

FINANCING THE ENVIRONMENTAL CAPACITY THROUGH INCOME TAX

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Introduction

Over the past decade several economists have discussed the issue of optimal public investment in pollution control capital in dynamic optimization models.[1] Their principal focus has been to analyze the optimal mix between private (or directly productive) capital and public capital employed in waste treatment. Unfortunately, their analytical models have ignored the mechanics of financing the public investment. The purpose of this paper is to take up this issue in the context of a growing, decentralized, competitive economy.

Certainly, if the economy is completely centralized and if the government has perfect information on the production possibilities, then it is possible to implement any feasible growth path, even the optimal growth path. If, however, the economy is decentralized and if the government has only a limited set of policy instruments, then it is no longer clear that the optimal path can be implemented. Generally, one might ask two natural questions: (1) Can the publicly optimal investment policy be achieved by the environmental authority with a given set of policy instruments? (2) If the optimal policy is not achievable, what is the nature of the "second-best" policy?

The first question is one of controllability defined, for example, by Arrow and Kurz as follows: "A policy is said to be controllable by a given set of instruments varying over time, in general, which cause the private and government sectors together to realize that policy" [1, p. 120]. When specific policy instruments are introduced and are insufficient to control the publicly optimal policy, a second-best policy is followed, which is the best that can be achieved with the given set of instruments. The characteristics of a second-best policy obviously depend on what kind of policy instruments are used. For the present problem, since environmental quality is a public good by nature, so that its price cannot be suitably determined in the market and its costs cannot be fully recaptured, taxation of current income would seem to be a natural means for financing the public investment.

This paper will explore the above two questions for a prototype competitive economy. In the following section the decentralized model will be fully described. As a point of departure, we will amend Comolli's planning formulation [3] of the problem in order to be able to compare the optimal policy in a

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[1] [3], [4], [6], [8], [10], [11]

completely centralized economy with a second-best policy in the decentralized economy. A general analysis of the income-tax financing policy will be undertaken in Section III. A principal result is that such a policy exists in the long run, though it may not correspond to the optimal policy for the completely centralized economy. A full set of conclusions is provided in Section IV.

Model

We assume that the environmental authority operates with a balanced budget and finances the public investment, I_g, through an income tax levied at rate x, $0<x<1$. In the Keynesian tradition aggregate consumption, C, is taken to depend solely on current disposable income, and the ratio of private saving to current disposable income is taken to be a constant number s, $0<s<1$. Denoting aggregate output by Y, consumption my be written as:

(1) $C = (1\text{-}s)(1\text{-}x)Y$

Because private saving is allocated to private investment, I_p, we have

(2) $I_p = s(1\text{-}x)Y$

So that net investment in directly productive capital, $\dot{K}p$, stand as

(3) $\dot{K}_p = s(1\text{-}x)Y\text{-}mK_p$

where $m>0$ is a constant rate of depreciation.

All tax revenue is assumed to be allocated to public investment by the environmental authority, thus

(4) $I_g = xY$

Following Comolli [3] the stock of public capital, K_g, is assumed to be measurable, as a capacity for assimilating the economy's effluents, E. Under this interpretation a constant rate of depreciation seems overly simplistic. Therefore, let us posit that Kg depreciates exponentially at a rate dependent on its utilization E/Kg, say E/Kg to simplify notation. With this assumption, net public investment in the economy is given by

(5) $\dot{K}g = xY - mE$

Finally, it will be assumed throughout that the stock of labor (population) is constant.

