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THE DYNAMIC COST AND PERSISTENCE OF ASSET INEQUALITY IN AN AGRARIAN ECONOMY*

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Abstract

A growing literature suggests that inequality is economically costly. However, much of this literature depends on static analyses, begging the question of why a market system doesn't redress inequality over time if it is efficient to do so. We develop a dynamic model of asset accumulation and endogenous asset-price formation in an agrarian economy with multiple market imperfections. The model is parameterized to pre-revolutionary Nicaragua and solved numerically. The results suggest that although a free land market would eventually lead to an egalitarian land distribution, the process would take long enough, and involve sufficiently great factor-use inefficiencies along the way, that a redistributive policy would improve on the market's performance in both equity and efficiency criteria.

keywords: Inequality and efficiency, Structural evolution, dynamic equilibrium analysis

JEL codes: D91, O12, D31

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The Dynamic Cost and Persistence of Asset Inequality in an Agrarian Economy

1. Introduction

A rich and growing literature has begun to explore the notion that inequality is economically costly. This literature has been in part spurred by the observation that the fastest growing nations of the world over the last few decades have been decidedly more egalitarian than other nations within their living standards cohort. It has also been spurred by the continuing development of the economics of imperfect information. When work effort is unobservable and non-contractible, economic productivity and efficiency are potentially sensitive to the distribution of endowments. Bardhan, Bowles and Gintis (1998), for example, show that at least over some range, economic efficiency and productivity decrease as the distribution of wealth and productive assets becomes less equal.[1]

The idea that an economy might exhibit this sort of endowment sensitivity has deep roots in the theory of agrarian economy. Writing in the early, 20th century, the Russian economist A.V. Chayanov argued that farm households with distinct endowments of productive resources would use those resources in different proportions, with different productivities. Subsequent writings have clarified when such behavioral differentiation occurs (see Kevane, 1996, for a thorough synthesis), and when it creates an endowment sensitive economy. Particularly noteworthy in this latter respect is Eswaran and Kotwal (1986) who follow Roemer (1982a) and Bardhan (1984) by analyzing agents arrayed according to their endowment of a productive asset (land). They show that when labor effort is not contractable and access to the capital needed to finance a roundabout production process is wealth dependent, behavioral differentiation emerges along the endowment continuum. They also show that under these

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[1] The potential for costly inequality that is rooted in non-contractability of labor effort becomes actualized in the Bardhan *et al.* model because financial market constraints prevent workers from forming their own firms that would circumvent the labor problem.

(intrinsic) multiple market failures, the economy *in equilibrium* is endowment sensitive, and that output and efficiency can be enhanced by egalitarian redistributions of land endowments.[2]

However, an important question that the static models of Eswaran and Kotwal and Bardhan *et al.* do not address is whether the economically costly inequality that they model will persist over time. Time and inter-temporal choice give agents an additional degree of freedom to work around multiple market imperfections through the accumulation of private savings that can be used to both finance production and fund its expansion through the purchase of productive assets. This observation poses two, inter-related questions about the static endowment sensitive economy:

1. Dynamic Persistence of Inequality

Will market institutions (specifically asset markets) serve to transfer assets from rich to poor, dampening inequality and eliminating its costs over time—i.e., does the distribution of endowments matter over the longer term?[3]; and,

2. Dynamic Costs of Inequality and the Prospects for Market Led versus State Managed Redistribution

If markets do tend to eliminate costly inequality, do they do it quickly enough that they dominate affirmative state policies designed to reach the same end?

This paper explores these two questions using a dynamic, general equilibrium multiple market failures model. In moving this family of models into a dynamic framework where consumption out of current income can be traded off against the accumulation of assets (and future consumption), three sorts of complexity emerge. First, given the centrality of thin or missing financial markets to the creation of

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[2] While not cast in an endowment continuum framework, Feder (1985) similarly shows that combinations of multiple market failures will result in persistent (post-exchange) behavioral differentiation.

[3] This question effectively asks if a Coaseian economy (meaing one that it is insensitive to the distribution and definition of property rights) can be salvaged dynamically from an economy that violates the Coase thereom statically.

static endowment sensitivity, the model must incorporate an accumulable financial asset (money) which, like land, can be accumulated over time and used to offset imperfect financial markets. The one-dimensional endowment continuum of Eswaran and Kotwal (1986) thus becomes here a two-dimensional state space over which dynamic choice takes place. In a two-dimensional analogue to what Roemer (1982b) calls a class correspondence, we will explore behavioral differentiation and economic class over this space, utilizing "class maps" to illustrate the correspondence between endowment positions and the static *and dynamic* behaviors that characterize those positions.

Second, because prices are endogenous (clearing all factor and asset markets every period), we need a concept of rational expectations about the evolution of future prices in order to prevent agents from taking decisions that are inconsistent with subsequent economic events. To solve this problem we use an iterative procedure that is similar in spirit to Imrohoroglu, Imrohoroglu and Joines (1993).

Third and finally, in order to identify the dynamic costs of inequality, it is necessary to track the full dynamic path along which the economy travels toward its steady state. Characterizing how inequality and productivity evolve as the economy undergoes a transition from one steady state to another is an important objective of our economic analysis—not just the character of the final state.[4] Such a characterization of the dynamic path is particularly important when transition to the steady state is long. Accordingly, the numerical methods developed here provide insight into both the length

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[4] In work that is similar in spirit to that presented here, Ray and Streufert (1993) ask if the costly inequality identified in the static efficiency wage model of Dasgupta and Ray (1986) will persist in a dynamic model. While Ray and Streufert's successfully identify the (locally stable) steady state endowment positions toward which agents can gravitate over time, their analytical approach does not permit them to identify the dynamic costs of inequality because it cannot characterize the field of attraction for those steady state points, nor the time it takes agents to reach them.

of the transition as well as the character of the economy along the way, including the costs of inequality. Within our focus here on an agrarian economy, we offer methods that go some distance towards answering Atkinson's (1997) call for economics to fully address the dynamics of income distribution, by modeling not only the factor distribution of income, but also modeling the evolving distribution of factors across households over time.

The remainder of this paper is organized as follows. Section 2 develops the structure of the model in detail. It presents first a single period model of agricultural production that incorporates the labor and credit market imperfections described above. Because of these imperfections, the profit-maximizing production strategy pursued by any given farmer can be sorted into a one of a number of production regimes, or classes. In each production period, factor prices arise endogenously out of the interacting demands and supplies for factors by a "community" of differentially endowed agents. Once production is realized, individual households must then determine how much to consume, how much to save, and in what sort of portfolio (land and money) to hold. To make this accumulation decision, households solve a dynamic programming problem that involves the numerical estimation of a true value function, relating the future value of agricultural production from the asset (land or liquid) to the present. Land prices are endogenous, again arising out of the competitive demands and supplies of land by the individual households. The true value function takes into account not only the multiple market imperfections of the static model, but also household's perfect foresight about future factor prices, which influence the profit implications of the market imperfections.

