Frontier production functions and technical efficiency: a survey of empirical applications in agricultural economics

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ABSTRACT

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The modelling and estimation of frontier production functions has been an important area of econometric research during the last two decades. Försund, Lovell and Schmidt (1980) and Schmidt (1986) present reviews of the concepts and models involved and cite some of the empirical applications which had appeared to their respective times of publication.

This paper seeks to update the econometric modelling of frontier production functions associated with the estimation of technical efficiency of individual firms. A survey of empirical applications in agricultural economics is an important part of the paper.

1. INTRODUCTION

In microeconomic theory a production function is defined in terms of the *maximum* output that can be produced from a specified set of inputs, given the existing technology available to the firms involved. However, up until the late 1960's, most empirical studies used traditional least-squares meth-
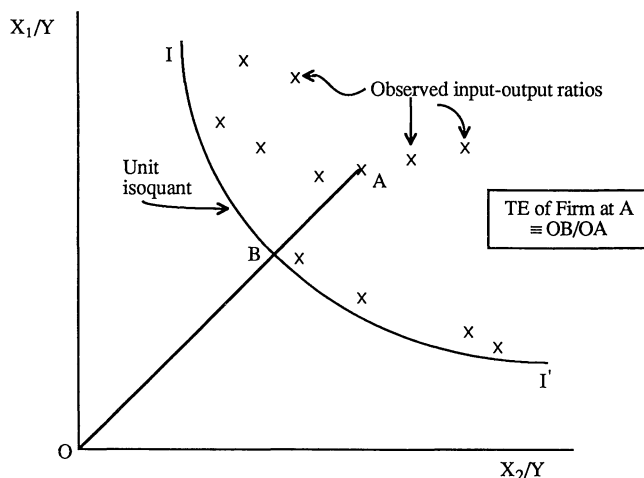
Fig. 1. Technical efficiency of firms in relative input space.

ods to estimate production functions. Hence the estimated functions could be more appropriately described as *response* (or *average*) *functions*.

Econometric modelling of production functions, as traditionally defined, was stimulated by the seminal paper of Farrell (1957). Given that the production function to be estimated had constant returns to scale, Farrell (1957) assumed that observed input-per-unit-of-output values for firms would be above the so-called *unit isoquant*. Figure 1 depicts the situation in which firms use two inputs of production, X_1 and X_2, to produce their output, Y, such that the points, defined by the input-per-unit-of-output ratios, $(X_1/Y, X_2/Y)$, are above the curve, II'. The unit isoquant defines the input-per-unit-of-output ratios associated with the *most efficient* use of the inputs to produce the output involved. The deviation of observed input-per-unit-of-output ratios from the unit isoquant was considered to be associated with *technical inefficiency* of the firms involved. Farrell (1957) defined the ratio, OB/OA, to be the *technical efficiency* of the firm with input-per-unit-of-output values at point A.

Farrell (1957) suggested that the efficient unit isoquant be estimated by programming methods such that the convex function involved was never above any of the observed input-per-unit-of-output ratios.

A more general presentation of Farrell's concept of the production function (or *frontier*) is depicted in Fig. 2 involving the original input and output values. The horizontal axis represents the (vector of) inputs, X, associated with producing the output, Y. The observed input–output values are *below* the production frontier, given that firms do not attain the maximum output possible for the inputs involved, given the technology
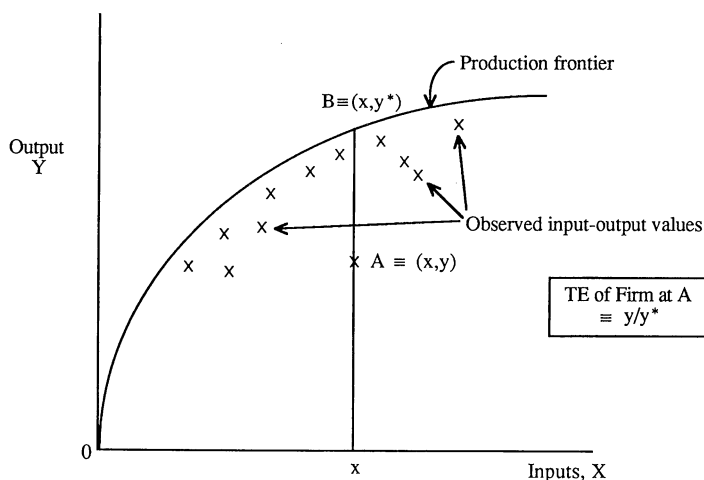
Fig. 2. Technical efficiency of firms in input–output space.

available. A measure of the technical efficiency of the firm which produces output, y, with inputs, x, denoted by point A, is given by y/y^*, where y^* is the 'frontier output' associated with the level of inputs, x (see point B). This is a measure of technical efficiency which is conditional on the levels of the inputs involved.

The existence of technical inefficiency of firms engaged in production has been a subject of considerable debate in economics. For example, Müller (1974) states (p. 731): "However, little is known about the role of non-physical inputs, especially information or knowledge, which influence the firm's ability to use its available technology set fully. ... This suggests how relative and artificial the concept of the *frontier* itself is. ... Once all inputs are taken into account, measured productivity differences should disappear except for random disturbances. In this case the *frontier* and the *average* function are identical. They only diverge if significant inputs have been left out in the estimation". Upton (1979) also raised important problems associated with empirical production function analysis. However, despite these criticisms, we believe that the econometric modelling of frontier production functions, which is surveyed below, provides useful insights into best-practice technology and measures by which the productive efficiency of different firms may be compared.

2. ECONOMETRIC MODELS

Production frontier models are reviewed in three sub-sections involving deterministic frontiers, stochastic frontiers and panel data models. For

convenience of exposition, these models are presented such that the dependent variable is the *original output* of the production process, denoted by Y, which is assumed to be expressed in terms of the product of a known function of a vector, x, of the inputs of production and a function of unobservable random variables and stochastic errors.