The environmental authority, as is common practice, will be taken to use direct control as a means of regulating polluting activities. Assuming that wastes are generated only in production processes, a direct control may mean giving individual firms permits to pollute at some specified levels and prohibiting them from exceeding these levels. Thus, if a firm violates this prohibition, whether by small or large amounts, it is considered a law-breaker subject to punitive action. It will be supposed that firms can internally reduce their effluents to legally acceptable levels by treatment (e.g., recycling of wastes). Mäler [9] has then shown how the aggregate production function

$$(6)\quad Y = f(K_p, E)$$

may be derived by technical optimization.[2] It will be assumed that (6) has the following properties:

A.1. $f(K_p, E)$ is concave function that possesses positive and diminishing marginal products and satisfy the Inada conditions.[3]

A.2. K_p and E are nonrivalrous, i.e., $f_{12} \geqq 0$

As Comolli [3] described, the level of environmental quality may be expressed by some numerical index Z, which seems, like temperature, to be cardinally measurable. It seems reasonable that this index, Z, has the following functional form

$$(7)\quad Z = Q(K_g, E)$$

for which the following assumptions can be made.

A.3. Q is concave, and has a positive partial derivative in K_g, a negative partial derivative in E, and zero cross partial derivatives.

A.4. Since the choice of zero is arbitrary, $Q(K_g,E) \gtreqless 0$ as $K_g \gtreqless E$.

The concavity assumption of Q in E is based on the threshold nature of environmental quality, which implies that the environmental quality would be degraded very sharply beyond some level of effluents. The assumption of zero cross partial derivatives is used for a purely technical purpose. This,

[2] The technical optimization problem is as follows:

$$\max Y = H(K^1_p)$$
$$\text{subject to}\quad K^1_p + K^2_p \leqq K_p$$
$$E \geqq G(K^1_p, K^2_p)$$
$$K^i_p \geqq 0,\ i = 1,2,\ 0 \leqq E \leqq E^*$$

where K^1_p and K^2_p are private capital stocks allocated to production and to waste treatment, respectively, and E^* is some specified level of wastes at which firms are allowed to discharge. The solution to their maximization problem gives output as a function of total available factor input, K_p and of the discharge of waste, E, or $Y = f(K_p,E)$. See Mäler [9, pp. 28-30] for more detail.

[3] That is, $f_1 > 0,\ f_2 > 0,\ f_{11} < 0,\ f_{22} < 0,\ f_{11}f_{22} - f^2_{12} \geqq 0$
$$f_1(0,E) = \infty,\ f_1(\infty,E) = 0,\ \forall E \geqq 0$$
$$f_2(K_p,0) = \infty,\ f_2(K_p,\infty) = 0\ \forall K_p \geqq 0$$

however, may be reasonably assumed since there is no reason to say that marginal environmental quality with respect to K_g(or E) is changed due to one unit change in E(or K_g). This environmental quality plays an important role on the social welfare. Thus an aggregate utility function, which is presented to exist for the society, will be U(C,Z), where

$$(8) \quad U(C,Z) = u(C) + v(Z)$$

for computational convenience. We introduce two assumptions.

A.5. U has positive and diminishing marginal utilities.

A.6. $u'(0) = +\infty = v'(0)$, and $v(Z) = -\infty$ for all Z, $Z < 0$.

In A.6. the former is reasonable with respect to human health, while the latter serves a purely technical purpose.[4] Social welfare is taken to be measured by the utilitarian criterion

$$(9) \quad J = \int_0^\infty U(C,Z)e^{-\rho t}\,dt$$

with $\rho>0$ reflecting the society's constant rate of time preference.

Then the problem of the environmental authority is to maximize (9) subject to (1), (3), (5), (6), and (7) and given initial conditions: $K_p(0) = K_p^o > 0$ and $K_g(0) = K_p^o > 0$. The admissable control variables are measurable functions, x(t), E(t) satisfying $0 \leqq + \leqq 1$ and $0 \leqq E \leqq K_g$. The latter is the consequence of our assumptions, A.4. and A.6. that $v(Z) = -\infty$ for all Z, $Z<0$ if and only if $E > K_g$. Thus $E > K_g$ is obvicusly infeasible.

