THE CONCEPTS OF QUALITY, CONSUMPTION AND PRODUCTION: A THEORETICAL EXPOSITION AND IMPLICATIONS TO RESEARCH EVALUATION *

by

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1. INTRODUCTION

This paper attempts to extend the consumption and production theories, by incorporating the quality of a commodity, defined in terms of its characteristics, in the context of a simple general equilibrium model. On the consumption side, consumer maximisation is specified in terms of commodity characteristics which is the main feature of the product characteristics approach to consumer theory now associated with Lancaster (1971). The utility of a commodity is viewed in the dual space of commodity and quality characteristics. On the production side, a commodity is viewed as comprising characteristics which are produced by a given technology. A classical production function is used in explaining the relationship between quality characteristics and production.

The treatment of characteristics in consumption and production is confined to explaining their relationship, with and without the occurence of research-induced technical change, and the implications to research evaluation.

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2. CONSUMPTION, PRODUCTION AND CHARACTERISTICS MODEL

A contribution to analysing the interrelationship between characteristics, consumption and production in terms of a simple general equilibrium for the whole economy is provided by Hazari, Lubulwa and Sgro (1986). The authors used a standard "two-by-two" (two-sector, two-factor) closed economy general equilibrium model in portraying that relationship. Their study so far, has not considered the effects of technical change. The framework used in the preceding study is hereby modified, and extended in allowing for technical change and its effects on characteristics, consumption and production.

For ease of analysis in the present study, the multiple characteristics approach of Lancaster (1971) is modified to a single-characteristic commodity for each commodity sector. There are two commodities Q_1 and Q_2[1] each one with identical characteristics Z_1 and Z_2 (Z_1), respectively, providing it at different levels. The commodities are produced from two primary factors of production, capital (K) and labour (L).

The other assumptions of the model are as follows: constant returns to scale, but diminishing factor returns; full employment of production factors, and these factors are inelastically supplied; perfect mobility of factors between commodities and characteristics; different capital/labour ratios for each commodity, and a commodity is intensive in the use of the same factor at all factor prices; and perfect competition in product and factor markets.

2.1 Commodity Space

The production functions for the two commodities are as follows:

$$(1)\quad Q_1 = F_1(K_1, L_1) = Q_{L1} f_1(K_1)$$

$$(2)\quad Q_2 = F_2(K_2, L_2) = Q_{L2} f_2(K_2)$$

or

$$(3)\quad Q_1 = g(Q_2)$$

The marginal rate of substitution between the two commodities is

$$(4)\quad \frac{dQ_1}{dQ_2} = g' < 0$$

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[1] Q_i (i= 1,2) is identified with the ith commodity as well as its output. Also, in this paper the terms "commodity", "industry", "sector" are used synonymously.

where K_i and L_i ($i = 1,2$) are the allocations of capital and labour to the Q_i commodity sectors; k_1 and k_2 are the capital-labour ratios in each sector,

$$k_i = \frac{K_i}{L_i}$$

Alternatively, the production functions can be defined in terms of the transformation curve X_1X_2 (quadrant I of Figure 1).

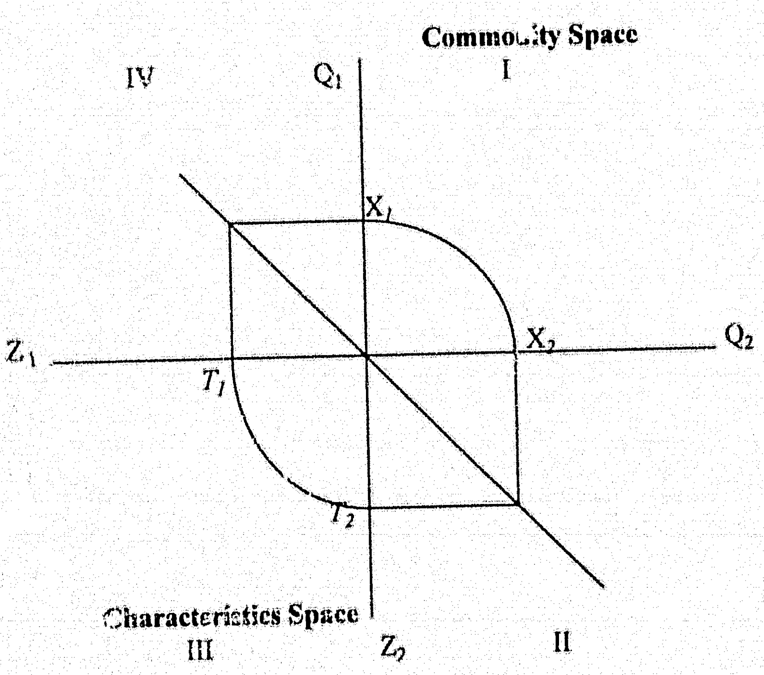

Figure 1. Transformation Curves in the Commodity and Characteristics Space

The characteristic endowment of each commodity is produced by a given technology:

$$(5)\quad Z_1 = \hat{a}_{z1} Q_{L1} f_1(k_1) = \hat{a}_{z1} Q_1$$

$$(6)\quad Z_2 = \hat{a}_{z2} Q_{L2} f_2(k_2) = \hat{a}_{z2} Q_2$$

where $\hat{a}_{zj}$ ($j = 1,2$) indicate fixed characteristic input-output coefficients. For example, $\hat{a}_{z1} = (Z_1/Q_1)$ so that $Z_1 = \hat{a}_{z1} Q_1$.