2.1 Deterministic frontiers

The deterministic frontier model is defined by:

$$Y_i = f(x_i; \beta) \exp(-U_i) \qquad i = 1, 2, \ldots, N \tag{1}$$

where Y_i represents the possible production level for the ith sample firm; $f(x_i; \beta)$ is a suitable function (e.g., Cobb–Douglas or TRANSLOG) of the vector, x_i, of inputs for the ith firm and a vector, β, of unknown parameters; U_i is a *non-negative* random variable associated with firm-specific factors which contribute to the ith firm not attaining maximum efficiency of production; and N represents the number of firms involved in a cross-sectional survey of the industry.

The presence of the non-negative random variable, U_i, in model (1), is associated with the technical inefficiency of the firm and implies that the random variable, $\exp(-U_i)$, has values between zero and one. Thus it follows that the possible production, Y_i, is bounded above by the non-stochastic (i.e., deterministic) quantity, $f(x_i; \beta)$. Hence the model (1) is referred to as a *deterministic frontier* production function. The inequality relationships:

$$Y_i \leq f(x_i; \beta) \qquad i = 1, 2, \ldots, N \tag{2}$$

were first specified by Aigner and Chu (1968) in the context of a Cobb–Douglas model. It was suggested that the parameters of the model be estimated by applying linear or quadratic programming algorithms. Aigner and Chu (1968) suggested (p. 838) that chance-constrained programming could be applied to the inequality restrictions (2) so that some output observations could be permitted to lie above the estimated frontier. Timmer (1971) took up this suggestion to obtain the so-called *probabilistic frontier* production functions, for which a small proportion of the observations is permitted to exceed the frontier. Although this feature was considered desirable because of the likely incidence of outlier observations, it obviously lacks any statistical or economic rationale.

The frontier model (1) was first presented by Afriat (1972, p. 576). Richmond (1974) further considered the model under the assumption that U_i had gamma distribution with parameters, $r = n$ and $\lambda = 1$ [see Mood, Graybill and Boes (1974, p. 112)]. Schmidt (1976) stated that the

maximum-likelihood estimates for the β-parameters of the model could be obtained by linear and quadratic programming techniques if the random variables had exponential or half-normal distributions, respectively. [1]

The *technical efficiency* of a given firm is defined to be the factor by which the level of production for the firm is less than its *frontier* output. Given the deterministic frontier model (1), the frontier output for the ith firm is $Y_i^* = f(x_i; \beta)$ and so the technical efficiency for the ith firm, denoted by TE_i, is:

$$\text{TE}_i = Y_i/Y_i^*$$

$$= f(x_i; \beta) \exp(-U_i)/f(x_i; \beta)$$

$$= \exp(-U_i) \tag{3}$$

Technical efficiencies for individual firms in the context of the deterministic frontier production function (1) are predicted by obtaining the ratio of the observed production values to the corresponding estimated frontier values, $\hat{\text{TE}}_i = Y_i/f(x_i; \hat{\beta})$, where $\hat{\beta}$ is either the maximum-likelihood estimator or the corrected ordinary least-squares (COLS) estimator for β. [2]

If the U_i-random variables of the deterministic frontier (1) have exponential or half-normal distribution, inference about the β-parameters cannot be obtained from the maximum-likelihood estimators because the well-known regularity conditions (see Theil, 1971, p. 392) are not satisfied. Greene (1980) presented sufficient conditions for the distribution of the U_i's for which the maximum-likelihood estimators have the usual asymptotic properties, upon which large-sample inference for the β-parameters can be obtained. Greene (1980) proved that if the U_i's were independent

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[1] Given that the β-parameters of model (1) are expressible as a linear function when logarithms are taken, it follows that the maximum-likelihood estimates for the β-parameters when U_i has exponential or half-normal distributions are defined by minimizing the absolute sum or the sum of squares of the deviations of the logarithms of production from the corresponding frontier values, subject to the linear constraints obtained by applying logarithms to (2). However, the *non-negativity restrictions* on the parameter estimates, which are normally associated with linear and quadratic programming problems, *are not required*. Although non-negative estimates for the partial elasticities in Cobb–Douglas models are generally reasonable, it does not follow that non-negativity restrictions should be applied for such functional forms as the TRANSLOG model.

[2] Given that the model (1) has the form of a linear model (with an intercept) when logarithms are taken, then the COLS estimator for β is defined by the OLS estimators for the coefficients of β, except the intercept, and the OLS estimator for the intercept *plus* the largest residual required to make all deviations of the production observations from the estimated frontier non-positive. Greene (1980) showed that the COLS estimator is consistent, given that the U_i-random variables are independent and identically distributed.

and identically distributed as gamma random variables, with parameters $r > 2$ and $\lambda > 0$, then the required regularity conditions are satisfied.

2.2 Stochastic frontiers

The stochastic frontier production function is defined by:

$$Y_i = f(x_i; \beta) \exp(V_i - U_i) \qquad i = 1, 2, \ldots, N \tag{4}$$

where V_i is a random error having zero mean, which is associated with random factors (e.g., measurement errors in production, weather, industrial action, etc.) not under the control of the firm.

This stochastic frontier model was independently proposed by Aigner, Lovell and Schmidt (1977) and Meeusen and Van den Broeck (1977). The model is such that the possible production, Y_i, is bounded above by the stochastic quantity, $f(x_i; \beta) \exp(V_i)$; hence the term *stochastic frontier*. The random errors, V_i, $i = 1, 2, \ldots, N$, were assumed to be independently and identically distributed as $N(0, \sigma_V^2)$ random variables, independent of the U_i's, which were assumed to be non-negative truncations of the $N(0, \sigma^2)$ distribution (i.e., half-normal distribution) or have exponential distribution. Meeusen and Van den Broeck (1977) considered only the case in which the U_i's had exponential distribution (i.e., gamma distribution with parameters $r = 1$ and $\lambda > 0$) and noted that the model was not as restrictive as the one-parameter gamma distribution (i.e., gamma distribution with parameters $r = n$ and $\lambda = 1$) considered by Richmond (1974).

