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Spatial allocation and the shadow pricing of product characteristics

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Abstract

The paper considers an industry transforming primary commodities (farm products) into processed commodities (food products). It focuses on the allocation of embedded characteristics (carbohydrate, protein, etc.) both across space and among commodities. The approach generates a spatial competitive market equilibrium of production, consumption, transformation, and trade for both primary and processed commodities, along with the spatial distribution of shadow prices for the product characteristics. The model provides a basis for analyzing the allocation and pricing of agricultural products, food products, and characteristics in spatial markets. The empirical usefulness of the model is illustrated in the context of regional resource allocation in the U.S. dairy sector. © 1998 Elsevier Science B.V.

1. Introduction

The development of agricultural markets and trade has stimulated interest in the spatial allocation of resources in the agricultural and food sector. Agricultural markets involve both primary agricultural products and food products. Primary agricultural products are outputs generated by geographically dispersed farms. Food products are processed products obtained from transforming primary farm outputs into wholesale and retail food items that are consumed by geographically dispersed households. In this context, primary agricultural products are the raw materials for the food processing, manufacturing and distribution industry that produces food commodities for consumers. This raises the issue of the efficiency in the allocation, pricing, and distribution of primary agricultural products and food products across space.

A related issue is the allocation of farm product characteristics (carbohydrate, protein, etc.) in the food sector. Primary agricultural products are the source (and often the only source) of nutrients that become part of the consumers' diet. The food processing industry is in the business of 'rearranging' these nutrients through the transformation of farm outputs into various food products. What is the efficient allocation of these nutrient characteristics both across space and among food products? This suggests a need to investigate the spatial allocation of food and the associated nutrient characteristics in the food sector.

Nutrient characteristics are basic components [1] of both agricultural and food products. They are typically nonmarket goods. While they are always embedded in marketed agricultural and food commodities, they generally are not subject to explicit market

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[1] The terms 'characteristics' and 'components' are used interchangeably throughout the paper.

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transactions and have no explicit market price. As a result, nutrient characteristics have only implicit (shadow) prices. Starting with the work of Gorman (1956), Becker (1965) and Lancaster, a growing literature on the implicit pricing of nonmarket goods (e.g., Griliches, 1971; Dhrymes, 1971; Deaton and Muellbauer, 1980; Stigler and Becker, 1975) has emerged. The shadow pricing of the nonmarket characteristics is of interest since it can reflect differences in quality as well as prices among market goods. Rosen (1974) has investigated the shadow pricing of characteristics under competitive market equilibrium. He showed that shadow prices reflect both marginal rates of substitution (on the demand side) and marginal rates of transformation (on the supply side) among nonmarket characteristics. Rosen's results stimulated much research on the implicit pricing of nonmarket charateristics for differentiated products (e.g., Lucas, 1975; Ball and Kirwan, 1977; Palmquist, 1984; Epple, 1987).

At this point, little research has been conducted on the multimarket aspects of shadow pricing. This is a situation of interest whenever nonmarket characteristics are allocated among several markets. Our focus here is on the nutrient characteristics of primary agricultural products and their allocation both across food products and over space. There has been considerable research on the trade of market commodities under spatially dispersed competitive markets (e.g., following Samuelson, 1952; Takayama and Judge, 1971). Also, the economic analysis of vertical market equilibrium (including farm, wholesale, and retail levels) is now well established (e.g., Gardner, 1975), but it has also focused on the allocation of market goods. This suggests a need to extend this analysis to include the allocation of nonmarket characteristics in a multimarket framework, including both spatial and vertical markets.

The objective of this paper is to develop a spatial trade model in a vertical sector, allowing for an explicit analysis of nonmarket characteristics. The analysis is illustrated in an application to the spatial allocation and shadow pricing of nutrient characteristics in the farm and food sector. The conceptual model helps bridge the gap between the Samuelson–Takayama–Judge (STJ) approach to commodity trade modelling, and Rosen's analysis of market allocation involving differentiated products.

The paper is organized as follows. Section 2 presents a generic model of spatial markets, allowing for explicit vertical market linkages. Expanding on the STJ model, it considers a two-stage vertical sector, where primary commodities are used in the production of processed commodities that are eventually consumed. Both primary and processed commodities can be produced, consumed, and traded in spatial markets. This provides a basis for formulating a model of competitive spatial market equilibrium, reflecting the effects of production cost for the primary and processed commodities, of transportation cost, and of the spatial distribution of consumer demands. In Section 3, this model is refined to include the allocation of nonmarket characteristics (nutrients) across both spatial markets and successive stages of the vertical sector (the food marketing channel). It relies on a Lancasterian-type model explicitly linking the market commodities (primary agricultural outputs and processed food commodities) with the embedded nonmarket goods (nutrients). This allows for an evaluation of the spatial shadow pricing of the nonmarket nutrient characteristics.

The usefulness of the model is illustrated in Section 4 that centers on the regional structure of production, consumption, and marketing in the U.S. dairy sector. The investigation focuses on the allocation of farm milk production both spatially (among 14 producing regions) and vertically (through the production of 9 dairy products). The nonmarket characteristics are the basic nutrient components of milk (fat, protein, and carbohydrate) allocated among the 9 dairy products and the 14 regions. In contrast with previous research (e.g., McDowell et al., 1990), our model uses a disaggregate analysis of the demand for 'non-fluid milk'. An important innovation is the modelling of spatial market equilibrium incorporating regional milk component balance. This is a significant contribution in that it allows the explicit analysis of the allocation of milk components among dairy products and across regions. The model evaluates regional component shadow pricing of fat, protein and carbohydrate under alternative market scenarios. The scenarios include competitive markets, the government price support program, and milk marketing orders. This allows for an investigation of the effects of dairy policy on the pricing, production, marketing and consumption of milk and dairy products, on the implicit shadow pricing of milk components, and on regional welfare distribution in the U.S. dairy sector. Finally, concluding remarks are presented in Section 5.

2. The model

In this section, a generic model of competitive spatial resource allocation among J regions is presented. The section also sets the stage for the rest of the paper. Our approach expands on the work of Samuelson, and Takayama and Judge (STJ), by considering also vertical markets. Resources consist of primary commodities and processed commodities, which can all be traded in markets assumed to be competitive. The primary commodities are not consumer goods; they are exclusively used as inputs in the production of the processed commodities that are consumer goods. Each region may be: (1) a producer of the primary commodities; (2) a producer of the processed commodities; (3) a consumer of the processed commodities; or some combination of the three possibilities. Also, each region can trade both primary and processed commodities with any other region. The question, then, is how to analyze the corresponding competitive spatial market equilibrium. This is done here by developing a market equilibrium model of resource allocation and trade over the J regions.

We begin our model development with some notation and definitions. Let N be the number of primary commodities, with w_{in} denoting the quantity of the n-th primary commodity produced in the i-th region, and x_{in} being the quantity of the n-th primary commodity used as an input in the production of processed commodities in region i, $n = 1,\ldots,N$, $i = 1,\ldots,J$. Let K be the number of processed commodities, and denote the production level of the k-th processed commodity in the i-th region by y_{ik}, $k = 1,\ldots,K$, $i = 1,\ldots,J$. The consumption level of the k-th commodity in region i is denoted by z_{ik}, $k = 1,\ldots,K$, $i = 1,\ldots,J$.

