Measurement of Farm Credit Risk: SUR Model and Simulation Approach

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Abstract

The study addresses problems in measuring credit risk under the structure model, and then proposes a seemingly unrelated regression model (SUR) to predict farms’ ability in meeting their current and anticipated obligations in the next 12 months. The empirical model accounts for both the dependence structure and the dynamic feature of the structure model, and is used for estimating asset correlation using FBFM data for 1995-2004. Farm risk is then predicted by copula based simulation process with historical default rates as benchmark. Results are reported and compared to previous studies on farm default.

Keyword: Credit Risk Measurement, Seemingly Unrelated Regression Model, Simulation
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Introduction

In a value-at-risk (VaR) framework, expected loss and unexpected loss at portfolio level are determined by probability of default (PD), loss given default (LGD), exposure at default (EAD), and default correlation (Barry 2001 and 2004, Saunders and Allen 1999, Caouette et al 1998). When Merton’s structure model (1974) is applied in credit risk measurement, default probability is often measured as the probability of an agent’s asset value falling below a threshold point, say total debt (Crouhy and Galai 1986, Crouhy et al. 2000, Gordy and Heitfield 2001). Default correlation is then determined by each agent’s probability of default and joint default for any two agents when a default event follows Bernoulli distribution.

Since marginal probability of default and default correlation are in practice closely associated with asset correlation, credit risk measurement under the approach generally relies on both the structure model and joint normal assumption of asset returns (Grouhy et al 2000).

Asset correlation is often calibrated by factor models that relate change in asset values to changes in a small number of common economic factors for an industry, region and/or country (Gordy 2003, Koyluoglu and Hickman1998). For example, Akhavein and Kocagil (2005) showed that average five-year intra-industry asset correlation for US issuers from a multi-factor model is 24.09% for 1970-2004. In agricultural lending, the reported average assets correlation is around 10.05% by applying a single factor model to farm reported asset returns for 1995-2001 (Katchova and Barry 2005). The main reason for adopting the approach is to reduce “dramatically the number of asset correlations to be calculated” (Crouhy et al 2000). Although default correlation, marginal probability of default and asset correlation are closely related, the connection is not emphasized by the approach.
Most popular credit risk models based on the structure model in commercial lending, such as KMV’s\(^1\) expected default frequency model (EDF), pre-require information on macroeconomic factors and long-time loss data for computing asset correlation and default rates. However, the prerequisites are hard to meet with farm records on which majority of the lending decisions are based. In this sense, if only farm records are available for measuring credit risk, it is impossible to directly use these models to assess asset correlation and then default risk.

Of the econometric methods, seemingly unrelated regression (SUR) is a technique for analyzing a system of multiple equations with cross-equation parameter restrictions and correlated error terms. When covariance matrix of disturbance is unknown, the parameters and correlation coefficients are estimated simultaneously by feasible generalized least square method (FGLS) (Zellner 1962, Zellner and Huang 1962), while “the least squares residuals may be used (of course) to estimate consistently the elements of covariance matrix of disturbance” (Greene 2000). In addition, if working with a panel data, we may use statistical classification to reduce computing size while keeping similarities within each class. Under the approach, individual agents within each group are said to have the same or similar characters. By combine the classing and the SUR approach together with the structure model, we may then have an appropriate empirical econometric model that not only considers asset correlation but also can take full use of farms’ accounting information in describing potential risk at group level.

It is noted that the correlation matrix obtained from the empirical model actually represents asset correlation among the groups. As mentioned before, marginal probability of default and joint default are in practice associated with asset correlation. One way to connect them together is by copula based simulation process that has been recently introduced in credit risk measurement (Bouyé et al). In the paper, we will illustrate that the estimated
correlation matrix under the structure model and multi-normality assumption for the error terms is actually comparable to that by Gaussian copula. Thus, with the asset correlation obtained from the econometric model, it is straightforward to apply Gaussian copula based simulation procedure to predict marginal probability of default and joint default.

As mentioned earlier, one of the difficulties in measuring farm credit risk is lack of long time loss data from which default thresholds used in default simulation are derived. To address the issue, historical default rates used in the study are from two different sources instead. The first one is from investigating farm records by reconsidering definition of default, and the other one is the default rates implied by popular credit score models. In agricultural lending, credit scoring is a generally accepted method in borrower’s rating, and there are several studies connecting rating criteria to default rate for each risk class (Barry et al 2004, Featherstone et al 2006).

In the study, asset correlation between farm groups is first estimated based on a previous empirical study on determinants of farm capital structure under the structure model (Yan et al 2008). An approach to integrate the model with Gaussian copula based simulation procedure is then proposed for predicting farm default at group level. Default definition is investigated and default thresholds are inferred from farm records. On the basis, probability of default (PD), expected loss (EL) and unexpected loss (EL) are predicted by Gaussian copula based simulation procedure. Empirical analysis will apply Annual Farm Business Farm Management (FBFM) for 1995-2004, and the results will be reported and compared to previous studies on farm default.

**Merton Model and Distribution of Farm Financial Position**

Under the framework of Merton’s model (1974), the value of any farm $i$’s asset $A_i$ at time $t$ is assumed to follow a standard geometric Brownian motion
where $A_{i0}$ is the initial asset value, $\mu_i$ is the instantaneous expected rate of return, $\sigma_i$ is the standard deviation of the return on the underlying asset, and $\omega_i$ is of $N(0,1)$. In the model, $\omega_i$ actually represents the normalized asset return at the time that is equal to

$$\omega_i = \frac{\ln(A_i/A_i)}{\frac{\sigma_i^2}{2}}$$

For a known debt value $D_i$ at the time, it is noted that the farm’s financial position can be represented by asset-to-debt ratio ($A_i/D_i$). In agricultural section, low asset-to-debt ratios are often interpreted as an indicator of “farm financial stress”. For example, in 1988, the U.S. Department of Agriculture (USDA) indicated that farms with debt-to-asset ratio above 0.7 ($A_i/D_i < 1.43$) were likely to experience a very high level of financial stress, and may have to liquidate certain assets in order to improve their financial position. In this sense, the probability of asset value falls below its debt value actually measures severity of financial stress for the farm. Given quantile, the probability is

$$P(A_i \leq D_i) = P\left(\omega_i \leq -\frac{\ln(A_i/D_i) + (\mu_i - \frac{\sigma_i^2}{2})t}{\frac{\sigma_i}{\sqrt{t}}}\right) = \Phi(z_i)$$

where $z_i = -\frac{\ln(A_i/D_i) + (\mu_i - \frac{\sigma_i^2}{2})t}{\frac{\sigma_i}{\sqrt{t}}}$, $\Phi(\cdot)$ is the standard normal cumulative density function.