The basic structure of the stochastic frontier model (4) is depicted in Fig. 3 in which the productive activities of two firms, represented by i and j, are considered. Firm i uses inputs with values given by (the vector) x_i and obtains the output, Y_i, but the frontier output, Y_i^*, exceeds the value on the deterministic production function, $f(x_i; \beta)$, because its productive activity is associated with 'favourable' conditions for which the random error, V_i, is positive. However, firm j uses inputs with values given by (the vector) x_j and obtains the output, Y_j, which has corresponding frontier output, Y_j^*, which is less than the value on the deterministic production function, $f(x_j; \beta)$, because its productive activity is associated with 'un-favourable' conditions for which the random error, V_j, is negative. In both cases the observed production values are less than the corresponding frontier values, but the (unobservable) frontier production values would lie around the deterministic production function associated with the firms involved. [3]

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[3] It is possible that *both* the observed and frontier production values, Y_i and $Y_i^* \equiv f(x_i; \beta) \exp(V_i)$, lie above the corresponding value of the deterministic production function, $f(x_i; \beta)$, if $V_i > U_i$. This case is not depicted in Fig. 3.
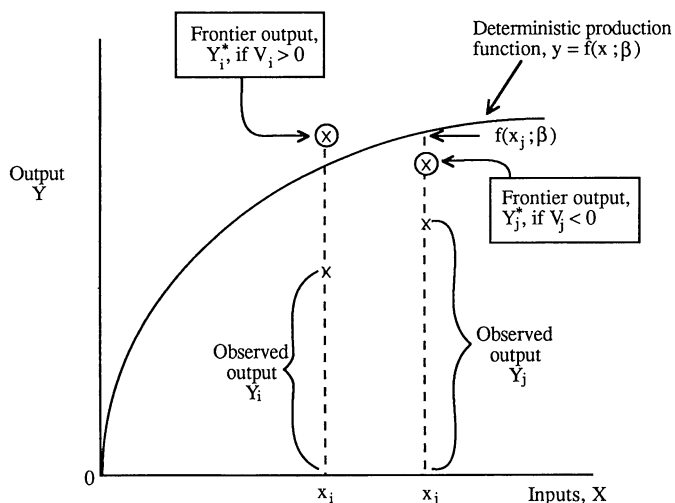
Fig. 3. Stochastic frontier production function.

Given the assumptions of the stochastic frontier model (4), inference about the parameters of the model can be based on the maximum-likelihood estimators because the standard regularity conditions hold. Aigner, Lovell and Schmidt (1977) suggested that the maximum-likelihood estimates of the parameters of the model be obtained in terms of the parameterization, $\sigma_V^2 + \sigma^2 \equiv \sigma_S^2$ and $\lambda \equiv \sigma/\sigma_V$. Rather than use the non-negative parameter, λ (i.e., the ratio of the standard deviation of the $N(0, \sigma^2)$ distribution involved in specifying the distribution of the non-negative U_i's to the standard deviation of the symmetric errors, V_i), Battese and Corra (1977) considered the parameter, $\gamma \equiv \sigma^2/(\sigma_V^2 + \sigma^2)$, which is bounded between zero and one.

Technical efficiency of an individual firm is defined in terms of the ratio of the observed output to the corresponding frontier output, conditional on the levels of inputs used by that firm. [4] Thus the technical efficiency of firm i in the context of the stochastic frontier production function (4) is the

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[4] Battese and Coelli (1988) suggest (p. 389) that the technical efficiency of firm i, associated with a panel data model with time-invariant firm effects, be defined as the ratio of its mean production, conditional on its levels of inputs and its realized firm effect, U_i, to the corresponding mean production if the firm effect, U_i, had value zero (and so the firm was fully efficient). This definition yields the same measure of technical efficiency as that given in the text.

same expression as for the deterministic frontier model (1), namely $\text{TE}_i = \exp(-U_i)$, i.e.

$$\text{TE}_i = Y_i/Y_i^*$$

$$= f(x_i; \beta) \exp(V_i - U_i)/f(x_i; \beta) \exp(V_i)$$

$$= \exp(-U_i)$$

Although the technical efficiency of a firm associated with the deterministic and stochastic frontier models are the same, it is important to note that they have different values for the two models. Considering Fig. 3, it is evident that the technical efficiency of firm j is greater under the stochastic frontier model than for the deterministic frontier, i.e., $(Y_j/Y_j^*) > [Y_j/f(x_j; \beta)]$. That is, firm j is judged technically more efficient relative to the unfavourable conditions associated with its productive activity (i.e., $V_j < 0$) than if its production is judged relative to the maximum associated with the value of the deterministic function, $f(x_j; \beta)$. Further, firm i is judged technically less efficient relative to its favourable conditions than if its production is judged relative to the maximum associated with the value of the deterministic function, $f(x_i; \beta)$. However, for a given set of data, the estimated technical efficiencies obtained by fitting a deterministic frontier will be *less* than those obtained by fitting a stochastic frontier, because the deterministic frontier will be estimated so that no output values will exceed it.

Stevenson (1980) suggested that an alternative model for the U_i's in the stochastic frontier (4) was the non-negative truncation of the $N(\mu, \sigma^2)$ distribution. This generalization includes the cases in which there may be low probability of obtaining U_i's close to zero (i.e., when there is considerable technical inefficiency present in the firms involved).

Aigner and Schmidt (1980) contains several other important papers dealing with the deterministic and stochastic frontier models.

