The Changing Distribution of Farms by Size: A Markov Analysis

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Abstract

Farm numbers and average farm size in the United States have held about constant since the 1974 Census, but the proportion of mid-sized farms has decreased This pattern follows four decades of a strong trend toward fewer and larger farms Markov analysis is a standard procedure for projecting changes in the number and distribution of firms in an industry based on observations of recent changes Previous applications to the U S farm sector have met with difficulty because of a lack of appropriate data This article applies Markov analysis to a recently available longitudinal data set for 1974-78 from the Census of Agriculture The model predicts reasonably well the actual changes during 1978-82 and indicates that the future distribution of farms by acres per farm will be more like the present than the present is like the past

Keywords

Markov, agriculture, distribution, projection, size of farm, structure

The number of farms in the United States reached a peak in the thirties and then declined The 1935 Census of Agriculture reported 6 81 million farms, by 1974 the number had dropped to 2 31 million, an average annual decrease of 2 73 percent If the 1935-74 trend is projected to 2000, the number of farms decreases substantially to about 1 13 million Total land in farms changed little, so the average farm size increased rapidly during 1935-74, and the distribution of farms by acres per farm shifted steadily toward the larger size classes The number of farms between 50 and 259 acres declined from 1935 on, the number of farms between 260 and 499 acres continued to increase until the midfifties and then began to decline, and the number of farms between 500 and 999 acres peaked in the 1969 Census of Agriculture The trend during 1935-74 characterized an agricultural industry whose firms were steadily becoming fewer in number and larger in size

During the seventies this pattern changed The last three Censuses of Agriculture, 1974, 1978, and 1982, show little change in farm numbers with no appreciable change in average farm size between 1974 and 1982 In 1982, there were 2 24 million farms, an average annual rate of decrease of only 0 4 percent since 1974 If the 1974-82 trend is projected to 2000, the number of farms moderately decreases to about 2 08 million

The distribution of farms by size continued to evolve, however The number of farms of 1,000-1,999 acres peaked in 1978 and then declined in 1982, but the number of farms of 2,000 acres or more continued to increase Farms of fewer than 50 acres began increasing in number in 1974, reversing the longstanding decline [1] The experience of the seven-

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[1] All U S summary data for 1978 used in this analysis were adjusted by the Census of Agriculture from totals published in the 1978 Census of Agriculture to account for the effects of the direct enumeration of sample areas conducted in 1978 The adjustments make the data from the 1978 Census more nearly comparable to those from prior and subsequent Censuses Without the adjustments, the number of farms in 1978 was slightly larger, and farms with fewer than 50 acres declined between 1978 and 1982 All 1978 summary data used in this analysis were drawn from the adjusted totals published in the 1982 Census volumes

ties thus suggested a somewhat different future for U S agriculture a relatively stable number of farms moving toward a bimodal structure with a large and increasing proportion of small farms, a small but increasing proportion of large farms, and a decreasing proportion of midsized farms

This article analyzes changes in size among individual U S farms during 1974-78 to explore the process of structural change in U S agriculture How strong a trend toward bimodality is reflected by recent data? What sort of future structure do the changes imply?

Markov Analysis of Structural Change

A variety of methods may be used to project the structure of an industry on the basis of historical data. Among these are simple trend extrapolation (linear or nonlinear), age cohort analysis, dynamic systems simulation, and Markov analysis Each of these procedures offers advantages and disadvantages depending on the context of inquiry, the nature of the system under study, and the data available (*10*)[2] Markov analysis is well suited to examining shifts among classes of farm size However, the data requirement is stringent, and most Markov analyses of U S agriculture have employed imputed data This study applies Markov analysis to unique, recently available data from the Census of Agriculture covering 1974-78

A finite Markov chain is one in which a population at time t has the distribution S^t over the discrete states, S_1, S_2, S_n, and in which the probability P_{ij} of moving from state S_i at one point in time to state S_j at a later time is dependent only on the initial state S_i and not on any prior state The transition probabilities P_{ij} form the transition probability matrix P, where $\sum_j P_{ij} = 1$ and $P_{ij} \geq 0$ for all i and j Together with an initial distribution of states S^t, these properties completely define a finite Markov chain (*8*)[3]

One can obtain the distribution of states after one time interval S^{t+1} by multiplying the initial distribution vector S^t by the transition probability matrix P Let S^t be a row vector, then $S^{t+1} = S^tP$ One can obtain the distribution of states after k intervals S^{t+k} by multiplying the initial distribution vector S^t by the matrix P^k, that is, P raised to the k^{th} power, $S^{t+k} = S^tP^k$ The appendix shows how to evaluate S^{t+k} when k is any rational fraction The system converges toward an equilibrium distribution as k approaches infinity In a Markov process the equilibrium distribution depends only on the transition probability matrix and is independent of the initial distribution

In economic analysis, use of the Markov chain carries several important assumptions First, a continuous variable such as farm size may reasonably be classified into discrete states, and the choice of states does not appreciably affect the results Second, the specified transition probabilities remain constant over time Third, a process that is continuous may be modeled as occurring at discrete points in time, and the choice of time intervals does not appreciably affect the results Markovian projections represent the implications of behavior observed during a given period persisting into the future This representation implies that the exogenous conditions affecting the observed behavior—for example, shocks from a food or energy crisis, or relative rates of unemployment and wages affecting entry and exit—would also persist

Markov analysis has been used frequently in agricultural economics research Farris and Padberg projected the structure of the Florida citrus packing industry, based on actual longitudinal data (*6*)

Several researchers have employed Markov analysis to investigate the implications of structural change in the U S farm sector. Krenz projected the distribution of farms in North Dakota by size in acres to the year 2000 (*9*) Daly, Dempsey, and Cobb projected U S farms by sales class to the year 2000 (*3*). Lin, Coffman, and Penn projected U S farms by both acreage and sales class, also to the year 2000 (*10*) Those three farm structure studies were not based on longitudinal data Transition probabilities were imputed from published Census data The assumptions made in imputing transition probabilities have a substantial impact on the behavior of the resulting Markov system In each study the transition matrices were assumed to be upper triangular, that is, farms were assumed to grow or

[2]Italicized numbers in parentheses refer to the items cited in the References at the end of this article

[3]Some Markov analyses use the transpose of matrix P in which case the columns, not the rows, sum to unity

to exit the industry, but never to contract in size (8, 9, 10). Although these assumptions appeared reasonable on the basis of aggregate trends and the limited available data on individual farm behavior, they led to modeling structural change as a Markov process in which the largest size class and the exit from agriculture are absorbing states This necessarily implies a longrun equilibrium distribution with all surviving farms in the largest size class This implication is consistent with the popular characterization of the 1935-74 trend that U S agriculture will eventually become one (or a few) very large farm(s)