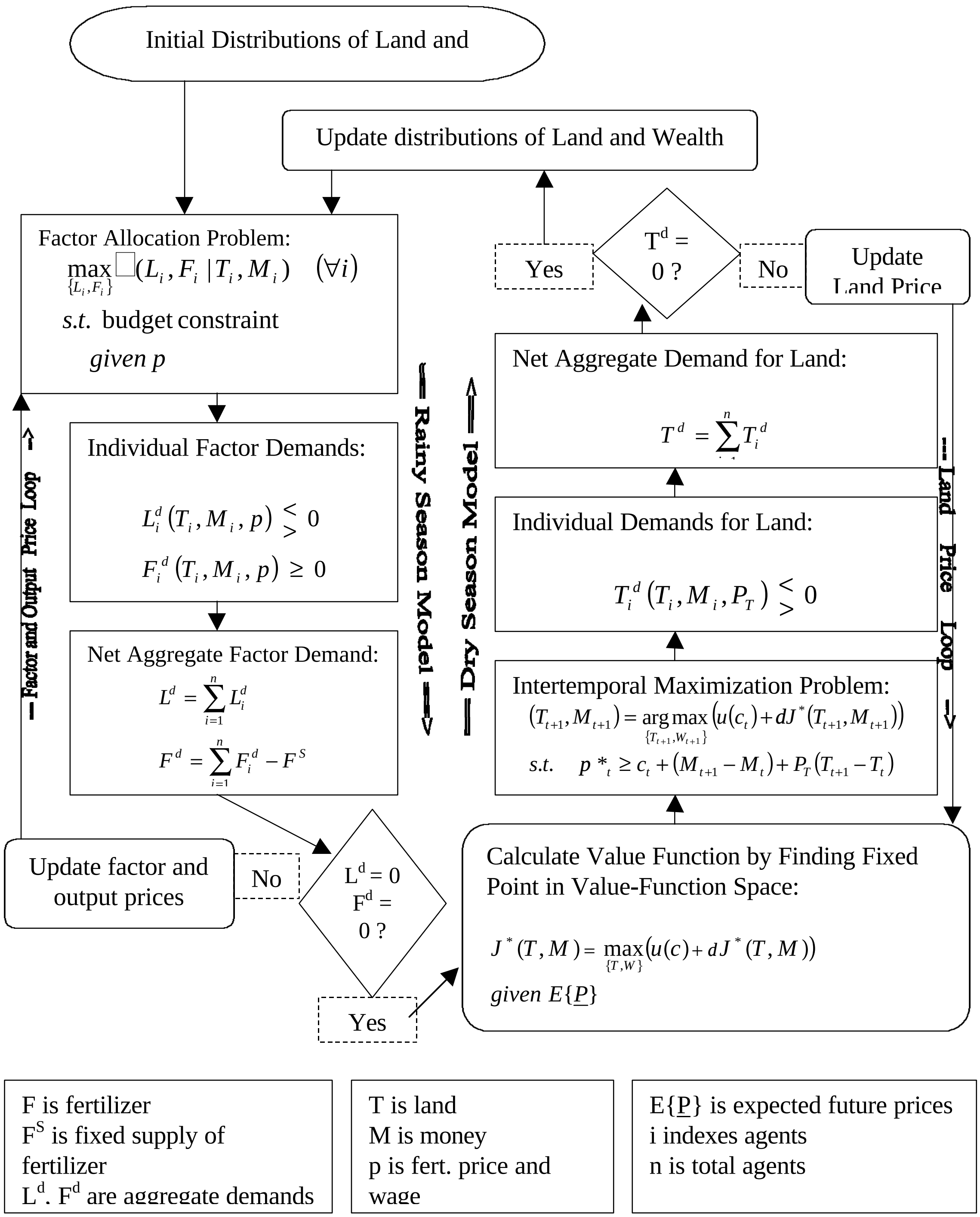
Section 3 presents and discusses the results of a simulation based on the above dynamic programming model. Analysis of the distinctive trajectories of accumulation pursued by agents identifies a dynamic class map that links initial endowment positions to dynamic behavior. The results also show that a convergence process is at work in the sense that asset inequality diminishes over time, and in response aggregate output steadily increases. While inequality thus does not persist indefinitely, its dynamic costs are considerable, and the paper's conclusion returns to the policy question about the options for market-led versus state managed redistribution.

2. A dynamic model of the agrarian economy with heterogeneous agents, multiple-market failures and endogenous prices

This section develops an infinite-horizon utility maximization model to explore intertemporally rational accumulation decisions within the context of imperfect factor markets. The model proceeds by discrete periods or years. Each year contains a rainy season when production occurs, and a post-harvest or dry season during when consumption and accumulation decisions are made. Under the assumptions made, production choices are separable from consumption and accumulation decisions. As a prelude to the details of the model, Chart 1 gives a quick overview of the overall structure of the model, and the individual and global variables that are determined at various times.

At the beginning of each rainy season, agents have predetermined (but ultimately endogenous) endowments of land and money. They maximize profits through borrowing, labor allocation and variable input ("fertilizer") purchase decisions. In so doing, they determine aggregate demand and supply for factors and goods. Following a standard Walrasian tâtonnement process, the rainy season model iterates on the model's endogenous factor prices (labor and fertilizer) until agents' choices are mutually consistent in the sense that supply equals

Chart 1: Model Structure



demand in input markets. Production then occurs, completing the rainy season session of the model.

Incomes realized, agents then enter the post-harvest or dry season in which they choose current consumption levels and adjust their land and money stocks by solving an infinite horizon dynamic programming problem. A tâtonnement process, similar to that used in the rainy season, is used to clear the land market every period. However, accumulation decisions are complicated by the expectations about future prices. Not only do expectations about future land prices matter, but also expectations about future fertilizer prices and wages play a role in the value to agents of accumulating land. Therefore, agents are assumed to be endowed with rational expectations (or in this riskless model, perfect foresight) about future land and fertilizer prices and wages. To implement this assumption, we run the entire simulation several times over, each time generating a realized set of prices. Those realized set of prices then becomes the expected values of prices for the next run. A rational expectations equilibrium occurs when the realized and expected prices converge.

The results of the model's dry season, or accumulation, component are reflected in an evolution of the land distribution over time. Note that, as will be discussed below, this land distribution evolution is fully endogenous in terms of relevant prices and intertemporal rationality. The model we present below is therefore similar to a CGE model, but represents a departure by fully endogenizing asset accumulation decisions at the individual household level, and by incorporating market imperfections. Our model has much in common with that of Imrohoroglu, Imrohoroglu and Joines (1993). Their model also incorporates individual rationality, dynamic decision-making, multiple market imperfections, and endogenous economic aggregates. But while we are concerned with endogenous structural evolution in agriculture, they are concerned with the distribution of money under alternative social security regimes. The main methodological differences are that where their model has life-cycle effects ours does not, and where our model incorporates rational expectations over endogenous prices, theirs does not.

2.1. The production period model and the static class map

Given endowments of land (T), Labor (L^0), and money (M), we assume that each agent attempts to maximize household income defined as:

$$p \equiv \{ p_c Q - w L^d - F\ P_f - 1[z + irB]\} + \{w f(L^s)\} + \{ irS\} \qquad (1)$$

where the first term in curly brackets gives the net-income from agricultural production, the second term gives labor market earnings, and the third gives returns to money invested in a bank over the production cycle. Agricultural output is produced with a simple Cobb-Douglas technology,

$$Q = DF^a T^a L^a \qquad (2)$$

where F measures inputs purchased at a price P_f, T is the land stock, and L is labor measured in quality-adjusted, efficiency units. Efficiency labor is produced according to the following technology:

$$L = L^h + l\,[g\,(T, L^h) L^d] + [1 - l][-n\,(L^d) + g_0 L^d] \qquad (3)$$

where L^h is family labor devoted to home production, L^d is hours of hired labor and the endogenous indicator variable λ takes the value of 1 if the household uses informal, family labor supervision and equals one if the agent formally supervises its labor force with hired supervisors. The employment function, $\phi(L^s)$ gives days employed as a function of days of labor supplied to the off-farm job market. We assume that $f(L^s) \to L^s$ as $L^s \to 0$, and that $0 < f' < 1$ to capture the notion that employment becomes increasingly difficult to obtain as increased desire to sell labor forces one to search for employment in the slack season.

Maximization of (1) is further constrained by an *ex ante* working capital constraint:

$$w L^d + P_f F + P_c R_0 \leq M - S + w f(L^s) + 1[B - z] \qquad (4)$$

which simply says that the agent needs sufficient cash on hand to finance cash costs of production plus family subsistence over the rainy season (P_xR_0).[5] Working capital can be obtained from money that is not saved (M-S), from contemporaneous off-farm wage earnings, and from the net proceeds of any loans taken out by the household, l[B-*z*], where B is the gross loan amount, *z* is a fixed transaction cost and l is a indicator taking the value of one if B is positive. Finally, each agent faces a borrowing ceiling tied to the amount of land owned,

$$B \leq bT \,, \tag{5}$$

and the following miscellaneous non-negativity restrictions:

$$(L_0 - L^h - L^h), S, L^d, B \geq 0 \tag{6}$$

The agents objective is thus to maximize (1) subject to (2)-(6) and we denote the optimum value function corresponding to this problem as $p\,^*(T, M)$ to emphasize its dependence on endowments.