The prediction of the technical efficiencies of individual firms associated with the stochastic frontier production function (4), defined by $\text{TE}_i = \exp(-U_i)$, $i = 1, 2 \ldots, N$, was considered impossible until the appearance of Jondrow, Lovell, Materov and Schmidt (1982). This paper focussed attention on the conditional distribution of the non-negative random variable, U_i, given that the random variable, $E_i \equiv V_i - U_i$, was observable. Jondrow et al. (1982) suggested that U_i be predicted by the conditional expectation of U_i, given the value of the random variable, $E_i \equiv V_i - U_i$. This expectation was derived for the cases that the U_i's had half-normal and exponential distributions. Jondrow et al. (1982) used $1 - E(U_i | V_i - U_i)$ to predict the technical inefficiency of the ith firm. However, given the multiplicative production frontier model (4), Battese and Coelli (1988)

pointed out that the technical efficiency of the ith firm, $\text{TE}_i \equiv \exp(-U_i)$, is best predicted by using the conditional expectation of $\exp(-U_i)$, given the value of the random variable, $E_i \equiv V_i - U_i$. This latter result was calculated for the more general stochastic frontier model involving panel data and the Stevenson (1980) model for the U_i's.

2.3 Panel data models

The deterministic and stochastic frontier production functions (1) and (4) are defined for cross-sectional data (i.e., data on a cross-section of N firms at some particular time period). If time-series observations are available for the sample firms involved, then the data are referred to as *panel data*. Pitt and Lee (1981) considered the estimation of a stochastic frontier production function associated with N firms over T time periods. The model is defined by:

$$Y_{it} = f(x_{it}; \beta) \exp(V_{it} - U_{it}) \quad \begin{aligned} i &= 1, 2, \ldots, N \\ t &= 1, 2, \ldots, T \end{aligned} \tag{5}$$

where Y_{it} represents the possible production for the ith firm at the tth time period.

Pitt and Lee (1981) considered three basic models, defined in terms of the assumptions made about the non-negative U_{it}'s. Model I assumed that the U_{it}'s were time-invariant effects, i.e., $U_{it} = U_i$, $t = 1, 2, \ldots, T$. Model II specified that the U_{it}'s were uncorrelated. Model III permitted the U_{it}'s to be correlated for given firms. [5]

The time-invariant model for the non-negative firm effects was considered by Battese and Coelli (1988) for the case in which the firm effects were non-negative truncations of the $N(\mu, \sigma^2)$ distribution. Battese, Coelli and Colby (1989) considered the case in which the numbers of time-series observations on the different firms were not all equal. Coelli (1989) wrote the computer program, FRONTIER, for obtaining the maximum-likelihood estimates and the predictions for the technical efficiencies of the firms involved.

More recently stochastic frontier models for panel data have been presented in which time-varying firm effects have been specified. Cornwell, Schmidt and Sickles (1990) considered a panel data model in which the firm effects at different time periods were a quadratic function of time, in

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[5] Pitt and Lee (1981, p. 46) incorrectly defined Model II as specifying $E(U_{it} U_{jt'}) = 0$ $i = j$ and $t = t'$. Model III is defined by $\text{Cov}(U_{it}, U_{it'}) = \sigma_{tt'}$ and $\text{Cov}(U_{it}, U_{jt'}) = 0$ if $i \neq j = 1, 2, \ldots, N$.

which the coefficients varied over firms according to the specifications of a multivariate distribution. The parameters of the model were estimated using instrumental-variable methods.

Kumbhakar (1990) presented a model in which the non-negative firm effects, U_{it}, were the product of an exponential function of time (involving two parameters) and a time-invariant (non-negative) random variable. This latter model permits the time-varying firm effects to be monotone decreasing (or increasing) or convex (or concave) functions over time [i.e., the technical efficiency of firms in the industry involved could monotonically increase (or decrease) or increase and then decrease (or vice versa)]. Battese and Coelli (1992) suggested a time-varying firm effects model for incomplete panel data, such that the technical efficiencies of firms either monotonically increased or decreased or remained constant over time. This time-varying model assumes that the firm effects are an exponential function of time which involves only one additional parameter to the time-invariant model of Battese and Coelli (1988).

3. EMPIRICAL APPLICATIONS

Frontier production function models have been applied in a considerable number of empirical studies in agricultural economics. Publications have appeared in the all major agricultural economics journals and a considerable number of other economic journals. *The Journal of Agricultural Economics* has published the most papers (at least seven, cited below) dealing with frontier production functions. Other journals which have published at least two applied production frontier papers are the *Canadian Journal of Agricultural Economics* (4), the *American Journal of Agricultural Economics* (2) and the *Southern Journal of Agricultural Economics* (2). At least one frontier production function paper involving farm-level data has appeared in the *Australian Journal of Agricultural Economics*, the *European Review of Agricultural Economics*, the *North Central Journal of Agricultural Economics* and the *Western Journal of Agricultural Economics*. Several papers have appeared in development economics journals as well as econometric and other applied economics journals.

The empirical studies are surveyed under the three headings involved in the above section, depending on the type of frontier production function estimated.

3.1 Deterministic frontiers

Russell and Young (1983) estimated a deterministic Cobb–Douglas frontier using corrected ordinary least-squares regression with a cross-sec-

tion of 56 farms in the North-West region of England during 1977–78. The dependent variable was total revenue obtained from the crop, livestock and miscellaneous activities on the farms involved. Technical efficiencies for the individual farms were obtained using both the Timmer and Kopp measures. [6] These two measures of technical efficiency gave approximately the same values and the same rankings for the 56 farms involved. The Timmer technical efficiencies ranged from 0.42 to 1.00, with average 0.73 and sample standard deviation 0.11. Russell and Young (1983) did not make any strong conclusions as to the policy implications of these results.