The Data

The data set used in this analysis consists of longitudinal records from the 1974 and 1978 Censuses of Agriculture (12) The Agriculture Division, Bureau of the Census, created the data set from the control file of the 1978 Census of Agriculture The control file, a normal part of recent censuses, aids in data collecting and processing It contains only a limited number of economic variables thought to be helpful in identifying farms and avoiding duplication Individual farm records were matched by the use of Census File Number (CFN) codes attached to each address label on the Census questionnaire CFN codes for 1978 were based largely on responses to the 1974 Census, farm records were included in the longitudinal set when a match was found between the two censuses All primary data processing was performed on Census Bureau computers under the supervision of Census Bureau employees so that the confidentiality of individual data was maintained

The longitudinal data may include some farms that underwent significant ownership, organizational, or management changes between 1974 and 1978 Changes could be missed if a new operator returned the 1978 questionnaire, addressed to the previous operator, with the mailing label uncorrected Similarly, the data set may exclude farms that continued in operation from 1974 to 1978, but for which a different CFN was assigned For example, a sole proprietorship becoming a partnership, a partnership incorporating, a different partner responding to the second census, a mailing address changing, duplicate questionnaires being received in 1978, or the mailing label provided not being used, all could have been cause for assigning a different CFN in 1978 than in 1974 Thus, the 1974-78 longitudinal data used in this analysis are neither a complete enumeration of all U S farms continuing in operation during the period nor a random sample of them

Nevertheless, a large number of farms were matched between the two censuses The total number of farms reported in 1974 was 2,314,013 (table 1) The 1,200,252 farms which were matched to the 1978 Census represent 52 percent of all farms enumerated in 1974. This leaves 1,113,761 of the 1974 farms for which the 1978 status is not known These farms are listed as nonlongitudinal in table 1 If exit rates in U S agriculture during this period were comparable to those in Canada, where 36 percent of all 1971 operators had exited by 1976 and 30 percent of 1976 operators had exited by 1981 (5), approximately one-quarter of all U S farm operators counted in the 1974 Census probably left agriculture by 1978 This conjecture suggests that the longitudinal data set may capture approximately two-thirds of all "true" longitudinal farms That is, about half the farms in the 1974 nonlongitudinal row of table 1 may actually have left agriculture. This sample represents the first comprehensive longitudinal data base ever available for U S farms

The proportion of 1974 farms represented in the 1974-78 longitudinal set varies by farm size, with medium- and large-sized farms more highly represented than smaller farms Among the 507,797 farms of fewer than 50 acres in 1974, 41 percent were included in the 1974-78 longitudinal set, while 60 percent of farms of 260 acres or more were included Similarly, the longitudinal set includes 43 percent of all farms with 1974 sales of less than $2,500 and 65 percent of farms with 1974 sales of $100,000 or more. Varying rates of inclusion by farm size may approximately reflect the farm sector In Canada, small farms have much higher entry and exit rates than do large farms (5)

Four economic variables were collected in the longitudinal set farm size by acres per farm, by value of sales, by tenure, and by standard industrial classification The measure of farm size in acres avoids problems posed by inflation when constructing intertemporal farm size classes based on sales, and it allows U S farm structure to be thought of

Table 1—Longitudinal and nonlongitudinal farms, 1974 and 1978, and allocation of nonlongitudinal farms between continuing and entering/exiting states

Item	Unit	Acres per farm								Total farms
		1-49 acres	50-99 acres	100-179 acres	180-259 acres	260-499 acres	500 999 acres	1,000-1,999 acres	2,000 plus acres	
1974										
All farms	Number	507,797	384,762	443,122	253,232	362,866	207,297	92,712	62,225	2,314,013
Longitudinal	do	209,987	180,175	230,473	143,539	217,189	126,881	55,718	36,290	1,200,252
Nonlongitudinal	do	297,810	204,587	212,649	109,693	145,677	80,416	36,994	25,935	1,113,761
MAX OUT	do	297,810	204,587	212,649	109,693	145,677	80,416	36,994	25,935	1,113,761
Nonlong/long	Ratio	1 4182	1 1355	0 9227	0 7642	0 6707	0 6338	0 6640	0 7147	0 9279
Continuing farms	Number	131,074	112,466	143,862	89,597	135,570	79,199	34,779	22,652	749,200
MIN OUT	do	166,736	92,121	68,787	20,096	10,107	1,217	2,215	3,283	364,561
1978										
All farms	do	542,787	355,755	403,292	233,854	347,777	213,209	97,800	63,301	2,257 775
Longitudinal	do	212,452	181,951	225,922	138,202	212,536	131,270	59,326	38,593	1,200,252
Nonlongitudinal	do	330,335	173,804	177,370	95,652	135,241	81,939	38,474	24,708	1,057,523
MAX IN	do	330,335	173,804	177,370	95,652	135,241	81,939	38,474	24,708	1,057,523
Nonlong/long	Ratio	1 5549	0 9552	0 7851	0 6921	0 6363	0 6242	0 6485	0 6402	0 8811
Continuing farms	Number	132,613	113,574	141,021	86,266	132,665	81,939	37 031	24,090	749,200
MIN IN	do	197,722	60,230	36,349	9,386	2,576	0	1,443	618	308,323
Net change, MAX	do	32,525	- 30,783	- 35,279	- 14,041	- 10,436	1,523	1,480	- 1,227	- 56,238
Net change, MIN	do	30,986	- 31,891	- 32,438	- 10,701	- 7,531	- 1,217	- 772	- 2,665	- 56,238

as a constantly changing number and mix of farms on a nearly fixed land base Total U S land in farms decreased by only 0 2 percent from 1974 to 1978 and by only 8 percent from 1940 to 1982

Results

Markov analysis was applied first to the 1 2 million longitudinal farms However, the longitudinal set alone fails to account for changes in the total number of farms during the period and does not reflect the size distributions in 1974 and 1978 of farms not included in the set, shown as nonlongitudinal farms in table 1 A subsequent reformulation of the problem accounts for the presence of continuing farms excluded from the longitudinal sample and for entry and exit

Longitudinal Farms Only

The Markov transition matrix for the longitudinal set appears in table 2 The table shows for each size class in 1974 how many farms moved into the various size classes by 1978 These are the only unpublished data used in this article Several points stand out in the transition matrix The matrix has near symmetry around the main diagonal Numbers on the diagonal are relatively large, 68 percent of the longitudinal farms were in the same class at the end of the period as at the beginning Numbers of farms off the diagonal approximately balance, symmetrically, cell by cell, thus indicating that growth in some farms is about offset by decline in others The upper right and lower left triangles are not empty, indicating that small farms can become very large from one census to the next and also that large farms can become very small For example, 432 farms went from under 50 acres to over 2,000, while 395 others went from over 2,000 acres to under 50 The central tendencies of the system are thus quite stable