On the other hand, if the farm’s financial position under the structure model is expressed as a dynamic regression model

$$y_i = x_i'\beta + \epsilon_i$$
where \( y_{it} = \ln \frac{A_{it}}{D_{it}} \), \( x_{it} \) is a vector of explanatory variables that contains some lagged values of \( y_{it} \), \( \beta_i \) is a vector of parameters to be estimated and \( \epsilon_{it} \) is the error term, an equivalent measurement of the probability in expression (3) is then

\[
P(A_{it} \leq D_{it}) = P(\epsilon_{it} \leq -x_{it}' \beta_i) = \Phi(v_{it})
\]

where \( v_{it} = -x_{it}' \beta_i \).

A major concern about the measurement is that the error terms between any two farms are actually correlated, i.e. \( E(\epsilon_{it}, \epsilon_{jt}) \neq 0 \) or \( E(z_{it}, z_{jt}) \neq 0 \) for \( i \neq j \). The correlation in fact represents asset correlation between the two farms according to the structure model. Obviously, the correlation is related to credit risk measurement. For a normal distribution, such correlation can be fully captured by correlation coefficient through seeming unrelated regression (SUR) model.

**Default Definition and Its Implication to Farm Credit Risk**

Crouhy and Galai (1986) suggested that default occurs once asset value falls below debt level. However, the scenario may not imply real default. Altman (1968) once pointed out that, for a firm with poor solvency, “because of its above average liquidity, the situation may not be considered serious”, i.e. the firm could actually enter a state of financial stress instead of default. Some studies on default, using the structure model under the definition, illustrated large differences between the predicted default rates and the reported values (Stein 2000, Crouhy et al 2000, Katchova and Barry 2005).

The Basel II\(^2\) (2004) also suggested a conservative definition of default for a bank, “a default is considered to have occurred with regard to a particular obligor when either or both of the two following events have taken place,
− The bank considers that the obligor is unlikely to pay its credit obligations to the banking group in full, without recourse by the bank to actions such as realizing security (if held).

− The obligor is past due more than 90 days on any material credit obligation to the banking group. Overdrafts will be considered as being past due once the customer has breached an advised limit or been advised of a limit smaller than current outstanding.”

The second event is close to the industry accepted standard that 90 days delinquency and assignment to non-accrual loans (Barry et al 2004, Stam et al 2003). However, since farm records are not likely to cover real loss information related to the standard, we pay more attention to the first scenario, i.e. unlikelihood of paying back on debt held by an obligor.

It is known that the farm financial crisis in 1980s is associated with rapid deterioration of farm return on asset and severe debt problems as measured by debt-to-asset ratio and interest rate. In corporate finance, on the other hand, if an issuer belongs to the speculative grade under Moody’s risk rating matrix, the firm will be also assigned a speculative-grade liquidity rating (SGL) as an assessment of its ability to cover its cash obligations by its projected cash flow over the coming 12 months. The ratings mainly assess an issuer’s operating income, current and anticipated cash balance, and internal and external sources of liquidity. Puchalla and Marshella (2007) showed that weak SGL is highly correlated with high probability of default, and “every company that has defaulted in roughly the last five years through a bankruptcy or missed payment was rated SGL-4 (weak SGL) at the time of default”. By incorporating with these findings and considering that only farm records are available in the study, a farm is defined as defaulted if it can not meet its current and anticipated cash balance over the coming 12 month (expected obligation) in combination with poor position in liquidity and return on asset (ROA) as well as heavy burden on interest
payment relative to its operating income. Specifically, a farm is in default for any given year if all of the following conditions are satisfied,

− Ratio of farm reported market value of asset over the expected obligation in the near future is less than 1;
− ROA is less than 0;
− Ratio of current debt to current asset is higher than 1.25;
− Ratio of farm reported interest expense and accrued interests over value of farm production (VFP) is higher than 10%.

When farm records indicate the current portion of intermediate and long-term liabilities (TLD) as well as the total balances of these categories of liabilities, a farm’s current liability plus one half of intermediate and long-term liabilities can be treated as a proper proxy for the expected obligation on debt. In statistics, when information on the TLD’s term structure is unavailable, the value close to maturity could then be assumed uniformly distributed between 0 and TLD, resulting in an expected value of TLD/2.

**SUR Model and Asset Correlation**

The empirical seemingly unrelated regression (SUR) model considered here differs from previous models for study of capital structure using farm records. The analysis emphases more on a farm’s ability to meet its financial obligation in the next 12 months, ratio of market value on farm asset-to-expected obligation is used as the dependent variable instead of total debt (Barry et al 2000) or leverage ratio (Jensen and Langemeier 1996, Yan et al 2008). Thus, the study extends beyond the verification of the structure model and theories on farm capital structure by identifying the linkage between farm’s financial position and credit risk as well as potential determinants among a set of farm attributes and credit risk factors. In the study, determinants of a farm’s ability to meet its financial obligation within the next 12 months are
selected based on the structure model, credit scoring models, and theories on optimal capital structure. The following factors, associated with a farm’s asset distribution, capital structure and credit risk, are considered as potential determinants of the strength of fulfillment.

**The Structure Model Factors**

The first two factors entering in the SUR model are directly from the structure model. They are lag of log of asset-to-expected obligation/debt ratio and the normalized return on farm assets (NROA). If the structure model is stable, a less than 1 estimated coefficient for lag of log of asset-to-expected debt ratio is expected. NROA is calculated by rescaling return on asset by its standard deviation for each farm record. A farm’s asset-to-expected debt ratio will be negatively influenced by NROA if the farm tends to make offsetting adjustments in its capital structure in response to modifications of business risk as measured by the standard deviation of return on farm asset (Barry and Robison 1987, Gabriel and Baker 1980).