Kontos and Young (1983) conducted similar frontier analyses to those of Russell and Young (1983) for a data set for 83 Greek farms for the 1980–81 harvest year. Kontos and Young (1983) applied a Box–Cox transformation to the variables of the model and obtained similar elasticities to those obtained by estimating the Cobb–Douglas production function by ordinary least-squares regression. Since the likelihood ratio test indicated that the Box–Cox model was not significantly different from the traditional Cobb–Douglas model, the deterministic frontier model was estimated by corrected ordinary least-squares regression. The estimated frontier model was used to obtain the values of the Kopp measure of technical efficiency for the individual farms involved. These technical efficiencies ranged from about 0.30 to 1.00, with an average of 0.57, indicating that considerable technical inefficiencies existed in the Greek farms surveyed.

Dawson (1985) analysed 4 years of data for the 56 farms involved in the paper by Russell and Young (1983). Three estimators for the technical efficiency of the individual farms were presented which involved a two-step, ordinary least-squares procedure, an analysis-of-covariance method and the linear programming procedure suggested by Aigner and Chu (1968). The technical efficiency measures obtained by the three methods exhibited wide variation and the estimated correlation coefficients were quite small. Dawson (1985) claimed that there was indication that the technical efficiencies were directly related to the size of the farm operation.

Taylor, Drummond and Gomes (1986) considered a deterministic Cobb–Douglas frontier production function for Brazilian farmers to investigate the effectiveness of a World Bank sponsored agricultural credit programme in the State of Minas Gerais. The parameters of the frontier

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[6] The Timmer measure of technical efficiency is the measure discussed above in Section 3. The Kopp measure of technical efficiency, introduced by Kopp (1981), involves the ratio of the frontier input levels which would be required to produce the observed level of output (the input ratios being constant) if the farm was fully technically efficient, to the actual input levels used. These two measures are not equivalent unless the production frontier has constant returns to scale.

model were estimated by corrected ordinary least-squares regression and the maximum-likelihood method under the assumption that the non-negative farm effects had gamma distribution. The authors did not report estimates for different frontier functions for participant and non-participant farmers in the agricultural credit programme and test if the frontiers were homogeneous. It appears that the technical efficiencies of participant and non-participant farmers were estimated from the common production frontier reported in the paper. The average technical efficiencies for participant and non-participant farmers were reported to be 0.18 and 0.17, respectively. [7] The authors concluded that these values were not significantly different and that the agricultural credit programme did not appear to have any significant effect on the technical efficiencies of participant farmers.

Bravo-Ureta (1986) estimated the technical efficiencies of dairy farms in the New England region of the United States using a deterministic Cobb-Douglas frontier production function. The parameters of the production frontier were estimated by linear programming methods involving the probabilistic frontier approach. Using the 96% probabilistic frontier estimates, Bravo-Ureta (1986) obtained technical efficiencies which ranged from 0.58 to 1.00, with an average of 0.82. He concluded that technical efficiency of individual farms was statistically independent of size of the dairy farm operation, as measured by the number of cows.

Aly, Belbase, Grabowski and Kraft (1987) investigated the technical efficiency of a sample of Illinois grain farms by using a deterministic frontier production function of ray-homothetic type. The authors presented a concise summary of the different approaches to frontier production functions, including stochastic frontiers. The deterministic ray-homothetic frontier, which was estimated by corrected ordinary least-squares regression, had the output and input variables expressed in revenue terms rather than in physical units. Hence the technical efficiencies also reflected allocative efficiencies. The mean technical efficiency for the 88 grain farms involved was 0.58 which indicated that considerable inefficiency existed in Illinois grain farms. The authors found that larger farms tended to be more technically efficient than smaller ones, irrespective of whether acreage cultivated or gross revenue was used to classify the farms by size of operation.

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[7] If Taylor, Drummond and Gomes (1986) had estimated separate production frontiers for participant and non-participant farmers, then the mean technical efficiencies of the farmers in the different groups could be estimated by $[\lambda/(\lambda+1)]^r$, where λ and r are the parameters of the gamma distribution involved.

Ali and Chaudhry (1990) estimated deterministic frontier production functions in their analyses of a cross-section of farms in four regions of Pakistan's Punjab. The parameters of the Cobb–Douglas frontier functions for the four regions were estimated by linear programming methods. Although the frontier functions were not homogeneous among the different regions, the technical efficiencies in the four regions ranged from 0.80 to 0.87 but did not appear to be significantly different.

3.2 Stochastic frontiers

Aigner, Lovell and Schmidt (1977) applied the stochastic frontier production function in the analysis of aggregative data on the U.S. primary metals industry (involving 28 states) and US agricultural data for 6 years and the 48 coterminous states. For these applications, the stochastic frontier was not significantly different from the average response function. Similar results were obtained by Meeusen and Van den Broeck (1977) in their analyses for ten French manufacturing industries. [8]

The first application of the stochastic frontier model to farm-level agricultural data was presented by Battese and Corra (1977). Data from the 1973–74 Australian Grazing Industry Survey were used to estimate deterministic and stochastic Cobb–Douglas production frontiers for the three states included in the Pastoral Zone of Eastern Australia. The variance of the farm effects was found to be a highly significant proportion of the total variability of the logarithm of the value of sheep production in all states. The γ-parameter estimates exceeded 0.95 in all cases. Hence the stochastic frontier production functions were significantly different from their corresponding deterministic frontiers. Technical efficiency of farms in the regions was not addressed in Battese and Corra (1977).