The transition probability matrix in table 3 differs significantly from the transition matrices imputed in the studies reviewed earlier, which were assumed to be upper triangular The flows indicated in the table are greater than one would expect from the low rates of farm real estate sales, implying that most of the large fluctuations, both up and down, were accomplished via land rental rather

Table 2—Transition matrix, farm size in acres per farm, 1974-78

1974 acres per farm	1978 acres per farm								Total farms 1978
	1-49 acres	50-99 acres	100 179 acres	180-259 acres	260 499 acres	500-999 acres	1,000-1,999 acres	2,000 plus acres	
	Number of farms								
1-49	163,914	22,985	12,040	4,385	4,066	1,592	573	432	209,987
50-99	24,385	122,100	21,819	5,922	4,237	1,324	277	111	180,175
100-179	12,664	25,134	154,083	20,960	13,477	3,237	683	235	230,473
180-259	4,494	6,185	21,563	82,386	24,092	3,997	639	183	143,539
260-499	4,322	4,126	13,097	20,850	144,220	27,080	2,860	634	217,189
500-999	1,705	1,040	2,527	3,028	20,004	83,550	13,456	1,571	126,881
1,000-1,999	573	267	556	478	1,933	9,277	36,724	5,910	55,718
2,000 plus	395	114	237	193	507	1,213	4,114	29,517	36,290
Total, 1974	212,452	181,951	225,922	138,202	212,536	131,270	59,326	38,593	1,200,252

Source Special longitudinal tabulation, 1974 and 1978 Censuses of Agriculture, Bureau of the Census, U S Department of Commerce

than purchase This conclusion is consistent with the observations that 41 percent of U S farmland was operated by someone other than the owner in 1982 and that there are more farmland owners than operators

If the 1 2 million farms in the longitudinal sample were to have moved again during 1978-82 as they did during 1974-78 and then were to move again and again in subsequent 4-year intervals according to the probabilities in table 3, a steady state would eventually be reached in which additional moves will each bring the system back to the same distribution it had before the additional move Table 4 compares the steady-state, longrun equilibrium distribution of farms implied by the 1974-78 transition probability matrix with the actual distributions of farms in the longitudinal set in 1974 and 1978

These distributions for the longitudinal sample suggest that the tendency among the longitudinal farms was toward a moderately lower proportion of farms under 500 acres and a higher proportion of farms with more than 500 acres And, the longitudinal data reflect a slight trend toward a shrinking of the middle-sized classes of farms However, these tendencies observed for 1974-78 are not dramatic, they imply a longrun equilibrium distribution not very different from the original 1974 distribution

Allowance for Entry, Exit, and Continuing Farms Excluded from the Sample

"Neither economic theory nor applied economic studies in agriculture adequately consider the subject of exit and entry of firms," according to Conneman and Harrington (2, p 40) They emphasize the importance of reliable data on exit and entry for Markov analysis The longitudinal data set includes only about half the farms in U S agriculture in 1974-78 The other half is composed of (1) farms present in 1974, but not present in 1978 (exiting farms), (2) farms not present in 1974, but present in 1978 (entering farms), and (3) farms present in both years, but not picked up in the longitudinal sample (continuing/excluded farms) Dealing with entry and exit raises two issues how to allocate the nonlongitudinal farms between the continuing/excluded and entry/exit states, and how to model the population of potential and former farmers

In earlier studies, Farris and Padberg (*6*) had complete information on entry and exit, so there were no imputational problems The following studies imputed transition matrices from aggregate data, and they assumed that there were no entrants so there was no need to model the population of potential farmers Krenz (*9*), Daly, Dempsey, and Cobb (*3*), and Lin, Coffman, and Penn (*10*) assumed farms either remained in the initial state, moved up one or two size classes, or exited Dean, Johnson, and

Table 3—Transition probability matrix, farm size in acres per farm, 1974-78

1974 acres per farm	1978 acres per farm								Total farms 1978
	1-49 acres	50-99 acres	100-179 acres	180-259 acres	260 499 acres	500-999 acres	1,000-1,999 acres	2,000 plus acres	
	Probability								
1-49	0 7806	0 1095	0 0573	0 0209	0 0194	0 0076	0 0027	0 0021	1 0000
50-99	1353	6777	1211	0329	0235	0073	0015	0006	1 0000
100-179	0549	1091	6686	0909	0585	0140	0030	0010	1 0000
180-259	0313	0431	1502	5740	1678	0278	0045	0013	1 0000
260 499	0199	0190	0603	0960	6640	1247	0132	0029	1 0000
500-999	0134	0082	0199	0239	1577	6585	1061	0124	1 0000
1,000-1,999	0103	0048	0100	0086	0347	1665	6591	1061	1 0000
2,000 plus	0109	0031	0065	0053	0140	0334	1134	8134	1 0000

Table 4—Relative distributions of longitudinal farms, by size of farm, 1974 1978, and projected equilibrium

Year	Acres per farm								Total farms
	1 49 acres	50 99 acres	100 179 acres	180 259 acres	260 499 acres	500 999 acres	1,000-1,999 acres	2,000 plus acres	
	Percent								
1974	17 5	15 0	19 2	12 0	18 1	10 6	4 6	3 0	100 0
1978	17 7	15 2	18 8	11 5	17 7	10 9	4 9	3 2	100 0
Equilibrium	17 5	14 5	17 1	10 3	16 7	12 1	6 6	5 2	100 0

Carter (4) used similiar assumptions, but showed, in addition, some moves to the next smaller size class

The entry, exit, and nonfarm population constraints can be treated by the addition of a row and a column to the matrices in tables 2 and 3 One can compute the gross flows of nonlongitudinal farms by farm size from published Census data by subtracting longitudinal farms from all farms in each size class We used two sets of assumptions about the nonlongitudinal farms First, the longitudinal farms are a complete count of all continuing farms so that the nonlongitudinal farms represent solely entry and exit (tables 5 and 6) This assumption overestimates turnover, it indicates the maximum that could have entered or exited each farm class during 1974-78 Second, the number of continuing/excluded farms is maximized (and the number of entries and exits minimized) for each farm size subject to the restriction that the distribution of continuing/excluded farms among the farm size classes is identical to the distribution of longitudinal farms (tables 7 and 8) The calculations pertaining to the second assumption are explained below In this case, entry and exit by size class are at a minimum subject to the proportionality assumption This assumption probably underestimates actual turnover