In equilibrium, profit maximising firms will utilise the factors of production when the value of marginal product of factors equal the reward of these factors, and marginal product is the same in the commodities.

$$(7)\ r = f'_1 = pf'_2$$

$$(8)\ w = (f_1 - k_1 f'_1) = p(f_2 - k_2 f'_2)$$

where f'i are the marginal productivity of the primary factors (K, L) in the ith commodity (i = 1,2); commodity real prices are specified in terms of a numeraire, r is capital rent and w is the labour wage rate. Each commodity price can be defined in terms of the marginal implicit price of the characteristic of each commodity. In simple linear equations,

$$(9)\ p_1 = \hat{a}_{z1} p_{z1}$$

$$(10)\ p_2 = \hat{a}_{z2} p_{z2}$$

Both factors of production are fully employed,

$$(11)\ \hat{a}_{z1} L_1 Q_1 + \hat{a}_{z2} L_2 Q_2 = L^*$$

$$(12)\ \hat{a}_{z1} K_1 Q_1 + \hat{a}_{z2} K_2 Q_2 = K^*$$

where L^* and K^* are fixed factor endowments.

The factor demand functions can be specified as functions of commodity prices and factor rewards.

$$(13)\ L = L(p, w, r)$$

$$(14)\ K = K(p, w, r)$$

The factor demand function is substituted into the production function in deriving the supply function of each commodity (Q_i) as follows.

$$(15)\ Q^d_1 = F_1[Q_1(p,w,r)] = Q_1(p,w,r)$$

$$(16)\ Q^d_2 = F_2[Q_2(p,w,r)] = Q_2(p,w,r)$$

2.2 Characteristics Space

The characteristics endowment of Q_1 and Q_2 in equations (5) and (6) are translated in the characteristic transformation curve T_1T_2 (quadrant III of Figure 1) by,

(17) $dZ_1 = \hat{a}_{z1}dQ_1$

(18) $dZ_2 - \hat{a}_{z2}dQ_2$

(19) $\frac{dZ_1}{dZ_2} = \frac{aZ_1}{aZ_2}\frac{dQ_1}{dQ_2} = \frac{aZ_1}{aZ_2}g' < 0$

The utility function is expressed in the characteristics space as,

(20) $U_z = U_z(Z_1, Z_2)$

where U_z indicates aggregate characteristics utility.

The characteristics utility function is assumed to posses the properties of a standard utility function: well-behaved; continuous; has first- and second-order derivatives; strictly concave; and the partial derivatives are positive (both characteristics are socially desirable). The utility maximising condition is given by

(21) Maximise $U_z = U_z(Z_1, Z_2)$

subject to the characteristics transformation frontier

(22) $Z_1 = g_1(Z_2)$

The consumer derives the highest level of satisfaction through the standard optimisation rule derived from equations (21) and (22). To form the first-order conditions for a local maximum, we form the Lagrangian function

(23) $L = U[Z_1, Z_2] - \lambda[Z_1 - g_1(Z_2)]$

where λ is the Lagrange multiplier. By differentiating L with respect to Z_1 and Z_2 we obtain

(24) $\frac{\partial U}{\partial Z_1} - \lambda = 0$

(25) $\frac{\partial U}{\partial Z_2} + \lambda g'_1(Z_2) = 0$

The solutions for Z_1 and Z_2, and for Q_1 and Q_2 are known as demand functions and depend on relative prices (defined in terms of implicit prices of commodity characteristics in equations 9 and 10) and income. Consider a production point R in the commodity space and correspondingly, point R' in the characteristics space. Geometrically, the solution in the characteristics space is given by the point of tangency between the characteristic indifference curve and the characteristic transformation curve (point e_z in quadrant III of Figure 2); and in the commodity space between the product transformation curve and the price line (point e_q in quadrant I of Figure 2).

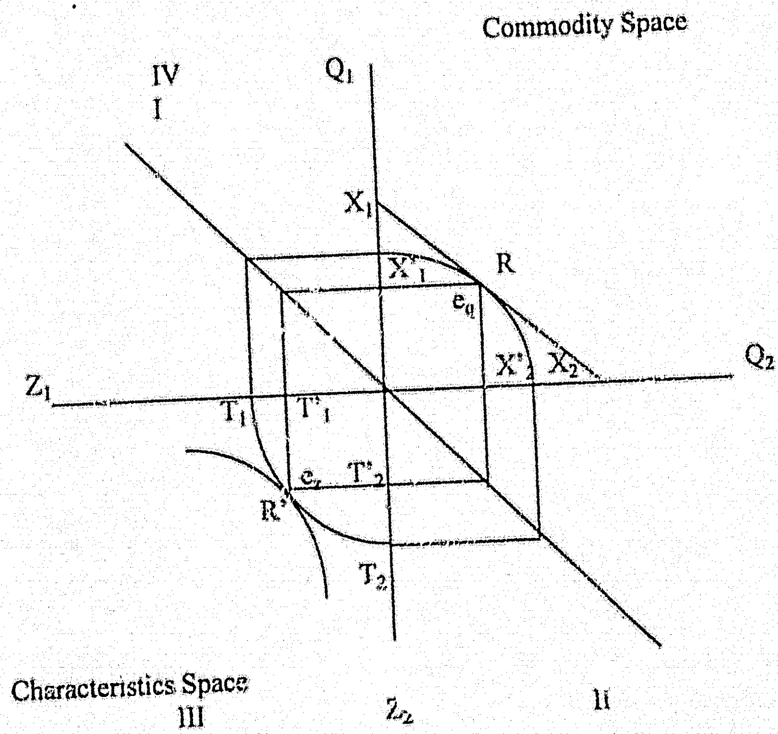

Figure 2. Equilibrium in the Commodity and Characteristics Spaces

In equilibrium the consumer maximises his utility by choosing combinations of Z_1 and Z_2 and hence, Q_1 and Q_2 such that the marginal rate of substitutions are

$$(26) \quad \frac{\partial U}{\partial Z_1} \frac{\partial U}{\partial Z_2} = - \frac{1}{g'(Z_2)}$$

$$(27) \quad \frac{dQ_1}{dQ_2} = p$$

Given the first- and second-order conditions for utility maximisation, the solutions for the demand functions are obtained.