Kalirajan (1981) estimated a stochastic frontier Cobb–Douglas production function using data from 70 rice farmers for the rabi season in a district in India. The variance of farm effects was found to be a highly significant component in describing the variability of rice yields (the estimate for the γ-parameter was 0.81). Kalirajan (1981) proceeded to investigate the relationship between the difference between the estimated 'maximum yield function' and the observed rice yields and such variables as farmer's experience, educational level, number of visits by extension work-

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[8] Since that time there have been a large number of empirical applications of the stochastic frontier model in production and cost functions involving industrial and manufacturing industries in which the model was significantly different from the average response function and the corresponding deterministic frontier. These are not included in this survey.

ers, etc. [9] In this second-stage analysis, [10] Kalirajan (1981) noted the policy implications of these findings for improving crop yields of farmers.

Kalirajan (1982) estimated a similar stochastic frontier production function to that in Kalirajan (1981) in the analysis of data from 91 rice farmers for the kharif season in the same district of India as in his earlier paper. The farm effects in the model were again found to be very highly significant (with $\hat{\gamma} = 0.93$).

Bagi (1982a) used the stochastic frontier Cobb–Douglas production function model to determine whether there were any significant differences in the technical efficiencies of small and large crop and mixed-enterprise farms in West Tennessee. The variability of farm effects were found to be highly significant and the mean technical efficiency of mixed-enterprise farms was smaller than that for crop farms (about 0.76 versus 0.85, respectively). However, there did not appear to be significant differences in mean technical efficiency for small and large farms, irrespective of whether the farms were classified according to acreage or value of farm sales. [11] Bagi (1984) considered the same data set as in Bagi (1982a) to investigate whether there were any significant differences in the mean technical efficiencies of part-time and full-time farmers. No significant differences were apparent, irrespective of whether the part-time and full-time farmers were engaged in mixed farming or crop-only farms.

Bagi and Huang (1983) estimated a translogarithmic stochastic frontier production function using the same data on the Tennessee farms considered in Bagi (1982a). The Cobb–Douglas stochastic frontier model was found not to be an adequate representation of the data, given the specifications of the translog model for both crop and mixed farms. The parameters

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[9] It is possible for observed yield to exceed the corresponding value of the 'maximum yield function' because the latter is obtained by using the estimated β-parameters of the stochastic frontier production function. Negative differences are explicitly reported in Kalirajan (1982) in Table 2 (p. 233). Under the assumptions of the stochastic frontier production function (4) the observed yields cannot exceed the corresponding stochastic frontier yields, but the latter are not observable values.

[10] Kalirajan (1981, p. 289) states that the parameters of the second-stage model involving differences between estimated maximum yields and observed yields were estimated by the maximum-likelihood method associated with the stochastic frontier model. However, the assumptions of the stochastic model (4) would not hold when the estimated yield function from the first-stage analysis is involved.

[11] Bagi erroneously (p. 142) claimed that the estimate for the γ-parameter in the stochastic frontier model [see the reference to Battese and Corra (1977) in Section 2(ii) above] of 0.72 implies that 72% of the discrepancy between the observed and maximum (frontier) output results from technical inefficiency.

of the model were estimated by corrected ordinary least-squares regression. The mean technical efficiencies of crop and mixed farms were estimated to be 0.73 and 0.67, respectively. Individual technical efficiencies of the farms were predicted using the predictor $\exp(-\hat{U}_i)$, where $\hat{U}_i$ is the estimated conditional mean of the ith farm effect (suggested by Jondrow, Lovell, Materov and Schmidt, 1982). These technical efficiencies varied from 0.35 to 0.92 for mixed farms and 0.52 to 0.91 for crop farms.

Bagi (1982b) included empirical results on the estimation of a translog stochastic frontier production function using data from 34 share cropping farms in India. The parameters of the model were estimated using corrected ordinary least-squares regression. The Cobb–Douglas functional form was judged not to be an adequate representation of the data given the assumptions of the translog model. For these Indian farm data, the variance of the non-negative farm effects was only a small proportion of the total variance of farm outputs ($\hat{\gamma} = 0.15$). The individual farm technical efficiencies were predicted to be between 0.92 and 0.95. These high technical efficiencies are consistent with the relatively low variance of farm effects which implies that the stochastic frontier and the average production function are expected to be quite similar.

Kalirajan and Flinn (1983) outlined the methodology by which the individual firm effects can be predicted (as discussed above with reference to Jondrow et al., 1982) and applied the approach in their analysis of data on 79 rice farmers in the Philippines. A translog stochastic frontier production function was assumed to explain the variation in rice output in terms of several input variables. The parameters of the model were estimated by the method of maximum likelihood. The Cobb–Douglas model was found to be an inadequate representation for the farm-level data. The individual technical efficiencies ranged from 0.38 to 0.91. The predicted technical efficiencies were regressed on several farm-level variables and farmer-specific characteristics. It was concluded that the practice of transplanting rice seedlings, incidence of fertilization, years of farming and number of extension contacts had significant influence on the variation of the estimated farm technical efficiencies.

Huang and Bagi (1984) assumed a modified translogarithmic stochastic frontier production function to estimate the technical efficiencies of individual farms in India. It was found that the Cobb–Douglas stochastic frontier was not an adequate representation for describing the value of farm products, given the specifications of the translog model. The variance of the random effects was a significant component of the variability of value of farm outputs. Individual technical efficiencies ranged from about 0.75 to 0.95, but there appeared to be no significant differences in the technical efficiencies of small and large farms.