An Analysis

The second-best optimality. The planning problem of our economy reduces to maximize the current-value Lagrangian with respect to x and E. That is,

$$\max L = u[(1-s)(1-x)f(K_p,E)] + v[Q(K_g,E)] + P_p[s(1-x)f(K_p,E) - mK_p]$$
$$+ P_g[xf(K_p,E)-mE] + \mu_1 x + \mu_2(1-x) + \gamma_1 E + \gamma_2(K_g-E)$$

where the costate variables, P_p and P_g are the current-value shadow prices of private and public investment, respectively, and μ_1, μ_2, γ_1 and γ_2 are Lagrange multipliers resulting from the inequality constraints on x, E, respectively.

Necessary conditions for optimality (in this case, second-best optimality) are as follows:

[4] This assumpton suggests that the choice of zero for Z is no longer arbitrary. Nevertheless, there is no loss in generality in our assumption, granted that $v(Z) = -\infty$, $Z < \bar{Z}$, for some $\bar{Z}$ constant.

(10.a) $x[(1-s)u' + sP_p - P_g] = 0, (1-s)u' + sP_p - P_g \gtreqless 0; x \gtreqless 0$

(10.b) $(1-x)[(1-s)u' + sP_p - P_g] = 0, (1-s)u' + sP_p - P_g \gtreqless 0; x \gtreqless 1$

(11.a) $[(1-x)\{(1-s)u' + sP_p\} + xP_g]f_2 - mP_g + v'Q_2 \leqq 0;$ $E[[(1-x)\{(1-s)u' + sP_p\} + xP_g]f_2 - mP_g + v'Q_2] = 0; E \geqq 0$ $[(1-x)\{(1-s)u' + sP_p\} + xP_g]f_2 - mP_g + v'Q_2 \gtreqless 0$

(11.b) $(k_g - E)[[(1-x)\{(1-s)u' + sP_p\} + xP_g] f_2 - mP_g + v'Q_2] = 0; E \leqq K_g$

(12.a) $\dot{K}_p = s(1-x)f(K_p,E) - mK_p$

(12.b) $\dot{K}_g = xf(K_p,E) - mE$

(13.a) $\dot{P}_p = (\rho + m)P_p - f_1[(1-x)\{(1-s)u' + sP_p\} + xP_g]$

(13.b) $\dot{P}_g = \rho P_g - v'Q_1 - \gamma_2$

We note that u' is the shadow price of consumption and $(1-s)u' + sP_p$ can be interpreted as the shadow price of disposable income since disposable income is allocated between consumption and private investment in the proportions 1–s, s, respectively. Let $P_I = (1-s)u' + sP_p$. In (10.b), $x=1$ is optimal if $P_I \leqq P_g$. But for $x=1$, $C=0$, so that $u'(0) = +\infty$, by assumption A.6 and the $P_I = +\infty$ and $\partial L/\partial x = -\infty$, a contradiction. This asserts that income tax rate should be less than 100 percent (i.e., $x<1$). Once $x>1$, $P_I = P_g$. In (10.a), $x = 0$ is optimal if $P_I \gtreqless P_g$. This cannot be excluded because consumption is positive from (1). If $x>0$, again $P_I = P_g$. In summary,

(14) if $x = 0$, $P_I \gtreqless P_g$
and if $0 < x < 1$, $P_I = P_g$

These results imply that in order to maximize L, if the shadow price of disposable income is greater than that of public investment, then the income tax should be zero, and in the case of positive income tax, the optimal tax could be achieved when both shadow prices of disposable income and public investment are the same. From (11.a), $E = 0$ is optimal if $\partial L/\partial E \leqq 0$. But by Inada condition, $f_2(K_p,0) = +\infty$ and hence $\partial L/\partial E = +\infty$, a contradiction. This contradiction reminds us of the Ayres and Kneese's [2] material balance view.[5] From (11.b), $E = K_g$ is optimal if $\partial L/\partial E = 0$. However, for $E = K_g$, $Z = 0$, so that $v'(0) = +\infty$ by assumption A.6 and then $v'Q_2 = -\infty$, a contraction. For these reasons, E should be strictly between zero and K_g (i.e., $0 < E < K_g$). Then (11) and (13.b) become, respectively,

[5] This view indicates that tonnage of raw materials extraction utilized by an economy is approximately equal to waste residuals generated by the economy in the long run. This contradiction becomes obvious from this view.