This income maximization problem gives prominence to the intrinsic (asymmetric information-based) capital and labor market imperfections that have been extensively discussed in the context of developing country agriculture. The working capital constraint (3) makes the specification of the rules of access to capital of primary importance. While some would argue that because of asymmetric information, small farmers are completely rationed out of credit markets (e.g., see Eswaran and Kotwal, 1986 and Carter, 1988), we more conservatively assume that all agents have equal access to credit at a given market rate of interest. Borrowers do, however, face a fixed transactions cost, *z*, that

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[5] Although this consumption requirement constrains farmers in their use of money for production, we have adopted the conservative assumption that it does not bind on the intertemporal accumulation problem: households can choose to spend up to their full agricultural income acquiring land. In the numerical parameterization of this model, the wage is high enough so that even a full time wage worker can meet the subsistence requirement.

is associated with the cash and opportunity costs of loan application, investigation and approval. The fixity of z makes small loans unattractive for all agents (rich or poor). Note that because of these transactions costs and the consequent reluctance of some agents to borrow, the shadow price of the working capital constraint (3)—which we denote as "m"—will endogenously vary over the endowment space even though there is a parametrically given market rate of interest.

The second feature of the production problem is that output depends on inputs of labor effort, not just labor time. The non-contractability of labor effort in spatially disperse, biologically based production process has a long history in agricultural economics (e.g., see Brewster, 1950), and we follow Bowles (1985) in specifying labor extraction technologies (2) that transform labor power or time into labor effort. Family labor may be used for supervision, but consistent with the findings of Frisvold (1994), the efficacy of family labor supervision diminishes as farm size grows and family labor becomes spread too thinly over a large area. Specifically we assume that $g(T, L^h) < 1$ and that $\partial g / \partial T < 0$. Because of the diminishing efficacy of informal, family labor supervision, larger farmers are thus likely to switch to hierarchical supervisory technology, thereby releasing the farm-size constraint on labor extraction, but adding significantly to labor costs (Bowles, 1985; Carter and Kalfayan, 1989).[6] Like hired labor supervised informally, hierarchically supervised hired labor is never more productive than family labor ($g_0 < 1$) and we assume further assume that the supervisory costs, $n(L^d)$, contains both a fixed and a variable component.

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[6] In addition, agents wishing to hire out their labor must pay a search cost in terms of time that is related to the amount of time they wish to sell. Without this assumption, the shadow price of labor would be constant for households that do not hire in labor. With this assumption, the shadow price of labor for non-labor hiring households increases with farm size and decreases with family size similar to a Chayanovian specification.

As with the shadow price of capital, the combined effects of this labor market specification is to make the effective or shadow price of labor endogenous to the individual's choices and, ultimately, their endowments. The end result is an analogue to the Chayanovian world in which the opportunity cost of labor is subjectively (or endogenously) determined. While the more recent literature on household models has tends to characterize an endogenous shadow price of labor as reflecting a non-separability between consumption and production decisions (e.g., see Singh, Squire and Strauss, 1986), it results here, as in those household models, from the fact that labor markets are thin or otherwise imperfect.

Maximization problem (1) admits a variety of solution regimes, depending on which constraints bind at the optimum. These solutions can be neatly divided into 12 solution regimes—or classes—based on labor and capital use on the farm.[7] Chart 2 summarizes these regimes or classes. The differences among the solution regimes may be highlighted by understanding what happens to demands for two key factors, labor and capital. Because of the search costs inherent in hiring out labor, a wage premium is necessary to draw household labor off farm, even if its marginal product there is below the market wage. Because of the difference in labor supervision strategies, at a certain level of land holding, it becomes rational to jump to the hierarchical supervision regime. These non-convexities generate four labor-use regimes which we straightforwardly designate as semi-proletarian, peasant, capitalist family and hierarchical capitalist. Each regime can be characterized

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[7] Note that like Eswaran and Kotwal (1986) we can rule out several possible solution regimes, including those which would have households simultaneously hire and sell labor in the market.

Chart 2: Static Class Regimes

		Labor Regimes			
		Hire labor out: $L^s > 0$; $\omega=(1+\mu)w\phi'$	**Peasant Production: $L^s = L^d = 0$; $\omega=(1+\mu)w\phi'$**	**Hire labor in, supervised by family: $L^d>0$; $\lambda = 1$; $\omega=(1+\mu)w/\gamma(T, L^h)$**	**Hire laborers and supervisors: $L^d>0$; $\lambda = 0$; $\omega=(1+\mu)w/\gamma_0$**
Credit Regimes	**Unconstrained in capital use; no borrowing: μ =ir ; B = 0**	*Independent Semi-Proletariat*	*Independent Peasants*	*Independent Family Farmers*	*Independent Hierarchical Capitalists*
	Constrained in capital use; no borrowing: μ > ir ; B = 0	*Capital-Constrained Semi-Proletariat*	*Capital-Constrained Peasants*	*Capital-Constrained Family Farmers*	*Capital-Constrained Hierarchical Capitalists*
	Positive levels of borrowing: $\mu \geq$ ir ; B > 0	*Indebted Semi-Proletariat*	*Indebted Peasants*	*Indebted Family Farmers*	*Indebted Hierarchical Capitalists*

by its shadow price of labor, meaning the endogenous price to which the marginal value product of labor in production. For example, for semi-proletarian agents that cultivate some land and supply some labor to the market, the first order conditions defined by (1) above imply a shadow price of labor, w, given by:

$$w = w(\partial f / \partial L^s)(1+ m)$$

where w is the market price of labor, m is the shadow price associated with the capital constraint (3). Note that the shadow price for this class is the expected marginal wage that could be earned in the off-farm labor market, marked up by the shadow price of capital. The shadow price for the other labor regimes is given across the tops of the columns in Chart 2. For the labor-hiring classes, the shadow wage is given by the cost of labor measured in efficiency terms, again marked up by the shadow price of capital.

Because of the fixed cost of borrowing capital, a wedge is created between those who have a shadow price of capital below the interest rate, and those who can profitably borrow. Letting "*m*" denote the shadow price of the *ex ante* capital constraint in (1) and "ir" the market rate of interest, we can designate the capital use regimes as "unconstrained" (*m*=ir, B=0), "constrained" (*m* >ir, B=0) and "indebted" (*m*=ir, B>0). Together the labor and capital market imperfections generate 12 endogenous classes shown in Chart 2.

Which of these 12 regimes is optimal for an individual agent will depend on his or her stocks or endowments of land and money. Following the lead of Roemer and Eswaran and Kotwal, we can explore how the optimal regime or class systematically varies over the two-dimensional endowment space, identifying "class boundaries" in land-money space. For example, the boundary between the Free Peasants and the Constrained Peasants under our parametric specification is defined by equation (2),

$$T(M) = \frac{(M - P_c R_0)^2 P_f}{(a_1 P_c D)^3} \qquad (7)$$

This boundary is obtained by finding first-order conditions of both Free Peasants and Constrained Peasants, and then identifying the combination of T and M that enables the first-order conditions for both sets of agents to hold simultaneously. Peasants with land endowments just below that boundary would be able to fully self-finance production and still have money to put in the bank during the rainy season to earn the interest rate. Peasants with land stocks just above that boundary would exhaust their funds financing production costs and would be unable to drive the return on funds invested in production to the market rate of interest. While these individuals would like to borrow money, the fixed transactions will discourage them from doing so.

While most of the class boundaries are not analytically tractable, the numerical parameterization of the model given in Appendix A can be used to fill out the entire class map over (T, M) space, as shown in Figure 1v. Land is measured in the Central American unit of a manzana (1.7 manzanas= 1 hectare), and production parameters and resulting income levels have been estimated to approximate real levels. As can be seen, agents with small amounts of money relative to land tend to borrow money and fall into the indebted classes. As their stock of money relative to land increases, their shadow price of capital without borrowing falls, eventually pushing them into the capital constrained classes, for whom the shadow price of capital is higher than the nominal interest rate, but not so high that agents are willing to pay the fixed transactions cost to borrow. Finally, as money relative to land increases yet more, agents enter the self-financing, capital-unconstrained, for whom the shadow cost of capital is less than the nominal interest rate.[8] A similar analysis of labor regimes can be articulated, as agents move toward hiring in more labor as their land endowments increase. The increases in labor-hiring come not gradually, but at discrete points in the asset continuum.