Taylor and Shonkwiler (1986) estimated both deterministic and stochastic production frontiers of Cobb–Douglas type for participants and non-participants of the World Bank sponsored credit programme (PRODEMATA) for farmers in Brazil. The parameters of the frontiers involved were estimated by maximum-likelihood methods, given the assumptions that the farm effects had gamma distribution in the deterministic frontier and half-normal for the stochastic frontier. The authors did not report that statistical tests had been conducted on the homogeneity of the frontiers for participants and non-participant farmers. Farm-level technical efficiencies were estimated for all the frontiers, as suggested by Jondrow et al. (1982). Given the stochastic frontiers, the average technical efficiencies for participants and non-participants were 0.714 and 0.704, respectively, and were not significantly different. However, given the assumptions of the deterministic frontiers, the average technical efficiencies were 0.185 and 0.059, respectively, and were significantly different. Taylor and Shonkwiler (1986) concluded that their results indicated somewhat confusing results as to the impact of the PRODEMATA programme on participant farmers in Brazil. [12]

Huang, Tang and Bagi (1986) adopted a stochastic *profit* function approach to investigate the economic efficiency of small and large farms in two states in India. The variability of farm effects was highly significant and individual farm economic efficiencies tended to be greater for large farms than small farms (the average economic efficiencies being 0.84 and 0.80 for large and small farms, respectively). The authors also considered the determination of optimal demand for hired labour under conditions of uncertainty.

Kalirajan and Shand (1986) investigated the technical efficiency of rice farmers within and without the Kemubu Irrigation Project in Malaysia during 1980. Given the specifications of a translog stochastic frontier production function for the output of the rice farmers, the Cobb–Douglas model was not an adequate representation of the data. Maximum-likelihood methods were used for estimation of the parameters of the models and the frontiers for the two groups of farmers were significantly different. Kalirajan and Shand (1986) reported that the individual technical efficiencies ranged from about 0.40 to 0.90, such that the efficiencies for those

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[12] However, given the relatively large estimated standard errors for the variances of the random errors in the stochastic frontiers, it may be the case that the stochastic model is not significantly different from the deterministic model. Hence this would suggest that the results obtained from the deterministic frontiers are more encouraging as to the positive impact of the credit programme on participant farmers, even though the absolute levels of technical efficiencies were quite small.

outside the Kemubu Irrigation Project were slightly narrower. They concluded that their results indicated that the introduction of new technology for farmers does not necessarily result in significantly increased technical efficiencies over those for traditional farmers.

Ekanayake and Jayasuriya (1987) estimated both deterministic and stochastic frontier production functions of Cobb–Douglas type for two groups of rice farmers in an irrigated area in Sri Lanka. The parameters of the two frontiers were estimated by maximum-likelihood and corrected ordinary least-squares methods. In only the 'tail reach' irrigated area, the stochastic frontier appeared to be significantly different from the deterministic model. Individual farm technical efficiencies were estimated for both regions. The estimates obtained for the farms in the 'head reach' area (for which the stochastic frontier appeared not to be significantly different from the deterministic frontier) were vastly different for the two different stochastic frontiers. These results are not intuitively reasonable.

Ekayanake (1987) [13] further discusses the data considered by Ekanayake and Jayasuriya (1987) and used regression analysis to determine the farmer-specific variables which had significant effects in describing the variability in the individual farm technical efficiencies in the 'tail reach' of the irrigation area involved. Allocative efficiency was also considered in the empirical analysis.

Kalirajan (1989) predicted technical efficiencies of individual farmers (which he called "human capital") involved in rice production in two regions in the Philippines in 1984–85. A Cobb–Douglas stochastic frontier model was assumed to be appropriate in the empirical analysis. The predicted technical efficiencies were regressed on several farm- and farmer-specific variables to discover what variables had significant effects on the variation in the technical efficiencies.

Ali and Flinn (1989) estimated a stochastic *profit* frontier of modified translog type [14] for Basmati rice farmers in Pakistan's Punjab. After estimating the technical efficiency of individual farmers, the losses in profit due to technical inefficiency were obtained and regressed on various farmer- and farm-specific variables. Factors which were significant in describing the variability in profit losses were level of education, off-farm employment, unavailability of credit and various constraints associated with irrigation and fertilizer application.

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[13] The author was incorrectly listed as S.A.B. Ekayanake by the *Journal of Development Studies*. The correct spelling of the author's name is "Ekanayake".

[14] Ali and Flinn (1989) deleted variables in the translog stochastic profit frontier which had coefficiencies which were *not individually significantly different from zero*. This is not a recommended applied econometric methodology.

Dawson and Lingard (1989) used a Cobb–Douglas stochastic frontier production function to estimate technical efficiencies of Philippine rice farmers using four years of data. The four stochastic frontiers estimated were significantly different from the corresponding deterministic frontiers, but the authors did not adopt any panel-data approach or test if the frontiers had homogeneous elasticities. The individual technical efficiencies ranged between 0.10 and 0.99, with the means between 0.60 and 0.70 for the 4 years involved.

Bailey, Biswas, Kumbhakar and Schulthies (1989) estimated a stochastic model involving technical, allocative and scale inefficiencies for cross-sectional data on 68 Ecuadorian dairy farms. The technical inefficiencies of individual farms were about 12%, with little variation being displayed by individual farms. However, the authors found that the losses in profits due to technical inefficiencies ranged from 20% to 25%.

Kumbhakar, Biswas and Bailey (1989) used a system approach to estimate technical, allocative and scale inefficiencies for Utah dairy farmers. The stochastic frontier production function which was specified included both endogenous and exogenous variables. The endogenous variables included were labour (including family and hired labour) and capital (the opportunity cost of capital expenses on the farm), whereas the exogenous variables included level of formal education, off-farm income and measures of farm size for the farmers involved. Both types of explanatory variables were found to have significant effects on the variation of farm production. Technical efficiency of farms was found to be positively related to farm size.

Bravo-Ureta and Rieger (1990) estimated both deterministic and stochastic frontier production functions for a large sample of dairy farms in the northeastern states of the U.S.A. for the years 1982 and 1983. The Cobb–Douglas functional form was assumed to be appropriate. The parameters of the deterministic frontiers were estimated by linear programming, corrected ordinary least-squares regression and maximum-likelihood methods (assuming that the non-negative farm effects had gamma distribution). The stochastic frontier model was estimated by maximum-likelihood techniques (given that the farm effects had half-normal distribution). The stochastic frontier model had significant farm effects for 1982 but it was apparently not significantly different from the deterministic frontier in 1983. The estimated technical efficiencies of farms obtained from the three different methods used for the deterministic model showed considerable variability but were generally less than those obtained by use of the stochastic frontier model. However, Bravo-Ureta and Rieger (1990) found that the technical efficiencies obtained by the different methods were highly correlated and gave similar ordinal rankings of the farms.

