POLICY SIMULATIONS OF ALTERNATIVE FUTURES*

Jerald R. Barnard and Warren T. Dent**

INTRODUCTION

With the increasing complexity of the economy, simulation models have become recognized as useful planning tools for both public and private planning needs. Indeed, in spite of the criticism that has been leveled at simulation models (for example, the Club of Rome's Limits to Growth Report [4]) the understanding of public and private decision makers has increased to the point where simulation modeling is gaining an active audience. Policy makers are becoming increasingly active in raising "what if" questions in their information processing and decision making and look with keen interest at the possible development paths explored.

This paper presents a simulation model developed for the Office for Planning & Programming, State of Iowa, to be used for simulating various policies as they may come up in the course of state economic planning. The simulation model is demonstrated by examining seven policies directed at state economic development:

(1) A change in the industrial mix toward more manufacturing and exporting in those industries for which Iowa has a comparative advantage;

(2) An increase in the degree of processing and packaging of agricultural products;

(3) The promotion of human resources development through more education and training, thus reducing out-migration and increasing productivity;

(4) An improvement in the transportation system of the State through completion of the freeway system and branch line maintenance;

(5) The development of coal resources to the fullest extent possible under Department of Environmental Quality standards;

---

*Research reported in this paper was supported in part by funds from the Office for Planning & Programming, State of Iowa.

**Professors, Department of Economics and The Institute for Economic Research, The University of Iowa. We gratefully acknowledge the assistance given to this project by Dennis Beckmann and John Paquette.

(6) The implementation of a land use policy which restricts confinement feeding of cattle and new industrial development;

(7) The promotion of industrial development through readily available supplies of electrical energy from coal and nuclear power.

The basic methodology employed in the economic simulation of these policies is a dynamic input-output analysis of the State of Iowa. Each policy action is interpreted in the light of changes in the parameters of the simulation model. Effects of specific changes are contrasted with projections made in the absence of major policy changes. A "base-run" set of projections represents the latter position, and describes the Iowa economy in terms of major economic variables under the action of maintaining the status quo.

THE SIMULATION MODEL

The simulation model is formulated around the basic Leontief input-output model as a recursive system.[1] The structure of the model and its recursive properties is depicted by the diagram of the causal ordering of the variables of the model in Figure 1. The 0 order variables at the left form the set of exogenous variables that drive the model. Six of the exogenous variables have a t-1 subscript indicating they are a lagged relationship. Exports (E_0) and federal government expenditures (F_0) are given exogenously as a base with a growth rate applied. Capital stock of year t (K_t) is given as the amount available at the beginning of the year (the level at the end of the previous year). Thus, the model needs only the initial levels of the lagged exogenous variables to proceed and the model will generate a time path over any specified number of years.

The exogenous variables form the basis for generating the set of final demands that are multiplied by the Leontief inverse $(I-A)^{-1}$ to determine the level of output for the 13 producing sectors. Three equations are of special importance in the estimation of output, namely, equation 2.3 which places a bound on the level of investment,

$$A_2K_t \leq I_t \leq A_5K_t$$

where the lower bound A_2K_t is replacement investment indicating that investment must be greater than capital depreciation so that the sector's capacity is not allowed to decrease. The upper boundary coefficients in the A_5 matrix represent financial and technical constraints that limit the growth of capital stock.

Growth of the labor force is bounded in a way similar to capital, i.e.,

$$A_{14}L_{t-1} \leq L_t \leq A_{15}L_{t-1}$$

---

[1] The simulation model follows the earlier model of the Iowa economy developed by Maki, Suttor, and Barnard [3].

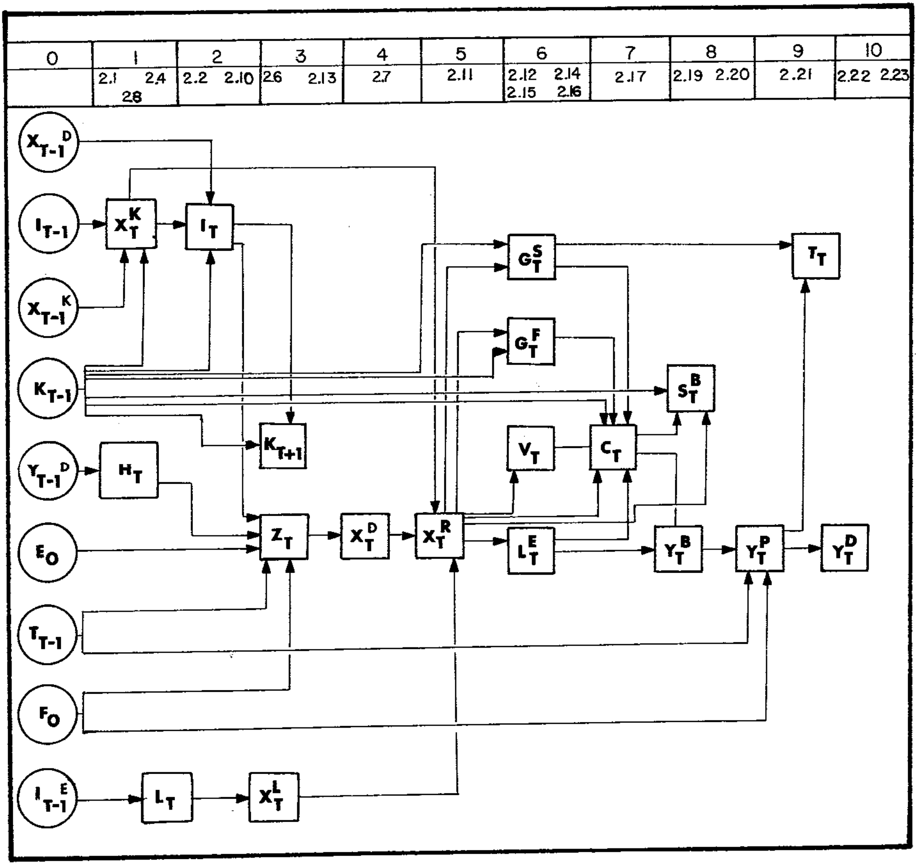


Fig. 1 Causal ordering of variables in the economic model.

where A_{14} and A_{15} represent lower and upper bounds respectively that reflect institutional restrictions on the percentage change in the labor force from year to year.