Table 1 presents the maximum and minimum flows of entry and exit computed under the above assumptions In the first case, when the number of continuing/excluded farms is assumed to be zero, entries and exits are labeled MAX OUT and MAX IN and are equal to the number of nonlongitudinal farms in 1974 and 1978, respectively In the second case, imputed entries and exits are labeled MIN OUT and MIN IN, and the implied continuing/excluded farms for each year are also identified We derived these values as follows The smallest ratio of nonlongitudinal to longitudinal farms (labeled Nonlong/long in the table) was for the 500-999 acre class in 1978, the ratio was 0 6242 For each class, the estimated number of continuing/excluded farms in 1978 is 62 42 percent of the number of longitudinal farms, and the MIN IN row is the residual This calcula tion yields zero entrants for the 500-999-acre class and positive levels of entry for each of the other classes To distribute the 749,200 continuing farms among size classes in 1974, we made parallel computations For each class, the estimated number of continuing/excluded farms is 62 42 percent of the 1974 distribution of longitudinal farms, and the MIN OUT row is the residual. This calculation yields positive levels of exit for each size class The last two rows of table 1 show the net changes in each size class under the two sets of assumptions These net changes are similar despite differences in the gross flows from which they were derived Both show most of the net entries under 50 acres For farms of 500-1,999 acres, the maximum case shows net entries, whereas the minimum case shows net exits

Table 5 shows an expanded transition probability matrix reflecting the assumption of maximum flows in and out of agriculture The matrix in table 5 is derived from a transition matrix which has the MAX IN row from table 1 as a new top row and the MAX OUT row as a new first column Similarly, table 7 shows an expanded transition probability matrix reflecting the minimum flow assumptions The matrix in table 7 is derived from a transition matrix which has the MIN IN and MIN OUT rows of table 1 as an extra row and column However, in this case we increased the 8-by-8 portion of the new transition matrix by 749,200 farms, to reflect the farms assumed to be continuing/excluded, by raising each entry in table 2 by the ratio of all continuing farms to longitudinal sample farms—that is, by multiplying by 1 6242 This procedure treated about two-thirds of the nonlongitudinal farms as continuing and the other one third as entry and exit This set of calculations associated with our second assumption implies more stability than suggested by the Canadian experience cited above, whereas the MAX IN and MAX OUT assumption clearly implies too much turnover

One further problem remains in accounting for farms outside the longitudinal data set The logic of Markov analysis requires information about the total size of the nonfarm population of potential farm oprators from which entrants come and to which exiters go This problem did not arise in earlier studies using imputed transition probabilities and with entries to agriculture assumed to be zero, in such cases the number of potential entrants is irrelevant Farris and Padberg (*6*) arbitrarily assumed a population of potential entrants over three times larger than the actual number of firms in the industry From the logical viewpoint, the population of potential entrants can be any finite, nonnegative number, for example, it can be zero Or, for a study of this type, one might suppose it equal to the number of nonfarm households in the United States or to the number of households in rural areas of the United States This arbitrary choice has no effect on the longrun equilibrium

Table 5—Transition probability matrix assuming maximum flows of entry and exit, 1974-78

1974 acres per farm	Nonfarm population	1978 acres per farm							
		1 49 acres	50-99 acres	100-179 acres	180-259 acres	260-499 acres	500-999 acres	1,000-1,999 acres	2,000 plus acres
	Probability								
Nonfarm	0 7885	0 0661	0 0348	0 0355	0 0191	0 0270	0 0164	0 0077	0 0049
1-49	5865	3228	0453	0237	0086	0080	0031	0011	0009
50-99	5317	0634	3173	0567	0154	0110	0034	0007	0003
100-179	4799	0286	0567	3477	0473	0304	0073	0015	0005
180-259	4332	0177	0244	0852	3253	0951	0158	0025	0007
260-499	4015	0119	0114	0361	0575	3974	0746	0079	0017
500-999	3879	0082	0050	0122	0146	0965	4030	0649	0076
1,000-1,999	3990	0062	0029	0060	0052	0208	1001	3961	0637
2,000 plus	4168	0063	0018	0038	0031	0081	0195	0661	4744

percentage distribution (1, pp 899, 901) However, the shortrun time path of distributions is sensitive to the choice, as is the total number of farms in equilibrium Stanton and Kettunen show algebraically that the equilibrium number of farms is a function of the number of potential entrants, a larger nonfarm population results in a larger equilibrium farm population (11) However, as Stanton and Kettunen explain, as the number of potential entrants is increased, the net effect on the resulting projections decreases at a decreasing rate They add that a larger choice may suit a competitive market situation, but that a smaller choice may better represent oligopoly By experiment, we found that the shortrun time path was particularly sensitive to smaller numbers, such as zero, or 1 million, but that choices above 5 million made little difference after the first few transitions Consequently, we chose to complete the modification of table 2 by assuming an initial nonfarm population of 5 million potential operators in 1974 Appending the new first row and column reflecting the gross flow assumptions to table 2 and assuming a 1974 nonfarm population of 5 million produce the transition probability matrix in table 5

The number of farms that entered agriculture during 1974 78 failed to offset the number that left, so the augmented probability matrix suggests a moderately decreasing number of farms However, the projected decrease is slow, tending toward a longrun equilibrium only slightly below the initial level The number of farms entering at the smaller and larger sizes exceeded the number leaving, whereas the number leaving at the middle sizes exceeded the number entering This situation indicates a stronger tendency toward bimodality than appeared in projections using farms from the longitudinal sample alone, with more farms under 50 acres and over 500 acres and with fewer in between The tendency is not great, however, and the overall stability implied by the longitudinal data alone continues to hold (table 6)

The above analysis assumes that all continuing farms were captured in the longitudinal sample In the alternative formulation, the maximum number of continuing farms, consistent with the distribution of the longitudinal set, was assumed to have been excluded from the longitudinal set Minimum entrants are appended as a new first row to table 2, and minimum exiters are appended as a new first column The continuing/excluded farms were incorporated into the remaining eight rows and columns of the transition matrix in the same proportion as the farms in the longitudinal sample The result is the new transition probability matrix shown in table 7

The first row of table 7 shows no farms entering the 500-999-acre class and very few in any size class over 260 acres The first column of the table shows that the proportion of farms exiting is highest at sizes below 180 acres and that it rises slightly for farms above 1,000 acres Table 8 shows the past and projected distributions

Compared with the earlier analysis, this one suggests a longrun equilibrium with somewhat fewer farms, about 15 percent below the present number

Table 6—Projected number of farms, by size for 1982, 1990, 2000, and equilibrium when maximum flows of entry and exit are assumed