An important exception to the above pattern of borrowing and capital constraints occurs for agents whose land endowments are small in absolute terms. As can be seen, for a small enough land endowment (less than about 2 manzanas), even an agent with no money will be in the capital unconstrained, semi-proletarian class. For such individuals, the wage earnings are sufficient to cover both subsistence costs and fully finance production. Equally important, however, incremental additions to the land stock of these agents pushes them into the capital constrained, rather than indebted classes. Again because of their small absolute size and the fixed transactions costs of

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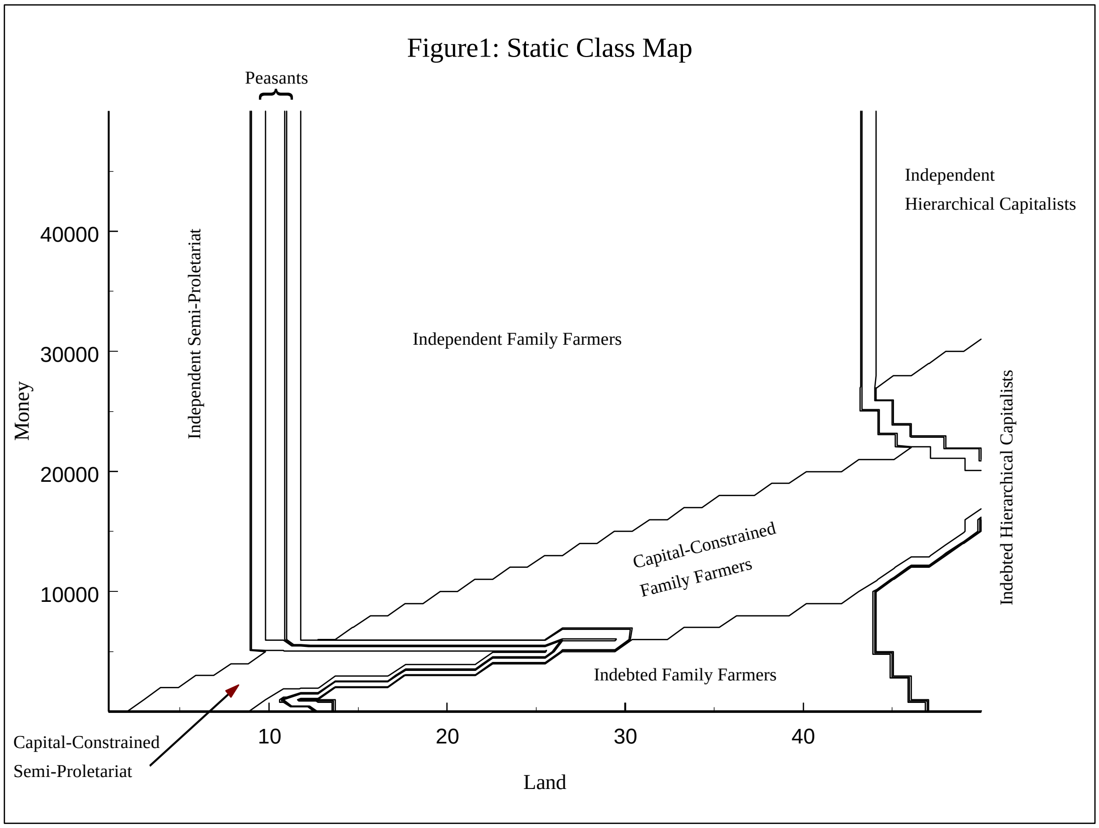
[8] The same analysis would be valid if, as is commonly observed, there were size-stratified interest rates, rather than a fixed portfolio fee.



borrowing, such individuals do not find it rational to borrow and they are hampered in production by a shadow price of capital that exceeds the market rate of interest.

The various classes that arise out of problem (1) and displayed in Figure 1 do not themselves determine behavior, but rather are themselves determined by optimal behavior. While this concept of class is at odds with some definitions, the solution regimes in Figure 1 do correspond to Elster's (1994) concept of class as defined by "endowment-necessitated behavior."[9] Class does not determine

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[9] We leave aside here questions about class consciousness and political behavior, though Wright (1997) finds that the mapping between endowment position and political behavior is quite strong inmost countries, with the United States standing as an important exception.

production behavior, but rather describes behavior that is determined by endowments, as in the previous analyses of Eswaran and Kotwal (1986), DeJanvry, Fafchamps and Sadoulet (1991), and Moene (1992).[10] The rich class map implied in Figure 1 suggests a wide variety of production strategies—each of which is optimal within the context of the multiple market imperfections faced by the agents.

What the class map does not address, however, is the extent to which agents can make themselves better off by migrating toward other positions on the class map. Profits per hectare vary systematically across the class map, and some regions are more materially desirable than others. Binswanger, Deininger and Feder (1995) pose the question to the literature on peasant classes that if indeed there are more and less desirable class positions, why don't agents forego some consumption now in order to facilitate asset accumulation and migration to better class positions. Before turning to the specification of the full dynamic model needed to address that fully question, we garner some insights from the static production model about the impact of multiple market failures on accumulation incentives and eventual asset market competitiveness of different classes of agents.

2.2. Insights from the production model about the economics of accumulation in inegalitarian agrarian economies

The labor and capital market imperfections that lie at the heart of the production model create the class-differentiated shadow prices for capital and labor summarized in Chart 2 above.
Because these marginal shadow prices guide variable input choice, they shape the marginal income that an agent would receive from an incremental increases in either land or money endowments. An agent

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[10] Although class does not determine production behavior, it does reflect interests, and therefore may condition political behavior.

with access to effectively cheap labor and capital will obviously cultivate an incremental unit of land more intensively, and realize greater income increases with it, than would an agent with correspondingly higher priced variable factors of production. Other things equal—and leaving aside for the moment the strategic complexities of dynamically rational decisionmaking—we might expect the first agent to be willing to pay more than the high cost second agent for the incremental unit of land.

There is, however, no easy way of unambiguously determining which agents and classes have access to the cheapest factors of production. The labor market failures tend to advantage agents with low land endowments. It is precisely this privileged access to cheap family labor that led Chayanov and latter day advocates of family labor farms, to postulate the long term stability of peasant farming. However, standing against this "Chayanovian advantage" are the fixed transactions cost in the capital market which tend to increase the shadow price of capital for smaller scale producers. Such countervailing disadvantages seem to underlie the view of Patnaik (1979) who disputes the notion that peasant poverty and weak labor market opportunities suffice to make small scale producers for long term expansion and survival.[11]

One way to explore these countervailing market failures and aggregate their cross-cutting economic impacts is to examine the marginal net present production value (NPPV) of land defined as:

$$r = \sum_{t=1}^{\infty} [\Delta(T,M)]/(1+m(T,M))^t \qquad (8)$$

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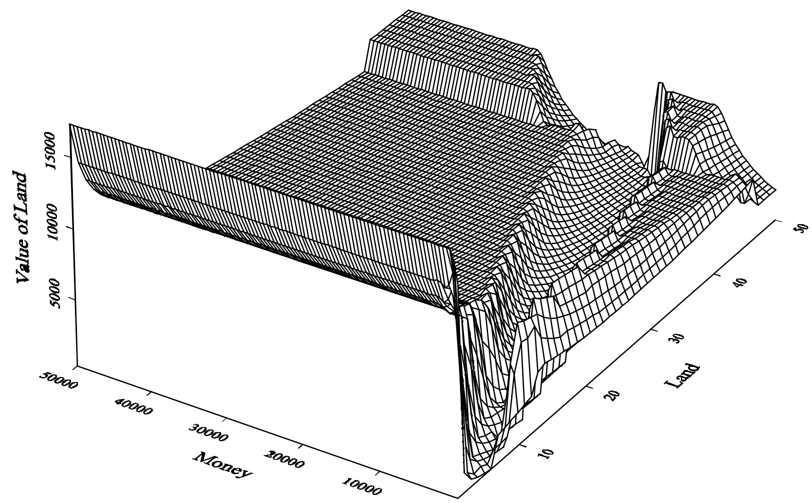
[11] While Patnaik (1979) notes that "capitalist producers" may use more capital-intensive, and more profitable techniques of production than peasant producers, she does not herself explain the different choices. Her approach thus seems to rely on an exogenous notion of class (i.e., class explains behavior), in contrast to the approach here that models class as an endogenous outcome of the same rational choice process (and market structures) that generate choices of technique.

where $\Delta = \partial p^*(T,M)/\partial T$ and the discount rate, m is the shadow price of the capital constraint given in expression (3) above. Note that this expression will not in general be independent of relative factor prices and technology, as these two things effectively shape how costly any particular market failure is. As written, expression (8) assumes that prices and technologies persist indefinitely into the future.