Production of the processed commodities will be influenced by interregional trade in the primary commodities and by processing technologies. The consumption of processed commodities will be influenced by their production and by the interregional trade in them. Denote by $T_{ijn} \geq 0$ the export of the n-th primary commodity from region i to region j (or alternatively the import of the n-th primary commodity into region j from region i). Similarly, denote by $t_{ijk} \geq 0$ the export of the k-th processed commodity from region i to region j (or alternatively the import of the k-th processed commodity into region j from region i). Using this notation, $T_{iin} \geq 0$ is the quantity of the n-th primary commodity that is both produced and used in the production of the processed commodities within the i-th region. Similarly, $t_{iik} \geq 0$ is the quantity of the k-th processed commodity that is both produced and consumed in the i-th region. The allocation process is illustrated in Fig. 1.

The production of the processed commodities, y, involves two categories of inputs: the primary commodities, x, and other inputs denoted by the vector $\boldsymbol{v}$. The transformation of the primary inputs x into the processed outputs y in region i is given by the production possibility set F_i:

$$(v_i, x_i, y_i) \in F_i \tag{1}$$

where $\boldsymbol{x}_i = \{x_{in}\colon n = 1,\ldots,N\}$ is the vector of primary inputs, $\boldsymbol{y}_i = \{y_{ik}\colon k = 1,\ldots,K\}$ is the vector of processed outputs, and $\boldsymbol{v}_i$ is the vector of other inputs (besides $\boldsymbol{x}_i$) used in the production of $\boldsymbol{y}_i$, $i = 1,\ldots,J$. Expression (1) establishes the technolog-

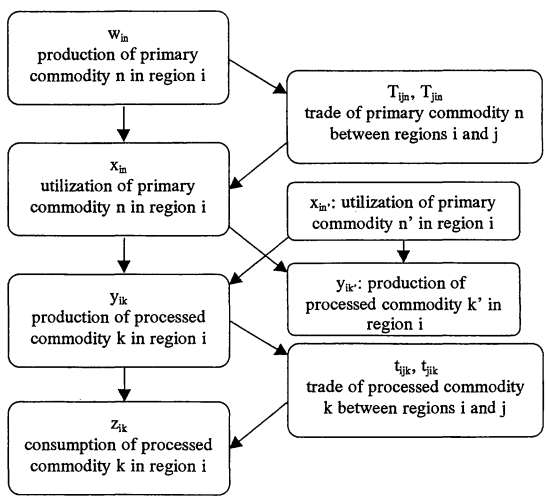

Fig. 1. The allocation process of primary and processed commodities.

ical relationship between available inputs $(\boldsymbol{v}_i, \boldsymbol{x}_i)$ and feasible processed outputs $\boldsymbol{y}_i$ in each region. We assume that the production possibility set F_i is nonempty, closed, and convex.

Assuming competition, let $\boldsymbol{h}_i$ denote the vector of market prices for the other inputs v_i, $i = 1,\ldots,J$. Then efficient use of the inputs v_i requires that they are chosen in a cost minimizing way as follows:

$$G_i(x_i, y_i) = \min_{v_i} \{h_i' v_i : (v_i, x_i, y_i) \in F_i\} \qquad (2)$$

where $G_i(x_i, y_i)$ is a restricted cost function measuring the cost of the optimal use of other inputs v_i, conditional on primary input use, x_i, and on output levels, y_i, for $i = 1,\ldots,J$. We will assume throughout that $G_i(x_i, y_i)$ is a decreasing function of x_i, and an increasing function of y_i.

The trade flow constraints across regions take the form:

$$w_{in} \geq \sum_{j=1}^{J} T_{ijn} \qquad (3a)$$

$$\sum_{j=1}^{J} T_{jin} \geq x_{in} \qquad (3b)$$

$$y_{ik} \geq \sum_{j=1}^{J} t_{ijk} \qquad (3c)$$

$$\sum_{j=1}^{J} t_{jik} \geq z_{ik} \qquad (3d)$$

For any region i, these constraints guarantee that exports plus domestic [2] use cannot be larger than domestic production, and that domestic consumption cannot exceed domestic production plus imports, $i = 1,2,\ldots,J$. This holds for primary commodities (Eqs. (3a) and (3b)) as well as processed commodities (Eqs. (3c) and (3d)).

A competitive market equilibrium satisfies the technology constraints Eq. (1) and the trade flow constraints Eqs. (3a), (3b), (3c) and (3d). It also allocates resources in an efficient manner both across commodities and across space. One way of capturing this efficiency is to consider the following quasi-welfare function:

$$V(w,x,y,z) = \sum_{i=1}^{J} \{D_i(z_i) - S_i(w_i) - G_i(x_i, y_i)\} \qquad (4)$$

where $w = \{w_{in}: i = 1,\ldots,J, n = 1,\ldots,N\}$, $x = \{x_{in}: i = 1,\ldots,J, n = 1,\ldots,N\}$, $y = \{y_{ik}: i = 1,\ldots,J, k = 1,\ldots,K\}$, $z = \{z_{ik}: i = 1,\ldots,J, k = 1,\ldots,K\}$, and $G_i(x_i, y_i)$ is the cost function defined in Eq. (2).

The quasi-welfare function V defined in Eq. (4) involves three sets of terms: D, S and G. Following Takayama and Judge, the terms D are interpreted as a measure of the total benefits to the consumers purchasing the processed goods z. And the terms S are interpreted as the cost of producing the primary commodities w. Given the cost function G defined in Eq. (2), it follows $(S + G)$ is the total cost of production of the processed goods z in the absence of trade. Then, the quasi-welfare function V in Eq. (4) is a measure of net social benefits (i.e., consumer benefits (D) minus total production cost $(S + G)$) in the absence of trade.

We make the following assumption:

Assumption A: The function $V(w, x, y, z)$ is differentiable and concave in (w, x, y, z), and satisfies:

$$\partial S_i / \partial w_{in} = p_{in}^s \geq 0, n = 1,\ldots,N$$
$$\partial D_i / \partial z_{ik} = p_{ik}^d \geq 0, k = 1,\ldots,K$$

where p_{in}^s is the price received by the producers of the n-th primary commodity in region i, and p_{ik}^d is the price paid by the consumers of the k-th processed commodity in region i, $i = 1,\ldots,J$.

Assumption A ensures that the quasi-welfare function is well-behaved, that the market prices of the primary commodities are equal to their marginal cost of production, and that the market prices of the processed commodities are equal to their marginal consumer benefit. As in Takayama and Judge, these conditions are consistent with competitive market equilibrium, where prices reflect the marginal valuation of the corresponding goods.

Let $C_{ijn} \geq 0$ be the unit cost of transportation of the n-th primary commodity from region i to region j. Similarly, let $c_{ijk} \geq 0$ be the unit cost of transportation of the k-th processed commodity from

[2] Borrowing from the trade literature, the term 'domestic' refers to activities taking place within a given region.

region i to region j. We assume throughout that $C_{iin} = 0$ and $c_{iik} = 0$, i.e., that transportation costs are zero in the absence of trade. Now, consider the following optimization model:

$$\max_{w,x,y,z,T,t} \{V(w,x,y,z) - \Sigma_{i,j,n} T_{ijn} C_{ijn} - \Sigma_{i,j,k} t_{ijk} c_{ijk} \text{: Eqs.(3a - 3d)}, w \geq 0, x \geq 0, y \geq 0, z \geq 0, T \geq 0, t \geq 0\} \quad (5)$$

Expression (5) maximizes the quasi-welfare function $V(w, x, y, z)$ net of transportation cost, subject to the trade flow constraints Eqs. (3a), (3b), (3c) and (3d), and non-negativity on the variables w, x, y, z, T, and t. Next, we show that, under assumption A, the optimization problem Eq. (5) generates the competitive spatial market equilibrium.