**Credit Risk Factors**

Three key financial ratios employed by rating agencies and credit risk model for farm lending are considered here, including liquidity, financial efficiency and VFP/Debt. Liquidity is calculated by dividing working capital by value of farm production (WC/VFP), and is expected to have positive impact on a farm’s ability to make timely payment, and thus stay liquid. A farm’s financial efficiency is represented by ratio of net income to value of farm production (NETINC/VFP). Since great financial efficiency could strengthen farms’ risk-bearing capacity and thus lower risk of financial stress, it is expected to vary positively with the asset-to-expected debt ratio. VFP/Debt is defined as log of value of farm production to total debts ratio and is an indicator of farm profitability. It is reasonably to say that farms with higher level of profitability should be less likely to default on their expected obligations.
**Capital Structure Factors**

Five structure factors than has been indentified as important determinants of capital structure are also included in the model. They are size (log of farm cash sale), tenure (owned land to total tillable land ratio), NGTA (annual growth in total assets divided by its volatility), collateral ratio (value of farmland plus machinery and equipment to total assets ratio) and non-debt tax shield (earning before depreciation divided by total assets).

If farms adjust to long term financial target of leverage ratio with additional financial needs following pecking order theory and/or agency theory, the risk of falling short on its financial obligation in the near future would be positively influenced by profitability and tenure position while it would be negatively correlated to farm cash sale (size). The low debt carrying capacity of farmland also justifies the expected positive tenure effect.

The non-depreciation property of farmland implies higher liquidation value, and thus it would be much easier for a farm with higher collateral position to meet its obligation in the next 12 months than otherwise. In this sense, we would expect a positive relationship between farm collateral ratio and the dependent variable. On the other hand, if farms with large non-debt tax shield tend to include less debt in their capital structures as predicted by trade off theory, the ability to pay back in full will increases as non-debt tax shield increases.

It is noted that the empirical SUR model includes lag value of the dependent variable and thus is a dynamic model. On the other hand, farm records are often characterized by short time period and large number of farms. To address the two issues, farms are grouped such that each group has enough degree of freedom. On the basis, a specific semi-parametric 3SLS estimator is then applied for the dynamic SUR model ((Yan et al 2008). Given the regression results and let the correlation matrix among the farm groups be $\Sigma$, the consistently estimated elements of $\hat{\Sigma}$ is then given by
6) \[ \hat{\rho}_{ij} = \frac{e_i' e_j}{T} \]

where \( e_i \) is the least square residuals from equation or group \( i \), and \( T \) is the total number of observations in each equation/group (Greene 2000).

**Farm Credit Risk Measurement**

It is noted that the calculated probability of \( P(A_{it} \leq D_{it}) \) is not necessarily equivalent to probability of default and an adjustment is often needed for more accurate prediction (Crouhy et al 2000, Altman 2002). For example, the reported default probability for an issuer by KMV is obtained by “mapping the DD \( z_{it} \) to the actual probabilities of default for a given time horizon” (Crouhy et al 2000). The actual probabilities of default, called default thresholds or historic default rates, are inferred from KMV’s default database while the mapped probability is called expected default frequency (EDF).

As pointed out earlier, marginal probabilities of farm default are closely associated with asset correlation. A popular way to connect both together is by copula approach (Nelson 1999). That is, given marginal distributions, we can derive correlation structure by choosing a copula, while given a copula, marginal probability of default for each agent can be predicted by simulation. Of the copulas, Gaussian copula is fully characterized by correlation matrix \( \Sigma_{G} \) as multivariate normal distribution does. In addition, when time series data of farm assets are available, maximum likelihood estimate (MLE) of the elements of \( \Sigma_{G} \) under Gaussian copula and the structure model is given by \( \hat{\delta}_j = \frac{\hat{v}_j' \hat{v}_j}{T} \), where \( \hat{v}_j \) is a vector of estimates generally obtained by applying kernel density function to the corresponding values calculated under the structure model with respect to farm \( i \), for example \( \omega_{it} \) in expression (2), and \( T \) is the total number of observations for the farm (Magnus and Neudecker 1988). Clearly,
\( \hat{\delta}_{ij} \) is comparable to \( \hat{\rho}_{ij} \) in expression (6) under the multivariate normal distribution assumption. In this sense, marginal probabilities of farm default and joint defaults can be predicted by Monte Carlo simulation procedure for Gaussian copula with the estimated correlation matrix from the SUR model.

With the predicted farm default and joint defaults from simulation and assuming that the default event follows Bernoulli distribution, default correlation \( \tau_{ij} \) for a typical farm \( i \) in group \( i \) and a typical farm \( j \) in group \( j \) is given by

\[
7) \quad \tau_{ij} = \frac{P_{ij} - P_i P_j}{\sqrt{P_i(1-P_i)P_j(1-P_j)}}
\]

In the equation, \( P(\cdot) \) denotes the predicted marginal probability of default for each farm and \( P_{ij} \) refers to the predicted joint probability of default for farm \( i \) and farm \( j \). The standard deviation (std.) of farm \( i \)'s default and the joint probability of default \( P_{ij} \) equal to

\[
8) \quad \text{Std}(\text{farm}\text{ } i \text{ in default}) = \sqrt{P_i(1-P_i)}
\]

\[
P_{ij} = P_i P_{j/i}
\]

where \( P_{j/i} \) denotes the conditional probability of default for farm \( j \) given that farm \( i \) is in default and is easy to calculate with the simulation results.

Given probability of default, loss-given-default, and default correlation matrix, the expected loss (EL) and unexpected loss (UL) at portfolio level is then computed by mean-variance method, in which

\[
9) \quad EL = \sum_{i=1}^{n} w_i P_i LGD_i
\]

\[
UL = \frac{\sum_{i=1}^{n} w_i^2 UL_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \rho_{ij} UL_i UL_j}{\sum_{i=1}^{n} w_i}
\]

where \( UL_i = LGD_i \sqrt{P_i(1-P_i)} \), and \( w_i \) is the weight for farm \( i \) and \( n \) is the total number of farms in the portfolio.
Historical Default Rates

As mentioned above, one of the difficulties in measuring farm credit risk is lack of long-term loss data from which historical default rates are inferred. To address the issue, FCS (Farm Credit System) default guideline and historical default frequencies from farm records based on default definition are used instead.

FCS Default Guideline

FCS default guideline is summarized based on two previous studies on credit risk model and default probability for farm lending (Barry et al 2004, Featherstone et al 2006). The credit score model was developed by Barry et al (2004) to distinguish between low credit risk (less financially constrained) and high credit risk (more financially constrained) farms. The model contains financial ratios recommended by the Farm Financial Standard Council, representing a farm’s solvency, liquidity, repayment capacity, profitability, and financial solvency.