(11.c) $[(1-x)P_l + xP_g]f_2 - mP_g = -v'Q_2$

(13.c) and $\dot{P}_g = \rho P_g - v'Q_1$

From (14), it is known that both cases of zero and positive income taxes are feasible. Hence we now consider each case separately.

Case 1: $x = 0$

The nature of this case is equivalent to what Keeler, Spence, and Zeckhauser [7] called the "Murky Age" during which no money is spent on pollution control. This case might occur in the economy, either where K_g is initially large enough to keep acceptable level of environmental quality without making any further public investment, or where, as in many underdeveloped countries, the central planner concentrates on economic development without regard to environmental quality. In any case, the opportunity cost of disposable income is greater than that of public investment. From (10.a) and (11.c), $(f_2-m)P_g \leqq -v'Q_2$. This implies that when assimilative investments are not undertaken, firms are also allowed to discharge effluents into the environment even though the value of their net marginal product is less than the value of the deterioration in environmental quality. This might support the authority's neglectful policy on pollution control. However, this might cause serious degradation in environmental quality. In fact, as time goes by, the stock of pollution may increase without any public investment for abating pollution. Since we assume that public capital depreciates at a rate depending upon the level of pollution, the stock of it will approach zero in the long run. This becomes obvious from (12.b), $\dot{K}_g = -mE$, so that, defining $\mu \equiv E/K_g$ where $0<\mu<1$, $K_g(t) = K_g^0 e^{-m\int_0^t \mu(\tau)d\tau}$ and $\lim_{t\to\infty} K_g(t) = 0$. This process asserts that the environmental authority should start to invest in environmental capacity after a while. Otherwise, a point of time will come after which environmental capacity is not enough for assimilating pollution to keep a certain level of environmental quality. That is, after some time period, $E \geqq K_g$, which is not feasible. Thus, the income tax rate will become positive in the long run. Besides, this is not rather interesting case because a steady state doesn't exist. Again, from (12.b), $\dot{K}_g = 0$ at a steady state, so that $E = 0$, a contradiction.

Proposition 1: Under the specification of the model, there does not exist a steady state with zero income tax. Instead, there does exist time t beyond which income tax becomes positive.

Case 2: $0 < x < 1$

This is the case of what Keeler, et al. [7] called the "Golden Age," which is characterized by lower private investment, consumption and better environmental quality than the "Murky Age." In this case, (11.c) becomes

(11.d) $(F_2-m)P_g = -v'Q_2$

This asserts that optimal increase in effluents should equate the value of their "net" marginal product with the value of the deterioration in environmental quality. That is, in order to maximize social welfare, the environmental authority should allow the firm to discharge effluents up to the specified level at which the value of effluents' net marginal product equals the value of the deterioration in environmental quality.

Totally differentiating (14) and (11.d),

$$(15)\quad -(1-s)^2u''fdx + (1-s)^2(1-x)f_2dE = -(1-s)^2(1-x)u''f_1dkp - sdP_p + dP_g \quad -(f_{22}P_g + v''Q_2^2 + v'Q_{22})dE = f_{12}P_gdKp + (v''Q_1Q_2 + v'Q_{12})dKg \frac{v'}{P_g} Q_2dP_g$$

The matrix of coefficients on the left-hand side of the above system is P – matrix since all principal minors are positive. Thus, by theorem 4 of Gale and Nikaido [5] there is a unique solution of the functions, $\hat{x}(K_p,K_g,P_p,P_g)$, $\hat{E}(K_p,K_g,P_p,P_g)$. Table 1 shows the probable signs of the various partial derivatives obtained from (15) as well as (1) and (6).