Under assumption A, the maximization problem in Eq. (5) is a standard concave programming problem, subject to linear constraints. Provided that it has a bounded solution, it can be alternatively characterized as the saddle point of the following Lagrangean:

$$\begin{aligned} L = {} & V(w,x,y,z) - \Sigma_{i,j,n} T_{ijn} C_{ijn} - \Sigma_{i,j,k} t_{ijk} c_{ijk} \\ & + \Sigma_{i,n} \alpha_{in} [w_{in} - \Sigma_j T_{ijn}] \\ & + \Sigma_{i,n} \beta_{in} [\Sigma_j T_{jin} - x_{in}] + \Sigma_{i,k} \gamma_{ik} [y_{ik} - \Sigma_j t_{ijk}] \\ & + \Sigma_{i,k} \delta_{ik} [\Sigma_j t_{jik} - z_{ik}] \end{aligned}$$

where $\alpha \geq 0$, $\beta \geq 0$, $\gamma 0$ and $\delta \geq 0$ are Lagrange multipliers corresponding to the constraints in Eqs. (3a), (3b), (3c) and (3d). Under assumption A, Kuhn–Tucker (K–T) conditions provide necessary and sufficient conditions for the solution to Eq. (5). These K–T conditions are:

$$\begin{aligned} \partial L/\partial w_{in} &= -\partial S_i/\partial w_{in} + \alpha_{in} \leq 0, w_{in} = 0 \\ &= 0, w_{in} > 0 \end{aligned} \quad (6a)$$

$$\begin{aligned} \partial L/\partial x_{in} &= -\partial G_i/\partial x_{in} - \beta_{in} \leq 0, x_{in} = 0 \\ &= 0, x_{in} > 0 \end{aligned} \quad (6b)$$

$$\begin{aligned} \partial L/\partial y_{ik} &= -\partial G_i/\partial y_{ik} + \gamma_{ik} \leq 0, y_{ik} = 0 \\ &= 0, y_{ik} > 0 \end{aligned} \quad (6c)$$

$$\begin{aligned} \partial L/\partial z_{ik} &= \partial D_i/\partial z_{ik} - \delta_{ik} \leq 0, z_{ik} = 0 \\ &= 0, z_{ik} > 0 \end{aligned} \quad (6d)$$

$$\begin{aligned} \partial L/\partial T_{ijn} &= -C_{ijn} + \beta_{jn} - \alpha_{in} \leq 0, T_{ijn} = 0 \\ &= 0, T_{ijn} > 0 \end{aligned} \quad (6e)$$

$$\begin{aligned} \partial L/\partial t_{ijk} &= -c_{ijk} + \delta_{jk} - \gamma_{ik} \leq 0, t_{ijk} = 0 \\ &= 0, t_{ijk} > 0 \end{aligned} \quad (6f)$$

$$\begin{aligned} \partial L/\partial \alpha_{in} &= w_{in} - \Sigma_j T_{ijn} \leq 0, \alpha_{in} = 0 \\ &= 0, \alpha_{in} > 0 \end{aligned} \quad (6g)$$

$$\begin{aligned} \partial L/\partial \beta_{in} &= \Sigma_j T_{jin} - x_{in} \geq 0, \beta_{in} = 0 \\ &= 0, \beta_{in} > 0 \end{aligned} \quad (6h)$$

$$\begin{aligned} \partial L/\partial \gamma_{ik} &= y_{ik} - \Sigma_j t_{ijk} \geq 0, \gamma_{ik} = 0 \\ &= 0, \gamma_{ik} > 0 \end{aligned} \quad (6i)$$

$$\begin{aligned} \partial L/\partial \delta_{ik} &= \Sigma_j t_{jik} - z_{ik} \geq 0, \delta_{ik} = 0 \\ &= 0, \delta_{ik} > 0 \end{aligned} \quad (6j)$$

From assumption A and Eq. (6a), it follows that α_{in} can be interpreted as the market price for the primary commodity w_{in} in region i. Indeed, given $w_{in} > 0$, Eq. (6a) and assumption A imply that $\alpha_{in} = p^s_{in}$. Similarly, δ_{ik} can be interpreted as the market price for the processed commodities z_{ik} in region i since, given $z_{ik} > 0$, Eq. (6d) and assumption A imply that $\delta_{ik} = p^d_{ik}$.

Eqs. (6e) and (6f) characterize the transportation arbitrage conditions expressed in terms of spatial prices. Note that, given $C_{iin} = 0$, it follows from Eq. (6e) that $\beta_{in} = \alpha_{in}$ whenever $T_{iin} > 0$. When $\alpha_{in} = p^s_{in}$, β_{in} can thus be interpreted as the market price for the n-th primary commodity x_{in} in region i. And given $c_{iik} = 0$, Eq. (6f) implies that $\gamma_{ik} = \delta_{ik}$ whenever $t_{iik} > 0$. When $\delta_{ik} = p^d_{ik}$, γ_{ik} can thus be interpreted as the market price for the k-th processed commodity y_{ik} in region i. Eqs. (6e) and (6f) state that commodity prices between any two regions cannot differ by more than the corresponding unit transportation cost. And in the case where trade takes place (i.e., $T_{ijn} > 0$, $t_{ijk} > 0$, for $i \neq j$), then the spatial price difference between the importing region and the exporting region must be exactly equal to the unit transportation cost. Note that an implication of Eqs. (6e) and (6f) is:

$$[p^s_{jn} - p^s_{in} - C_{ijn}] T_{ijn} = 0 \text{ for all } i, j, \text{and } n \quad (7a)$$

and:

$$\left[p^d_{jk} - p^d_{ik} - c_{ijk} \right] t_{ijk} = 0 \text{ for all } i, j, \text{and } k \tag{7b}$$

Eqs. (7a) and (7b) mean that the equilibrium conditions for trade necessarily imply zero profit from transportation activities. Thus, any departure from Eqs. (6e) and (6f) cannot correspond to an equilibrium situation since it would provide incentives for transportation firms to alter trade patterns. In this sense, Eqs. (6e) and (6f) characterize trade efficiency.

The Lagrange multipliers β and γ measure the shadow price of the trade constraints Eqs. (3b) and (3c). More specifically, β_{in} measures the marginal social cost of one unit of the primary commodity x_{in}, $i = 1,\ldots,J$, $n = 1,\ldots,N$. Then, Eq. (6b) states that, at the optimum, the marginal value of the commodity ($\beta_{in} \geq 0$) is equal to its marginal cost ($-\partial G_i / \partial x_{in} \geq 0$) whenever x_{in} is positive. But we have seen that β_{in} can be interpreted as the market price of x_{in} in region i. It follows that the model is consistent with a competitive market equilibrium, where market price is equal to the marginal cost of each commodity at the optimum.