Finally, the third relationship deals with realized output X_t^R. Realized output is the minimum of the maximum output allowed by the capacity of the capital stock X_t^K, maximum output given the labor force X_t^L, or output demanded X_t^D, i.e.,

$$X_t^R = \text{minimum of } (X_t^K, X_t^L, X_t^D)$$

Once output has been estimated, the model proceeds to determine state and local taxes, value added, and personal income. These variables in turn provide the basis for the determination of personal consumption and governmental expenditures which enter as the lagged exogenous variables. (A listing of the equations and the definitions of the coefficient matrices, vectors of variables, and scalar numbers and variables of the model is found in the Appendix.)

INPUT DATA

The base year (subscript zero) for the analysis is 1969, and all monetary variables are measured in 1970 dollar equivalent terms. The primary data source is Barnard [2]. The 13 sectors used in analyzing the economy are:

Sector 1	Livestock agriculture
Sector 2	Other agriculture
Sector 3	Construction and mining
Sector 4	Food and kindred products
Sector 5	Other nondurable goods
Sector 6	Farm machinery
Sector 7	Other machinery
Sector 8	Other durable goods
Sector 9	Transportation
Sector 10	Communications
Sector 11	Trade
Sector 12	Finance, real estate and insurance
Sector 13	Services

The model was calibrated against a set of base run projections of population, employment and income for years 1970-2020 prepared for the Iowa economy [1]. These economic projection series were derived in part from the national projection series prepared for the U. S. Water Resources Council by the Bureau of Economic Analysis, U. S. Department of Commerce, and the Economic Research Service, U. S. Department of Agriculture. They are consistent with the national and regional projections for the United States to 2020 based on the series E population projections of the U. S. Bureau of the Census.

POLICY SIMULATIONS

In the first section, seven policy actions were listed for examination. In this section, we report projections and growth implications for the Iowa economy ssuming implementation of these policies and contrast changes with base run no change) projections. For each policy simulated, we state the changes effected n the dynamic input-output model in order to examine the impact of the policy. 'he actual impact of these changes cannot be known, nor can one typically find in .he literature empirical measurement of similar type impacts that have been ıeasured ex post. The simulation process is carried out in a ceteris paribus :nvironment where policy changes are examined against a baseline series. The ›aseline assumes the economic relationships and growth paths of the economy 'ollow closely those of the past. Economic theory guides the changes in paraıeters and relationships explored in the simulations, but the extent of change n the parameters and relationships is carried out by necessity on a judgmental ›asis. The policy changes explored in the simulations on the Iowa economy were lone on a sequential basis so as to observe the impact of changes in the indivilual parameter changes and/or relationships.

Policy changes by a given state government that may influence industrial 'evelopment may also invoke changes by contiguous states as they attempt to remain .ompetitive in the industrial location and development process. Whether other ,tates may respond in such a way as to limit the impact of a policy is a possibility ınd the subject for further simulations. The computational speed of the simuation model allows numerous runs with various sets and sub-sets of assumptions to ıe examined in terms of their impact on the major economic aggregates. Additionally, :he sensitivity of various assumptions and parameter changes on the economy can ılso be examined. In summary, the simulation of the complex system adds to the nformation of the impact of policy changes, and provides the opportunity of ıoning in on most reasonable policy changes and strategies. The impact of the ›olicy simulation is graphically depicted by comparing growth patterns of employnent under the policy with the base run.[2]

Industrial Mix Policy

A change in the industrial mix toward more manufacturing and export in those industries for which Iowa has a comparative advantage.

Logic of Policy Change. The impact is primarily directed towards the agricultural processing and farm input sectors. The impact is introduced into the simulation through the following vectors:

1. An increase in the export demand growth rates (A_8) in food and kindred products, farm machinery, and other machinery sectors (sectors 4, 6, 7).

2. An increase in the maximum allowed investment per unit of capital stock (A_5) in sectors 4, 6, 7.

---

[2]A computer routine was developed as part of this project to print the major economic data series of the model as depicted in Table 1 and to plot comparisons of the base run with the policy simulation as depicted in Figure 2.

TABLE 1: Base Run of Simulation Model, 1970 and Projections to 2020

Year 1970

	Producing Sectors												
	1	2	3	4	5	6	7	8	9	10	11	12	13
Capacity,Plant-Equipt	3915.6	2339.0	1604.5	5651.9	1854.1	995.4	1525.6	1937.6	816.6	829.0	2957.7	2732.6	1715.1
Output Demanded	3194.4	1894.6	1439.8	4687.8	2760.7	934.6	1351.6	3020.3	936.9	856.9	2757.6	2834.8	2260.9
Maximum Output,Labor	3767.7	2245.2	1576.8	5387.1	1844.8	929.7	1476.9	1927.6	821.7	793.4	2839.3	2652.2	1689.3
Realized Output	3194.4	1894.6	1439.8	4687.8	1844.8	929.7	1351.6	1927.6	816.6	793.4	2757.6	2652.2	1689.3
Value Added	1038.8	1070.5	594.5	1023.7	1005.4	301.7	650.6	775.7	558.5	542.1	2137.7	2069.8	1276.9
Gross Investment	488.3	291.6	32.6	112.5	71.0	26.8	36.3	65.0	234.9	177.8	115.8	200.3	154.2
Capital Stock,Year End	4097.5	2447.0	408.2	1440.7	911.8	344.6	468.1	839.8	2960.4	2319.9	1477.5	2951.6	1984.2
Final Demands	921.3	833.2	1049.6	3581.7	999.1	864.7	801.2	1190.7	301.8	347.6	1995.2	1642.7	1359.6
Labor Force	119.3	47.7	62.7	50.7	39.4	25.1	78.8	60.5	18.0	25.8	237.4	47.7	230.1
Employment	101.2	40.3	57.2	44.1	39.4	25.1	72.1	60.5	17.9	25.8	230.6	47.7	230.1
Taxes,State and Local	75.6	108.0	8.6	31.7	24.0	8.7	11.2	10.2	54.0	60.9	408.4	312.3	80.5
Taxes,Federal	0.0	0.0	6.7	27.5	20.8	7.4	9.5	6.8	43.9	51.0	31.8	24.3	44.8
Unallocated Value Added	417.8	549.3	162.8	284.3	508.5	44.9	0.0	227.1	99.1	124.7	512.1	995.8	376.6
Business Savings	522.1	473.9	51.1	813.9	520.9	52.0	55.2	231.0	257.5	257.4	150.0	554.2	105.7
Personal Income,Bus.	483.2	575.3	536.1	657.4	574.8	246.2	625.5	587.0	214.6	207.2	1561.8	1276.4	1048.1

