Fondazione Eni Enrico Mattei

NOTA DI LAVORO

02.2011

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By **Alexander Golub,** Environmental Defense Fund

Daiju Narita, Kiel Institute for the World Economy

Matthias G.W. Schmidt, Potsdam Institute for Climate Impact Research

SUSTAINABLE DEVELOPMENT Series
Editor: Carlo Carraro

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Summary

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JEL Classification: D81, Q54, C61

Address for correspondence:

Alexander Golub
Environmental Defense Fund
1875 Connecticut Ave.
NW, Washington
DC 20009
USA
E-mail: agolub@edf.org

Uncertainty in integrated assessment models of climate change:

alternative analytical approaches

Alexander Golub[1*], Daiju Narita[2], Matthias G.W. Schmidt[3]

Abstract

Uncertainty plays a key role in the economics of climate change, and the discussions surrounding its implications for climate policy are far from settled. We give an overview of the literature on uncertainty in integrated assessment models of climate change and identify some future research needs. In the paper, we pay particular attention to three different and complementary approaches that model uncertainty in association with integrated assessment models: the discrete uncertainty modeling, the most common way to incorporate uncertainty in complex climate-economy models: the real options analysis, a simplified way to identify and value flexibility: the continuous-time stochastic dynamic programming, which is computationally most challenging but necessary if persistent stochasticity is considered.

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1 Introduction

There is broad recognition that uncertainty is an essential aspect of the economics of climate change (see Heal and Kriström, 2002, for a general overview). In fact, all parts of the cause-effect chain are highly uncertain, from greenhouse gas emissions and mitigation costs to the response of the climate system and the resulting impacts. The uncertainty is both parametric[4] and stochastic. Uncertainty about climate sensitivity and the damage function, for instance, is parametric. Uncertainty about temperature and economic output, for instance, contains a significant additional stochastic component due to unresolved processes.

It can be expected that uncertainty associated with climate change will at least partly be resolved in the future. Parametric uncertainty will be reduced by additional and improved

[1*]Corresponding author: Environmental Defense Fund; 1875 Connecticut Ave., NW, Washington, DC 20009, USA
Email: agolub@edf.org

[2] Kiel Institute for the World Economy; Hindenburgufer 66, 24105 Kiel, Germany
Email: daiju.narita@ifw-kiel.de

[3] Potsdam Institute for Climate Impact Research; Telegraphenberg A31, 14473 Potsdam, Germany
Email: schmidt@pik-potsdam.de

[4] or more generally structural uncertainty

measurements and modeling. Past realizations of the stochastic shocks will also become known. It will be possible to adjust climate policy to this information. Since decisions such as investments and emissions are at least partly irreversible, it is also important to anticipate future learning about uncertainty and to adjust today's decisions accordingly.

The key question is how uncertainty influences the optimal timing and stringency of climate policy compared to a deterministic world, where all uncertainties are replaced by their expected value. A closely related question is whether mitigation effort should be deferred until more information is received or whether mitigation should be done preemptively. Indeed, the argument to delay action, or of keeping non-commitment, in the face of uncertain climate change has some traction in the policy debate, as witnessed by skeptics' attention to certainty of scientific facts (e.g., certainty of the rising trend of the global mean surface temperature).

Theoretical studies have shown that the answers to these questions are ambiguous. Therefore, an increasing number of studies have dealt with them in numerical integrated assessment models (IAMs), the primary tool to investigate complex climate-economy interactions. Although many insights have been obtained, there is no general agreement on the policy implications of uncertainty, and a number of research gaps exist. In Section 2, we provide a brief overview of the literature.