Year	Nonfarm population	Acres per farm								Total farms
		1-49 acres	50-99 acres	100 179 acres	180-259 acres	260-499 acres	500-999 acres	1,000-1,999 acres	2,000 plus acres	
	People	*Number of farms*								
1974	5,000,000	507,797	384,762	443,122	253,232	362,866	207,297	92,712	62,225	2,314,013
1978	5,056,238	542,787	355,755	403,292	233,854	347,777	213,209	97,800	63,301	2,257,775
1982	5,076,885	554,384	347,231	388,534	225,846	340,892	215,331	100,492	64,418	2,237,128
1990	5,087,286	559,732	343,893	380,797	221,212	336,368	216,328	102,494	65,721	2,226,726
2000	5,088,762	560,437	343,537	379,811	220,368	335,383	216,443	102,999	66,226	2,225,203
Equilibrium	5,088,921	560,496	343,504	379,681	220,260	335,236	216,454	103,097	66,366	2,225,094

Table 7—Transition probability matrix assuming minimum flows of entry and exit, 1974-78

1974 acres per farm	Nonfarm population	1978 acres per farm							
		1-49 acres	50 99 acres	100-179 acres	180-259 acres	260 499 acres	500-999 acres	1,000 1,999 acres	2,000 plus acres
	Probability								
Nonfarm	0 9383	0 0396	0 0121	0 0073	0 0019	0 0005	0	0 0003	0 0001
1 49	3284	5243	0735	0385	0140	0130	0051	0018	0014
50 99	2395	1029	5154	0921	0250	0179	0056	0012	0005
100-179	1554	0464	0921	5647	0768	0494	0119	0025	0009
180-259	0794	0288	0397	1383	5284	1545	0256	0041	0012
260-499	0279	0193	0185	0586	0933	6455	1212	0128	0028
500-999	0060	0134	0081	0198	0237	1567	6545	1054	0123
1,000-1,999	0242	0100	0047	0097	0084	0339	1625	6432	1035
2,000 plus	0533	0103	0030	0062	0050	0132	0316	1073	7700

In the intermediate run, from 1978 to 2000, the reduction in farm numbers is projected at an average annual rate of 0 4 percent, which, by coincidence, is the same average annual rate observed from Census data during 1974-82 The implied equilibrium distribution shows a greater tendency toward bimodality than the other projections, with a much larger proportion of farms under 100 acres and a slight increase in farms over 1,000 acres The projected rate of concentration by farm size is not great even in the long run, however, and implies relatively little change from the initial distribution [4]

Projections to 1982 Based on 1974-78 Transition Probabilities

Both the augmented Markov transition matrices produced reasonable estimates of 1982 from the 1974 distribution by using 1974-78 probabilities Table 9 compares the projections with the actual 1982 distribution The minimum flow matrix came a bit closer to the actual 1982 distribution than the maximum flow matrix did In both projections to 1982, the number of farms under 50 acres was underestimated and the number of farms in each of the other size classes was slightly overestimated [5] More farms were estimated toward the center of the distribution than were actually there, indicating that the trend toward bimodality was somewhat more pronounced in 1978-82 than in 1974-78

Table 10 shows the relative distributions associated with the data in table 9 It also shows a distribution quotient measuring how close one relative distribution is to another The distribution quotient is the sum of the positive first differences between the the elements of a pair of relative distributions (7, pp 252-53) The quotients are calculated with actual 1982 data as the base distribution, so each quotient compares a distribution with the actual 1982 distribution Distribution quotients computed in this way range from zero to unity A zero value indicates two relative distribu-

[4]Under both sets of assumptions, projected distributions of farms by size in acres to the year 2000 imply a total acreage in farms within recent historical levels, assuming 1982 average farm sizes in each class

[5]One factor almost certainly contributing to the wide fluctuations observed in the number of farms in the smallest size class from census to census is the practice of defining as farms all places meeting the minimum sales threshold ($1,000 in 1974, 1978, and 1982) on the basis of potential as well as actual sales of agricultural products Using a point system derived by the Agricultural Research Service, the Census imputes potential sales values to each place on the basis of features such as cropland not harvested, pasture, and number of animals Because of fluctuating product values, the number of points assigned to each item also varies from census to census As a result, even with no change in the characteristics of a given place, changing point allocations may classify it as a farm in one census and as a nonfarm place in another In addition, the inclusion in the point system of some common animals such as horses, for the first time in 1982 raises further difficulties for year to-year comparisons In 1982, for example, farms with actual sales of less than $1,000 increased by about 95 000 from 1978 the only sales class below $80 000 to show an increase in numbers Although the Census does not publish data on the acreage distribution of farms classified under the point system, a significant portion of the increase in farms of fewer than 50 acres reported in 1982 was probably due to the point system

Table 8—Projected number of farms, by size for 1982, 1990, 2000, and equilibrium when minimum flows of entry and exit are assumed

Year	Nonfarm population	Acres per farm								Total farms
		1-49 acres	50-99 acres	100-179 acres	180-259 acres	260-499 acres	500-999 acres	1,000-1,999 acres	2,000 plus acres	
	People	*Number of farms*								
1974	5,000,000	507,797	384,762	443,122	253,232	362,866	207,297	92,712	62,225	2,314,013
1978	5,056,238	542,787	355,755	403,292	233,854	347,777	213,209	97,800	63,301	2,257,775
1982	5,105,636	557,846	339,406	376,525	219,268	334,161	215,111	101,488	64,571	2,208,377
1990	5,182,459	566,707	322,901	345,105	199,738	311,747	212,818	105,356	67,182	2,131,554
2000	5,246,947	568,238	313,986	326,019	185,987	291,703	205,776	106,030	69,326	2,067,066
Equilibrium	5 412,574	572,957	305,201	300,520	162,444	242,159	168,187	89,078	60,893	1,901,439

Table 9—Actual and projected number of farms, by acres per farm, 1974, 1978, 1982

Year	Acres per farm								Total farms
	1-49 acres	50-99 acres	100-179 acres	180-259 acres	260-499 acres	500-999 acres	1,000-1,999 acres	2,000 plus acres	
	Number of farms								
1974 actual	507,797	384,762	443,122	253,232	362,866	207,297	92,712	62,225	2,314,013
1978 actual	542,787	355,755	403,292	233,854	347,777	213,209	97,800	63,301	2,257,775
1982 actual	636,917	343,775	367,877	211,485	315,025	203,925	97,395	64,577	2,240,976
1982 maximum	554,384	347,231	388,534	225,846	340,892	215,331	100,492	64,418	2,237,128
1982 minimum	557,846	339,406	376,525	219,268	334,161	215,111	101,488	64,571	2,208,377

Table 10—Relative distributions and quotients for actual and projected farms, 1974, 1978, 1982