Total Household Expenditures	8318.2	$1970 millions
Total Taxes,State and Local	1456.6	"
Total Taxes,Federal	1547.4	"
Total Personal Savings	824.5	"
Total Savings	4869.4	"
Gross Area Product	15130.5	"
Net Area Product	13577.9	"
Total Personal Income	10678.2	"
Disposable Income	9142.7	"
		"
		"
Total Population	2931.8	thousands
Total Labor Force	1160.3	thousands
Total Employment	1109.0	thousands

SECTOR 1	Livestock Agriculture
SECTOR 2	Other Agriculture
SECTOR 3	Construction and Mining
SECTOR 4	Food and Kindred Products
SECTOR 5	Other Nondurable Goods
SECTOR 6	Farm Machinery
SECTOR 7	Other Machinery
SECTOR 8	Other Durable Goods
SECTOR 9	Transportation
SECTOR 10	Communications
SECTOR 11	Trade
SECTOR 12	Finance, Real Estate and Insurance
SECTOR 13	Services

Sector values in $1970 millions except for Labor Force, Employment, and Populations in thousands.

TABLE 1 - Continued

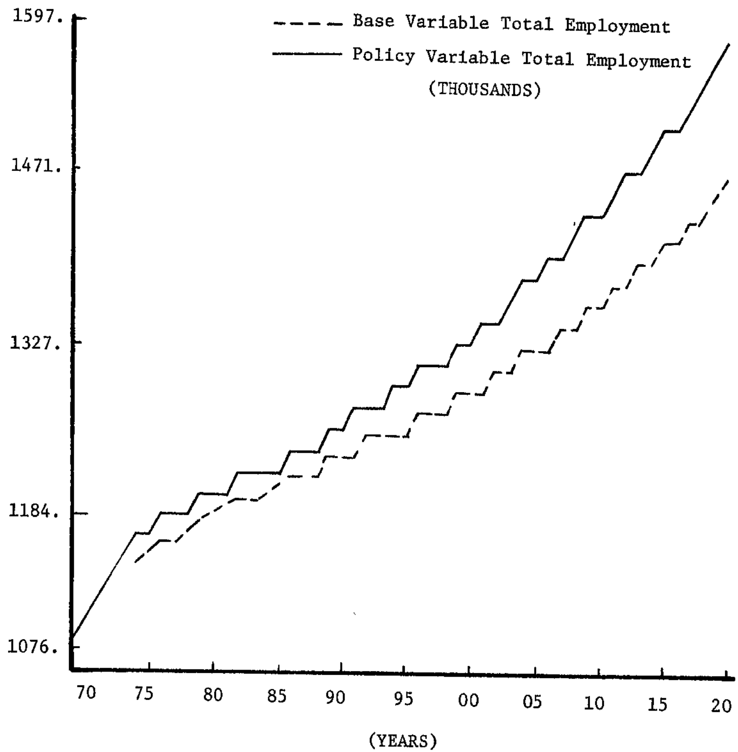
Year 2020

Producing Sectors

	1	2	3	4	5	6	7	8	9	10	11	12	13
Capacity,Plant-Equipt	5332.0	3184.9	1843.8	14408.7	7087.2	3381.4	4690.3	8225.5	1427.8	5344.8	7813.5	12373.5	7126.7
Output Demanded	11605.2	6962.1	5444.2	18877.8	12593.8	3105.7	6261.7	12159.6	4046.3	3897.3	12593.8	13222.9	10717.6
Maximum Output,Labor	5630.2	3363.1	1994.3	14884.4	7529.1	3195.8	4853.0	8816.0	1518.8	4075.4	8164.4	12959.9	7628.8
Realized Output	5332.0	3184.9	1843.8	14408.7	7087.2	3105.7	4690.3	8225.5	1427.8	3897.3	7813.5	12373.5	7126.7
Value Added	1733.9	1799.6	761.3	3146.4	3862.5	1007.7	2257.6	3309.9	976.5	2663.1	6057.1	9656.3	5386.7
Gross Investment	687.2	410.4	37.8	371.8	282.3	110.3	167.9	329.8	355.1	322.0	318.0	935.4	666.0
Capital Stock,Year End	5766.9	3444.1	474.2	4762.3	3627.2	1418.3	2164.7	4258.0	4475.3	5409.7	4055.6	13781.5	8572.7
Final Demands	2601.5	2972.7	3740.4	14524.8	4985.9	2866.0	4020.2	4813.0	1432.4	1649.6	9460.9	8166.7	6862.7
Labor Force	54.5	21.8	28.0	52.0	46.8	49.9	149.9	88.8	16.6	12.7	415.1	90.9	386.1
Employment	51.6	20.7	25.9	50.4	44.0	48.5	144.8	82.8	15.6	12.2	397.3	86.8	360.6
Taxes,State and Local	126.2	181.5	11.0	97.5	92.0	29.1	38.8	43.4	94.4	299.1	1157.2	1456.9	339.5
Taxes,Federal	0.0	0.0	8.6	84.4	79.9	24.7	32.9	28.9	76.8	250.6	90.2	113.3	188.9
Unallocated Value Added	827.9	1002.6	0.0	0.0	1361.1	0.0	0.0	0.0	0.0	1161.2	0.0	3236.5	0.0
Business Savings	751.1	721.5	53.9	2249.7	1800.2	133.2	217.1	616.5	382.5	1193.1	417.4	2586.0	455.8
Personal Income,Bus.	926.8	1042.3	1163.9	2939.0	2409.2	2348.0	7015.3	3467.2	613.2	1089.0	6215.7	5954.3	6167.8