Getting answers to the questions above from IAMs is challenging. Modeling all relevant processes and uncertainties would demand dynamic stochastic optimization in a complex climate-economy model. However, numerical stochastic dynamic optimization generally necessitates a large number of iterations to compute an entire set of contingencies, and the size of computation easily becomes prohibitively large in the modeling of complex systems such as climate-economy interactions – the challenge lies not only in computational time but also in the attainment of solution convergence. To date, major full-scale IAMs have not adopted the stochastic dynamic optimization method to model uncertainty. Even if such models could be solved, reduced models would remain worthwhile for transparency and isolation of effects. Simpler models are obtained by reducing the information dynamic complexity or the system dynamic complexity. Information dynamic complexity includes the uncertainties considered, the sampling of the distributions, and the number of points in time where additional information is received. System dynamic complexity refers to the detail with which the climate and the economy are modeled. In this paper, we discuss approaches of simplification, corresponding methods and results.

Particularly, we discuss three complementary approaches in greater detail. Parameter uncertainty is normally treated with reduced information dynamic complexity and greater system dynamic complexity. Discrete uncertainty modeling (DUM) (or sequential decision-making under uncertainty) assumes that information is received only once or a few times, and that the distributions are discrete. We also discuss a special kind of real options analysis (ROA) that uses methods from option pricing to estimate the costs and benefits from climate policy taking continuous distributions and potentially tails into account. Meanwhile, stochasticity has to be treated in a stochastic dynamic programming (SDP) framework, which is only tractable in simple system dynamics. All three approaches are based on a Ramsey-type growth model and analyze the tradeoff between present consumption (utility) and future consumption. In the model, future consumption is influenced by climate change resulting from previous emissions from production.

The paper is structured as follows. Section 2 gives an overview of the integrated assessment literature on uncertainty. Section 3 is about parametric uncertainty. It discusses DUM (3.1) and ROA (3.2). Section 4 discusses stochasticity in a SDP framework. Section 5 contains the conclusions and some future research needs.

2 Literature review

In the economics of climate change, numerical analysis plays an important role because of the complexity of the problem. A number of studies have dealt with the issue of uncertainty using IAMs, the primary numerical tool to investigate climate-economy interactions, and various findings have been obtained (see also Peterson, 2006, for a review of results). At the same time, research efforts are still fragmentary, and a number of research gaps exist.

The uncertainties involved in climate change can be grouped into two types, parametric uncertainty and stochasticity (Kelly and Kolstad, 1999a; Kann and Weyant, 2000; Peterson, 2006). The former describes the current incomplete knowledge of relevant parameters (e.g., the climate sensitivity to the increase of carbon dioxide concentrations), and the latter describes persistent randomness of the climate and other systems never to be completely resolved in the future. Most existing studies of IAMs examine the first category of uncertainty.

Methodologically, the simplest numerical approach to model uncertainty is uncertainty propagation (also called stochastic simulation), which is essentially a Monte-Carlo analysis utilizing (joint) probability distributions of input parameters. The probability distributions are normally set up based on empirical data. Because of relative methodological tractability especially in large IAMs, many IAM studies adopt this approach to simulate uncertainty. Major studies in this group include Cline (1992), Dowlatabadi and Morgan (1993), Nordhaus (1994, 2008), Ha-Duong et al. (1997), Zapert et al. (1998), Gjerde et al. (1999), Pizer (1999), Scott et al. (1999), and Tol (1999). The advantage of this method is the capacity to include multiple sources of uncertainty simultaneously, and it is therefore useful for obtaining a general sense about relative importance of different uncertainties for the outcome. A general finding from this strand of modeling studies is that it is ambiguous how the inclusion of uncertainties affects the desirable reduction paths of greenhouse gas emissions – for example, a long upper tail of the probability distribution for the climate sensitivity parameter may favor enhanced mitigation, but a long upper tail for the uncertainty of total factor productivity may bring down the justifiable levels of mitigation.