Year	Acres per farm								Distribution quotient
	1-49 acres	50-99 acres	100-179 acres	180-259 acres	260-499 acres	500-999 acres	1,000-1,999 acres	2,000 plus acres	
	Distribution								*Quotient*
1974 actual	0 2194	0 1663	0 1915	0 1094	0 1568	0 0896	0 0401	0 0269	0 0715
1978 actual	2404	1576	1786	1036	1540	0944	0433	0280	0447
1982 actual	2842	1534	1642	0944	1406	0910	0435	0288	0000
1982 maximum	2478	1552	1737	1010	1524	0963	0449	0288	0365
1982 minimum	2526	1537	1705	0993	1513	0974	0460	0292	0316

tions are identical, unity indicates they are quite different

The distribution quotient for the minimum flow projection to 1982 is smaller than that for the maximum flow projection, indicating that the minimum flow projection more closely approximated the actual The quotient which compares the actual 1974 with the actual 1982 distribution is 0 0715, indicating that the distribution of U S farms by acres per farm did not change much during the 8-year interval Both projections from 1974 to 1982 are closer to the actual 1982 distribution than to the 1974 distribution, indicating that the 1974-78 trend forms a useful basis for characterizing the entire 1974-82 period

Assessing the Applicability of the Transition Matrix to Earlier Periods

The pattern of structural change described by the 1974-78 probabilities explains changes between previous censuses reasonably well Three censuses, 1974, 1969, and 1964, were projected from their previous censuses The distribution quotients compare the actual distribution with the associated projection in each of the 3 years The minimum flow matrix consistently made better predictions than the maximum flow matrix (table 11)

In each case, the actual proportion of farms under 50 acres was below the projected level enough to account for most of the value of the quotient, the projections also consistently overestimated farms above 500 acres and underestimated farms from 50 to 500 acres. These divergences are consistent with the view that the trend toward a reduced proportion of farms in the middle-sized classes has accelerated since the sixties. That acceleration is reflected in the pattern of divergences encountered in the projections to 1982, which underestimated the proportion of farms in the smaller size class and overestimated the proportion of farms in the middle range However, the divergences between the actual and projected proportions are relatively small in all cases and seem to be closely related to fluctuations in the smallest size class which, as noted earlier, appears to be very sensitive to definitional changes

Table 11—Distribution quotients for 1974, 1969, and 1964 projected from the previous Census

Projection	Maximum flow	Minimum flow
	Quotient	
1974 from 1969	0 0266	0 0237
1969 from 1964	0301	0222
1964 from 1959	0281	0088

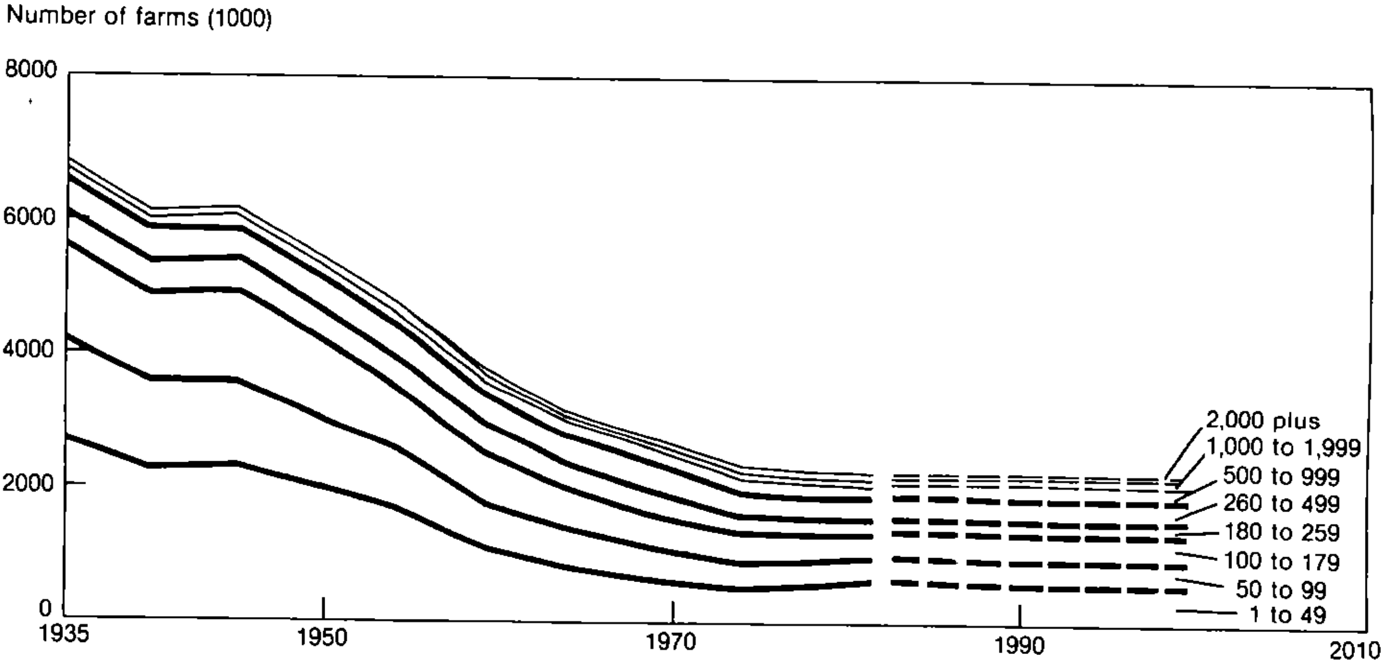
Several things affect the outcome for shortrun projections the structure of the longitudinal farms, the structure of the nonlongitudinal farms, the arbitrary assumption of the size of the pool of potential operators from which entrants come and to which exiters go, the size classes and time interval selected for analysis, and the initial distribution of farms All these factors changed during the 1959-74 interval under consideration, and each doubtless had an effect on the outcome However, the last mentioned—the initial distribution of farms—affects the shortrun path in a predictable way Any initial distribution will be moved toward equilibrium Inasmuch as the 1964 distribution is closer to equilibrium than the 1959 distribution, it is not surprising that the projection from 1959 to 1964 moved the distribution in the correct direction The same phenomenon occurs with projections from the actual 1935 distribution Each projected distribution is closer to the projected longrun equilibrium, which in turn is not far from the actual 1974 distribution That is, the Markov chain estimated for 1974-78 moves the actual 1935 distribution toward the actual 1974 distribution The projections make the adjustment more rapidly than actually occurred, however, in about half the actual number of years

A Longrun Perspective

Figure 1 shows the number of farms by size in acres per farm since 1935 The figure makes clear the rapid descent in farm numbers during 1935-74 and the subsequent leveling It also indicates projections to 2000 using the maximum flow matrix Figure 2 shows the relative distributions associated with the data in figure 1, and the data appear in table 12 Table 12 also reports the distribution quotient which compares each distribution with the actual 1982 distribution As one traces these quotients