Similarly, γ_{ik} measures the marginal social value of one unit of the processed commodity y_{ik}, $i = 1,\ldots,J$, $k = 1,\ldots,K$. Then, Eq. (6c) states that, at the optimum, the marginal value of the commodity ($\gamma_{ik} \geq 0$) is equal to its marginal cost ($\partial G_i / \partial y_{ik} \geq 0$) whenever y_{ik} is positive. We have seen that γ_{ik} can be interpreted as the market price of y_{ik}. Thus, the model is consistent with a competitive market equilibrium, where market price is equal to the marginal cost of each commodity at the optimum.

Finally, Eqs. (6g), (6h), (6i) and (6j), together with the complementary slackness conditions with respect to the corresponding Lagrange multipliers, are trade flow constraints, representing the feasibility conditions for interregional trade.

These results indicate that the optimization problem in Eq. (5) provides a representation of a competitive market equilibrium both across commodities and over space. They extend the Samuelson–Judge–Takayama approach to spatial market equilibrium (see Samuelson, 1952; Takayama and Judge, 1971, pp. 107–121) by considering both trade and the transformation of primary commodities into processed commodities. As such, they appear useful in the analysis of spatial resource allocation in a vertical marketing sector.

3. Spatial shadow pricing of product characteristics

The model developed in Section 2 can be refined when the production of processed commodities from primary commodities involves nonmarket characteristics. Our interest here is the food sector. In this context, primary products are farm outputs, processed products are food commodities, and the nonmarket characteristics as nutrient components embedded in both farm and food commodities. The spatial allocation and shadow pricing of the nutrient components of farm and food products under competitive markets and trade are analyzed here in the context of a Lancasterian-type model.

We assume the N primary commodities involve S nutrient characteristics, where the s-th nutrient is denoted by r_s, $s = 1,\ldots,S$. Each primary as well as each processed commodity in each region has a given composition in terms of these underlying nutrients. In the i-th region, let $a_{ins} \geq 0$ denote the quantity of the s-th nutrient per unit of n-th primary commodity x_{in}, and let $b_{iks} \geq 0$ denote the quantity of the s-th nutrient per unit of the k-th processed commodity y_{ik}. We also assume that the nutrient composition of each commodity is constant, i.e., that a_{ins} and b_{iks} are constant. Under this assumption, consider that the production technology F_i in region i (as given in Eq. (1)) takes the specific form:

$$y_{ik} = \min\left\{ \sum_{n=1}^{N} x_{ink} a_{in1} / b_{ik1}, \sum_{n=1}^{N} x_{ink} a_{in2} / b_{ik2}, \ldots \right.$$
$$\left. \times \sum_{n=1}^{N} x_{ink} a_{inS} / b_{ikS}, \; f_{ik}(v_{ik}, x_{ik}) \right\}, \text{for all } i \text{ and } k \tag{8}$$

where x_{ink} is the quantity of the n-th primary input used in the production of the k-th processed output in region i, which satisfies the identity: $x_{in} = \sum_{k=i}^{K} x_{ink}$, $i = 1,2,\ldots,J$, and $n = 1,2,\ldots,N$. The production technology Eq. (8) assumes fixed proportions with respect to each of the nutrient characteristics used in the production of the processed output

y_{ik}, $\Sigma_{n=1}^{N} x_{ink} a_{ins}$, $s = 1, \ldots, S$. However, given the general function $f_{ik}(v_{ik}, x_{ik})$, no a priori restriction on the elasticities of substitution among the various inputs, (v_{ik}, x_{ik}), are imposed. Under the technology in Eq. (8), the cost function (Eq. (2)) becomes:

$$g_i(x_i, y_i) = \min_{v_i} \{h_i' v_i : y_{ik} \le f_{ik}(v_{ik}, x_{ik})\}$$

$$\text{for all } k = 1,2,\ldots,K \tag{9a}$$

subject to:

$$\sum_{k=1}^{K} y_{ik} b_{iks} \le \sum_{n=1}^{N} x_{in} a_{ins}, \text{ for all } s = 1,2,\ldots,S \tag{9b}$$

$i = 1, \ldots, J$. The relationship in Eq. (9b) ensures the balanced allocation of the s-th nutrient component in the i-th region. It corresponds to a linear Lancasterian model where each commodity exhibits fixed component proportions, but where the components are perfect substitutes in their allocation among commodities (Lancaster, 1966, 1971). [3] The optimization problem in Eq. (5) now becomes:

$$\begin{aligned} \max_{w,x,y,z,T,t} \ & \{\Sigma_i[D_i(z_i) - S_i(w_i) - g_i(x_i, y_i)] \\ & - \Sigma_{i,j,n} T_{ijn} C_{ijn} - \Sigma_{i,j,k} t_{ijk} c_{ijk} : \\ & \text{Eqs.}(3\text{a} - 3\text{d}), \\ & \text{Eq.}(9\text{b}), w \ge 0, x \ge 0, y \ge 0, \\ & z \ge 0, T \ge 0, t \ge 0\} \end{aligned} \tag{10}$$

and the corresponding Lagrangean is:

$$\begin{aligned} L = \ & \Sigma_i[D_i(z_i) - S_i(w_i) - g_i(x_i, y_i)] \\ & - \Sigma_{i,j,n} T_{ijn} C_{ijn} - \Sigma_{i,j,k} t_{ijk} c_{ijk} \\ & + \Sigma_{i,s} \lambda_{is}[\Sigma_n x_{in} a_{ins} - \Sigma_k y_{ik} b_{iks}] \\ & + \Sigma_{i,n} \alpha_{in}[w_{in} - \Sigma_j T_{ijn}] \\ & + \Sigma_{i,n} \beta_{in}[\Sigma_j T_{jin} - x_{in}] \\ & + \Sigma_{i,k} \gamma_{ik}[y_{ik} - \Sigma_j t_{ijk}] \\ & + \Sigma_{i,k} \delta_{ik}[\Sigma_j t_{jik} - z_{ik}] \end{aligned}$$

where $\lambda_{is} \ge 0$ is the Lagrange multiplier for the s-th nutrient constraint Eq. (9b) in region i. At the optimum, $\lambda = \{\lambda_{is}: i = 1,2,\ldots,J;\ s = 1,2,\ldots,S\}$ provides a measure of the shadow prices, or implicit prices, of the S nutrient components in the j regions. This provides a convenient basis for evaluating component pricing for nutrients in a spatial market equilibrium framework.