(28) $Z^d_1 = Q^d_1 = Q^d_1 (p, Y)$ (29) $Z^d_2 = Q^d_2 = Q^d_2 (p, Y)$

2.3 Equilibrium

In equilibrium the first-order condition for utility maximisation in consumption and first-order condition for profit maximisation in production are satisfied. In a closed economy, in the charcteristics space at the point of equilibrium the total characteristics of each commodity consumed (Z^d_j) is equal to the total characteristic endowment of each commodity (Z_j). In the commodity space, the consumption of each commodity (Q^d_j) must equal the amount of each commodity produced (Q_j). Equilibrium in the characteristics and commodity spaces are depicted further in Figures 3 and 4.

(30) $Z^d_1 = Z_1$ (32) $Q^d_1 = Q_1$

(31) $Z^d_2 = Z_2$ (33) $Q^d_2 = Q_2$

Equilibrium in the commodity and characteristics spaces in the first sector is explained as follows. As defined earlier, commodity price is expressed in terms of the implicit prices of characteristics. Assume a one-to-one relationship between price ratios (p, p_z) where p_z = (p_{z2}/p_{z1}), wage rate (w); and rent of capital (r). With given price ratios, w and r can be determined which in turn determines the factor intensities in each sector (k_1, k_2). Given full employment conditions, labour requirements (Q_{L_1} and Q_{L_2}) are known. By varying p (and hence p_z), the supply curve for Q_1 and its characteristic endowment (Z_1) are known. Given any price ratio, Y can also be determined ($Y = Q_1 + pQ_2$), since any p and p_z has corresponding unique levels of supply of output (Q_1 and Q_2) and their characteristics (Z_1, Z_2) if $k_1 \neq k_2$. If p and Y are known, the demand for the first commodity (Q^d_1) and its characteristic (Z^d_1) is obtained. The demand curve in the first sector (in the goods or in the characteristics space) is determined also by varying p (and hence p_z). The equilibrium prices p and p_z give solutions for all the variables in the supply and demand systems in the commodity and characteristics space. By Walras law the equilibrium in the first sector also clears the market in the second sector[2].

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[2] The process of obtaining equilibrium in the commodity and characteristics spaces is an extension of the explanation by Hazari (1986). The concept of obtaining equilibrium follows the standard micro-economic theory.

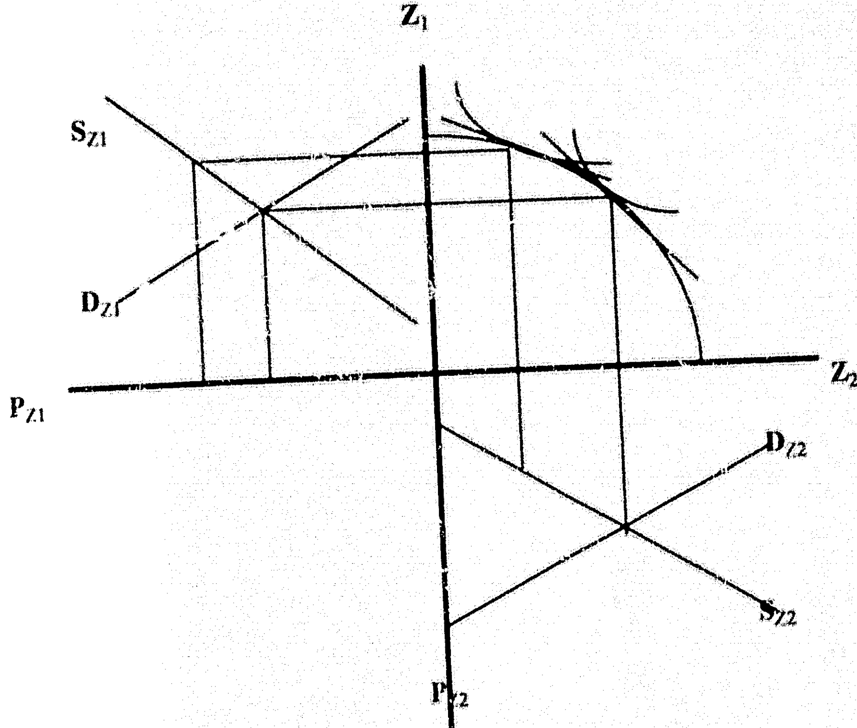
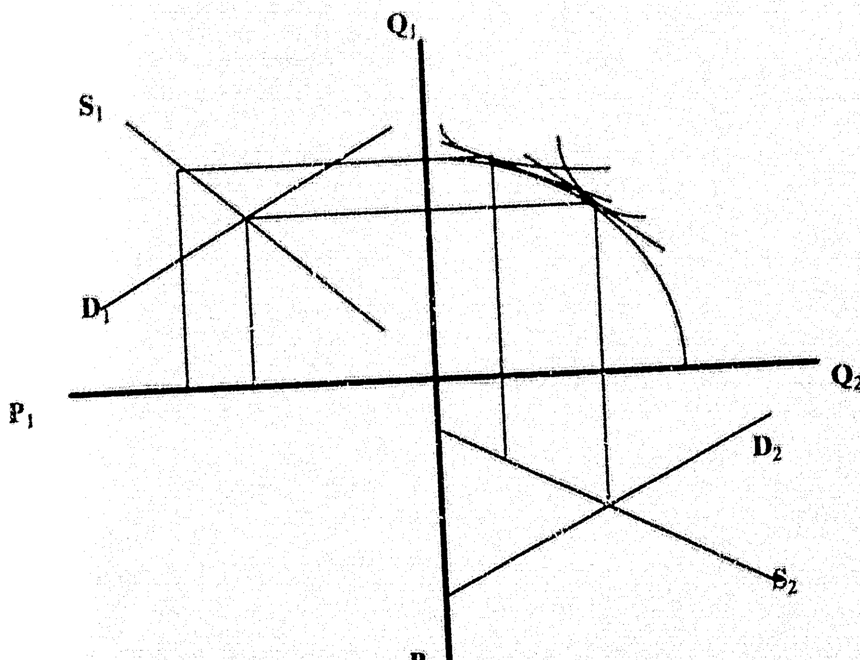