Using the numerical specification of the production model given in Appendix A and the prices from the final period of the dynamic simulation to be discussed below, Figure 3 graphs expression (8) evaluated over the same endowment or state space used for the static class map (Figure 2) above. As can be seen, the net present production value of land is very high for very small land endowments regardless of the money endowment. However, a land poor agent with little financial wealth who tried to accumulate land would quickly run into the trench that cuts across the NPPV surface. This trench corresponds to the band of capital-constrained classes shown in Figure 1. For agents in the trench, the shadow price of capital is extremely high (as they neither have the funds to self-finance production, nor is it worth their while to borrow given transactions costs), and their resulting factor intensities and productivities are low. Assuming a market price above the level of the trench (which it is in the dynamic simulation below), any agent traversing the trench through land accumulation will not only have to sacrifice current consumption, they will also have to sacrifice some future income as well given that the land price will exceed the NPPV of the land obtained.

This trench, and the interacting market imperfections that underlie it, thus stand as non-trivial barriers to land accumulation by low wealth agents and to the elimination of economically costly asset inequality. At best strategies to traverse the trench, or to circumnavigate it, may take substantial time and sacrifice of consumption. Whether or not agents will find it worth their while to

Figure 2: Net Present Production Value of Land



undertake such sacrifice requires proper specification of the dynamic choice problem to which we now turn.

2.3. The Intertemporal Model

Following the end of the production period, agents make their consumption and accumulation decisions in order to maximize the following infinite horizon utility problem:

$$
\begin{aligned}
&\max U_0(\underline{c}) = \sum_{t=0}^{\infty} u(c_t) d^{\,t} \\
&s.t. \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (9) \\
&c_t \leq p_t^{*} + M_t - M_{t+1} - (T_{t+1} - T_t) P_{T_t} \qquad (\forall t)
\end{aligned}
$$

where p_t^* is the (pre-determined) optimum value function defined by the production problem (1). Because there is no risk in the model, and because the form of the utility function is time-separable, it is possible for households to make production decisions independently from accumulation decisions. They first choose inputs to maximize profits, and then use those profits as income out of which they maximize intertemporal utility through their asset accumulation decisions. Households therefore face two sets of tradeoffs: in their production decisions, households must choose to what extent at the margin to invest their money in fertilizer or labor, or whether to save it off farm for a specified interest rate. In their intertemporal decisions, households decide to what extent at the margin to invest money in land accumulation or money accumulation, or whether to consume the money for its current utility value.

Note that as written, the infinite horizon model makes a number of simplifying assumptions about the operation of the land market. In particular, we ignore transactions costs in the land market that evidence (and government policies) suggest miliate against the purchase of land by poor people from rich people (see Carter and Zegarra, forthcoming). The dynamic specification also does not impose any minimum subsistence requirement, allowing poor households to save as much of their income as is dynamically rational, even it means reducing consumption to levels that in the reality are inconsistent with maintaining work capacity and health (see Ray and Streufert, 1993). Finally, the dynamic problem follows the static production problem in assuming a riskless world. The net effect of all these assumptions is to increase the likelihood that the land market can in fact effectively eliminate costly inequality over time.[12]

Corresponding to the infinite horizon problem is the value function,

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[12] Using a much simpler model of production, Zimmerman and Carter (1997) explore the impact of risk and subsistence constraints on trajectories of accumulation in a multi-asset model.

$$J^*(T_t, M_t) = \max_{T_{t+1}, M_{t+1}} \{u(c_t) + d J^*(T_{t+1}, M_{t+1})\} \qquad (10)$$

which expresses more directly the tradeoffs between current consumption and the value of accumulating the state variables, land and money. The true value function, J*, self-confirms its optimality, as in equation (10) (Bellman's equation). Understanding how the value of J*(T,M) changes over its domain helps in understanding the structural evolution generated by the model's multiple market imperfections. Note that like the net present production value (NPPV) measure examined in the previous section, the true value function is defined over the state or endowment space. But whereas the NPPV measure captures the value of accumulating land at the margin and holding just that increment forever, the true dynamic value of accumulating land lies as much in the possibility of accumulating a larger quantity of land in order to facilitate further accumulation. The difference between the NPPV of land and the true value function lies precisely in this distinction: the true value function is conveys a similar sort of information, but expresses the value of a stock of land, not only for its own value, but also for the sort of accumulation it permits along an entire dynamic accumulation trajectory. Accordingly, the true value function smoothens out many of the rougher features of the reservation price of land.[13]

Unfortunately, the value function for this problem is extremely complicated. If we removed from the model the capital market imperfections, the holding of money stocks would not impinge on production decisions, and the value function would be additively separable in its arguments: J(T,M) = $J_1(T) + J_2(M)$. If we then removed the labor market imperfections, the first derivative of J_1 with respect to T would be continuous and monotonic. These modifications would lead to an analytically tractable

---

[13] Indeed, when plotted as a three dimensional surface, the numerical approximation to the true value function is topographical uninteresting, though it does exhibit some subtle and important change in its curvature.

value function, which would imply straightforward accumulation trajectories for all the agents. Under the specified market imperfections, however, the value function is analytically intractable. We therefore turn to numerical estimation of the true value function.

2.4. Numerical dynamic programming methods

Numerical solutions of complex models have frequently been used in the literature dealing with multiple market imperfections and agent heterogeneity (Williams and Wright, 1991; Imrohoroghlu, Imrohoroglu and Joines, 1993; Eswaran and Kotwal, 1986). Numerical solutions are popular in such contexts because analytically tractable models often depend on very restrictive assumptions (such as representative agents or complete markets), and because appropriate, long-term panel data of microeconomic processes often do not exist. Furthermore, as Albers (1996) argues, when what is of interest is not just a long-term steady-state, but rather the values of particular economic data along the *transition* to that steady state, then numerical models are especially useful.

We find the true value function by a commonly-used iterative method, explained in greater depth in Zimmerman (1994) and Streufert (1990). We first estimate a value function, then use that value function in conjunction with equation (10) to find an updated, more accurate, estimate of the value function. We repeat this process as many times as necessary for our guess of the value function to converge (pointwise) to a value—its true value.

To know that our search for a fixed point in value function space in fact yields the true value function, we need only know that the utility function and the production function together constitute a dynamic problem that is biconvergent, a concept that is similar to tail insensitivity (see Streufert, 1990). Biconvergence also assures us that the transversality condition is met, and that a solution to Koopmans' equation (expression 10 above) is the same as a solution to the infinite horizon utility maximization problem (expression 9). The biconvergence of this problem is proven in Appendix B below. The intuition behind the proof is that since the profit function under ideal factor allocation is decreasing returns to scale, satiation (in the utility function) and impatience (in the discount rate) imply that agents will not want to infinitely accumulate land or money. We assume speculative price bubbles do not occur.

Because the value function depends on the indirect profit function, which is itself a function of prices, and because prices evolve over time, the value function is not stationary. Strictly speaking, we would like to find J*(T,M,**P**), but such a task would involve numerically searching over a huge value function space. Since in every period the agents know **P**, we therefore reduce that space by looking for J*(T,M) given the expected path of prices. We will save notation by writing simply J*(T,M) instead of $J^*_{\mathbf{P}}$(T,M), but it will be understood that the value function does depend on price expectations, and is different in every period.