The Kuhn–Tucker conditions associated with the above Lagrangean are identical to Eqs. (6a), (6d), (6e), (6f), (6g), (6h), (6i) and (6j), while Eqs. (6b) and (6c) take the forms:

$$\begin{aligned} \partial L / \partial x_{in} = & -\partial g_i / \partial x_{in} + \Sigma_s \lambda_{is} a_{ins} - \beta_{in} \le 0, \\ & x_{in} = 0, \\ & = 0,\ x_{in} > 0 \end{aligned} \tag{11a}$$

$$\begin{aligned} \partial L / \partial y_{ik} = & -\partial g_i / \partial y_{ik} - \Sigma_s \lambda_{is} b_{iks} + \gamma_{ik} \le 0, \\ & y_{ik} = 0, \\ & = 0,\ y_{ik} > 0 \end{aligned} \tag{11b}$$

At the optimum, λ_{is} can be interpreted as the *shadow price of the s-th nutrient component in the i-th region.* Expressions Eqs. (11a) and (11b) then indicate how the shadow valuation of nutrient components relates to market equilibrium. Eq. (11a) involves the marginal value (β_{in}) of the n-th primary input, x_{in}, which is equal to the marginal cost associated with inputs v_i $(-\partial g_i / \partial x_{in} \ge 0)$, plus the marginal cost of the S components $(\Sigma_s \lambda_{is} a_{ins} \ge 0)$. This states that, at the optimum, the marginal value (β_{in}) is equal to the marginal cost of the n-th primary commodity in the i-th region. Eq. (11b) involves the marginal value (γ_{ik}) of the k-th processed product y_{ik}, which is equal to the marginal cost associated with inputs v_i $(\partial g_i / \partial y_{ik} \ge 0)$, plus the marginal cost of the S components is $\Sigma_s \lambda_{is} b_{iks} \ge 0$). Again, this shows that, at the optimum, the marginal value (γ_{ik}) equals the marginal cost for the k-th processed commodity in the i-th region. To the extent that the marginal values β_{in} and γ_{ik} are equal to market prices ($\beta_{in} = p_{in}^s$ and $\gamma_{ik} = pik^d$), these results are consistent with resource allocation obtained under competitive market equilibrium.

[3] Note that this assumption of perfect substitutability among commodities can be relaxed by including appropriate constraints in addition to Eq. (9b). This will be illustrated in Section 4.

Finally, the following additional Kuhn–Tucker condition must be satisfied:

$$\partial L/\partial\lambda_{is} = \Sigma_n x_{in} a_{ins} - \Sigma_k y_{ik} b_{iks} \geq 0,\ \lambda_{is} = 0$$
$$= 0,\ \lambda_{is} > 0 \tag{11c}$$

which represents the component balance constraint for the s-th component in the i-th region, $i = 1,\dots,J$, $s = 1,\dots,S$. These equations provide a convenient characterization of spatial competitive equilibrium of nutrient allocation and their implicit pricing. The usefulness of these results is illustrated next in a regional analysis of the U.S. dairy industry.

4. Application to regional allocation in the U.S. dairy industry

In this section, we apply the model to a regional allocation in the U.S. dairy industry. While previous research has focused on aggregate analysis of fluid and 'non-fluid milk' (e.g., McDowell et al., 1988, 1990; Helmberger and Chen, 1994), our model uses a disaggregate analysis of the demand for 'non-fluid milk'. An important innovation is the modelling of spatial market equilibrium taking into consideration milk component balance. A significant contribution is the explicit analysis of milk component allocation both among dairy products and across regions. We also incorporate in the model dairy policy, policies like the milk price support program (implemented through federal government purchases); and milk marketing orders. This provides a basis for evaluating regional impacts of U.S. dairy pricing policy.

4.1. The basic model

We analyze the case of milk and its transformation into dairy products. We have a single primary commodity ($N = 1$): farm milk. Farm milk is transformed into nine categories of dairy products ($K = 9$): (1) fluid milk; (2) soft dairy products; (3) American cheese; (4) Italian cheese; (5) other cheese; (6) butter; (7) frozen dairy products; (8) all other manufactured dairy products; and (9) nonfat dry milk. We focus on a regional analysis, dividing the U.S. into 14 regions ($J = 14$).[4] Finally, building on Selinsky, Cox and Jesse, we consider the allocation of three nutrient characteristics of milk ($S = 3$): (1) fat; (2) protein; and (3) carbohydrate. Farm milk is assumed to contain 3.66% fat, 3.20% protein, and 4.65% carbohydrates. The composition of fluid milk is: 2.20% fat, 3.32% protein, and 4.73% carbohydrates. The composition of all dairy products was estimated in a way consistent with their average composition in 1990.[5] Our focus here is to investigate the spatial market equilibrium of the U.S. dairy sector that is consistent with the allocation and implicit pricing of the three nutrient components.

The objective function in Eq. (10) involves consumer benefits, D, the costs of milk production, S, the other costs, g, and transportation costs. Let $p_i^s(w_i)$ represent the price dependent supply function for milk in the i-th region, where $\partial p_i^s/\partial w_i > 0$, $i = 1,\dots,J$. And let $p_{ik}^d(z_{ik})$ represent the price dependent demand function for the k-th dairy product consumed in the i-th region, where $\partial p_{ik}^d/\partial z_{ik} < 0$, $i = 1,\dots,J$, $k = 1,\dots,K$. We choose:

$$D_i = \sum_{k=1}^{K} \int_0^{z_{ik}} p_{ik}^d(q)\,\mathrm{d}q \tag{12a}$$

and:

$$S_i = \int_0^{w_i} p_i^s(q)\,\mathrm{d}q \tag{12b}$$

[4] The 14 regions are: (1) New England (Maine, New Hampshire, Vermont, Massachusetts, Connecticut, Rhode Island); (2) Middle Atlantic (New York, Pennsylvania, New Jersey); (3) South Atlantic (Delaware, Maryland, West Virginia, Virginia); (4) South East (North Carolina, South Carolina, Georgia, Florida); (5) Central (Kentucky, Tennessee); (6) East South Central (Alabama, Mississippi, Arkansas, Louisiana); (7) West South Central (Oklahoma, Texas, New Mexico); (8) East North Central (Ohio, Indiana, Illinois, Michigan); (9) Wisconsin; (10) West North Central (Minnesota, South Dakota, North Dakota); (11) West Central (Missouri, Kansas, Iowa, Nebraska); (12) North West (Idaho, Oregon, Washington); (13) Mountain (Arizona, Colorado, Utah, Nevada, Wyoming, Montana); and (14) California.

[5] We neglect other components of milk (e.g., water and minerals). We implicitly assume that these other components have a zero shadow price and are disposable at no cost.

where q denotes the dummy of integration, $i = 1, \ldots, J$, a choice that satisfies assumption A.[6] As argued previously, the model in Eq. (10) provides a representation of a spatial competitive model for dairy products.

We consider the case where the demand function $p_{ik}^d(z_{ik})$ is the derived demand for the k-th dairy product at the wholesale level. In other words, p_{ik}^d is interpreted as the wholesale price of the k-th dairy product, and g_i as the marketing cost (excluding component cost and transportation cost) of milk and dairy products in the i-th region, $i = 1, \ldots, J$.

Eq. (12a) is the sum of the total areas under the K-derived demand curves in the i-th region. This area can be interpreted as a measure of benefits generated by the K commodities in the i-th region. Eq. (12b) is the area under the supply curve. Since the supply curve is also the marginal cost of production under competition, Eq. (12b) is a measure of milk production cost in the i-th region. The terms $[D_i - S_i]$ are then a measure of welfare obtained in region i: the sum of producer surplus and consumer surplus. Note that consumer surplus (as measured from a Marshallian demand function) is only an approximate welfare measure in the presence of income effects (Willig, 1976). In that sense, the objective function in either Eq. (5) or Eq. (10) cannot be interpreted as a true welfare measure. This motivates the characterization of the objective function as a 'quasi-welfare function', following Samuelson, and Takayama and Judge.[7]

The empirical use of the model requires estimates of the supply function for milk and wholesale demand functions for the K dairy products in each region. The regional milk supply elasticities are taken from the analysis of Buxton (1985). The product demand elasticities are obtained from Huang.[8] In the absence of strong prior information on their functional form, the price dependent supply and demand functions are assumed to be linear. Their intercept and slope values are set consistent with dairy market conditions (i.e., price and quantity) prevalent in 1990.