Note that an implicit assumption of uncertainty propagation is that the decision maker is unaware of the multiplicity of future contingencies and does not adjust his or her decisions according to her risk preferences. In fact, a major problem with uncertainty propagation is that it is generally unable to examine how uncertainty itself influences the optimal choices. A condition for uncertainty propagation to compute optimality under uncertainty is that the optimal decision is equivalent to the expected value of the decisions under each realization of the uncertain parameters (certainty equivalence). Due to the economic agents' risk aversion and other non-linearities in the model, however, this condition generally does not hold.

DUM, which is further discussed in Section 3.1, manages this problem by simplifying the representation of uncertainty. It thus can consider the optimality of actions under explicit uncertainty. Often there is only a single uncertain parameter, represented by a small sample, and uncertainty is resolved at a fixed time in the future. Optimal decisions are computed by taking the future revelation of uncertainty into account.

Three fairly robust conclusions have emerged from DUM in different models and for different uncertain parameters. The first result is that the value of information is significant. It is very valuable to learn about the key uncertainties of the climate problem, particularly climate sensitivity and climate impacts. This was shown in different IAMs by Nordhaus and Popp (1999), Peck and Teisberg (1993), and Lorenz et al. (2010).

The second result is that in cost-benefit analysis both static uncertainty and future learning often only have a small and ambiguous effect on the near-term optimal climate policy. In other words, there is little difference between the red and the green trajectory in Fig. 2, and little difference between the orange and the red trajectories. This is shown for uncertainty about climate sensitivity and damages by amongst others Peck and Teisberg (1993), Webster (2000), Webster et al. (2008), and Lorenz et al. (2010), and for uncertainty about population growth by O'Neill and Sanderson (2008). Nordhaus (1994) finds that learning is an argument for less abatement, whereas Ulph and Ulph (1997) show the reverse under certain circumstances. Studies performing cost-effectiveness analysis in general find lower optimal emissions with uncertainty and learning (parts of Webster et al. (2008); Bosetti et al (2009). But Schmidt et al. (2010) argue that these latter results should be taken with caution because of two reasons: The first reason is their controversial interpretation of climate targets under uncertainty as strict targets that have to be met with certainty (probabilistic targets are investigated in Held et al., 2009). The second reason is that they implicitly introduce an artificial cut-off in the distributions – in particular of climate sensitivity - in order to render such a strict 2°C target feasible.

The third result is that the inclusion of a potential irreversible climate threshold with uncertain corresponding damages can be an argument for stricter climate policy. This was shown by Keller et al. (2004), by Dumas and Ha-Duong (2005), and by Lorenz et al. (2010). The justification of stricter policy is that it keeps the option open to avoid the threshold if it is learned to be severe.

The IAM studies using DUM have a conceptual link to the literature on irreversibility and the quasi-option value in the context of climate change. Some studies point out that early emission reductions carry an option value due to the irreversible nature of atmospheric greenhouse gas accumulation (Arrow and Fisher, 1974; Henry, 1974; Gollier et al., 2000). But they also show that capital investment in mitigation infrastructure is sunk or irreversible as well (i.e., infrastructure will be wasted if climate change proves to be inconsequential), which provides delayed action with an option value. Therefore, on balance, the effect of uncertainty on the optimal level of mitigation is ambiguous (Kolstad, 1996; Pindyck, 2000; Fisher and Narain, 2003).

A promising approach closely related to the quasi-option value literature is ROA (Dixit and Pindyck, 1994; Pindyck, 2000; Baranzinia et al. 2003; Lin et al., 2007; Anda et al. 2009; see Aslaksen and Synnestvedt, 2004, for a clarification of the relation between quasi-option values and option values in general), which is discussed in greater detail later. The real option value can be seen as an extension of the net present value (NPV), which is most commonly used in cost-benefit analysis. Unlike the NPV, the real option value can incorporate the value of flexibility in the presence of uncertainty. It is further discussed in Section 3.2.