TABLE 1
Summary of Probable Signs

Derivative of	with respect to			
	K_p	K_g	P_p	P_g
X	?	–	–	?
E	+	+	0	+
Y	+	+	0	+
C	+	+	+	?
Z	–	?	0	–

Conclusively, Pontryagin's necessary conditions indicate that the movement along the second-best optimal path is governed by the differential equations for the state variables, (12), and those for the costate variables, (13), with the control variables, x and E, being determined by (14) and (11.d).

Existence of a Steady State. In the previous section, we discussed that there does not exist a steady state in the "Murky Age." We now show the existence of a steady state in the "Golden Age." In a steady state, the relevant conditions can be summarized as follows:

$$(16)\quad (1-s)u' + sP_p = P_g$$

$$(17)\quad (f_2-m)P_g = -v'Q_2$$

$$(18)\quad \dot{K}_p = s(1-x)Y - mK_p = 0$$

$$(19)\quad \dot{K}_g = xY - mE = 0$$

$$(20)\quad \dot{P}_p = (\rho+m)P_p - f_1P_g = 0$$

$$(21)\quad \dot{P}_g = \rho P_g - v'Q_1 = 0$$

$$(22)\quad \text{and } Y = f(K_p,E)$$

$$(23)\quad C = (1-s)(1-x)Y$$

Note that all control and state variables in the above are stationary values. Thus, the existence problem of a steady state becomes that of a solution in the above equation system. Under assumption (A.3), we may reasonably assume that $Q(K_g,E) = \beta(K_g - E)$, $K_g>E$. This helps to make the proof of existence easier without loss of generality. Then (17) and (21) become

$$(17')\quad (f_2-m)P_g = \beta v'$$

$$(21')\quad \text{and } \rho P_g = \beta v'$$

It follows then from (17') and (21') that

$$(24)\quad f_2(K_p, E) = \rho + m$$

By Inada condition, there exists a solution $\tilde{E}(K_p)$, with

$$(25)\quad E'(K_p) = -{}^f12/{}^f22 \geqq 0, \text{ for any } K_p > 0$$

Eliminating x between (18) and (19) and substituting for Y in (22) yields

$$(26)\quad sf[K_p, \tilde{E}(K_p)] = m[K_p + s\tilde{E}(K_p)]$$

If there exists a solution K_p for equation (26), the existence of a steady state will be realized. This is so because $\tilde{Y}$ is determined by (22) and then $\tilde{x}$ is detemined by either (18) or (19), the other being automatically satisfied by (26). Then $\tilde{C}$ is determined by (23) and thereby u', so that $\tilde{P}_p$ and $\tilde{P}_g$ are defined by (16) and (20). The $\tilde{K}_g$ is also determined by either (17') or (21'), in view of assumptions (A.3) and (A.5), the other being automatically satisfied by (24). Thus, the proof of the existence of a steady state is reduced to that of the existence of a solution for (26). For this, we shall make the following assumptions.

A.7 $f(K_p,E)/(K_p+E)$ can be made arbitrarily small for $K_p + E$ large.

A.8 If K_p converges to zero and (K_p,E) vary along $f_2 = \rho + m$, then K_p/E converges to zero.[6]

[6] Two more alternative assumptions can be made:
$f_2(O,E) = \rho + m$, some $E > 0$
or $f_1(0,0) > m/s$
The former is similar to A.8 and the latter is tenable in view of the Inada condition. The proof of the existence under each assumption can be done very similarly to Arrow and Kurz [1, p. 136].

A.7 implies that aggregate output per total inputs used converges to zero (for fixed labor forces) when K_p approaches to infinity.