Figure 1

Farms by Size, 1935-82, with Projections



Historical data from Bureau of the Census, projections from table 6

backwards through time from 1982, the difference from 1982 increases, indicating that the farther one looks into the past, the greater the difference in farm structure becomes

The proportion of farms under 50 acres increased in 1982 to about that of 1959 Hence, from 1964 to 1978, the proportion of such farms was smaller than in 1982, and from 1935 to 1954 the proportion was larger The absolute share of these farms was large, and the rate of change from one census to the next was rapid, so this difference is the most important single contributor to the size of the distribution quotient from 1935 through 1978, with the single exception of 1959 After 1959, the second major difference is that there were proportionately fewer farms of 50-180 acres than in 1982 Before 1959, the second major difference is that there were proportionately more farms of 260-500 acres than in 1982

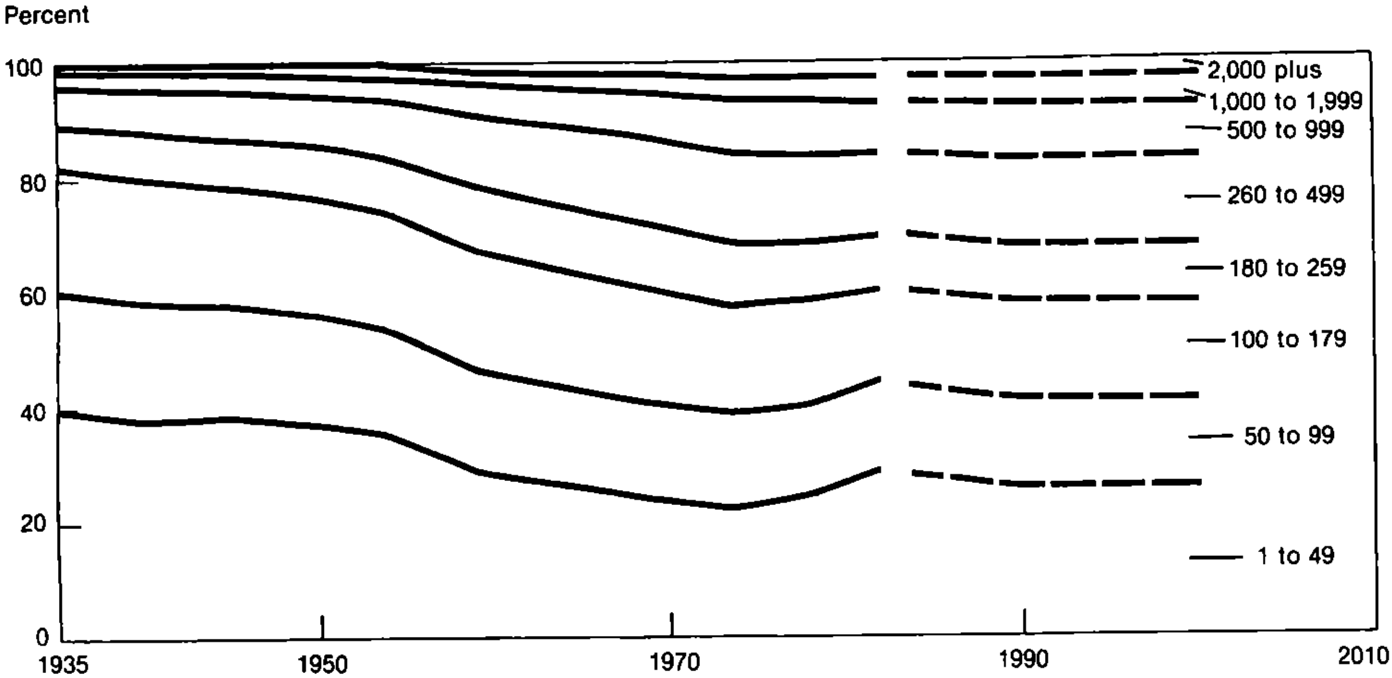
The distribution quotients for projected distributions for 1990, 2000, and longrun equilibrium, under both sets of assumptions, illustrate the extent to which the change in farm size distribution stabilized during 1974-78 (table 12 and see figs 1 and 2) When the 1974-78 trends are projected forward, they suggest that the future number and distribution of farms by acres per farm will be more like the present than the present is like the past

Conclusions

We analyzed longitudinal data on U S farms to evaluate changes in size by acres per farm during 1974-78 The data reveal considerable stability among these farms, both at the individual and aggregate levels For individual farms remaining in operation, the most likely outcome after 4 years was that each would remain in the same size class as before Among those changing size classes, most changed into an adjacent class Only a small fraction of continuing farms exhibited dramatic changes in acreage during 1974-78 However, the number of shifts in size was more than one would expect from

Figure 2

Percentage Distribution of Farms by Size, 1935-82, with Projections



Historical data from Bureau of the Census, projections from table 6

the number of transactions per year in farm real estate, suggesting that leasing is important in explaining the changing structure of farms Changes in farm size displayed a great deal of symmetry. For every farm moving up from a smaller to larger class, another farm was likely to move in the other direction Relative stability in the size distribution is suggested when the 1974-78 pattern of change is assumed to continue indefinitely This symmetry and stability suggest a substantially different view of structural change in agriculture than the 1935-74 trend toward fewer and larger farms would suggest

Tendencies toward a bimodal distribution are evident, but longrun projections suggest they are moderate The 1974-78 data do not support the view that the mid-sized farms will disappear. Based on the 1974-78 data, projections to 1982 also suggest that the comparative stabilization of structural change occurring in 1974-78 continued in 1978-82

One projection examined here uses longitudinal farms only, and two others make alternative assumptions about entry and exit of nonlongitudinal farms We experimented with other assumptions about nonlongitudinal farms, and found that all methods treating nonlongitudinal farms in a uniform and consistent manner led to approximately the same results Even so, none of the assumptions used exactly captures the actual distribution of nonlongitudinal farms, and projections were sensitive to nonuniform assumptions, such as that losses were concentrated among mid-sized farms

The longrun implications of this analysis turn on the stability of the transition probability matrix estimated for 1974-78 If the longrun transition probabilities remain close to those estimated here, then the structure of U S agriculture will change little from what it is today. However, the transition probabilities could change. The significantly changed conditions in U S. agriculture—from the

Table 12—Relative distribution by size in acres per farm 1935 to longrun equilibrium, and quotients with 1982 = base year