Figure 3. Equilibrium in the Characteristics Space



Fi_ re 4. Equilibrium in the Commodity Space

3. TECHNICAL CHANGE

The concept of technical change[3] is generally applied to a shift in the production function, i.e., a change in the technique of combining inputs. Most technical change is associated with a new process involving only changes in the input mix. Some technical change however, has the salient feature of providing a change in the nature or characteristics of a commodity. Alston, Norton and Pardey (1995) provides an example of a new process in refining crude gasoline that changes the input proportions, as well as introduce a qualitative change in the output, i.e., a higher octane gasoline. Also, technical change either affects the cost or quality of a commodity[4], or both.

3.1 Effects of Different Types of Technical Change

Technical change or progress and its effects on factor intensities, factor prices and outputs are well studied[5]. The following discussions on the effects of factor-saving and factor-neutral technical changes on factor intensities, factor prices, and outputs are provided. It is assumed that the technical change also affects the characteristics of a commodity. The discussion below relies heavily on Kemp (1974).

3.2.1 Technical Change in Q_1 Commodity Sector

Capital (K)-Saving Technical Change. Suppose Q_1 is capital intensive ($k_1 > k_2$). A K-saving[6] technical change in Q_1 which lowers its unit cost of production, increases the original commodity price ratio (p) and cost ratio w/r. In order to maintain the original commodity price and cost ratios, the rent on capital the factor used intensively in Q_1 should rise relative to the wage rate on labour, the non-intensive factor. The decrease in factor cost ratio induces the substitution of L for K in both Q_1 and Q_2 such that k_1 and k_2 decrease. To maintain full employment, the output of Q_1 the sector which can absorb capital more rapidly, will rise and the output of Q_2 the sector which is in a better position to release labour will decline. In the characteristics space, the marginal characteristic product of capital also increases relative to labour leading to an increase in characteristic endowment Z_1 and a decrease in the characteristic endowment Z_2.

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[3] The term is used interchangeably with "technical progress".

[4] In the literature on research assessment these are referred to as cost-reducing or quality-improving research (see Alston, Norton and Pardey, 1995 for a more recent review).

[5] Work on the topic is primarily done in relation to international trade. For a thorough discussion see Findlay and Grubert (1959,) Jones(1965), and Kemp (1974).

[6] Due to an increase in the marginal productivity of capital, less of this input is required per unit of the output.

On the other hand, if Q_1 is L-intensive ($k_2 > k_1$), the original cost ratio can be restored by a decline in the wage rate resulting in an increase in k_2. The marginal physical productivity of capital will fall, the effect of the K-saving improvement on k_1 is indeterminate.

Labour (L) -Saving Technical Change. Analogously, if Q_1 is L-intensive ($k_2 > k_1$) both k_1 and k_2 will increase, capital rent decreases, while wage rate increases. However, if Q_1 is K-intensive ($k_1 > k_2$), k_2 will fall, the effect on k_1 is indeterminate, wage rate will decrease while capital rent will rise. The output of Q_1 and its characteristic endowment will increase while that of Q_2 will decrease in both cases of factor intensities.

Factor-Neutral Technical Change. At the original factor prices, costs in Q_1 fall relatively to the costs of Q_2. However, if Q_1 is capital-intensive ($k_1 > k_2$) the original commodity price ratio will be maintained by a rise in the wage-rental ratio; k_1 and k_2 decrease. If Q_1 is labour-intensive ($k_2 > k_1$) wage rate declines, k_1 and k_2 rise.

In either case of factor intensity assumptions, a factor-neutral improvement in Q_1 results in increases in its output and characteristic endowment but a decrease in Q_2.

3.2 Technical Change in Q_2 Commodity Sector. The above discussion on the effects of technical improvement in sector Q_1 provides an analogy of the effects of technical improvement in sector Q_2.