Because the intertemporal problem depends on expectations of future prices, a word is necessary on how price expectations are formed. It is assumed that agents have rational expectations about future prices in the sense that they use all information available in a given period to predict prices from then forward, or in other words they calculate $E[\ \{\boldsymbol{P}\}\ |\ W_t]$, where W_t is the information set available in time t, and $\{\boldsymbol{P}\}$ is the sequence of future prices (including the wage, the fertilizer price and the land price--the output good price being the numéraire). This assumption is operationalized by running the simulation model a number of times over. When the price path from one run of the model conforms to people's expectations of it--i.e., the price path from the previous run--we will say that agents have rational expectations. Of course in practice it will be impossible or at least highly unlikely for a price path to exactly confirm itself. The confirmation criterion used is accordingly that of a net deviation of prices from expectations of not more than 0.1% for all prices for all periods.

We assume in this version of the model that the output price is fixed. This assumption is part of the laissez-faire set of policies that we wish to model. We assume that there is a world market which can provide food at a price which this small, open economy is powerless to affect.

3. The dynamics of asset inequality

In order to explore the dynamics of asset inequality, we parameterized an initial land distribution using agrarian census data from pre-revolutionary Nicaragua (Government of Nicaragua, 1975). Nicaragua at the time of this census (1972) had not yet undergone agrarian reform of any kind, in which respect it was similar to many Latin American agricultural sectors in terms of the striking degree of inequality in the land ownership distribution. While the Nicaraguan census covered only landed households, we assume that about 30% of all rural households were landless. Including these landless households, the richest 50% of the households had 70 times more land the poorest 50%, and the Gini coefficient was over 0.9. Under an egalitarian land distribution, each household would hold about 9.5 manzanas of land.

This distribution of land provides the initial distribution for the numerical model. Production parameters were also estimated using farm survey data collected in Nicaragua in 1982 (see Carter 1989). Because the Census did not provide information on holding of financial wealth, it was assumed that liquid assets (from here on "money") would have been proportional to land holdings. With the price of the output good as the numéraire, it was assumed that each household would have 175 units of money initially per manzana owned. This amount of money is equivalent to about 5% of the annual wage that emerges in the model. While this specification is admittedly arbitrary, by making initial financial wealth relatively low and correlated with land size, it captures the dynamic challenge facing low-wealth agents who try to accumulate their way around financial market imperfections.

The numerical model was run for 50 periods, creating a 50 year, dynamic general equilibrium history for this economy. The successive solutions toward a rational expectations equilibrium in prices converged in 5 iterations. In the final iteration, the mean absolute deviation of prices from expectations was 0.005%. The equilibrium wage and fertilizer price increase in the first 25 periods, with the fertilizer price stabilizing and the wage dropping off slightly thereafter. The general increase in these factor prices can be attributed to the more efficient use of labor and fertilizer as land is redistributed by the market. The wage ebbs somewhat later in the model as more of the large farms enter less labor-intensive forms of production. As would be expected in a dynamic model the price of land, which can be costlessly stored across periods, is relatively stable at about 7000 units of money per manzana.

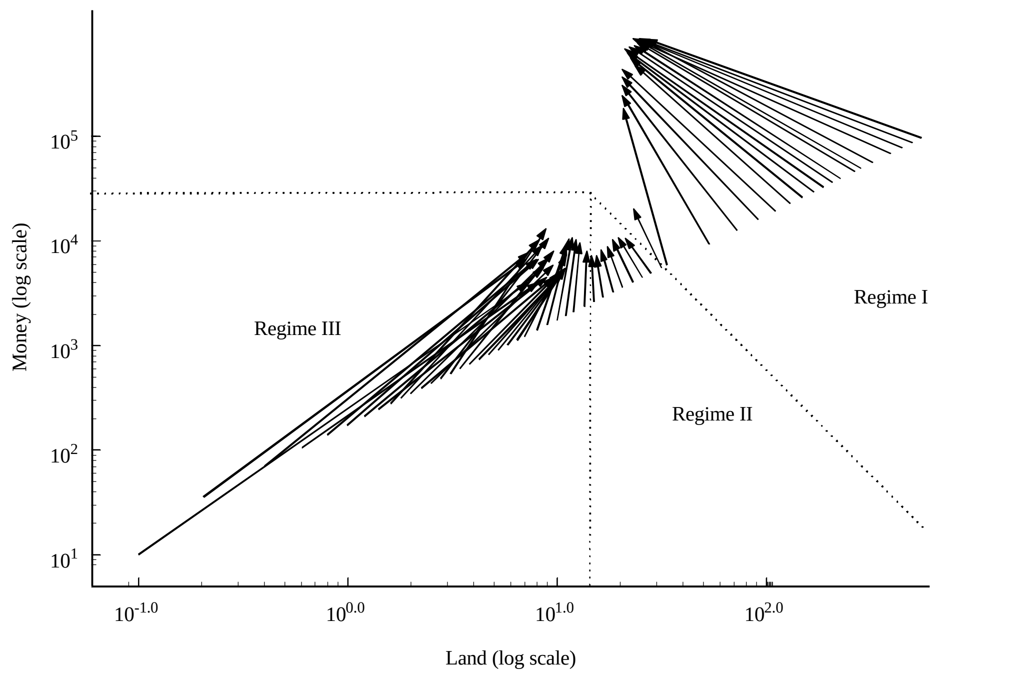
3.1. Trajectories of accumulation and the dynamic class map and temporary self-induced capital constraints.

Section 2.1 above portrayed the class map for the single period production problem, showing how regions of the endowment space systematically correspond to particular, individually rational behavioral patterns. An analogue exists for the dynamic behavior (or trajectories of accumulation and consumption) associated with the different endowment position. This correspondence between endowment position and dynamic behavior is also a class mapping in the sense that it represents "endowment necessitated behavior" in the sense of Elster (1994).

Figure 3 portrays the dynamic class map for this model. Each arrow summarizes the accumulation history for a particular agent, with the beginning point of the arrow fixed at the agent's initial endowment position, and the endpoint signifying the agent's portfolio holding of land and money in year 50. Arrows are shown only for a subset of agents in the model and have been selected to capture the full range of the endowment space from which agents are pulled onto similar accumulation trajectories. Class I, the Latifundistas, are enticed by the favorable market price for land to sell off large quantities of land over the 5 decades of simulated history, boosting both their consumption and money holdings along the way. Class II is comprised of a class of persistent capitalized family farmers. Over time, they modestly adjust their land holdings and build-up sufficient funds to internally finance the production process.

The third and final dynamic class is comprised of agents who were either initially assetless, or whose land holdings were below about 11 manzanas. Over 5 decades of simulated history these agents move—often dramatically so—toward the asset levels of the persistent capitalist family farmers. The specific, year-by-year trajectory of the land and money accumulation is more complex than Figure 3 shows, however. Agents in this group confront the brunt of the labor and capital market imperfections. Dependent on the labor market for a large portion of their livelihood, and with capital needs too small to make it worth their while to pay the fixed costs associated with

Figure 3: Dynamic Class Map of Accumulation Regimes



borrowing, these agents must devise a way to expand their holdings, while maintaining some minimal level of self-finance for the production they do have, while not sacrificing too much of current consumption. These Class III agents slowly accumulate land over the first several years, drawing down any initial money endowment to aid in land purchase and defense of their consumption standard. Still selling a large portion of their labor time on the market, these agents use their wage earnings in part to meet the upfront costs of production.