3.3 Panel data models

Battese and Coelli (1988) applied their panel-data model in the analysis of data for dairy farms in New South Wales and Victoria for the three years – 1978–79, 1979–80 and 1980–81. Given the specifications of the stochastic frontier Cobb–Douglas production function involved, the hypothesis that the non-negative farm effects had half-normal distribution was rejected for both states. Individual farm technical efficiencies ranged from 0.55 to 0.93 for New South Wales farms, whereas the range was 0.30 to 0.93 for Victorian farms.

Battese, Coelli and Colby (1989) estimated a stochastic frontier production function for farms in an Indian village for which data were available for up to ten years. Although the stochastic frontier was significantly different from the corresponding deterministic frontier, the hypothesis that the non-negative farm effects had half-normal distribution was not rejected. Technical efficiencies ranged from 0.66 to 0.91, with the mean efficiency estimated by 0.84.

Kalirajan and Shand (1989) estimated the time-invariant panel-data model using data for Indian rice farmers over five consecutive harvest periods. The farm effects were found to be a highly significant component of the variability of rice output, given the specifications of a translog stochastic frontier production function. Individual technical efficiencies were estimated to range from 0.64 to 0.91, with average 0.70. A regression of the estimated technical efficiencies on farm-specific variables indicated that farming experience, level of education, access to credit and extension contacts had significant influences on the variation of the farm efficiencies.

Battese and Coelli (1992) applied their panel-data model incorporating time-varying firm effects in the analysis of data for paddy farmers in an Indian village who were observed for up to ten years. Given the specifications of a stochastic frontier production function with time-invariant parameters, the hypothesis of time-invariant technical efficiencies of the paddy farmers was rejected. However, given that a linear time trend was included in the stochastic frontier model (Hicksian neutral technical change), then the hypothesis of time-invariant technical efficiencies was accepted. In addition, the stochastic frontier production function with the time trend included was not significantly different from the average response function (i.e., technical inefficiencies could be considered absent from the model. [15] The empirical results were obtained using the computer

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[15] However, in other empirical applications of the time-varying model which have been conducted, the inclusion of a time-trend variable in the stochastic frontier production function has not necessarily resulted in time-invariant technical inefficienies or the conclusion that technical inefficiency does not exist (see Battese and Tessema, 1992).

program, FRONTIER, which has been updated to estimate the time-vary-
ing model presented by Battese and Coelli (1991). [16]

4. CONCLUSIONS

Frontier production functions have been applied in the analysis of
farm-level data in a large number of developed and developing countries.
The survey of empirical applications in agricultural economics given in this
paper is obviously incomplete. Only publications in the major agricultural
economics journals and a few other economic journals are cited. Ali and
Byerlee (1991) cite several other publications involving technical efficiency
of farmers in developing countries. Ley (1990) presents an extensive bibli-
ography of papers dealing with production and efficiency analysis (659
references) and Beck (1991) gives a lengthy bibliography of empirical
applications of frontier functions (266 references). The publications listed
in these bibliographies are in refereed journals, working papers, research
reports and dissertations involving a wide range of areas of application,
including agricultural studies.

Frontier production functions have permitted sophisticated analyses of
technical efficiency. Many empirical studies may be described as illustrative
of the econometric models and estimators which have been suggested for
application. These studies may be open to criticism for not including some
relevant inputs or firm-specific variables or for not involving more appro-
priate functional forms. Superior empirical analyses must obviously involve
a careful specification of all these aspects. The stochastic behaviour of the
random variables involved in frontier production functions is obviously only
part of the modelling exercise.

The use of frontier production functions for the prediction of the
technical efficiencies of farmers involves several problems which require
further research. Although frontier production functions can be estimated
by different methods (other than maximum likelihood), it appears that
distributional assumptions are basic to obtaining predictions for technical
efficiencies. The robustness of predictors when different distributional
assumptions are made is an issue for further study. The precision of
predictors for individual technical efficiencies is also an area requiring
careful research.

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[16] FRONTIER was written in Fortran 77 by Tim Coelli. The source code and executable
program are available on a 5.25 inch disk from Tim Coelli, Department of Econometrics,
University of New England, Armidale, N.S.W. 2351 Australia.

Philosophical issues concerning the existence of technical inefficiency for individual farm operators and the measurement of managerial ability are likely to involve considerable debate among economists in the future. However, it is clear that frontier production functions are quite significant for the investigation of causes of inefficiency in production and indicating means by which inefficiency may be reduced and production increased, given the level of technology involved. Very high levels of technical efficiencies indicate that increasing production would require new innovations or a higher level of technology to be introduced.

The rather sophisticated nature of the econometric models for frontier production functions and the predictors for technical efficiencies is likely to discourage many researchers from applying them in empirical studies. However, the computer program, FRONTIER, can be readily used for estimating both time-invariant and time-varying models of technical efficiency with panel data or applied in the analysis of cross-sectional data (see Coelli, 1992). More general frontier models are being investigated [17] and will be incorporated into the program, FRONTIER, as they are developed. The development of less-restrictive frontier models and the corresponding computer software will offer significant stimulus to better empirical analysis of the efficiency of production.

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[17] A stochastic frontier production function with flexible risk properties has recently been suggested by Wan and Battese (1992). This model incorporates the risk properties which Just and Prope (1978) indicated as desirable for many applications in agricultural economics.

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