Total Household Expenditures	41532.9	$1970 millions	SECTOR 1	Livestock Agriculture
Total Taxes,State and Local	5279.9	"	SECTOR 2	Other Agriculture
Total Taxes,Federal	7342.8	"	SECTOR 3	Construction and Mining
Total Personal Savings	4174.6	"	SECTOR 4	Food and Kindred Products
Total Savings	15752.6	"	SECTOR 5	Other Nondurable Goods
Gross Area Product	54650.3	"	SECTOR 6	Farm Machinery
Net Area Product	51089.5	"	SECTOR 7	Other Machinery
Total Personal Income	53384.2	"	SECTOR 8	Other Durable Goods
Disposable Income	45707.5	"	SECTOR 9	Transportation
		"	SECTOR 10	Communications
		"	SECTOR 11	Trade
Total Population	3313.2	thousands	SECTOR 12	Finance, Real Estate and Insurance
Total Labor Force	1543.7	thousands	SECTOR 13	Services
Total Employment	1471.9	thousands		

FIGURE 2: Projected Total and Baseline Employment Under Industrial Mix Policy



3. An increase in the upper bounds on the percentage change in the labor force (A_{15}) in sectors 4, 6, 7.

4. An increase in the growth rates in employment (A_{13}) in sectors 4, 6, 7.

5. An increase in labor productivity (A_{17}) in sectors 4, 6, 7.

The process followed was to implement the various changes in stages. At the end of each stage, various constraints were noticed which led to additional changes. This continued until it was felt that the policy had been fully covered.

Results. The changes made reflect the desire to allow the three sectors to grow unhindered. Thus, the outcomes which resulted from these changes reflect the growth in these sectors as compared to the base run. The most substantial change was within the food and kindred products sector (4) where employment increased over 60 percent and capital stock by more than 200 percent.

Processing Agricultural Products Policy

An increase in the degree of processing and packaging of agricultural products.

Logic of Policy Change. The impact is reflected in the increase of product demanded (X_I^D) in Food and Kindred Products (sector 4) by 20 percent, beginning in the year 1980. Additional changes were made in the following matrices:

1. An increase in the maximum allowed investment per unit of capital stock (A_5) in the Food and Kindred Products sector (4).

2. An increase in labor productivity (A_{17}) in sector (4).

3. An increase in the ratio of value added to output (A_{18}) in sector (4) by 20 percent.

These changes show the increase in processing and packaging by means of price increases and purchases of additional equipment to implement the higher degree of processing. Labor productivity was increased to share, with the addition of capital, an increase in productivity associated with that capital.

Results. Only minor changes in total employment, population, personal income and gross area product resulted. However, substantial changes were noted within the Food and Kindred Products sector in those variables most indicative of growth.

Education Policy

The promotion of human resources development through more education and training, thus reducing out-migration and increasing productivity.

FIGURE 3: Projected Total and Baseline Employment Under Processing Agricultural Products Policy

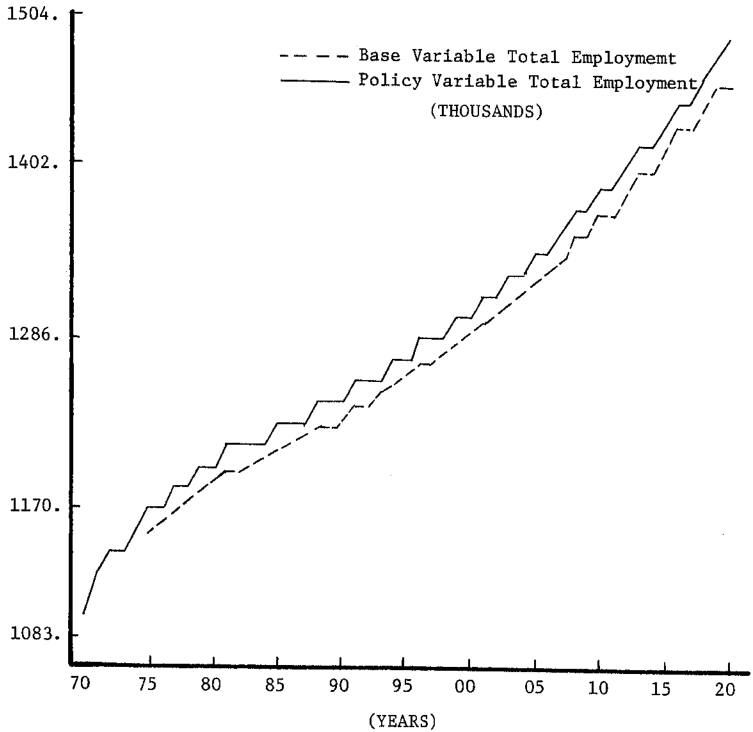
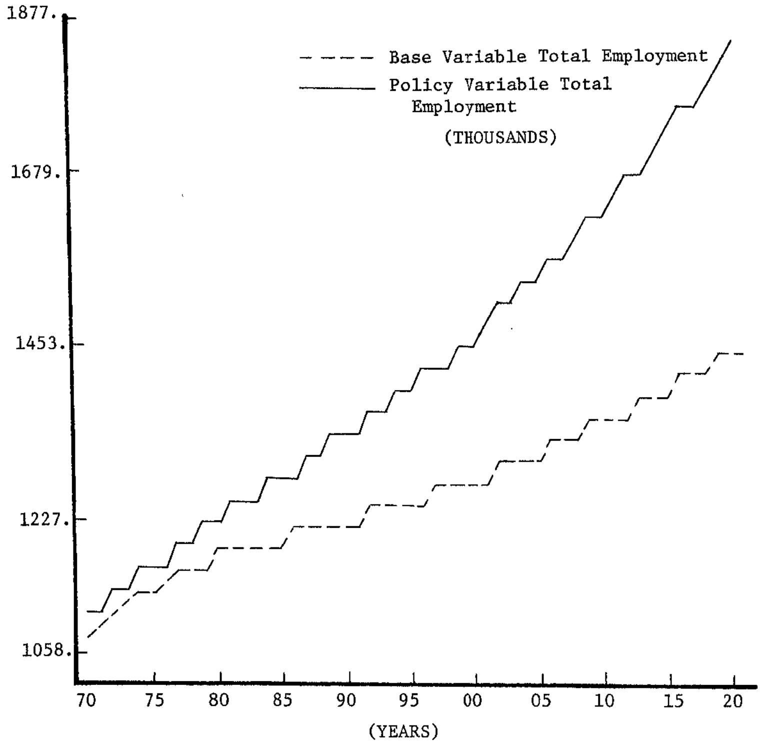