4. CHANGES IN PRODUCTION

Let t and t', respectively, represent the shift parameters in K and L. Initially, t = t'= 1. Technical progress is K-saving if dt > 0 and dt' = 0 or t/t' increases; L-saving if dt = 0 and dt' > 0 or t/t' decreases; and factor-neutral if dt = dt' > 0 or t/t' is constant. With factor-neutral technical change occuring only in the first commodity sector, the production relation for Q_1 in section 2 can be rewritten as:

$$(1') \quad Q'_1 = F_1(K_1, L_1, t/t') = t' Q_{L_1} f_1(t/t', k_1)$$

The production transformation curve equations become

$$(3') \quad Q'_1 = g(Q_1)$$

$$(4') \quad \frac{dQ'_1}{dQ'_2} = g' < 0$$

The new technology in producing the characteristics endowment of Q_1 becomes

$$(5') \quad Z_1 = a'_z t' Q_{L_1} f_1(t/_{t'_1}, k_1) = (t/_{t'}) a_{z_1} Q_1$$

The marginal productivity conditions are:

$$(7') \quad r' = tf_1 = p'f_2$$

$$(8') \quad w' = t'f_1 = tk_1 f_1 = p'(f_2 - k_2 f'_2)$$

The full employment conditions in equations (11) and (12) are retained.

The price of Q_1 in terms of marginal implicit price of its characteristic, is

$$(9') \quad p'_1 = \hat{a}'_{z1} p'_{z1}$$

The factor demand functions are,

$$(13') \quad L' = L'(p', w', r')$$

$$(14') \quad K' = K'(p', w', r')$$

The supply function of each commodity is,

$$(15') \quad Q'_1 = F'_1[Q'_1(p', w', r')] = Q'_1(p', w', r')$$

$$(16') \quad Q_2 = F_2[Q'_2(p', w', r')] = Q_2(p, w', r')$$

The output effect of factor-neutral technical change in Q_1 is depicted in a new transformation curve (quadrant I of Figure 5). The progressive sector and its characteristics endowment (Q_1, Z_1) increases at the expense of the other sector (Q_2, Z_2). This result is shown in the post-technological improvement production transformation curve $X'_1X'_2$ and in the characteristic transformation curve T'_1,T'_2. The output and characteristics endowment effects described here is termed - ultra-biased - in most of the literature (see for example Findlay and Grubert, 1959).

5. CHANGES IN THE CHARACTERISTICS SPACE

The post-technical progress characteristic endowments of Q_1 and Q_2 are translated into a new characteristics transformation curve $T'_1T'_2$ (quadrant III of Figure 5) by:

(17') $dZ'_1 = \hat{a}'_{z1}dQ'_1$

(18) $dZ_2 = \hat{a}_{z2}dQ_2$

$$(19')\ \frac{dZ'_1}{dZ_2} = \frac{aZ'_1}{aZ_2}\frac{aQ'_1}{aQ_2} = \frac{aZ'_1}{aZ_2}g' < 0$$

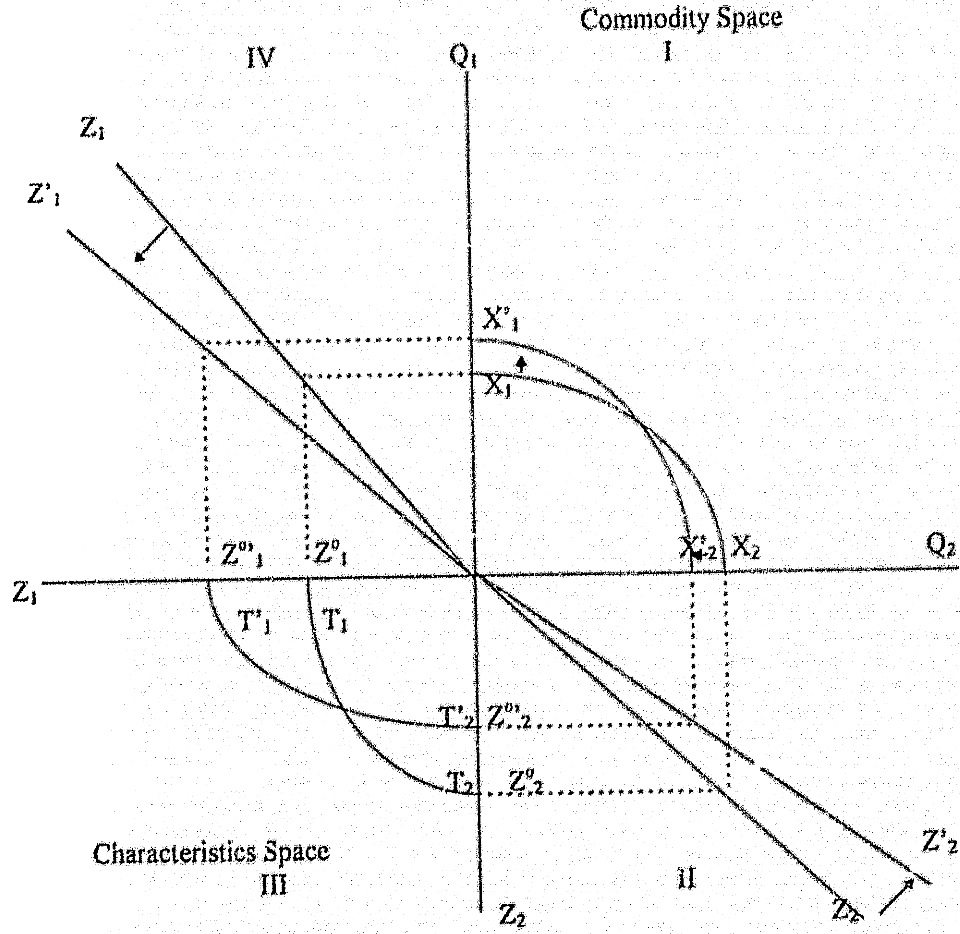

Figure 5. Commodities and Their Characteristic Endowments With Ultra-Biased Neutral Technical Progress in Q_1