Finally, there are yet very few economic studies that address the persistent stochasticity of climate change, though as mentioned above, it is already recognized as a distinct class of research subjects. An early attempt is Lontzek and Narita (2009), who estimate optimal emissions reductions taking the randomness of the climate system into account. Computation on this front is technically more demanding, it needs stochastic dynamic programming. Nevertheless, this area has a large research potential, as the climate change embodies many sources of persistent stochasticity. It is further discussed in Section 4.

3 Parametric Uncertainty

Large part of the uncertainty in climate change can be attributed to parameters in the IAMs. Key parameters are climate sensitivity, the parameters in the damage function, and the learning rates of technologies, amongst others. In contrast to stochasticity, this uncertainty could at least in principle be resolved completely. Most models assume an exogenous reduction of uncertainty justified by projected advances in measurements and modeling. Exceptions are Kelly and Kolstad (1999) and Leach (2007), who model the resolution of uncertainty endogenously.

We discuss two approaches to model climate policy under parametric uncertainty, the discrete uncertainty modeling (DUM) and a special form of real options analysis (ROA). Both approaches essentially tackle the same problem, but the solution concepts and necessary approximations are different. Firstly, DUM directly performs a policy optimization, whereas ROA only values different pre-specified policy options such as interim stabilization targets. Secondly, DUM can only take simple discretizations of the probability distribution into account, whereas ROA can handle continuous distributions with tails. Thirdly, in DUM risk aversion is considered explicitly by a utility function. This could also be done in ROA, but for simplicity the risk aversion is often neglected.

3.1 Discrete Uncertainty Modeling

DUM introduces several approximations and simplifications. Firstly, it discretizes the probability distributions, usually rather crudely, by a couple of sample points. Secondly it discretizes the updating process. Information arrives continuously, and in theory climate policy could be adjusted continuously. But information pooling and climate policy formation are slow and complex processes. The IPCC publishes its reports every 7 years, it took five years to negotiate the Kyoto Protocol, 15 years to build consensus on the 2^0C threshold as a long-term environmental target, and it may still take several years to get the major developing countries to commit to absolute emission targets. Therefore, it is not unrealistic to assume that an initial near-term climate policy up to 2030 or 2050, for instance, is revised only once or a couple of times. In accordance with most of the literature, we will focus on the case of only one learning step in the following. This greatly simplifies the information dynamic complexity and the calculation and interpretation of optimal policies and values of different policies as discussed in the remainder of this subsection. It would also be interesting to have a systematic study of how valuable more frequent adjustment would be. Thirdly, and in contrast to the approach in Section 4, it assumes a finite planning horizon. Most studies use a planning horizon of between 100 and 300 years.

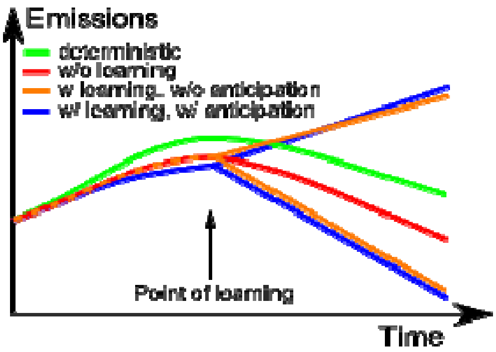
With these approximations, the stochastic dynamic problem in a generic IAM can be written as a static finite sequence problem. If we denote the finite horizon by T, felicity for consumption at time t in state of the world s, $c_{t,s}$, by $u(c_{t,s})$, the rate of pure time preference by ρ, the possible messages containing information about the uncertain parameters by m, their respective probability by q_m, the posterior probability after receipt of message m by $p_{s|m}$, investments at time t for receipt of message m by I_t^m and the state variables by $K_{t,s}$, we get:

$$\begin{array}{lllll} \max\limits_{\{I_t^m\}} & \sum\limits_{m=1}^{M} q_m \sum\limits_{s=1}^{S} p_{s|m} \sum\limits_{t=0}^{T} \frac{1}{1+\rho} u(c_{t,s}) & & & \\ & s.t. \qquad c_{t,s} & = & g(K_{t,s}, I_t^m, s) & \\ & \qquad\qquad K_{t+1,s} & = & f(K_{t,s}, I_t^m, s), & \\ \forall t < t_l, \forall m < M & \qquad\qquad I_t^m & = & I_t^{m=1}. & \end{array} \tag{1}$$