FIGURE 4: Projected Total and Baseline Employment Under Education Development Policy



Logic of Policy Change. The impact is introduced into the simulation through the following means.

1. Out-migration is reduced by raising the growth rate in wages (A_{24}) for all sectors.

2. It is assumed that increased training will result in increased worker productivity, therefore, the annual rates of growth of output/labor ratios (A_{17}) were increased for all sectors.

3. Increased productivity of the labor force is expected to enhance capital investment in the state. Accordingly, the upper bounds of investment per unit of capital stock (A_5) and percentage change in labor force (A_{15}) were increased for all sectors.

Results. The result of this policy change is a substantial increase in both employment and total population with moderate increases in gross area product and personal income.

Transportation Policy

An improvement in the transportation system of the State through completion of the freeway system and branch line maintenance.

Logic of Policy Change. The impact is directed at the construction and mining sector, the trade sector, and the machinery sectors. The impact is introduced into the simulation through the following means:

1. To reflect increased spending for road construction and railway maintenance, the ratio of government expenditures to tax collections (A_9) is increased for sectors 3, 6, 7, 11, and decreased for the remaining sectors.

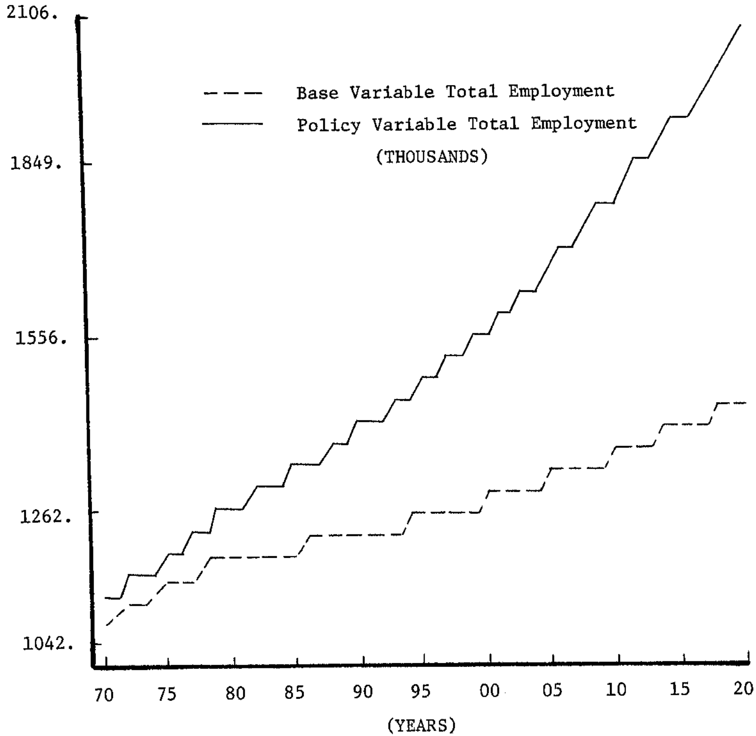
2. This increased spending would be financed by an increase in the road use tax; therefore, the ratios of state and local tax receipts (A_{19}) are raised for all sectors.

3. An improved transportation system will improve access to markets and lower relative cost, thereby increasing the demand for exports; therefore, the growth rate of export demand (A_8) is increased for those sectors 1, 2, 3, 4, 6, 7, and 11 where it is positive.

4. To allow growth to take place unhindered, the maximum amount of investment per unit of capital stock (A_5) is increased for all sectors.

Results. The results show sizable increases in employment and total population with smaller increases in gross area product and personal income. Of

FIGURE 5: Projected Total and Baseline Employment Under Transportation Development Policy



the four sectors receiving increased government expenditures, the construction mining sector shows the most response where there is a large gain in personal income and employment.

Coal Production Policy

The development of coal resources to the fullest extent possible under Department of Environmental Quality standards.

Logic of Policy Change. It was assumed that the development of coal resources would be directly related to an increase in purchases of local coal by the electric utilities. Also, it was assumed that capital formation within the utilities and mining sectors would be increased. These changes were made within the simulation in the following manner:

1. In the manipulation of the matrix of interindustry flow coefficients (A_{11}) before the inverse was taken, the matrix cells pertaining to mining and utilities were changed. These changes reflected the increase in purchases of coal by the utilities from the mining sector and a small decrease in the utilities intersectoral transactions.

2. An increase in the maximum amount of investment per unit of capital stock (A_5) in the mining and construction sector and in the communication and utilities sector (3, 10).

Results. Substantial increases in the construction and mining sector in areas of investment, employment and capital stock result. The policy change has a small impact on the communication and utilities sector variables. Only a relatively small change is then reflected in the aggregate variables of employment, population, personal income, and gross area product.

Land Use Policy

The implementation of a land use policy which restricts confinement feeding of cattle and new industrial development.