The utility function becomes

$$(21')\ \text{Max } U_2 = U_2(Z'_1, Z_2)$$

subject to:

$$(22')\ Z'_1 = g'_1(Z_2)$$

The first-order utility maximization condition is,

$$(23')\quad L' = U'[Z'_1, Z_2] - \lambda[Z'_1 - g'_1(Z_2)]$$

$$(24')\quad \frac{\partial U}{\partial Z'_1} - \lambda = 0$$

With technical improvement utility function in the characteristics space is maximized a point e'_z (quadrant III of Figure 6). At this point, the equilibrium solution is given, where the characteristics indifference curve is tangent to the transformation curve $T'_1\ T'_2$. In the commodity space, there is a corresponding new point of tangency between the transformation curve and the price line (point e'_q in quadrant I of Figure 6)

$$(26')\quad \frac{\partial U / \partial Z_1}{\partial U / \partial Z_2} = \frac{1}{g'_1(Z_2)}$$

$$(27')\quad \frac{d\,Q'_1}{d\,Q'_2} = p'$$

The new solution for the demand function in the characteristics and commodity spaces are:

$$(28')\ Z^{d'}_1 = Q^{d'}_1(p', Y, \psi_1)$$

$$(29')\ Z^d_2 = Q^d_2 = Q^d_2(p', Y, \psi_2)$$

where ψ_i (i = 1,2) represents either a change in consumption due to substitution effects or a demand shift due to a change in taste, an increase in income or in population. It is assumed that ψ_1 is positive and ψ_2 is negative.

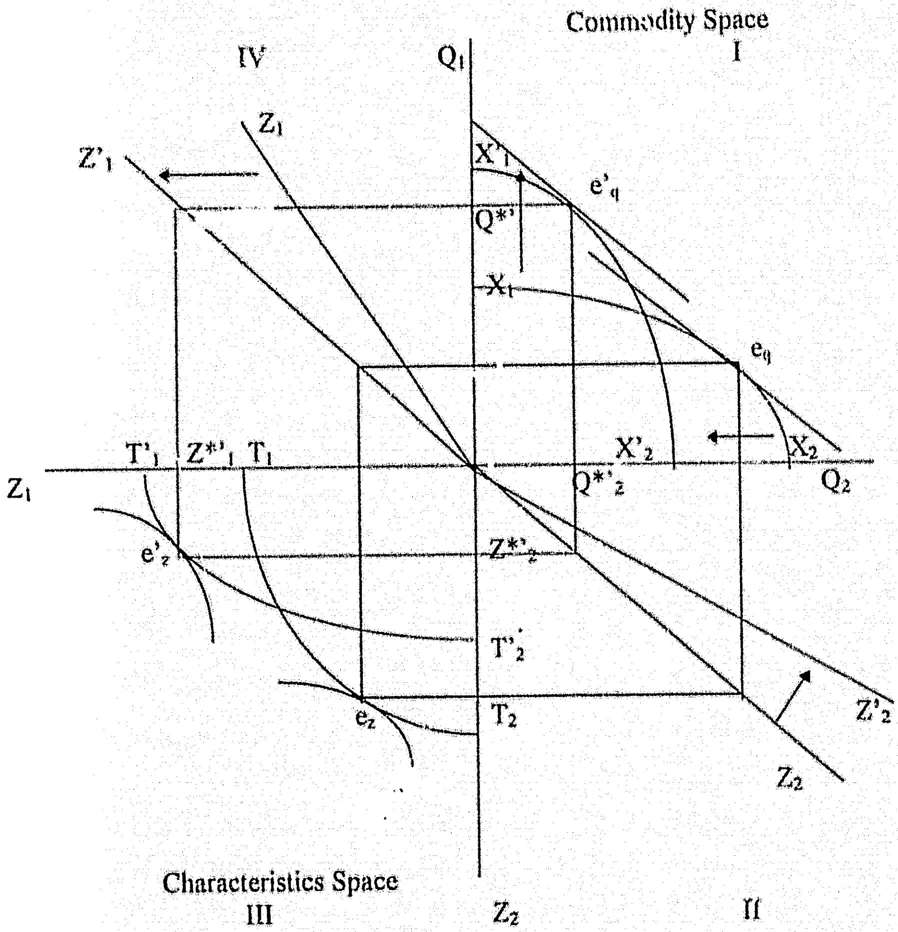

Figure 6. Equilibrium in the Commodity and Characteristics Spaces With Ultra-Biased Neutral Technical Progress in Q_1

5.1 Changes in Consumption. Assuming constant level of Z_2, in Q_1, constant prices and income , the effects of a change in the characteristic of Q_1 can be viewed from the substitution between Q_1 and Q_2. The effect of a change in Z_1, upon the consumption of Q_1 depends upon marginal utilities and substitution terms.

$$(34)\quad \frac{dQ_2}{dZ_2} = -\frac{1}{\lambda}\frac{dUQ_1}{dZ_1}\sigma q_1 q_2$$

where λ is as earlier defined. $\sigma q_1 q_2$ is the substitution term between the two commodities. The consumption of Q_2 may vary.