Year	Acres per farm								Distribution quotient
	1-49 acres	50 99 acres	100-179 acres	180-259 acres	260 499 acres	500-999 acres	1,000-1,999 acres[1]	2,000 plus acres	
	Distribution								*Quotient*
Actual									
1935	0 3955	0 2120	0 2111	0 0744	0 0695	0 0246	0 0130	0	0 2168
1940	3755	2116	2147	0797	0752	0268	0165	0	2000
1945	3838	1975	2048	0842	0808	0297	0193	0	1844
1950	3652	1945	2047	0905	0887	0338	0225	0	1627
1954	3549	1807	1993	0970	1008	0401	0273	0	1356
1959	2850	1773	2082	1117	1271	0539	0213	0154	0861
1964	2597	1718	2004	1126	1429	0666	0269	0191	0752
1969	2328	1685	1984	1124	1536	0790	0333	0219	0804
1974	2194	1663	1915	1094	1568	0896	0401	0269	0715
1978	2404	1576	1786	1036	1540	0944	0433	0280	0447
1982	2842	1534	1642	0944	1406	0910	0435	0288	0000
Maximum flow									
1990	2514	1544	1711	0993	1511	0972	0460	0295	0328
2000	2519	1544	1707	0990	1507	0973	0463	0298	0324
Equilibrium	2519	1544	1706	0990	1507	0973	0463	0298	0323
Minimum flow									
1990	2559	1515	1619	0937	1463	0998	0494	0315	0232
2000	2749	1519	1577	0900	1411	0996	0513	0335	0217
Equilibrium	3015	1606	1581	0850	1274	0885	0469	0320	0310

[1]Includes all farms of 1 000 acres or more, 1935 54

low real interest rates and rising asset values, exports, and farm income of the seventies to the high real interest rates, declining asset values, and lower exports and farm income of the eighties—suggest that the pattern of change since 1982 may differ from the pattern of 1974-82 Yet, the relative stability exhibited by U S agriculture during 1974-82 makes it less likely that its structure in the near future will be as radically different as had been expected based on 1935-74 trends For example, Lin, Coffman, and Penn projected on the basis of trends through 1974 that between then and the year 2000 the number of 100-499-acre farms would drop by 493,000, a 47-percent decline (*10*, p 11) Projections based on 1974-78 data suggest a drop in this size class of less than one-third that figure over the same period

The general stability observed during 1974-78 points to the critical role in structural change played by entry, exit, and the few continuing farms undergoing rapid change Relatively minor changes among these farms have potentially significant longrun implications We are now developing more detailed data on the characteristics of continuing as well as entering and exiting farms in the 1974-78 and 1978-82 periods from Census data These additional data will allow an exploration of questions such as the stability of the transition probabilities estimated here, the characteristics of changing versus stable farms, the patterns of change among other variables such as sales, tenure, and enterprise mix, and the interrelationships among changes in these variables

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Appendix: Raising a Matrix to Fractional Powers

One problem encountered in projecting farm size distributions was that of raising a probability transition matrix estimated on a 4-year interval to fractional powers in order to produce transition matrices describing periods not in 4-year multiples No generally available microcomputer software of which we are aware offers direct procedures for taking fractional powers of an asymmetric matrix This section briefly describes the method used here to project 5-, 16-, and 26-year intervals from a 4-year matrix, as well as some alternative methods

A method described by Waugh and Abel (*13*) adapts matrices to the binomial expansion and approximates the final value from the first few terms of the expansion. Their algorithm has the advantage of being relatively easy to write in a programming language such as BASIC if more efficient software is not available

A second method for calculating the square root of an asymmetric matrix P of rank n is to think of it as the product of two identical matrices B, where each element of the original matrix may be written

$$P_{ij} = \sum_{k=1}^{n} b_{ik} b_{kj}$$

This method yields a system of n^2 equations in n^2 unknowns Lloyd Teigen suggested to us that one can solve this system using commercially available microcomputer software for solving nonlinear simultaneous systems, such as TK!Solver We found the method to work well for a 3-by-3 test, but coding and iterating for the 9-by-9 problem became tedious.

A third approach allows us to write

$$P = A \Gamma A^{-1}$$

where A is a matrix of eigenvectors of P and Γ is the associated diagonal matrix of eigenvalues of P. The inverse of the matrix of eigenvectors will exist if the transition probability matrix (P) is not defective. The eigenvectors (A) will be linearly independent, and the inverse (A^{-1}) will therefore exist, if there are as many distinct eigenvalues as there are rows in the transition probability matrix (P). In the case of the asymmetric matrices used here, most commercial software packages do not offer direct solution procedures for calculating eigenvalues and eigenvectors. Most of the software for both mainframe and microcomputers calculate eigenvectors only for symmetric matrices. One exception is SPEAKEASY, in both the mainframe and microcomputer versions.

The integer power P^2 can be written

$$\begin{aligned} P^2 &= A \Gamma A^{-1} A \Gamma A^{-1} \\ &= A \Gamma I \Gamma A^{-1} \\ &= A \Gamma^2 A^{-1} \end{aligned}$$

Similarly

$$P^r = A \Gamma^r A^{-1}$$

for any integer r. Therefore, once A and Γ have been derived, P^r can easily be obtained by taking powers of scalars on the diagonal of Γ.

Consider the square root $P^{0.5}$ written as

$$P^{0.5} = A \Gamma^{0.5} A^{-1}$$

To show that $P^{0.5}$ is indeed the square root, multiply the right hand side by itself.

$$\begin{aligned} P &= A \Gamma^{0.5} A^{-1} A \Gamma^{0.5} A^{-1} \\ &= A \Gamma^{0.5} \Gamma^{0.5} A^{-1} \\ &= A \Gamma A^{-1} \end{aligned}$$

The procedure can be extended to the q^{th} root for any integer q.

$$P^{1/q} = A \Gamma^{1/q} A^{-1}$$

Complex roots will not arise so long as the eigenvalues are positive. P can be raised to any rational power $k = r \div q$ for any integer r and q by raising the scalar eigenvalues to the desired fractional power.

$$P^{r/q} = A \Gamma^{r/q} A^{-1}$$

Four-year transition probability matrices estimated for this study were reduced to the fourth root to approximate 1-year transition matrices. Complex roots were not encountered; positive roots of the eigenvalues were used. For both the 9-by-9 matrices developed to account for farms not included in the longitudinal set, the 1-year transition matrices contained negative elements. The average annual move was, therefore, not a true Markov process. One interpretation is that the actual annual transition probabilities may not have been constant during 1974-78; it would take at least two different Markov chains with nonnegative probabilities to move annually from the 1974 to 1978 distribution. Projections incorporating annual patterns of farm growth, decline, entry, and exit, such as those reported here, imply that the apparent cycles within the 4-year observation period will recur indefinitely. The matrix $P^{1.25}$ does behave as a Markov process, however, with all probabilities positive. It was used to project the behavior of the system over 5-year intervals. Similarly, P raised to the 6.5 power was used to project from 1974 to 2000.