Logic of Policy Change. The impact is directed at the livestock sector and all manufacturing sectors. The impact is introduced into the simulation through the following means:

1. The output of the livestock sector is fixed in year 1980 by holding capital stock and labor fixed.

2. Land use restrictions on new industrial development are assumed to be implemented in 1985. Accordingly, it is assumed there would be no new industrial parks or additions to existing industrial parks. The assumed result is that there would still be some vacant land in

FIGURE 6: Projected Total and Baseline Employment Under Coal Production Development Policy

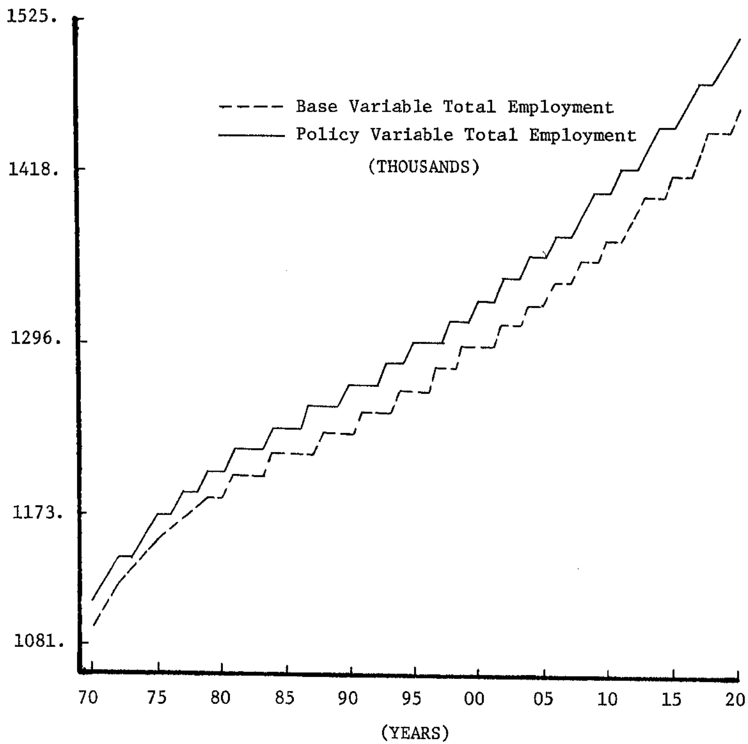
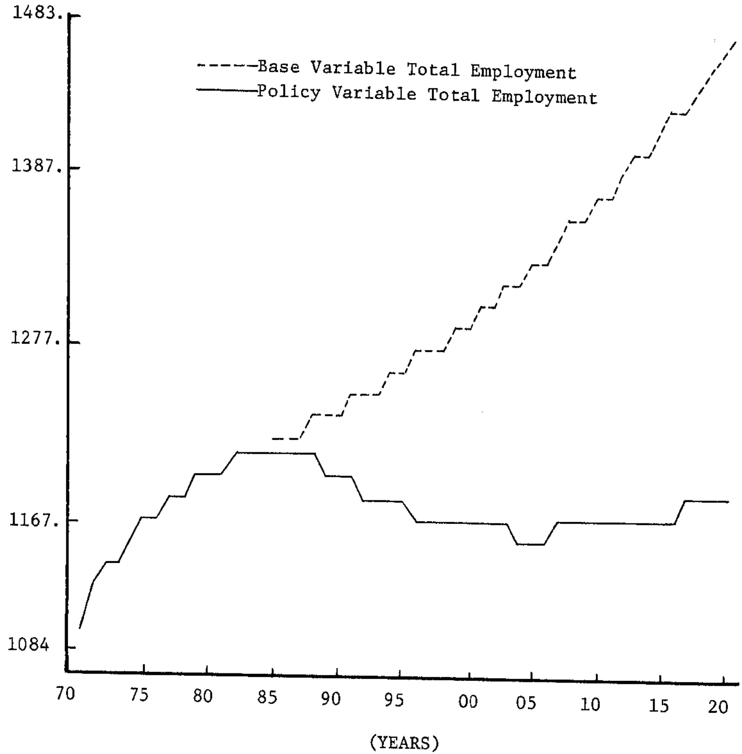


FIGURE 7: Projected Total and Baseline Employment Under Restrictive Land Use Policy



existing parks for new industrial expansions and that land may be used more intensively, but that the result of such action is to cut the rate of capital formation in the manufacturing sector to one-half. This is implemented by cutting the expected growth of capital stock (investment, I_t) by one-half (.5) of what is called for within the model.

Results. The result of these two policy changes is to impose a substantial reduction in economic growth on the Iowa economy. All major variables, population, employment, income, gross area product, and capital formation fall below the baseline levels.

Energy Policy

The promotion of industrial development through readily available supplies of electrical energy from coal and nuclear power.

Logic. It was assumed that a policy of reasonable accommodation to power plant siting and related environmental factors would allow electric utilities to expand that capacity and the electrical supply would thus become less expensive relative to other forms of energy from other parts of the United States. It was then assumed that this would lead to an increase in capital formation within the manufacturing sectors. This was implemented within the simulation as follows:

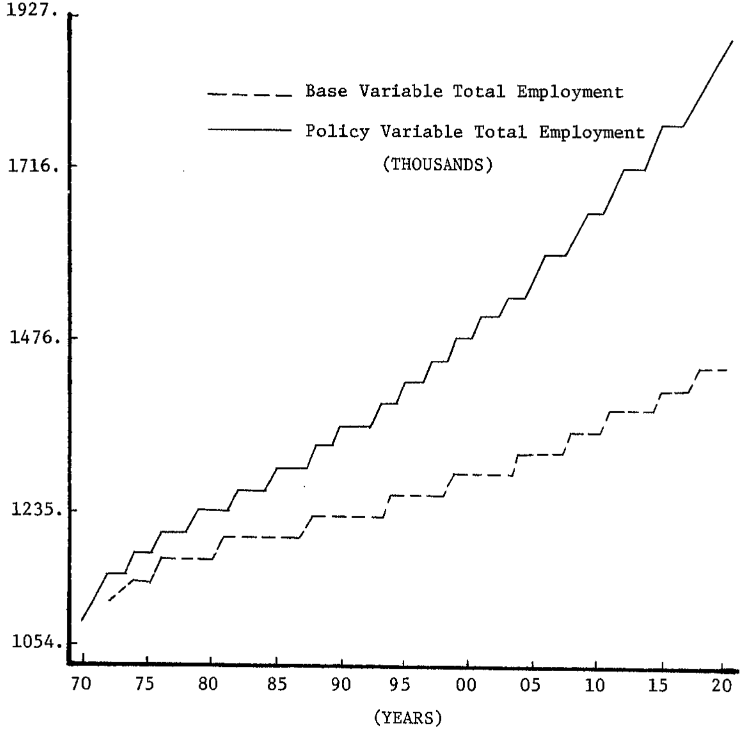
1. An increase in the export demand growth rate (A_8) for sectors 4, 5, 6, 7, and 8.

2. An increase in the expected rate of growth in demand (A_3) for sectors 4, 5, 6, 7, and 8.

3. An increase in the maximum amount of investment per unit of capital stock (A_5) for all sectors except sectors 1 and 2.

Results. Substantial increases in the manufacturing sectors' employment and capital stock as well as the aggregate increases in employment, population, personal income, and gross area product result.

FIGURE 8: Projected Total and Baseline Employment Under Available Energy and Industrial Development Policy



APPENDIX

Table A.1 Coefficient matrices in state development model

Matrix symbol	Description
A_1	Diagonal matrix of incremental capital-output ratios; or, when given as the inverse of A, a diagonal matrix of output capital ratios.
A_2	Diagonal matrix of depreciation ratios.
A_3	Diagonal matrix with elements being one plus the anticipated rate of growth in demand for the specified sector's output.
A_4	Diagonal matrix of output-capacity ratios that businessmen try to maintain.
A_5	Diagonal matrix with elements being the maximum amount of investment per unit of capital stock.
A_6	Column vector of parameters where the coefficients are the proportion of total household expenditures spent for specified outputs.
A_7	Matrix of capital input-output ratios where the element in the i-th row and j-th column is the proportion of sector j's capital purchases from sector i.
A_8	Diagonal matrix with elements being one plus the sector's export demand growth rate.
A_9	Column vector of parameters where the coefficients express the relationship between state and local tax collections in year t-1 and state and local government expenditures in year t.
A_{10}	Column vector of parameters where the i-th coefficient is the proportion of federal expenditures for the output of sector i.
A_{11}	Inverse matrix $(I-A)^{-1}$ where A is the matrix of interindustry technical coefficient.
A_{12}	Diagonal matrix with elements being the equilibrium labor force-employment ratios.
A_{13}	Diagonal matrix with elements being one plus the growth rates in employment.
A_{14}	Diagonal matrix representing lower bonds on percentage change in labor force.
A_{15}	Diagonal matrix representing upper bounds on percentage change in labor force.
A_{16}	Diagonal matrix of output labor ratios in year 0.
A_{17}	Diagonal matrix with elements being one plus the annual rate of growth in the corresponding output labor ratios.
A_{18}	Diagonal matrix with the i-th diagonal element being the ratio of value added to output in sector i.
A_{19}	Diagonal matrix of state and local tax receipts per unit of output.
A_{20}	Diagonal matrix of state and local tax receipts per unit of capital stock at beginning of period.

Table A.1 continued

A_{21}	Diagonal matrix of federal tax receipts per unit of output.
A_{22}	Diagonal matrix of federal tax receipts per unit of capital stock at beginning of period.
A_{23}	Diagonal matrix with i-th element being the wage rate in the i-th industry in year 0.
A_{24}	Diagonal matrix of growth rates in wages by sector.
A_{25}	Diagonal matrix with i-th element being the ratio of autonomous retained earnings to value added in i-th sector.
A_{26}	Diagonal matrix with i-th element being the proportion of unallocated value added which is allocated to business saving in sector i.
A_{27}	Diagonal matrix with i-th element being the ratio of imports to output for sector i.
A_{28}	Diagonal matrix in which the i-th element is the population-labor ratio for sector i in the first year of the simulation.
A_{29}	Diagonal matrix with i-th element being one plus the rate of growth in the corresponding element in A_{28}.

Table A.2 Vectors of variables in state development model

Symbol	Description
1. X_t^K	Capacity of plant and equipment, t-th year.
2. X_t^D	Output demanded, t-th year.
3. X_t^L	Maximum outputs with a given labor force L_t, t-th year.
4. X_t^R	Realized output, t-th year.
5. V_t	Value added, t-th year.
6. I_t	Gross investment, t-th year.
7. K_t	Capital stock, beginning of t-th year.
8. Z_t	Final demands, t-th year.
9. L_t	Labor force, t-th year.
10. L_t^E	Employment, t-th year.
11. G_t^S	State and local tax collections, t-th year.
12. G_t^F	Federal tax collections, t-th year.
13. C_t	Unallocated value added, t-th year.
14. S_t^B	Business savings, t-th year.
15. Y_t^B	Personal income received from business, t-th year.
16. E_0	Vector of export demands in year 0.