The transportation cost for farm milk and fluid milk is assumed to be US\$0.35/cwt/100 miles. Transportation costs for other dairy products are estimated from actual transportation costs prevalent in 1990 for refrigerated products (soft dairy products, cheeses, butter, frozen products, and manufactured products) and nonrefrigerated products (nonfat dry milk). These were obtained from a sample of actual, 1990 negotiated rates reported by the Interstate Commerce Commission. The use of actual transportation rates allows for asymmetric rates, where the unit transportation costs of a given commodity between two regions can differ for imports versus exports (e.g., because of backhauling opportunities).

The optimization model in Eq. (10) is subject to constraints shown in Eqs. (3a), (3b), (3c), (3d) and (9b)). Imposing Eqs. (3a), (3b), (3c) and (3d) in the dairy model is straightforward. Eq. (9b) required some adjustments for components that never reach the consumers. For example, whey is a byproduct of cheese production. Although some whey is recovered and utilized in dairy products, a significant proportion is typically discarded. Also, a small percentage of farm milk production is consumed on farm and therefore never reaches the market place. Appropriate adjustments in Eq. (9b) were made to reflect these characteristics of the dairy industry.

Finally, linear equation Eq. (9b) implicitly assumes that nutrient components are perfect substitutes in their allocation among the different processed commodities. This may not be an appropriate

[6] Note that the specification Eq. (12a) neglects possible cross-price demand effects across dairy commodities. This simplification is motivated by the current absence of reliable information on the nature and magnitude of these cross-price effects at the farm gate. If such information became available, it could be easily incorporated in the model.

[7] However, these approximations do not affect the validity of the arguments presented earlier that our model generates a competitive market equilibrium.

[8] Note that these elasticities may be upward biased since wholesale demand tends to be less elastic than retail demand (Kinnucan and Forker, 1987; McDowell et al., 1990). They may also be downward biased if we consider an intermediate run scenario, since long run elasticities tend to be larger (in absolute value) than short run elasticities. Our choice of elasticities may be appropriate to the extent that these two biases cancel each other. A sensitivity analysis indicated that most of the results presented below were not affected much by our elasticity estimates.

assumption for some dairy commodities. In particular, there are technological constraints that prevent perfect substitution of components across commodities. Such constraints are typically associated with specialized plants that can use components only in the production of selected dairy commodities. First, because of the difference in fat composition, the production of fluid milk from raw milk results in fat byproduct that are typically used only in the production of soft products, frozen products, or butter. Second, butter is a residual commodity using fat surpluses from two sources: (1) fat in whey associated with cheese production; and (2) fat surpluses due to production of butter and nonfat dry milk from 'reserve fluid milk' that is needed to smooth seasonal fluctuations and uneven weekly bottling schedules in the fluid milk market. Two sets of constraints further restricting the allocation of components across commodities have been added to the model to incorporate these specific attributes.

The model is a well-behaved nonlinear programming problem, with a strictly concave objective function and linear constraints. It can be solved numerically using standard optimization software. [9]

4.2. Government milk price support

The model discussed above can be modified to account for the government milk price support program, implemented through federal purchases of dairy commodities. Such purchases are designed to stimulate aggregate demand for milk and maintain the price received by dairy farmers above a minimum level set by government. Those purchases are limited to storable dairy products. In 1990, the U.S. government purchased 44 million lb of American cheese, 404 million lb of butter, and 100 million lb of nonfat dry milk.

In order to include government purchases in the model, an 'additional region' was created to account for government demand. The quantity demanded by government was treated as exogenous and set at the 1990 levels as reported above. Again, the model incorporating government purchases is a well-behaved nonlinear programming problem, with a strictly concave objective function and linear constraints.

4.3. Milk marketing orders

U.S. milk marketing orders influence the regional allocation and pricing of milk and dairy products. They include both federal and California marketing orders. Through classified pricing, the marketing orders implement a price discrimination scheme that increases the price of fluid milk relative to the prices of non-fluid dairy products. Since the demand for fluid milk is very inelastic (Huang, 1993; Haidacher et al., 1988), this generates increased revenues that can be passed on to producers in the form of higher farm milk price (Helmberger, 1991). Federal milk marketing order are implemented in several ways. First, they impose a 'minimum class I differential' between fluid milk price and 'manufactured milk price' in Eau Claire, WI. [10] Second, in other regions affected by federal orders, they impose a lower bound on fluid milk price: [11] the regional fluid milk price is restricted to be at least as large as the Wisconsin fluid milk price, plus a differential of US$0.21/cwt/100 miles distance from Wisconsin. Finally, federal orders implement 'blend pricing' for milk at the farm level. This involves paying farmers a weighted average price based on the prices of fluid and of manufactured dairy products in each region. The 1990 class I differential in the California marketing order was estimated to be US$1.61/cwt (3.5% fat), with US$0/cwt of over-order premium.

These characteristics of milk marketing orders are incorporated in the model as follows. The 1990 class I price differential in each region (along with a possible over-order premium) is treated as a price wedge that is equivalent to an increase in the cost of producing fluid milk. This is done by introducing an additional term in the objective function of Eq. (10),

[9] The empirical analysis presented below relies on GAMS-MINOS for optimization software.

[10] In 1990, this class I differential was US$1.25/cwt (for 3.5% fat milk) in the Chicago federal order.

[11] When the actual fluid milk price exceeds this minimum price, the difference is known as the 'over-order premium'. It has been interpreted as a premium paid to manage the perishability of fluid milk, and/or as a premium due to the exercise of market power by large cooperatives (Jesse and Johnson, 1985).

Table 1
Milk price received by farmers (US$/cwt)

Region	(1) 1990 Actual prices	(2) Price support and marketing order	(3) Price support (no marketing order)	(4) Marketing order (no price support)	(5) No marketing order and no price support
North East	14.62	15.20	14.34	14.78	13.76
Mid Atlantic	14.79	14.42	13.99	14.01	13.67
South Atlantic	14.91	15.23	14.37	14.82	13.70
South East	16.20	16.62	15.66	16.17	14.82
Central	14.50	14.54	14.33	14.14	14.04
East South Central	15.30	15.86	15.17	15.30	4.25
West South Central	14.40	15.05	14.30	14.44	13.78
East North Central	13.81	13.82	13.73	13.58	13.39
Wisconsin	13.47	13.17	13.55	12.75	13.20
West North Central	13.13	13.01	13.41	12.59	13.07
West Central	13.36	13.60	13.58	13.17	13.24
North West	12.92	13.81	13.14	12.38	12.80
Mountain	13.78	13.60	13.49	13.18	13.16
California	12.02	12.77	13.22	12.34	12.88
U.S. average	13.73	13.82	13.77	13.40	13.37

Table 2
Farm level milk production (million lb)