Now, let us define $F(K_p) = sf[K_p,\tilde{E}(K_p)] - m[K_p + s\tilde{E}(K_p)] = 0$ from (26). To guarantee that F has a solution in K_p, we require

(27) $\lim_{K_p \to \infty} F(K_p) < 0$

(28) and $F(K_p) > 0$, for some $K_p < +\infty$

By assumption A.7, as K_p tends to infinity,

$$sF[K_p,\tilde{E}(K_p)] < ms[K_p + \tilde{E}(K_p)] < m[K_p + s\tilde{E}(K_p)],$$

so that (27) holds. Since f is concave, we have the well-known inequality

$$f_1[K_p,\tilde{E}(K_p)](K_p - 0) + f_2[K_p,\tilde{E}(K_p)](\tilde{E}(K_p) - 0) \leqq f(K_p,\tilde{E}(K_p)] - f(0,0)$$

Since $f(0,0) \geqq 0$ and $f_2[K_p,\tilde{E}(K_p)] = \rho + m$, $f[K_p,\tilde{E}(K_p)] \geqq f_1[K_p,E(K_p] + (\rho + m)\tilde{E}(K_p)$. Then it is sufficient for (28) to hold that

$$sf_1[K_p,\tilde{E}(K_p)]K_p + s(\rho + m)\tilde{E}(K_p) - m[K_p + s\tilde{E}(K_p)] \geqq 0.$$

(29) That is $s\rho \geqq (m - sf_1)\ [K_p/\tilde{E}(K_p)]$

Since $s > 0$ and $\rho > 0$, the LHS is positive, so that a sufficient condition for (29) is that

(30) $\lim_{K_p \to 0} p\ (m - sf_1)[K_p/\tilde{E}(K_p)] \leqq 0$

Here, since $f_1 > 0$, $m - sf_1 < m$. Hence, by A.8

$$\limsup_{K_p \to 0}\ (m - sf_1)[K_p/\tilde{E}(K_p)] < \lim_{K_p \to 0}$$

$\sup m[K_p/\tilde{E}(K_p)] = 0$ so that (30) certainly holds.

Uniqueness of the Steady State. We have just shown that the solution to the equations (24) and (26), say K_p and E does exist under the assumptions A.1, A.7 and A.8 when Q is linear function. We also know that once K_p and E are determined, all other variables — such as x, C, Y, P_p, P_g and K_g — are uniquely determined. Thus, the uniqueness question of the steady state boils down to that of a solution for equations (24) and (26). That is, it is whether these curves intersect only once in $K_p - E$ plane.

Now let us define

(31) $\phi(K_p,E) \equiv (\rho + m) - f_2(K_p,E)$

(32) and $\psi(K_p,E) \equiv sf(K_p,E) - m(K_p + sE)$

In fact, the function $\phi(K_p,E) = 0$ represents $\dot{P}_g/P_g = 0$ in view of (21), (17') and (21'), while the function $\psi(K_p,E) = 0$ represents $\dot{K}^p + s\dot{K}_g = 0$ from (18), (19) and (22). Both of these functions may be regarded as the long-run functions. The long-run function $\phi(K_p,E) = 0$, with derivatives

$$\partial\phi/\partial K_p = -f_{21} \lesseqqgtr 0$$

$$\text{and } \partial\phi/\partial E = -f_{22} > 0$$

may be represented by a curve in $K_p - E$ phase plane with slope

$$dE/dK_p|_\phi(K_p,E) = 0 = -(\partial\phi/\partial K_p)/(\partial\phi/\partial E) = -f_{12}/f_{22} \gtreqqless 0$$

where this curve starts from some positive E because $\phi(K_p,E) = 0$ would, otherwise, not be satisfied in view of Inada condition. Similarly, the curve in the plane for $\psi(K_p,E) = 0$, with derivatives,

$$\partial\psi/\partial K_p = sf_1 - m \text{ and } \partial\psi/\partial E = s\rho > 0$$

has the slope

$$dE/dK_p|\psi K_p,E) = 0 = -(\partial\psi/\partial K_p)/\partial\psi/\partial E) = (m - sf_1)/s\rho$$

whose sign depends on the sign of $m - sf_1$. Now for our purpose, we need to check