The first constraint is the budget constraint, the second describes the system dynamics and the

Figure 1:

Scheme of optimal emissions in different scenarios. The cases with learning are depicted for only two messages.



third ensures that decisions can only be tailored to the information of a message after receipt at time t_l .

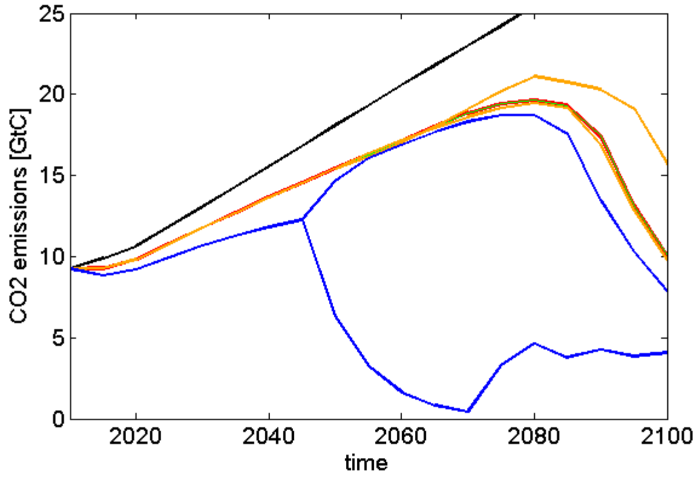
The advantage of a sequence formulation as in Eq. (1) as opposed to a recursive formulation is that it can be solved using efficient large-scale non-linear optimization solvers in modeling systems such as AMPL and GAMS. For more than on learning step, the number of decision variables and constraints increases exponentially, which quickly renders the problem unsolvable. For several learning steps, solution methods based on a recursive formulation are superior (see Section 4). The number of sample points and messages increases the number of constraints and decision variables linearly.

The sequence formulation Eq. (3) could in principle be used both for problems with parametric uncertainty and problems with stochasticity. Stochasticity, though, represented by an AR(1) as in Section 4, would introduce a random variable (shock) for each time-step and thereby blow up the calculation of the expectation in the sequence problem. Therefore, recursive methods are a better choice if stochasticity is considered.

Since there is only one learning point, the distinction between the following scenarios is particularly intuitive: (i) deterministic, where the uncertain parameters are fixed at their expected value; (ii) no-learning, where the parameters are uncertain and uncertainty is not resolved; (iii) non-anticipated learning, where the uncertainty is at least partly resolved but this is not anticipated. Decisions before learning coincide with decisions without learning; (iv) anticipated learning, where additionally learning is anticipated potentially leading to different optimal pre-learning decisions. The key results of DUM are differences in optimal policies in these scenarios and the associated welfare differences. A schematic of optimal emissions policies for only two possible messages is shown in Fig. (1).

We can then distinguish two effects: Firstly, static uncertainty has an effect on optimal emissions and associated welfare as compared to the deterministic scenario. This is the difference between the red and the green line in Fig. (1). This effect stems from non-linearity of the

Figure 2:
Optimal emissions in MIND for different scenarios described in the text. The color-coding is the same as in Fig. 1. The green and red lines essentially coincide.