Figure 4 shows the evolving static class positions of these Class III agents. After the first few years of land accumulation, these agents arrive at farms sizes beyond which they can no longer self-finance production. By year 15 of the simulation, most of these agents fall into the capital-constrained semi-proletarian class. Because of their undercapitalized production processes, the net present production value of land (equation 8 above) to these agents falls drastically. In terms of

Figure 4: Evolution of Classes for Regime III Households

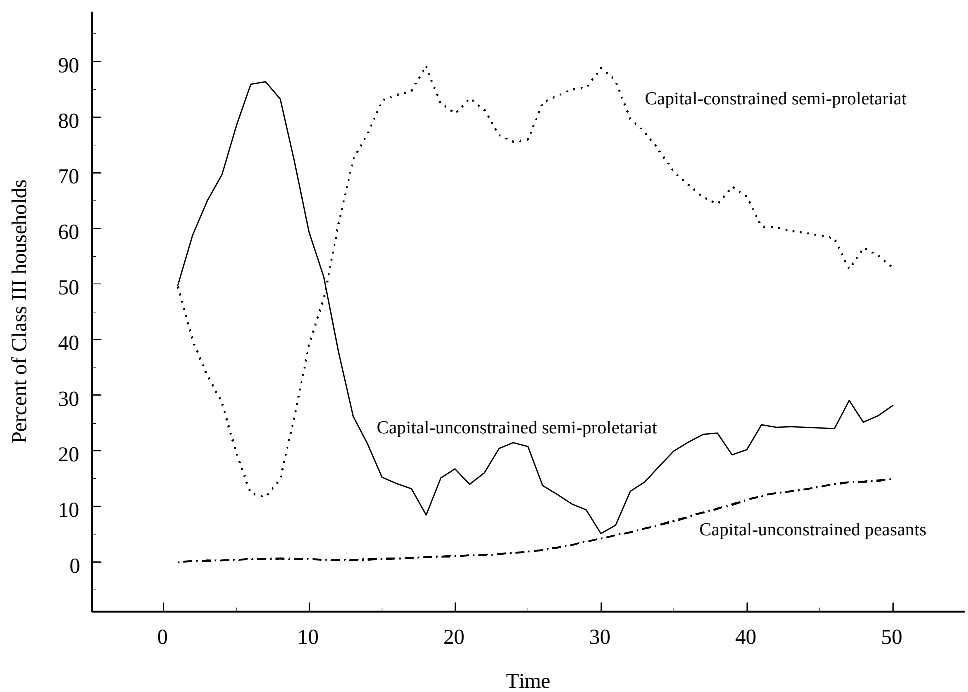


Figure 2 above, these agents find themselves in the trench that cuts across asset space. Despite facing land prices well in excess of their marginal net present production value of land, Group III agents continue to accumulate land, though they move onto a balanced accumulation trajectory in which they acquire both land and money. Still by the end of the simulation, nearly half of Class III agents remain capital constrained, though moderately so, as they work their way out of the trench. The other half of the Class III agents have, by history's end, reached an economic position where they can self-finance a production process which equates the shadow price of the capital constraint to the market rate of interest.

In addition to its social importance to be discussed below, the (dynamically rational) behavior of Class III farmers illustrates the value-added of the dynamic modeling strategy developed here. The static, net present value of land measure (equation 8 and Figure 2) would indicate that agents would stop short of accumulation decisions that moved them into the trench in the net-present-value of land surface. The fact that they do move into and slowly across the trench in the dynamic programming problem indicates that in an environment of multiple market imperfections, asset accumulation has not only an immediate income value, but also has a longer term strategic value as it moves the agent toward a portfolio mix and scale which will permit him or her to circumvent market imperfections. Although this evolution is of course rational on the part of the accumulating household, it may mean a loss to society as the household's production becomes temporarily constrained by a shortage of important factors.[14]

3.2. Costs and persistence of asset inequality

As seen in Figure 3 above, over the 50 years of simulated agrarian history there is a strong tendency toward an egalitarian land distribution as all agents converge toward a single holding size,

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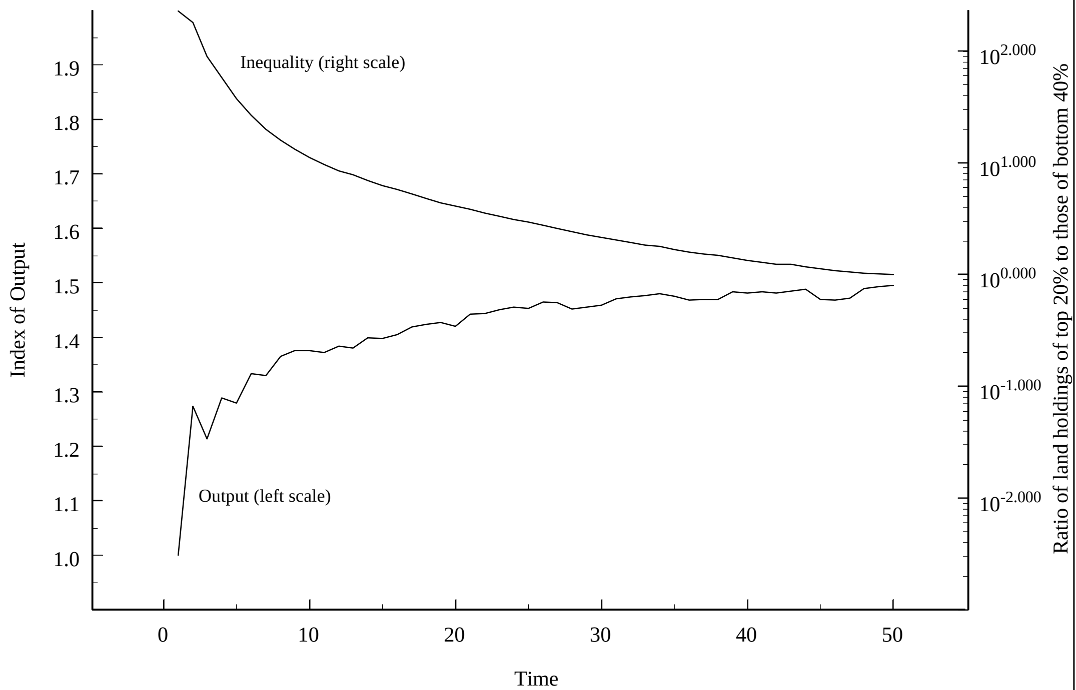
[14] The notion of tenacious, but undercapitalized producers was symbolized by one of South Africa's premier wine makers who attended a Ministerial meeting on land reform, at the end of which he presented the Minister of Land Reform a wooden wine case. The delighted minister opened the case, only to find it filled with vegetables. While the winemakers not so subtle message about the potential impact of land reform presumably had its own motivations, it does symbolize the idea that undercapitalized trajectories of land accumulation can carry economic costs even if it eliminates costly inequality in the long term.

irrespective of their initial endowment. Figure 5 graphs a single, summary inequality measure—the ratio of the land of the top 20% to that of the bottom 40%—over the simulation period. As can be seen, land inequality by this measure has nearly evaporated by the final year, declining from 230 to 1.0.

Figure 5: The Dynamic Cost and Persistence and of Asset Inequality



This finding suggests that time and inter-temporal choice suffice to dampen the endowment sensitivity in an agrarian economy with the sorts of multiple market imperfections highlighted in the agrarian institutions literature. Left to its own devices, the market will, in fact, redress inequalities in the land distribution, even in the presence of multiple factor market imperfections which, as discussed earlier, place significant barriers in the way of land accumulation by low wealth agents.

This market-driven redistribution is not without a cost, however. Output is considerably lower in the earlier periods than it would be under a fully equal distribution of land. The output index, also shown in Figure 5, rises steadily over time as the land distribution slowly becomes more equal. By the end of the simulation, the more egalitarian economy produces nearly 50% more output off the same aggregate land-labor endowment. The present value of the cumulative output cost of inequality over the 50 years is 2 ½ times the total output of the first period. A government-led redistribution early on would therefore generate a considerable social surplus. Such findings provide an important numerical calibration of the theoretical observations of Bardhan, Bowles and Gintis (1998) and others, who argue that equity and efficiency need not be substitutes in social policy.

4. Conclusion

Using numerical dynamic programming methods, this paper contributes a dynamic component to our growing microeconomic understanding of the economic costs of inequality. Beginning with a multiple market failures model of an agrarian economy in which asset inequality at any point in time is costly, this paper goes on to ask whether time and dynamically rational individual accumulation decisions will (1) eliminate costly inequality; and (2) do so at a pace that trivializes the static costs of inequality.