The risk rating definition, interval ranges, and implied default rates are illustrated in table 1. In the risk rating, each farm has five scores ranging from 1 to 10 on solvency, liquidity, repayment, profitability and efficiency accordingly. The five scores are then weighted to generate a final score between 1 and 10 by expression (10), where each farm is then grouped to a rating class with respect to the final score.

\[
\text{score} = 30\% \times \text{solvency} + 20\% \times \text{liquidity} + 20\% \times \text{repayment} \\
+ 20\% \times \text{profitability} + 10\% \times \text{efficiency}
\]

10)

Default Frequencies from Farm Records

Historical default frequencies are inferred from Farm Business Farm Management Association (FBFM) data that contains farm accounting information, such as income and
cash flow statement, as well as farm reported market value on assets and liabilities during the period of 1995-2004. Consistent with Katchova and Barry (2005), farms with no debt are excluded, resulting in approximately 1,670 farms with 11,745 farm observations.

Following the default criterion and the notion for expected debt, a discrete time approximation of the nonparametric continuous-time hazard rate approach (cohort method) is used to infer marginal default rates from the FBFM data. The nonparametric continuous-time hazard rate approach was first proposed by Cutler and Ederer (1958) and has been commonly used by the rating agencies like Moody and Fitch Ratings for default and migrating analysis. A pool of farms, called a cohort, is formed on the basis of their risk ratings held in a given calendar year, and the default status for the farms of the cohort is tracked over some stated time horizon. In each time interval or horizon, some fraction of the cohort that has survived up to that time may default. The marginal default rate is the probability that a farm survived in the cohort up to the beginning of a particular time interval will default by the end of the time interval. The cohort method assumes that withdraws occurs randomly during the interval, and the probability of survival/default at one interval, though conditional on surviving previous intervals, is independent of the probability of survival at the prior interval(s). Moreover, the interval is often set evenly distributed when long time data is available. For example, the default rates calculated by most rating agencies under the method are based on more than 30 years’ annual or monthly observations. In this study, since only 10 years’ annual data is available, the cohort spacing is selected based on data availability instead. In total, 9 cohorts are formed from the data with time interval ranging from 9 to 1.

Consistent with FCS default guideline, the farms are grouped with respect to their risk ratings defined in expression (10). Since relatively fewer farms are rated 7 and above, the farms with risking rating greater than 7 are grouped together, the 9 cohorts are then created.
for each of the 7 risk rating classes. The results are listed in table 2. Overall, average default rate across all cohorts for each risk rating class is lower than the corresponding value in table 1. Small sample size may be the reason.

Loss-given-default (LGD) for each cohort is calculated as the average LGD for the defaulted farms by using reported marked values on asset and the expected obligations in the next 12 months and a 10% recovery rate (Featherstone and Boessen 1994, Featherstone et al 1993). The average values on LGD across all cohorts for each rating class are also reported in table 2. On average, LGD is 23.91% for 1995-2004, which is similar to previous reported values on farm LGD. For example, Stam et al (2003) reported that average LGD for all farm loans issued by commercial banks, the FCS, life insurance companies and the Farm Service Agency was 24.26% for 1995-2001. According to Federal Deposit Insurance Corporation (FDIC)\(^4\), average LGD on farm loans issued by all commercial banks in Illinois is around 18.26% for 1995-2004.

**Data and Estimation**

Consistent with previous studies, the empirical estimation and prediction uses a subset of the FBFM annual farm data that includes only farm records with a minimum time range of 10 years and the farms are grouped with respect to their credit risk ratings in 1995 prior to the estimation period of 1996-2004 ((Barry et al 2004, Yan et al 2008). In total, we have 5,346 farm observations with 635 farms, and these farms are grouped into 7 risk classes. Table 3 shows definition and summary statistics of the variables included in the dynamic SUR model.

On average, the log of asset-to-expected debt ratio for a typical farm is 1.60 with log of farm cash sale (size) equal to 12.26. The tenure position and collateral ratio for the typical farm are 0.19 and 0.54 respectively. The non-debt tax shield (shield) for the farm is 0.078 with a liquidity position of 0.57. The values of efficiency ( NETINC / VFP) and
profitability (VFP/Debt) for the average farm are 0.20 and -0.08 respectively. In addition, the normalized values of return on asset (NROA) and growth in total assets (NGTA) for the typical farm are 1.26 and 0.34 respectively. Since mean values for most of the selected variables are higher than their corresponding medians, above half farms are ranked low in values as compared to the typical farm.

The regression results are listed in Table 4. Overall, most of the variables are significant at better than the conventional level of 5%. In addition, the coefficients for the lag of the dependent variables are all less than 1, indicating that the estimated dynamic model is stable. Results show that the ability of paying back within the next 12 months is negatively associated with NROA, size and non-debt tax shield, while is positively influenced by NGTA, tenure, collateral ratio, VFP/Debt (profitability), WC/VFP and NETINC/VFP. The positive significant signs for the credit risk components suggest that the probabilities of a farm’s ability to meet its current and anticipated financial obligations over the coming 12 months are related to its liquidity, financial efficiency, profitability as well as availability of secured assets.

The estimated correlation matrix is listed in table 5. Overall, average correlation coefficient among the 7 risk rating classes is 20% with a standard deviation of 5.6%, which is clearly higher than the reported average asset correlation of 16% by KMV’s risk classing (Lopez 2002). Since KMV’s risk classing is for public firms, the result indicates that agricultural production is more likely to move in the same direction than other industries, and thus comparatively the systematic risk plays a more important role in agricultural production. In addition, the estimated correlation is also close to the reported intra-industry average asset correlation of 24.09% by Akhavein and Kocagil (2005) while higher than the reported value of 10.05% with a similar FBFM data by Katchova and Barry (2005).
It is noted that the matrix illustrates correlation between farm groups; the order of farm observations within each group is ignored. To further verify that the estimated asset correlation matrix in table 5 actually represents population correlation, two statistical tests, the log likelihood ratio test and the Jennrich’s test for equality of correlation matrices, are applied in the study. The tests indicate that the estimated correlation metrics is valid and can be used to predict farm default and default correlation at group level (Appendix).