Region	(1) 1990 Actual output		(2) Price support and marketing order	(3) Price support (no marketing order)	(4) Marketing order (no price support)	(5) No marketing order and no price support
	Quantity	Share (%)	Quantity	Quantity	Quantity	Quantity
North East	4235	3	4296	4206	4252	4144
Mid Atlantic	21,090	14	20,764	20,385	20,403	20,110
South Atlantic	3710	3	3765	3616	3695	3503
South East	5853	4	5967	5705	5845	5478
Central	4390	3	4401	4338	4282	4255
East South Central	2990	2	3061	2974	2990	2856
West South Central	8192	6	8488	8145	8209	7908
East North Central	14,617	10	14,620	14,590	14,539	14,474
Wisconsin	24,059	16	23,660	24,159	23,101	23,700
West North Central	12,646	9	12,615	12,721	12,503	12,630
West Central	9821	7	9862	9858	9789	9801
North West	8833	6	8804	8892	8690	8802
Mountain	4953	3	4914	4893	4827	4824
California	20,661	14	20,943	21,112	20,780	20,984
U.S. total	146,049	100	146,158	145,595	143,905	143,468

Table 3
Regional implicit component prices (US$/cwt) for fat (in all products except butter, soft and frozen), protein and carbohydrates under alternative scenarios[a]

Region	Fat				Protein				Carbohydrate			
	(2) ps and mo	(3) ps and mo	(4) mo and no ps	(5) No mo and no ps	(2) ps and mo	(3) ps and no mo	(4) mo and no ps	(5) No mo and no ps	(2) ps and mo	(3) ps no mo	(4) mo no ps	(5) No mo and no ps
North East	156	155	111	130	236	262	276	270	17	21	17	23
Mid Atlantic	158	143	112	126	234	267	273	276	20	19	20	20
South Atlantic	160	156	114	128	234	270	273	276	19	16	19	19
South East	163	192	117	156	229	277	266	289	22	10	22	13
Central	149	137	95	115	284	305	337	324	3	5	0	4
East South Central	175	178	126	142	240	279	277	287	14	9	15	13
West South Central	167	154	116	130	238	271	274	279	16	15	17	17
East North Central	151	138	110	119	243	273	286	285	14	14	12	14
Wisconsin	132	126	92	106	271	291	302	303	4	6	6	6
West North Central	126	120	86	100	280	299	310	311	1	3	2	3
West Central	145	134	101	113	249	277	287	291	11	11	11	11
North West	129	114	85	95	269	303	306	313	0	0	0	0
Mountain	139	125	95	107	256	287	293	298	9	9	9	9
California	130	116	86	98	270	302	308	312	0	1	0	0
U.S. average	149	142	103	119	252	283	291	294	11	10	11	11

[a]'ps' and 'mo' stand for 'price support' and 'marketing order', respectively.

Table 4
Aggregate U.S. wholesale commodity prices and consumption

Commodity	(1) 1990 Actual price	(2) Price support and marketing order	(3) Price support (no marketing order)	(4) Marketing order (no price support)	(5) No marketing order and no price support
Average U.S. wholesale commodity prices (US$/cwt)					
Fluid	14.89	14.72	13.75	15.87	14.88
Soft	29.00	29.28	30.05	27.01	28.66
American cheese	110.00	128.61	132.79	126.07	131.90
Italian cheese	120.00	107.84	111.50	111.19	116.26
Other cheese	125.00	108.34	112.20	102.15	106.81
Butter	82.89	84.71	87.92	1.76	1.46
Frozen	24.72	23.84	23.97	20.84	22.40
Other mfg	40.71	42.72	45.62	44.53	45.83
Nonfat dry milk	85.00	96.53	107.72	110.05	111.75
Aggregate U.S. wholesale level commodity consumption (million lb)					
Fluid	54,338	54,016	54,503	53,438	53,935
Soft	3735	3738	3696	3859	3770
American cheese	2741	2626	2601	2642	2606
Italian cheese	2231	2287	2270	2271	2248
Other cheese	1129	1166	1158	1180	1170
Butter	906	901	893	1121	1122
Frozen	7137	7157	7154	7225	7189
Other mfg	3536	3487	3418	3444	3412
Nonfat dry milk	706	680	654	648	645

Table 5
Regional surplus measures under alternative scenarios (US$, million)

Region	Producer surplus[a]			Consumer surplus[b]		
	(2) Price support and marketing order	(3) Price support (no marketing order)	(4) Marketing order (no price support)	(2) Price support and marketing order	(3) Price support (no marketing order)	(4) Marketing order (no price support)
North East	877	841	860	3236	3251	3263
Mid Atlantic	2460	2369	2373	9159	9244	9238
South Atlantic	415	383	399	3499	3509	3513
South East	653	597	626	6954	6917	6991
Central	327	318	310	2070	2074	2089
East North Central	367	346	350	3138	3136	3156
West South Central	793	731	742	5370	5424	5408
East North Central	3136	3123	3101	8412	8399	8484
Wisconsin	2095	2184	1997	2017	2028	2012
West North Central	3004	3055	2951	1427	1424	1438
West Central	2,901	2899	2859	2895	2914	2921
North West	1461	1490	1423	2214	2204	2232
Mountain	581	576	561	2844	2841	2866
California	5799	5894	5710	7889	7809	7949
Total	24,867	24,805	24,262	61,127	61,173	61,561

[a]Producer surplus measures the net return to dairy farmers. It does not include processing and marketing cost.
[b]Consumer surplus does not include the cost to the taxpayers.

representing this additional cost. From the discussion presented in Sections 2 and 3, this price wedge implies an equivalent increase in the price of fluid milk consumed in each region. The regional revenue generated by this price wedge is then redistributed to regional dairy farmers in terms of higher farm milk price (as implemented through blend pricing).

4.4. Results

The empirical results are summarized in Tables 1–5. Five sets of data are presented: (1) the actual 1990 data; (2) the simulation results obtained under the 1990 price support program and milk marketing orders; (3) the results under the price support program (as reflected by 1990 government purchases), but in the absence of marketing orders; [12] (4) the simulation results under federal marketing orders, but without the price support program; and (5) the results in the absence of both milk marketing orders and price support program. In the discussion that follows, scenario (2) is expected to represent the actual situation (1) present in 1990. As such, comparing (2) with (1) provides a means of validating the model. And comparing scenario (2) with scenarios (3), (4) and (5) gives useful information concerning the regional effects of the dairy price support program and marketing orders on the U.S. dairy industry.

Under these alternative scenarios, the model provides information on regional milk prices paid to farmers (Table 1), regional milk production (Table 2), regional implicit prices for milk components (Table 3), market equilibrium for dairy products (Table 4), and welfare distribution as measured by regional producer and consumer surpluses (Table 5).

Since the price support program and marketing orders were both in place in 1990, comparing scenario (2) with the 1990 actual data provides some basis for evaluating the accuracy of the model. As shown in Table 1, the model prediction error for the average U.S. farm milk price is US$0.09/cwt or 0.65%. The largest prediction error of the farm price of milk in any region is for California: US$0.75/cwt, or 5.44%. From Table 2, the model predicts U.S. milk production within 109 million lb, or 0.07% error. The maximum prediction error for milk production in any region is for Wisconsin: 1.62%. These fairly low relative errors suggest that the model provides a reasonably good representation of the U.S. dairy industry.

