \bar{Y}_o [1 - D] - P_h H - F - \frac{\lambda}{2} \sigma_\tau^2 P_y^2 (1 - D)^2 \quad (\text{B.20})$$

Then to find  $\pi_{CE}$  maximising herbicide rate

$$\text{let } F = \frac{\partial \pi_{CE}}{\partial H} = -P_y Y \frac{\partial D}{\partial H} - P_h + \lambda \sigma_y^2 P_y^2 (1 - D) \frac{\partial D}{\partial H} = 0 \quad (\text{B.21})$$

As in the previous section,  $\frac{\partial H^*}{\partial \lambda} > 0$  as  $F_\lambda > 0$ .

$$F_\lambda = \sigma_y^2 P_y^2 (1 - D) \frac{\partial D}{\partial H} < 0$$

so the greater the degree of risk aversion, the lower the optimal herbicide rate. Similarly it can be shown that  $F_{\sigma_y^2} < 0$  and therefore that the greater the variance of weed-free yield the lower the optimal herbicide rate.

#### References

- Hogg, R.V. and Craig, A.T. (1978), Introduction to Mathematical Statistics, MacMillan, New York.
- Robison, L.J. and Barry, P.J. (1987), The Competitive Firm's Response to Risk, MacMillan, New York.