**Prediction of Farm Credit Risk**

The default simulation is at a time horizon of 1 year. In the simulation, the default threshold for any farm group is set to the quantile of the corresponding historic default rate given in table 1 or table 2 under the standard normal distribution. For example, for a default rate of 1.73% for group 7 in table 1, the default threshold is equal to $\Phi^{-1}(1.73\%) = -2.11$. In each simulation run, seven correlated random variables corresponding to each of the 7 farm groups are created by way of Cholesky decomposition given the estimated asset correlation matrix in table 5 (Bouyé et al 2000). Each random variable represents a standardized asset return for the corresponding risk class, and a default will be registered if its value falls below the default threshold.

A total of 50,000 scenarios for each risk rating class are generated, and the default probability at group level is then defined as frequency of defaults out of the 50,000 simulation runs. In addition, joint default probability is computed with a procedure similar to that for default probability in which joint default for any two farms is defined as concurrence of default for the two farms in a single simulating run. The default correlation is then calculated using expression (7) given the predicted marginal probabilities of default and joint default probabilities.
Predicted probabilities of default are listed in table 6. In the table, threshold 1 and threshold 2 correspond to the historic default rates in table 1 and table 2 respectively. From table 6, the order of the predicted default probabilities under both default thresholds is consistent with the risk rating. Comparatively, the predicted default rates under threshold 1 (FCS default guideline) are relatively higher than those from threshold 2 (historical default benchmark from FBFM data), and they are more close to their corresponding benchmark rates than otherwise. Overall, the weighted average default probability, weighted by the average debt in each group, is 0.895% for threshold 1 and is 0.643% for threshold 2.

According to FDIC, the average default rate for the agricultural loans issued by commercial banks in Illinois is 0.83% for 1995-2004. Featherstone et al (2006) reported a default percentage of 1.83% for the loans issued by the Seventh District during 1995 to 2002, while Stam et al (2003) reported a default rate of 1.02% for agricultural banks during 1995-2001. Obviously, the predicted default rates are close to those from FDIC and Stam et al (2003).

Table 7 illustrates the estimated default correlation inferred from the predicted defaults and joint defaults. Although average asset correlation between the groups is around 20%, it is not surprising to observe that default correlation is much lower, implying that the two types of correlation matrix are not equivalent although both are closely associated. The result is consistent with a previous study by Crouhy et al (2000) who showed that for an asset correlation of 20% for two rated AA and B issuers by Moody’s risk rating, the default correlation is only around 1.9%, and thus, “the ratio of asset returns correlations to default correlations is approximately 10-1 for asset correlations in the range of 20-60%”. In addition, the joint defaults are more likely to occur among the higher risk groups. For example, when a farm of group 7 is in defaults, there is around 3% chance that another farm of group 6 will be also in default while the chance is less than 1% if “another farm” is from group 1 or group 2. These findings are consistent with a previous study by Hrvatin and Neugebauer (2004). They
showed that with the same asset correlation of 25%, the derived default correlation is 3.5% between two issuers with default rates of 1% and 4% respectively, and 8.1% if the default rates for the two issuers are 4% and 10% respectively.

Given probability of default, loss-given-default, and default correlation matrix, the expected loss (EL) and unexpected loss (UL) for the farm portfolio is then computed using equation (9). The EL and UL at portfolio level and 1-year horizon are listed in table 8. Overall, the average expected loss (EL) and unexpected loss (UL) for a typical farm portfolio is 0.19% and 0.98% respectively. The reported average loan loss allowance (EL) for agricultural banks at a national level during 1995-2001 is around 0.33% (Stem et al 2003). According to FDIC, the average loss for agricultural loans issued by commercial banks in Illinois is 0.18% for 1995-2004.

The data period for estimation of farm asset correlation and prediction of EL and UL is 1995-2004. The same period should be considered in comparison. According to FDIC, the average default rate of agricultural loans issued by commercial banks in Illinois is 0.84% for 1995-2004, 0.74% for 1995-2007 and 0.49% for 2005-2007, while the corresponding EL are 0.18, 0.14% and 0.04% respectively. Obviously, the predicted average PD and EL are close to the historical average values of the period 1995-2004, the same period for the sample data. On the other hand, it is noted that the reported EL and PD by Stem et al (2003) is at a national level and for 1995-2001 while the prediction in the study focus on farms in Illinois and most of the farms are grain farms. In this sense, it is not surprising to see the difference.

**Conclusion**

As the regulatory requirements are moving towards economic-based measures of risk, banks are urged to build sound internal measures of credit risk, in which prediction of a borrower or group of borrowers’ credit risk plays a dominant role. The study addresses problems in
measuring credit risk under the structure model, and then proposes a seemingly unrelated regression model (SUR) to predict farms’ ability in meeting their current and anticipated obligations in the next 12 months. The empirical model accounts for both the dependence structure and the dynamic feature of the structure model. On the basis, the SUR model is used for estimating asset correlation using FBFM data for 1995-2004. With the estimated asset correlation, farm risk is then predicted by copula based simulation process in which FCS default guidelines and historical default rates from the farm records are used to infer default thresholds, where definition of default is reconsidered and used for investigating historical default rates from farm records.

Regression results indicate that the dynamic model is stable, and the structure model is confirmed by most of the farm records. Results also show that a farm’s ability to meet its current and anticipated financial obligations in the next 12 months is associated with those factors related to the structural model, theories of farm capital structure, and the credit scoring model.

The estimated asset correlation is 20%, and the predicted default correlation is lower than the corresponding asset correlation. In addition, the predicted probability of default and expected loss at portfolio level are close to the reported values for the same period of 1995-2004 and same region. Results indicate that the predicted probabilities of default at high-risk groups are on average slightly higher than the corresponding FCS default guideline (Figure 1). The difference may be characterized by the approach. For example, some farms were grouped into group 5 by their own credit scores should be actually be classified as group 6 by the predicted default rate.