Table 3 reports the shadow prices of milk components. It shows that protein is the most valuable component (average of US$252/cwt in scenario (2)), followed by fat (US$149/cwt), and then carbohydrate (US$11/cwt). These differences in component prices reflect the value of different dairy products (Table 4). For example, cheeses (that are high in protein) are priced higher than butter or frozen dairy products (that are high in fat). Again, comparing scenarios (1) and (2) in Table 4 shows that the model provides a fairly accurate representation of dairy prices and dairy consumption for 1990.

The effects of the price support program can be evaluated by comparing scenarios (2) and (4) (in the presence of marketing orders), or scenarios (3) and (5) (in the absence of marketing orders). From scenarios (2) and (4), the results indicate that the price support program raises the price received by farmers by an average of 3.13% (see Table 1) and stimulates U.S. milk production by 1.56% (Table 2). Helmberger and Chen found similar long run effects: they estimated a 3.6% impact of the 1990 price support program on blend price, and a 2% impact on production. Government purchases are found to have a positive and large influence of the shadow price of fat (+US$46/cwt on average; see Table 3). [13] This is due to large government purchase of butter in 1990. Such effects are fairly uniform across regions. The influence of the price support program on the shadow price of protein is also fairly uniform across regions but negative (−US$39/cwt on average).

[12] The absence of marketing orders is represented by setting the class I price differential equal to zero in each region, holding the over-order premium at the 1990 levels.

[13] Note that the shadow price for fat reported in Table 3 does not concern fat in butter, soft, and frozen products. The price of fat in butter, soft, and frozen products tends to be lower than the one reported in Table 3 because of additional constraints imposed in the model reflecting the limited substitutability of fat between these products and other dairy products.

These different effects across components are driven by the fact that 1990 government purchases involve dairy products (i.e., American cheese, butter, and nonfat dry milk) with average compositions that differ from raw milk. As a result, the price support program differentially stimulates the demand for components, which influences their relative shadow prices.

These changing component values are linked with the associated impact of the price support program on the price of dairy commodities depending on their composition. Table 4 shows that government purchases tend to increase significantly the price of butter (with high fat content), but to depress the price of Italian cheese (with high protein and carbohydrate content). These results illustrate that the 1990 government purchases have important differential effects on both the shadow value of components and the relative prices of dairy commodities.

Since dairy production and consumption are not evenly distributed among regions, differing price effects across commodities have implications for the spatial welfare distribution of government purchases. As expected, the price support program tends to increase producer surplus while decreasing consumer surplus, thus making farmers better off at the expense of consumers (see scenarios (2) and (4) in Table 5). These effects are found to be relatively uniform across regions. The dead weight loss (measuring the net welfare loss to society, including the cost to taxpayers) due to the 1990 price support program is estimated at US$429 million. Helmberger and Chen found similar welfare effects: they estimated the long run social cost of 1990 dairy price support at US$514 million.

The effects of the milk marketing (both federal and California) orders can be evaluated by comparing scenarios (2) and (3) (in presence of the price support program), or scenarios (4) and (5) (in the absence of the price support program). Table 1 indicates that marketing orders increase the average farm milk price by just a few cents per cwt. This contrasts with the AAEA task force or Helmberger and Chen, who found larger impact of marketing orders on U.S. average blend price: 2–5% (AAEA Policy Task Force on Dairy Marketing Orders, 1986), 4.2% (Helmberger and Chen, 1994, short run) and 1.8% (Helmberger and Chen, 1994, long run). However, these studies did not consider the regional effects of marketing orders. Our small average effect on blend price hides large differences across regions. From scenarios (2) and (3), the effect of marketing orders on farm milk price goes from an *increase* of US$0.96/cwt in South East and US$0.86 in North East and South Atlantic, to a *decrease* of US$0.38/cwt in Wisconsin, US$0.40/cwt in West North Central and US$0.45/cwt in California. This reflects in part the federal pricing scheme based in Eau Claire, WI: the further from Wisconsin, the higher the fluid milk price. Hence, regions with high fluid milk utilization and distant from Eau Claire (e.g., South east, South Atlantic, North East) obtain higher farm milk price due to marketing orders, while regions with low fluid milk utilization and close to Eau Claire (e.g., Wisconsin, West North Central) obtain lower price.

Scenarios (2) and (3) (or (4) and (5)) in Table 3 indicate that marketing orders have, on the average, modest effects on component prices. Such effects are most important in South East, where marketing orders tend to depress the shadow prices of fat and protein, while increasing the price of carbohydrate. As expected, Table 4 shows that marketing orders increase fluid milk price while decreasing the price of most manufactured dairy products, with opposite changes in consumption levels.

Some of the welfare effects of marketing orders can be evaluated from Table 5 by comparing scenarios (2) and (3). They show that, on the average, marketing orders increase producer surplus but decrease consumer surplus. This is an expected result of the associated price discrimination scheme that transfers welfare from consumers to producers. However, such effects are not uniform across regions. For example, producer surplus *rises* significantly in South East, but *declines* in Wisconsin, West North Central and California. These results indicate that milk marketing orders tend to redistribute welfare with differential impacts across regions. This stresses the importance of a disaggregate and regional analysis of the distributional impact of current U.S. dairy policy. Finally, from scenarios (2) and (3), the dead weight loss (measuring the net welfare loss to society) due to milk marketing orders are estimated to be US$112 million. This compares with the estimated long run social cost of US$40 million obtained by

Helmberger and Chen for marketing orders under 1990 conditions.

In general, the results illustrate that the allocation and pricing of components influence the determination of market equilibrium under alternative pricing policies. They show the general usefulness of our approach to better understand the role of nonmarket goods in spatially and vertically linked markets.

5. Conclusion

A multimarket competitive model that represents spatial resource allocation in a vertical sector including both market commodities (farm and food commodities) and their nonmarket characteristics (nutrients) has been developed. It helps fill the gap between the Samuelson–Judge–Takayama approach to commodity trade modelling, and Rosen's market model of shadow pricing for embedded nonmarket characteristics. Using a Lancasterian-type approach, we specified a competitive market equilibrium for agricultural and food commodities, as well as the spatial allocation of the embedded nutrients and the distribution of their shadow prices. Moreover, we explored how to incorporate the effects of pricing policy interventions in the model.

The usefulness of the model is illustrated in the context of a regional analysis of the U.S. dairy sector. The empirical focus is the allocation of farm milk nutrient components (i.e., fat, carbohydrate, and protein) among nine categories of dairy products in 14 regions of the U.S. First, a validation exercise suggests that the proposed modelling approach provides a reasonable approximation to the U.S. dairy sector. Second, the allocation and shadow pricing of milk components is analyzed in a way that is consistent with trade efficiency and market equilibrium. The empirical estimates of shadow prices provide useful information on the regional allocation of milk nutrients. Third, we evaluate the impact of the milk price support program and milk marketing orders on the U.S. dairy industry. The effects of pricing policy on the shadow price of nutrients and on the distribution of farmers' and consumers' welfare across regions are estimated. It is shown that the price support program tends to benefit dairy farmers in all regions. But it has large influences on the relative prices of fat versus protein. In contrast, milk marketing orders are found to have a modest effect on the shadow price of milk components. However, their impact on the welfare of milk producers varies significantly across regions. These regional differences appear crucial in understanding the current debate related to U.S. dairy policy reform. This illustrates the valuable insights that can be obtained from our approach in the analysis of spatial and vertical allocation and pricing of market commodities and nutrients in the farm and food sector.

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