objective function in the uncertain parameters. Secondly, learning has an effect on optimal emissions as compared to the no-learning case. This is the difference between the blue and the red line. The associated welfare increase is called the expected value of information (EVOI). The effect of learning can be decomposed into two parts. Firstly, optimal policy after learning will depend on what is learned. This is the difference between the orange and the red line. The associated welfare difference can be called an option premium (see Subsection 3.2). Secondly, if decisions are at least partly irreversible, then anticipation of future learning changes optimal near-term climate policy before learning. This is the difference between the blue and the orange line. The associated welfare increase can be called the expected value of anticipation (EVOA). The EVOA results from choosing a near-term policy with a higher option value. Anticipation of learning is an argument for maintaining flexibility, i.e. against irreversible decisions. There are several counteracting inertias and irreversibilities involved in the climate problem. Investments in mitigation are mostly sunk, and emissions and climate processes such as the disintegration of the West-Antarctic ice-shield are essentially irreversible.

In Section 2, we discussed a few robust results that have emerged from studies applying DUM in IAMs. Here we shortly discuss some results on optimal emissions policy in the face of an uncertain damage threshold obtained in the MIND model (see Lorenz et al., 2010, for details). The threshold location is 2.3°C and assumed to be known. The implied damages are uncertain and either 20% of GDP or 0% of GDP with probability 7.5% and 92.5%, respectively, so that expected damages are 1.5%. Whether the threshold is bad or not is learned in 2050. Optimal emissions in the different scenarios are shown in Fig. 2. Static uncertainty (red line) does not change optimal policy compared to the deterministic case (green line). Non-anticipated learning (orange lines) only has a small effect, which is due to the fact that the threshold cannot be avoided anymore after learning if the learning wasn't anticipated. The small effect is due to the imposed damages, which change optimal consumption behavior. Anticipation of learning (blue lines) leads to significantly stronger emissions reductions. These emissions reductions lead to higher costs but provide the option to avoid the threshold after learning about the severity of the threshold. Lorenz et al. (2010) show that anticipation is only important for specific combinations of threshold locations and learning times, and that if it is important it argues for stronger emissions reductions.

3.2 Real Options Analysis

ROA uses methods from financial option pricing to value the managerial flexibility inherent in real investment decisions. As discussed in the previous subsection, future flexibility depends on near-term climate policy. If uncertainty is resolved in the future, flexible near-term policy carries an option premium above its NPV, which can be estimated by ROA. As opposed to DUM, ROA does not calculate an optimal policy but only calculates whether benefits including option values of a given policy exceed costs. Here, we focus on greenhouse gas concentration targets as available policies, and in particular on the 450ppm target as an example.

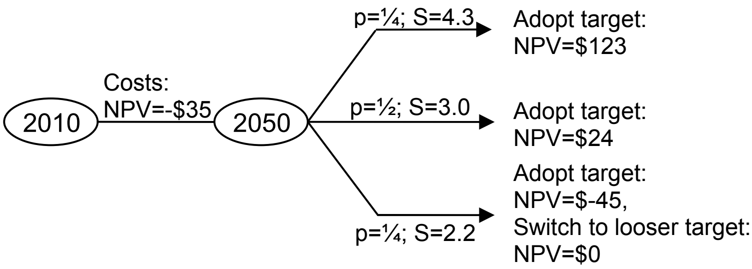
A simple example depicted in Fig. 2, demonstrates the option value concept. It shows that it can be economical to adopt an interim concentration target even if the NPV of the long-term target is negative. Uncertainty is about climate sensitivity (S), which can only take one of three possible values. The true value is learned in 2050. The mitigation costs of the interim target up to 2050 are \$35 trillion. Benefits in the form of avoided climate damages accrue only after 2050 and depend on the true value of climate sensitivity. Adoption of the long-term target for all S has a negative NPV of -\$3.5 trillion. But if we add the option to abandon the target for a looser one in case S is low, we get an expanded NPV of \$7.75 trillion. Thus, the option premium is \$11.25 trillion. The interim target can then be seen as providing a call option on the long-term target. The value (net its costs) of this option is positive, whereas the value of the one-shot long-term target itself is negative.