The study has important implications to farm credit risk management under the Basel II. First, although the study focuses on model testing, application and comparison, the approaches introduced in the study are applicable to farm credit risk management. For
example, given a farm’s accounting information, we can use one of the equations say the equation for group 5 in the SUR model to estimate its asset-to-expected debt ratio. By comparing the estimated value to the observed one, we can statistically test whether the farm belongs to group 5 or not. Second, dependence structure as well as level of the dependence for farm assets has significant impact in measuring farm credit risk, and thus should be emphasized in loan pricing and in risk diversification. Third, asset correlation as well as default correlation may change under different business cycles. Application of the approaches should pay attention to possible changes in economic factors and update the information accordingly.
Appendix

The log likelihood ratio (LR) test is to test whether the correlation matrix is diagonal or not (Greene 2000). If the estimated correlation matrix does not pass the LR test, implying that observations from any two different risk classes are independent, the problem of measuring credit risk would be much simplified than otherwise. Given the regression results for the SUR model, the likelihood ratio statistic is calculated by

\[ \lambda_{LR} = T \left[ \sum_{i=1}^{M} \log \hat{\sigma}_{i}^2 - \log |\hat{\Sigma}_{mle}| \right] \]

where \( \hat{\sigma}_{i}^2 = \frac{e'e}{T} \) and is calculated from the least square estimation, and \( \hat{\Sigma}_{mle} \) is the maximum likelihood estimator of the variance and covariance matrix for the regression. Under the null hypothesis of no asset correlation, the statistic has a limiting \( \chi^2 \) distribution with \( M (M-1)/2 \) degree of freedom. In the study, \( M \) is the total number of equations in the SUR model and is equal to 7.

Jennrich’s \( \chi^2 \) test for homogeneity of two correlation matrices is based on a study by Jennrich (1970). Let \( R \) denote the correlation matrix estimated by a sample of size \( n \) from a \( p \)-variate normal distribution with population correlation matrix \( P(r_{ij}) \), the test statistic is then

\[ \text{Jenn} = \sum_{i=1}^{n} \left( \frac{1}{2} \text{tr}(Z^2) - dg^*(Z)S^{-1}dg(Z) \right) \]

with \( Z = \sqrt{n}P^{-1}(R - P) \), \( S = (\delta_i + \delta_j \bar{r}^y) \) and \( \delta_i \) being the Kronecker delta. The null hypothesis is that there is no difference between the two correlation matrices. Under the null hypothesis, the test statistic has an asymptotic \( \chi^2 \) distribution with \( p (p-1)/2 \) degree of freedom. In the study, the population correlation matrix is assumed to be the matrix from the SUR process and is illustrated in table 5 with \( p \) equal to 7.
Empirical testing applies Bootstrap techniques (Hollander and Wolfe 1999). A total of 5,000 random samples of residuals for each equation were drawn from the least square regression results for the SUR model. Each sample is randomly selected by the 7 risk classes or equations, and each class has 9 observations randomly picked out with one for each year in the sample. In each sampling run, the random sample is then used to compute a value of $\lambda_{LR}$ and $Jenn$ respectively, in which the corresponding sample correlation matrix $R$ is obtained by using expression (6). Both statistics are of $\chi^2$ distribution with 21 degree of freedom. Overall, the mean value of $\lambda_{LR}$ is 39.02 out of the total 5,000 calculated values, greater than the 1 percent critical value of 38.93. So the null hypothesis of diagonal for the correlation matrix is rejected, and thus asset correlation among farm risk classes is confirmed statistically. In addition, the average value for $Jenn$ is 29.58 as compared to the same critical value of 38.93, implying that statistically, the order of farm observations within each group has no significant impact on the correlation matrix.
Notes:

1. KMV is a trademark of KMV Corporation. Stephen Kealhofer, John McQuown and Oldrich Vasicek founded KMV Corporation in 1989.

2. Basel II is the second of the Basel Accords, which are recommendations on banking laws and regulations issued by the Basel Committee on Banking Supervision. The purpose of Basel II is to create an international standard that requires financial institutions to maintain enough capital to cover risks incurred by operations.

3. A farm is removed completely from the panel if it has zero debt in any year. In total, 137 farms are excluded with a total of 650 observations.

4. All FDIC insured institutions are required to file consolidated Reports of Condition and Income (Call Report) as of the close of business on the last day of each calendar quarter. FDIC constructed a database from the Call Report, and the database is publicly available on the website starting 1998.

5. In calculating the standard deviations of return on asset (ROA) and annual growth in total assets (GTA) for each farm, a total of 10 records on farm asset and return are used. If there is any missing value, the missing value is replaced by imputed value through the multiple imputation method (Rubin 1987, Schafer 1997, Schafer and Schenker 2000, Reilly 1993, Li 1988).

6. A bank is defined as agricultural bank if its ratio of farm loans to total loans exceeds 14.97 percent (Stam et al 2003).

7. The LR test applied in the study is to test whether there is correlation among farms of different risk classes. As for the farms within each class, it is reasonable to assume that they are identical and are derived independently from the same population, but we did not test the assumption here.
It is noted that the correlation matrix in table 5 is calculated using all the farm observations in each group while only 9 observations in each group is used to calculate the sampled correlation matrix in each sampling run. To account for any impact of sample size, we also did a similar test with sample size being considered. The testing result illustrates no such influence.
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Figure 1 Predicted Average PD by the SUR Model vs. FSC Default Guideline
### Table 1 Credit Scoring Model and Proposed Probability of Default (PD) by Farm Credit System