In answering these questions, this paper builds on the contemporary analytical Marxist tradition, utilizing class maps defined over the model's two-dimensional endowment or state space to explore behavioral differentiation of heterogeneously endowed agents in both the spheres of production and accumulation. Class (understood as endowment necessitated behavior) emerges as a salient concept and helps clarify the barriers that multiple market failures place in the path of asset accumulation by low wealth agents. At a more conventional methodological level, this paper contributes ways to grapple with rational expectations-based dynamic equilibrium in a model that rests on complex microfoundations in which heterogeneously endowed agents rationally choose to behave in qualitatively different ways. Importantly, the resulting numerical simulation method permits us to not only characterize stable points of attraction in state space, but to also quantitatively characterize the field from which those points pull and their rate of attraction.

Beginning with an asset and technological structure based on Nicaragua *circa* 1980, the model is used to create a simulated 50 year dynamic rational expectations equilibrium history. The results reveal that the dynamics indeed matter as over this time period agents circumvent the static accumulation barriers created by multiple market failures and the economy moves toward greater asset equality. However, process is slow and the magnitude of the accumulated costs suggests that well-designed redistributive policies could potentially dominate the *laissez faire* operation of the asset market. It should be noted that these results are, if anything, biased in favor of the dynamic efficiency of asset markets. The model both ignores transactions costs in the land market and, more significantly, the impact of risk on desired portfolio mix that less well-off agents might desire to hold (e.g., see Rosenzweig and Binswanger, 1993 and Zimmerman and Carter, 1997).

The answers to the two questions posed at the beginning of this conclusion are thus mixed. A semblance of Coasean world in which the endowment distribution does not matter.is salvaged in the long run. But in the short and medium terms, inequality matters and class counts.

Appendix A: Parameterization of the Multiple Market Failure Model

The model has been parameterized using a combination of census and farm production data from Nicaragua (the latter data source is explained and analyzed in detail in Carter, 1989). Estimated parameter values include the starting distribution of assets, production parameters and initial prices, which, though endogenous, were calibrated by adjusting the total supply of fertilizer and some of the (unobservable) labor supervision parameters. Many of the specific parameter values are inherently unobservable. For these parameters (such as search costs in the labor market, hired labor supervision costs) an effort has been made to choose reasonable parameter values. The specifications and parameters used in the model are as follows:

Production

$Q = DT^{a_1} L^{a_2} F^{a_3}$, with D = 1750, and α_1, α_2, α_3=0.33;

Off Farm Employment Function

$f(L^s) = L^s - c(L^s)^2$, with c = 0.4;

Efficiency Labor Extraction under Informal Supervision

$g(T, L^h) = L^s + \left(\frac{g_l}{g_l + T}\right) L^d$, with g_l= 100;

Efficiency Labor Extraction under Hierarchical Supervision

$-n(L^d) + g_0 L^d = L^0 - a + b L^d + g^0 L^d$, with a = 0.25, b = 0.1, and γ^0 = 0.9;

Credit Access and Interest Rate

$B \leq bT$, with b = 500, the transactions costs of borrowing, *z*=1000, and the interest rate, *ir* = 0.05.;

Rainy Season Subsistence Parameter

R_0= 1000;

Utility Parameters

$u(c_t) = (c_t)^e$ ε=0.25, and the discount factor, d = 0.95.

Appendix B: Proof of Biconvergence of the Intertemporal Maximization Problem

Streufert (1990) shows that if a dynamic problem can be formally shown to be "biconvergent," a term he defines and which is composed of upperconvergence and lowerconvergence, then Koopmans' equation holds and the problem has a solution. This appendix demonstrates that both upper and lowerconvergence obtain for the model developed in this paper.

I. Upperconvergence.

Intuitively, upperconvergence will hold as long as agents do not have incentives to accumulate assets indefinitely and the transversality condition will be satisfied. Discounting and diminishing utility will assure that this condition is met as long as returns on assets do increase so sharply as to offset their accumulation-dampening effects. Upperconvergence holds for money trivially, since the return on money is the constant interest rate. For land, upperconvergence may be expected to hold since there is only a finite supply of land. If the returns were to increase sharply enough to create incentives for unbounded land accumulation and violate upperconvergence, then we might expect the equilibrium price of land to be correspondingly bid up sharply and to thereby diminish accumulation incentives and restore uppperconvergence. Along the lines of this intuition, we will now show that upperconvergence in fact holds for the problem and parameters at hand.

To facilitate what the proof, first define A* /P* to be the greatest ratio of per-hectare profit of land observed in the model to the land price in the same period. In considering incentives for unbounded accumulation, we need only be concerned with the hierarchical capitalist class, since any large accumulation of land implies moving beyond the range of peasant production and capitalist family farming. In order to prove upperconvergence we need to assume that A*/P* is low relative to the utility satiation parameter (ε) and the discount rate (δ) in the following well-defined sense:

A1. Assume that the value today of consuming in the next period one consumption unit, plus the per-hectare, one-period return to land, is always less than one:

$$d\left(1+\frac{A*}{P*}\right)^{e} < 1$$

This assumption is quite reasonable given the parameters of the model. Given the values of δ and ε, the quantity (A*/P*) would have to be less than 0.228. While A*/P* is of course endogenous, it maximum value over the course of the simulation is only 0.178 so that this condition is satisfied.

Now define the optimum value function that corresponds to the production period problem for the hierarchical capitalist class defined by equations (1)-(6) in section 2.1 above:

$$p_{t}^{HCF}(T,M) \equiv \max_{\{F,L^{d},S\}} DT^{a_1}F^{a_2}\left(L^{0}-a+bL^{d}+g^{0}L^{d}\right)^{a_3} - FP_{t}^{f} - w_{t}L^{d} + irS$$

where for a given [T,M], and subject to the maximization is subject to the following constraints:

$$\begin{aligned}
&w_{t}f(L^{0}-L^{h})+M \geq FP_{t}^{f}+w_{t}L^{d}+S+R_{0}\\
&L^{h} \leq L^{0}\\
&F,\, L^{d} > 0
\end{aligned}$$

The stream of utility from hierarchical capitalist production now can be defined as:

$$U^{*}(T,M) \equiv \max_{\{c^{\bullet},T^{\bullet},M^{\bullet}\}} \sum_{t=0}^{\infty} d^{t} c_{t}^{e}$$

subject to:

$$\begin{aligned}
&(T_{t+1}-T_{t})P_{Tt}+(M_{t+1}-M_{t}) \leq \Pi_{t}^{HCF} - c_{t}\\
&c^{\bullet} \equiv \langle c_{1},c_{2},c_{3},\ldots\rangle\\
&T^{\bullet} \equiv \langle T_{1},T_{2},T_{3},\ldots\rangle\\
&M^{\bullet} \equiv \langle M_{1},M_{2},M_{3},\ldots\rangle
\end{aligned}$$

We are now in a position to bound this utility. First note that,

$$(\forall t)\,(\exists A_t) \ni A_t T + irM \geq \Pi_t^{HCF}(T,M)$$

As defined above, let (A*/P*) be the least upper bound of

$$\left\{ \frac{A_1}{P_{T1}}, \frac{A_2}{P_{T2}}, \frac{A_3}{P_{T3}}, \ldots \right\}$$

Therefore,

$$\begin{aligned}
U^*(T,M) &\leq \sum_{t=0}^{\infty} d^t \left(\left(1 + \frac{A^*}{P^*}\right)^t T_0 + ir^t M_0 \right)^e \\
&\leq \sum_{t=0}^{\infty} d^t \left(\left(1 + \frac{A^*}{P^*}\right)^{et} T_0^e + ir^{et} M_0^e \right) \\
&= \frac{T_0^e}{1 - d\left(1 + \frac{A^*}{P^*}\right)^e} + \frac{M_0^e}{1 - (dr^e)} \\
&< +\infty \quad iff \quad dr^e < 1 \quad and \quad d\left(1 + \frac{A^*}{P^*}\right)^e < 1
\end{aligned}$$

Streufert (1992) proves that when the greatest feasible utility within the infinite product space of the production-consumption problem is bounded, then the intertemporal maximization problem is upperconvergent. As can be seen above, this condition holds here under assumption A1.

II. Lowerconvergence.

Lowerconvergence is satisfied automatically, because the utility within the product space is bounded from below (by zero) by the form of the utility function.

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