Suppose $\frac{dUQ_1}{dZ_1} > 0$ but $\frac{dUQ_2}{dZ_2} = 0$

for $Q_1 \neq Q_2$, increasing Z_1 increases the marginal utility of Q_1 without a change in the marginal utility of Q_2, then

$$(35)\quad \frac{dQ_2}{dZ_2} = -\frac{1}{\lambda}\left(\frac{dQ_1}{dZ_1}\right)\sigma q_1 q_2$$

If Q_1 and Q_2 are substitutes $(q_1 q_2 > 0)$, increasing Z_1 decreases the demand for Q_2; if Q_1 and Q_2 are complements $(\sigma q_1 q_2 < 0)$, increases the demand for Q_2. Setting $Q_1 = Q_2$

$$(36)\quad \frac{dQ_2}{dZ_2} = -\frac{1}{\lambda}\left(\frac{dQ_1}{dZ_1}\right)\sigma q_1 q_2 > 0$$

5. CHANGES IN EQUILIBRIUM

The new equilibrium in the characteristics and commodity spaces can be given by

(30') $Z^{d'}{}_1 = Z'_1$ (32') $Q^{d'}{}_1 = Q'_1$

(31') $Z^d{}_2 = Z_2$ (33') $Q^d{}_2 = Q_2$

The new equilibrium for Q_1 and Q_2 arising from the changes in consumption and production due to technical progress are derived using a similar procedure to that used in deriving the initial equilibrium discussed in section 2. With technical progress occuring only in Q_1 the supply curves in the characteristics and commodity spaces in that sector shift downwards (refer to Figures 7 and 8). The demand for commodity characteristics and hence the commodity itself may shift upwards. Following consumer demand theory, the conditions for a demand shift are: change in tastes, population or income. An ultra-biased factor neutral technical advance in the first sector has the following effects. In a two-sector economy