<table>
<thead>
<tr>
<th>Risk Rating Class</th>
<th>Proposed PD Guidelines (%)</th>
<th>FCS Guidelines</th>
<th>Solvency</th>
<th>Liquidity</th>
<th>Repayment</th>
<th>Earnings</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Equity to Assets Ratio</td>
<td>Current Debt Ratio</td>
<td>CDRC Ratio</td>
<td>Return on Assets (%)</td>
<td>Gross Farm Return Ratio</td>
</tr>
<tr>
<td>1</td>
<td>0.00-0.035</td>
<td>AAA to AA</td>
<td>&gt;0.80</td>
<td>&gt; 2.5</td>
<td>&gt; 1.6</td>
<td>&gt;10</td>
<td>&gt; 0.4</td>
</tr>
<tr>
<td>2</td>
<td>0.035-0.08</td>
<td>AA to A</td>
<td>0.75-0.80</td>
<td>2.00-2.50</td>
<td>1.50-1.60</td>
<td>8-10</td>
<td>0.25-0.40</td>
</tr>
<tr>
<td>3</td>
<td>0.08-0.014</td>
<td>A to A-</td>
<td>0.70-0.75</td>
<td>1.80-2.00</td>
<td>1.40-1.50</td>
<td>6-8</td>
<td>0.30-0.35</td>
</tr>
<tr>
<td>4</td>
<td>0.14-0.26</td>
<td>BBB+ to BBB</td>
<td>0.65-0.70</td>
<td>1.60-1.80</td>
<td>1.30-1.40</td>
<td>4-6</td>
<td>0.25-0.30</td>
</tr>
<tr>
<td>5</td>
<td>0.26-0.52</td>
<td>BBB to BBB-</td>
<td>0.60-0.65</td>
<td>1.40-1.60</td>
<td>1.20-1.30</td>
<td>2-4</td>
<td>0.20-0.25</td>
</tr>
<tr>
<td>6</td>
<td>0.52-0.69</td>
<td>BB+</td>
<td>0.50-0.60</td>
<td>1.20-1.40</td>
<td>1.10-1.20</td>
<td>0-2</td>
<td>0.15-0.20</td>
</tr>
<tr>
<td>7</td>
<td>0.69-1.145</td>
<td>BB+ to BB</td>
<td>0.40-0.50</td>
<td>1.00-1.20</td>
<td>1.00-1.10</td>
<td>-2-0</td>
<td>0.10-0.15</td>
</tr>
<tr>
<td>8</td>
<td>1.145-1.73</td>
<td>BB to BB-</td>
<td>0.30-0.40</td>
<td>0.80-1.00</td>
<td>0.901-1.00</td>
<td>-4 - 2</td>
<td>0.05-0.10</td>
</tr>
<tr>
<td>9</td>
<td>1.73-2.88</td>
<td>BB- to B+</td>
<td>0.20-0.30</td>
<td>0.60-0.80</td>
<td>0.80-0.90</td>
<td>-6 - 4</td>
<td>0.00-0.10</td>
</tr>
<tr>
<td>10</td>
<td>2.88 and up</td>
<td>B and down</td>
<td>&lt; 0.2</td>
<td>&lt; 0.5</td>
<td>&lt; 0.80</td>
<td>&lt; -6</td>
<td>&lt; 0</td>
</tr>
</tbody>
</table>

*CDRC ratio = (farm and nonfarm net income + depreciation + debt service-annual family expenditures-income taxes)/debt services

Source: Farm Credit System risk-rating guidelines definitions (Featherstone et al 2006) and Credit Risk Rating Systems: Tenth Farm Credit District (Barry et. al 2004)
Table 2  Historical Default Frequency and Loss-Give-Default (LGD) of FBFM Farms (1995-2004)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td># of Farms</td>
<td>Default Rate</td>
<td># of Farms</td>
<td>Default Rate</td>
<td># of Farms</td>
<td>Default Rate</td>
<td># of Farms</td>
<td>Default Rate</td>
<td># of Farms</td>
<td>Default Rate</td>
<td># of Farms</td>
</tr>
<tr>
<td>1</td>
<td>179</td>
<td>236</td>
<td>134</td>
<td>21</td>
<td>69</td>
<td>116</td>
<td>41</td>
<td>42</td>
<td>118</td>
<td>0.55%</td>
<td>188</td>
</tr>
<tr>
<td>2</td>
<td>146</td>
<td>183</td>
<td>198</td>
<td>105</td>
<td>182</td>
<td>193</td>
<td>153</td>
<td>155</td>
<td>205</td>
<td>0.54%</td>
<td>145</td>
</tr>
<tr>
<td>3</td>
<td>130</td>
<td>145</td>
<td>173</td>
<td>91</td>
<td>169</td>
<td>176</td>
<td>141</td>
<td>131</td>
<td>183</td>
<td>0.78%</td>
<td>148</td>
</tr>
<tr>
<td>4</td>
<td>130</td>
<td>148</td>
<td>184</td>
<td>154</td>
<td>182</td>
<td>223</td>
<td>0.45%</td>
<td>166</td>
<td>155</td>
<td>206</td>
<td>0.05%</td>
</tr>
<tr>
<td>5</td>
<td>126</td>
<td>125</td>
<td>149</td>
<td>210</td>
<td>243</td>
<td>205</td>
<td>196</td>
<td>181</td>
<td>188</td>
<td>0.26%</td>
<td>128</td>
</tr>
<tr>
<td>6</td>
<td>110</td>
<td>0.91%</td>
<td>90</td>
<td>134</td>
<td>193</td>
<td>185</td>
<td>0.54%</td>
<td>159</td>
<td>186</td>
<td>197</td>
<td>0.44%</td>
</tr>
<tr>
<td>7</td>
<td>113</td>
<td>1.77%</td>
<td>84</td>
<td>162</td>
<td>384</td>
<td>324</td>
<td>0.31%</td>
<td>210</td>
<td>0.29%</td>
<td>378</td>
<td>1.06%</td>
</tr>
</tbody>
</table>