$$dE/dK_p|\phi = 0 \gtreqless dE/dK_r|\psi = 0$$

which is equivalent to

(33) $-f_{12}/f_{22} \gtreqless (m - sf_1)/ s\rho$

Naturally, if $f_1(K_p,E) > m/s$ then the steady state is unique without further analysis; that is, $\phi(K_p,E) = 0$ is nondecreasing in $K_p - E$ phase plane, while $\psi(K_p,E) = 0$ is strictly decreasing. In view of (30), however, this is the case only when K_p approaches zero. Alternatively, the sufficient condition for uniqueness is that $f_{21}K_p + f_{22}E < 0$ from (29) and (33)[7]; both curves are nondecreasing in the phase plane, but the slope of $\psi(K_p,E) = 0$ is greater than that of $\phi(K_p,E) = 0$. This condition implies that effluents' marginal product is decreasing as K_p and E increase in any fixed proportion. This condition will not hold if there are increasing returns. Hence, it may be more reasonable to say

[7] This can be shown in the following way:
From (29), $\tilde{E}(K_p)/K_p \gtreqqless (m - sf_1)/s\rho$
Combining (33) with this, $\tilde{E}(K_p)/K_p \gtreqqless (m - sf_1)/s\rho > -f_{12}/f_{22}$
Hence $f_{21}K_p + f_{22}E < 0$.

that $f_1(K_p,E) > m/s$ for very small K_p and $f_1(K_p,E) < m/s$ for larger K_p. That is, $\psi(K_p,E) = 0$ curve falls and then rises in the phase plane. Thus, without additional restrictions, unfortunately, it is impossible to be specific about the direction of the inequality in (33). Hence, the $\phi = 0$ curve may intersect the $\psi = 0$ curve at numerous points.

Before concluding, it may be instructive to characterize some aspects of a steady state. Condition (17) has the same form as in Comolli's [3] publicly optimal policy. Moreover, equation (24), which implies that effluent's "net" marginal product equals the "social rate of interest" at any stationary point, is exactly the same as the one in Comolli's. This result asserts that the level of effluent could be controlled optimally by the environmental authority even in the decentralized economy just as by the one with full power over the economy. On the other hand, private capital's "net" marginal product necessarily differs from the "social rate of interest." To see this suppose $f_1 - m = \rho$. From (20), $P_p = P_g$. Let P be the common value of these shadow prices. Then (16) simplifies to $u' = P$. This notes that all the shadow prices of consumption, private and public capital are the same. If we let $K = K_p + K_g$, it follows from (18), (19), and (23) that $Y = C + m(E + K - K_g)$. This implies $\dot{K} = 0$, which defines a stationary locus in Comolli's publicly optimal model. The inequality among the shadow prices of consumption, private investment and public investment is an indicator of the failure to achieve the optimum possible in perfectly planned economy. Moreover, it may be noted from (20) and (24) that when Q is linear, the ratio between the shadow prices of private and public capital is equal to the marginal rate of technical substitution of private capital for the joint input of effluents at a stationary point.

The foregoing results are summarized succinctly in the following.

Proposition 2: Under the specification of the model, there exists a steady state with positive income tax if Q is linear, even though the uniqueness of it is uncertain. At any stationary point, effluent's net marginal product equals the "social rate of interest." Private capital's net marginal product, however, is not equal to the "social rate of interest" unless the stationary solution for the second-best policy happens to be that of the publicly optimal policy.

Concluding Remarks

We have seen that the optimal policy is not controllable when environmental authorities with balanced budgets impose only income tax and use direct control on a firm's polluting activities, and private savings are a fixed proportion of disposable income. This is caused by the differences among the shadow prices of consumption, private capital and public capital. We also saw that without making any public investment in environmental capacity, there does not exist a steady state, ending up with serious deterioration in environmental quality. These results warn any country against expanding its economy without taking serious consideration of its environment. It is suggested that in order to maximize the social welfare, environmental authorities should finance public investment with an optimal tax rate at which the shadow price of disposable income equals that of public capital, and allows firms to increase effluents up to the specified level at which the value of effluents' net marginal product equals the value of the deterioration in environmental quality.

In fact, this paper is an attempt to reduce a gap between literatures on public investment in environmental capacity and fiscal aspect of taxation, but we must admit that our results are restricted to a special case.

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