More generally, the expanded NPV of interim target E_t for time *t,* where uncertainty is resolved, is given by

$$F(E_t) = V(E_t) - Z(E_t)\,, \qquad (2)$$

where $V(E_t)$ denotes the present value of the European call option on the long-term target with expiration time *t* and strike price zero, and $Z(E_t)$ denotes the NPV of mitigation costs up to t. The zero strike price of the option stems from the assumption that when it turns out at time *t* that future costs of the long-term target exceed benefits then a zero NPV can be achieved by loosening the target. The option effectively cuts off the negative side of the net benefit distribution of the long-term target after *t*.

Figure 2 Simple demonstration of the option value of interim target. With probability ¼ climate sensitivity is equal to 4.3; with same probability ¼ S=2.2 and S=3.0 otherwise.



In order to determine the price of the option $V(E_t)$ using option pricing formulas or risk-neutral valuation, one needs the spot price of the underlying asset. For financial options and standard real options this price can be observed in markets, or at least the price of a traded twin security if the asset itself is not traded. There are no markets for long-term climate targets. Therefore, we have to price the asset from its payoffs. One possibility is to calculate the expected payoffs and discount them back at an appropriate risk-adjusted discount rate. Another possibility is to calculate the certainty equivalent payoffs and then discounting them back at the risk-free rate. Both require the utility function of a representative agent. Since our primary concern here is to demonstrate the option value of interim targets, we simply neglect the risk aversion and assume that the price of the asset equals its expected payoff discounted at the risk-free rate. In particular, the present value of net benefits after *t* is given by their expected value discounted at the risk-free rate, denoted by $\overline{D}_{\text{net}}(E_t)$. Then we compare the net benefits of climate policy including the option premium after t with costs before t.

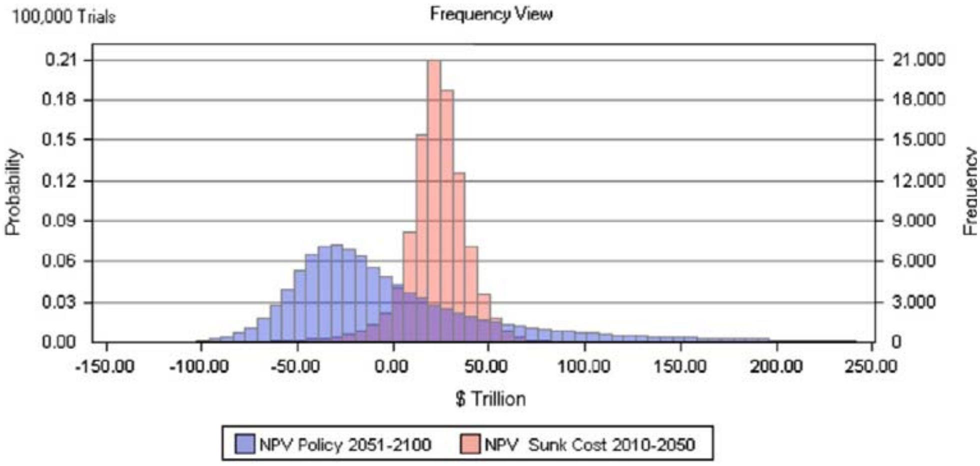
Most option pricing formulas such as Black-Scholes's presume non-negative prices. The price of a long-term target (i.e. under assumed risk neutrality, the discounted expected net benefits) can be negative. We first use Bachelier's model, in which prices follow a Brownian motion (instead of a geometric Brownian motion) and can be negative. We discuss how to use formulas for non-negative prices below as a simple extension. If we denote the spot price by P_0, the present value of the strike price by K, and the aggregate volatility of the asset (standard deviation of aggregate returns) by σ, Bachelier's formula for a call option reads

$$V^B(P_0, K, \sigma) = (P_0 - K)\Phi\left(\frac{P_0 - K}{P_0\sigma}\right) + P_0\sigma\phi\left(\frac{P_0 - K}{P_0\sigma}\right), \qquad (3)$$

where Φ and ϕ are the cumulative distribution function and the probability distribution function of the standard normal distribution, respectively. Since we used the aggregate volatility $\sigma = \sigma'\sqrt{t}$, where σ' is instantaneous volatility, and a discounted strike the time to expiration does not occur explicitly in the pricing formula. The value of the option on the long-term target is then given by $V(E_t) = V^B(\overline{D}_{\text{net}}(E_t), 0, \sigma_{\text{net}}(E_t))$..