Table A.3 Scalar numbers and variables in state development model

Symbol	Description
1. a_1	Desired ratio of expenditure to current disposable income.
2. a_2	One plus the expected rate of growth in disposable income.
3. a_3	Lag coefficient.
4. a_4	One plus the annual rate of growth in federal expenditures.
5. a_5	Coefficient relating state and local payments to households to lagged state and local taxes.
6. a_6	Proportion of federal expenditures paid to households.
7. a_7	Ratio of state and local personal taxes to personal income.
8. a_8	Federal personal income tax rate.
9. a_9	One plus the rate of growth of P_0^g.
10. a_{10}	Labor-population ratio for governmental employment.
11. a_{11}	Coefficient relating state and local government wage and salary payments to lagged state and local taxes.
12. a_{12}	Proportion of federal expenditures paid as wages and salaries.
13. h_t	Total household expenditures, t-th year.
14. t_t	State and local tax collections, t-th year.
15. t_t^F	Total federal tax collections, t-th year.
16. s_t^P	Total personal savings, t-th year.
17. s_t	Total savings, t-th year.
18. x_t^G	Gross area product, t-th year.
19. x_t^N	Net area product, t-th year.
20. y_t^P	Total personal income, t-th year.
21. y_t^D	Disposable income (i.e., personal income minus state, local and federal personal taxes), t-th year.
22. e_t	Total exports, t-th year.
23. m_t	Total imports, t-th year.
24. P_t	Total population, t-th year.

Table A.3 continued

25.	p_0^g	Total population associated with government employment, base year.
26.	E_0	Vector of export demands in year 0.
27.	f_0	Federal government expenditures in year 0.
28.	z_t	Fourteenth element of the vector Z_t.
29.	I_t	Total labor force.
30.	I_t^E	Total employment.

EQUATIONS

1. Input-Output Equations

The basic Leontief input-output equation is

$$X = AX + Z, \tag{1.1}$$

where

X = vector of sector outputs;

Z = vector of sector final demands;

A = matrix of interindustry flow coefficients.

Output is expressed as a function of final demand; i.e.,

$$X = (I-A)^{-1}Z \tag{1.2}$$

2. Basic Equations

$$X_t^K = X_{t-1}^K + (A_1)^{-1}(I_{t-1} - A_2K_t) \tag{2.1}$$

$$I_t = A_2K_t + A_1 \; [(A_3)^2X_{t-1}^D - A_4X^K{}_t] \tag{2.2}$$

$$A_2K_t \leq I_t \leq A_5K_t \tag{2.3}$$

$$h_t = a_1 a_2 (y_{t-1})^D \tag{2.4}$$

$$h_t + a_3(h_{t-1} - h_t), \; 0 < a_3 < 1 \tag{2.5}$$

$$Z_t = A_6 h_t + A_7 I_t + (A_8)^t \; E_0 + A_9 t_{t-1} + A_{10} (a_4)^t \; f_0 \tag{2.6}$$

$$X_t^D = A_{11} Z_t \tag{2.7}$$

$$L_t = A_{12} A_{13} \; (L_{t-1})^E \tag{2.8}$$

$$A_{14} \; L_{t-1} \leq L_t \leq A_{15} \; L_{t-1} \tag{2.9}$$

$$X_t^L = A_{16} (A_{17})^t \; L_t \tag{2.10}$$

$$X_t^R = \text{minimum of } (X_t^K, \; X_t^L, \; X_t^D) \tag{2.11}$$

$$L_t^E = [A_{16}(A_{17})^t]^{-1} X_t^R \tag{2.12}$$

$$K_{t+1} = K_t + I_t - A_2K_t \tag{2.13}$$

$$V_t = A_{18}X_t^R \tag{2.14}$$

$$G_t^S = A_{19}X_t^R + A_{20}K_t \tag{2.15}$$

$$G_t^F = A_{21}X_t^R + A_{22}K_t \tag{2.16}$$

$$C_t = V_t - A_2K_t - A_{23}(A_{24})^t \; L_t^E - G_t^S - G_t^F - A_{25}V_t \tag{2.17}$$

$$A_{26} = [(I_t - A_2K_t)I] \; [(I_t)I]^{-1} \tag{2.18}$$

$$S_t^B = A_2K_t + A_{25}X_t^R + A_{26}C_t \tag{2.19}$$

$$Y_t^B = A_{23}(A_{24})^t L_t^E + (I - A_{26})C_t \tag{2.20}$$

$$y_t^P = iY_t^B + a_5 t_{t-1} + (a_6)\ (a_4)^t\ f_0 \tag{2.21}$$

i is a unit row vector

$$t_t = iG_t^s + a_7 y_t^P \tag{2.22}$$

$$y_t^D = (1 - a_7 - a_8)\ y_t^P \tag{2.23}$$

3. Auxiliary Equations

$$e_t = i(A_8)^t\ E_0 - i(X_t^D - X_t^R) \tag{3.1}$$

$$m_t = iA_{27}\ X_t^R + z_t \tag{3.2}$$

$$t_t^F = iG_t^F + a_8\ y_t^P \tag{3.3}$$

$$s_t^P = y_t^P - h_t - a_7\ y_t^P - a_8\ y_t^P \tag{3.4}$$

$$x_t^G = iV_t + a_{11}t_{t-1} + a_{12}\ (a_4)^t\ f_0 \tag{3.5}$$

$$x_t^N = x_t^G - i\ A_2K_t \tag{3.6}$$

$$P_t = i\ A_{28}(A_{29})^t\ L_t + p_0^g\ (a_9)^t \tag{3.7}$$

$$I_t = i\ L_t + a_{10}p_0^g\ (a_9)^t \tag{3.8}$$

$$I_t^E = i\ L_t^E + a_{10}p_0^g\ (a_9)^t \tag{3.9}$$

$$s = i\ S_t^B + s_t^P \tag{3.10}$$

REFERENCES

1. Barnard, Jerald R. and Warren T. Dent. "Projections of Population, Employment, Income and Water Use for Iowa River Basins, 1975-2020," The Institute for Economic Research, The University of Iowa, 1976.

2. Barnard, Jerald R. "The Iowa Economy: Interindustry Structure and Accounts," Bureau of Business and Economic Research, The University of Iowa, 1974.

3. Maki, Wilbur R., Richard E. Suttor and Jerald R. Barnard. "Simulation of Regional Income and Product with Emphasis on Iowa, 1954-1974," Research Bulletin 548, Agricultural and Home Economics Experiment Station, Iowa State University, 1966.

4. Meadows, Donella H., Dennis L. Meadows, Jorgen Randers, and William W. Behrens, III. The Limits to Growth, New York: Universe Books, 1972.