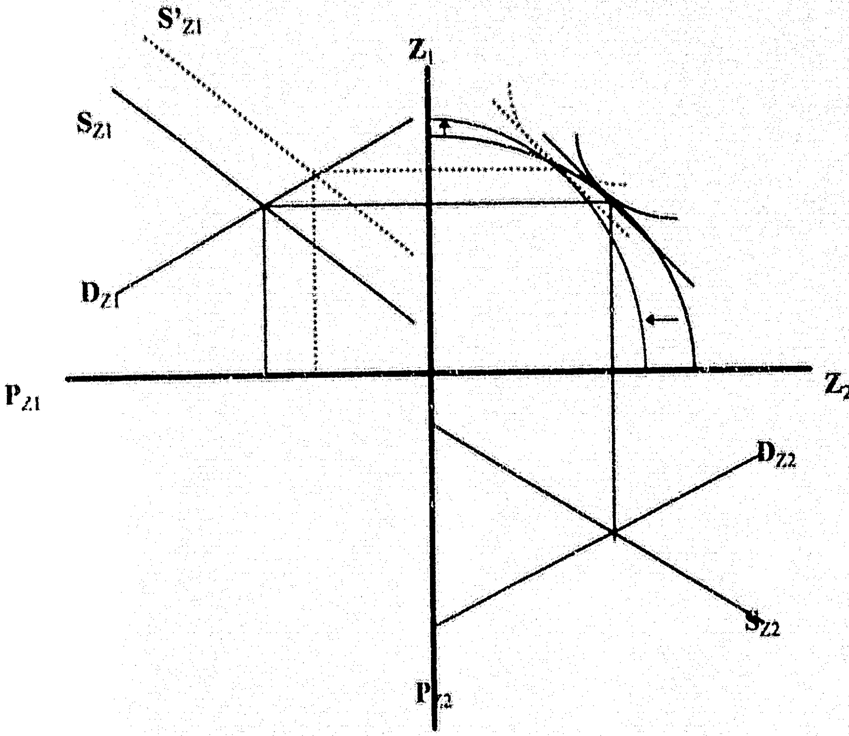

Figure 7. Supply Shift in the Characteristics Space

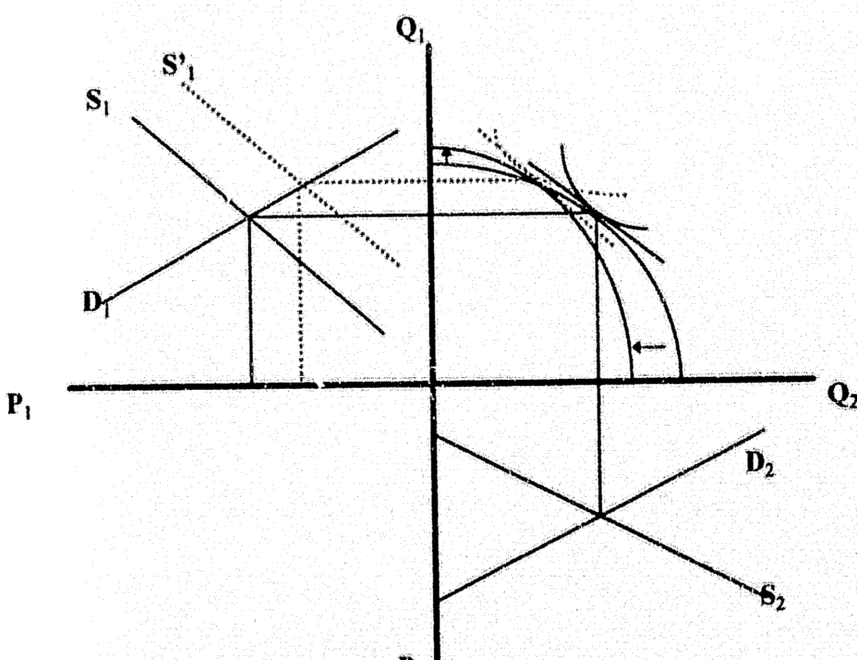

Figure 8. Supply Shift in the Commodity Space

model, the new equilibrium position may be a higher or lower point due to the changes in consumption and production in both sectors. If the expansion in the progressive sector more than offsets the contraction in the non-progressive sector, the equilibrium is at a higher point. However, a case of immiserising growth occurs if the contraction in the sector outweighs the gain in the first sector (Bhagwati, 1958).

When the technical advance occurs in both sectors, the supply curves in each sector shift downwards. The vertical shift however, may not be equal depending upon the extent of progress in each sector. The new equilibrium is at a higher point due to the expansion in both sectors. A neutral technical change in both sectors and changes in the commodity and characteristics spaces are illustrated in Figure 9.

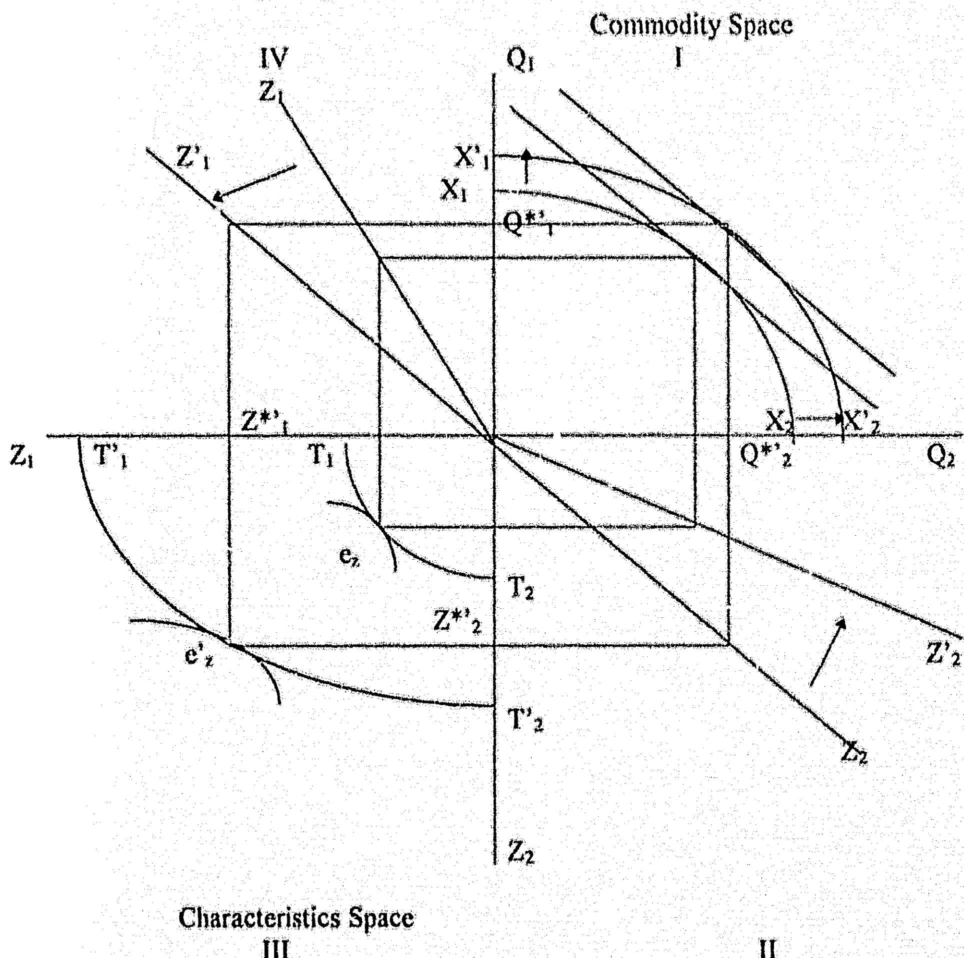

Figure 9. Equilibrium in the Commodity and Characteristics Spaces With Neutral Technical Progress in Q_1 and Q_2

8. SUMMARY AND IMPLICATIONS TO RESEARCH ASSESSMENT

In this chapter the relationship of quality characteristics of a commodity to its consumption and production is explained in terms of a simple two sector, two-factor general equilibrium model with considerations of technological progress. The new consumer demand theory, which takes into account the quality characteristics of a commodity in the demand for a commodity, provides a basis for analysing the effects on consumption of a change in quality characteristics resulting from technical change. By introducing utility functions in the characteristics and commodity spaces, this paper has demonstrated the duality of these functions.

On the production side, a commodity is seen as comprising characteristics produced by a given technology. The impacts of biased and neutral technical changes on commodity characteristics are explained in parallel with factor intensities, factor prices, and outputs. Technical improvements alters the relationship between inputs and their rewards, affecting the relationship between production costs and output, thus between supply and price. Some technical improvement provides qualitative changes in commodities which may result also in changes in consumption or demand. The consequences of research-induced technical progress can be evaluated by looking at the effects on production and consumption. The changes in consumption and production at the new equilibrium positions are used to measure the benefits from research-induced technical progress.

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