LGD: Loss-Give-Default
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Mean</th>
<th>Median</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (asset to debt ratio)</td>
<td>Log of (total assets to expected obligation ratio)</td>
<td>1.60</td>
<td>1.46</td>
<td>0.76</td>
</tr>
<tr>
<td>Size</td>
<td>Log of farm cash sale</td>
<td>12.26</td>
<td>12.27</td>
<td>0.60</td>
</tr>
<tr>
<td>NROA</td>
<td>Return on asset divided by its volatility</td>
<td>1.27</td>
<td>1.16</td>
<td>1.28</td>
</tr>
<tr>
<td>Tenure</td>
<td>Owned land over total tillable land</td>
<td>0.19</td>
<td>0.11</td>
<td>0.22</td>
</tr>
<tr>
<td>Collateral ratio</td>
<td>Machinery &amp; equipment plus farmland value over total assets</td>
<td>0.54</td>
<td>0.55</td>
<td>0.16</td>
</tr>
<tr>
<td>Shield</td>
<td>Earning before depreciation to total assets</td>
<td>0.078</td>
<td>0.169</td>
<td>0.055</td>
</tr>
<tr>
<td>Liquidity</td>
<td>Working capital over value of farm production (VFP)</td>
<td>0.57</td>
<td>0.36</td>
<td>1.08</td>
</tr>
<tr>
<td>NETINC/VFP</td>
<td>Netfarm income over VFP</td>
<td>0.20</td>
<td>0.21</td>
<td>0.16</td>
</tr>
<tr>
<td>VFP/Debt</td>
<td>Log of value of farm production to total liabilities ratio</td>
<td>-0.08</td>
<td>-0.21</td>
<td>0.73</td>
</tr>
<tr>
<td>NGTA</td>
<td>Annual growth in total assets (GTA) divided by its volatility</td>
<td>0.34</td>
<td>0.21</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Table 3 Basic Statistics for Selected Variables of FBFM Farms (1995-2004)
<table>
<thead>
<tr>
<th>Variable</th>
<th>Group1</th>
<th>Group2</th>
<th>Group3</th>
<th>Group4</th>
<th>Group5</th>
<th>Group6</th>
<th>Group7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (lag of assets to expected debt ratio)</td>
<td>-0.161* (0.02)</td>
<td>0.039 (0.01)</td>
<td>0.297** (0.01)</td>
<td>0.232** (0.01)</td>
<td>0.338* (0.01)</td>
<td>0.790* (0.01)</td>
<td>0.643** (0.01)</td>
</tr>
<tr>
<td>NROA</td>
<td>-0.029** (0.02)</td>
<td>-0.063* (0.01)</td>
<td>-0.018 (0.01)</td>
<td>-0.048* (0.01)</td>
<td>-0.017 (0.01)</td>
<td>-0.032 (0.01)</td>
<td>-0.036* (0.01)</td>
</tr>
<tr>
<td>Size</td>
<td>-0.185* (0.02)</td>
<td>-0.112* (0.02)</td>
<td>-0.116* (0.02)</td>
<td>-0.142* (0.02)</td>
<td>-0.155* (0.02)</td>
<td>-0.053* (0.02)</td>
<td>-0.103* (0.02)</td>
</tr>
<tr>
<td>NGTA</td>
<td>0.020* (0.01)</td>
<td>0.027* (0.01)</td>
<td>0.016* (0.01)</td>
<td>0.011** (0.01)</td>
<td>0.009*** (0.01)</td>
<td>0.013* (0.01)</td>
<td>0.019* (0.01)</td>
</tr>
<tr>
<td>Tenure</td>
<td>0.479* (0.07)</td>
<td>0.705* (0.08)</td>
<td>0.655* (0.07)</td>
<td>0.733* (0.08)</td>
<td>0.699* (0.06)</td>
<td>0.592* (0.06)</td>
<td>0.435* (0.09)</td>
</tr>
<tr>
<td>Collateral Ratio</td>
<td>0.468* (0.10)</td>
<td>0.314* (0.09)</td>
<td>0.213* (0.09)</td>
<td>0.092 (0.09)</td>
<td>0.575* (0.09)</td>
<td>0.222* (0.08)</td>
<td>0.428* (0.08)</td>
</tr>
<tr>
<td>Shield</td>
<td>-2.424* (0.27)</td>
<td>-2.62* (0.26)</td>
<td>-2.791* (0.28)</td>
<td>-2.698* (0.25)</td>
<td>-1.831* (0.23)</td>
<td>-1.924* (0.25)</td>
<td>-1.869* (0.26)</td>
</tr>
<tr>
<td>WC/VFP</td>
<td>0.300* (0.02)</td>
<td>0.172* (0.01)</td>
<td>0.308* (0.03)</td>
<td>0.266* (0.02)</td>
<td>0.268* (0.02)</td>
<td>0.230* (0.02)</td>
<td>0.272* (0.02)</td>
</tr>
<tr>
<td>VFP/Debt</td>
<td>0.699* (0.01)</td>
<td>0.698* (0.02)</td>
<td>0.517* (0.02)</td>
<td>0.543* (0.02)</td>
<td>0.443* (0.02)</td>
<td>0.328* (0.02)</td>
<td>0.310* (0.03)</td>
</tr>
<tr>
<td>NETINC/VFP</td>
<td>0.391* (0.09)</td>
<td>0.548* (0.09)</td>
<td>0.324* (0.08)</td>
<td>0.666* (0.09)</td>
<td>0.262* (0.08)</td>
<td>0.461* (0.07)</td>
<td>0.502* (0.09)</td>
</tr>
<tr>
<td>R-square</td>
<td>0.7629</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Note: single, double and triple asterisks (*) denote significance at 1%, 5% and 10% confidence level respectively.
Table 5  Asset Correlation Matrix By the SUR Model

<table>
<thead>
<tr>
<th>Risk Rating</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.289</td>
<td>0.306</td>
<td>0.222</td>
<td>0.235</td>
<td>0.180</td>
<td>0.238</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0.200</td>
<td>0.152</td>
<td>0.243</td>
<td>0.151</td>
<td>0.189</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>0.222</td>
<td>0.225</td>
<td>0.165</td>
<td>0.220</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.181</td>
<td>0.162</td>
<td>0.102</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>0.127</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>0.128</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6 Predicted Probability of Default (PD) by the SUR Model and Simulation

<table>
<thead>
<tr>
<th>Risk Rating</th>
<th>Threshold 1</th>
<th>Threshold 2</th>
<th>Average Debt</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>PD (%)</td>
<td>Std. of Default (%)</td>
<td>PD (%)</td>
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<td>1</td>
<td>0.01</td>
<td>1.00</td>
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</tr>
<tr>
<td>2</td>
<td>0.042</td>
<td>2.05</td>
<td>0.012</td>
</tr>
<tr>
<td>3</td>
<td>0.092</td>
<td>3.03</td>
<td>0.012</td>
</tr>
<tr>
<td>4</td>
<td>0.252</td>
<td>5.01</td>
<td>0.1</td>
</tr>
<tr>
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<td>0.816</td>
<td>9.00</td>
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<tr>
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<td>0.894</td>
<td>9.41</td>
<td>0.512</td>
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<td>7</td>
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<td>15.95</td>
<td>2.278</td>
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<td>Weighted Average</td>
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<td>0.643</td>
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</table>
### Table 7 Default Correlation by the SUR model and Simulation

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<th>Threshold 1</th>
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<th>Threshold 2</th>
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<tbody>
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<td>1 2 3 4 5 6 7</td>
<td></td>
<td>1 2 3 4 5 6 7</td>
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<td>0.004 0.028</td>
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<td>0.013 0.042</td>
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<td>0.015 0.023</td>
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<td>0.030</td>
<td>1</td>
<td>0.028</td>
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<td>7</td>
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</table>

### Table 8 Expected Loss (EL) and Unexpected Loss (UL)

<table>
<thead>
<tr>
<th>Type</th>
<th>Expected Loss (%)</th>
<th>Unexpected Loss (%)</th>
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</thead>
<tbody>
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<td>Threshold 1</td>
<td>0.218</td>
<td>1.051</td>
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<tr>
<td>Threshold 2</td>
<td>0.161</td>
<td>0.912</td>
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<tr>
<td>Average</td>
<td>0.190</td>
<td>0.982</td>
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