In the following we estimate the NPV of a long-term 450 ppm target and the option value of an interim target up to 2050 similar to Golub et al. (2008). The interim target is defined by the

Figure 3: Distribution of costs and net benefits before and after 2050, respectively. (Source: Anda et al., 2009).



concentration that is obtained in 2050 for the cost-effective policy achieving the 450 ppm long-term target. We assume the following: S is log-normally distributed with log-mean 1.09 and log-standard deviation 0.4. We assume damages are quadratic in temperature increase ($D(T) = a T^2$) and a is log-normal with log-mean 0.5 and log-standard deviation 0.25. Mitigation costs are calibrated based on IPCC's Fourth Assessment Report: 2% of the gross world product (GWP) in 2030; 4% in 2050 and 5% in 2100. We assume that the cost distribution has a standard deviation of 0.5%. All uncertainty is exogenously resolved in 2050.

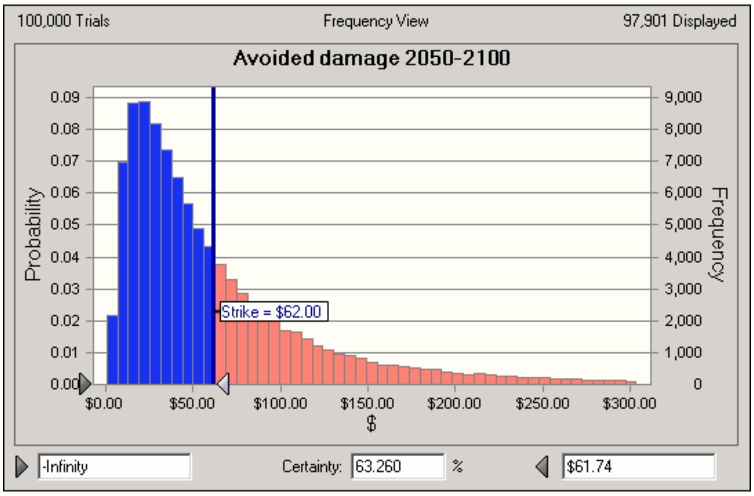
First, we estimate the NPV of the one-shot long-term 450 ppm target, i.e. if there is no flexibility to adjust the target in 2050. Fig. 5 shows the distributions of net benefits after 2050 and mitigation costs before 2050 of the 450 ppm target. They were obtained by Monte Carlo simulation using the parameter values described above. It is obvious that expected costs are larger than expected benefits: the NPV is -$14 trillion. Now we include the option to switch to a looser target in 2050. The mean and standard deviation of net benefits discounted back to 2010 are $7.33 trillion and $83.00 trillion, respectively. Plugging these values in the option pricing formula Eq. (3) and subsequently into Eq. (2), we get a positive expanded NPV of the interim target of $36.91 – $22 = $14.91 trillion.

As we can see in Fig. 3, the net benefits of climate policy show a significant skewness and kurtosis. Bachelier's model assumes a normal distribution of asset prices and cannot capture higher moments of the distribution. It therefore underestimates the option value of the underlying asset. Other option pricing formulas such as the one by Black and Scholes for log-normally distributed value of underlying asset or more sophisticated option pricing formulas such as the one by Gram-Charlier of Edgeworth binomial can take

Table 1 Costs and benefits of interim target

Net cost 2010-2050	22
Implementation cost 2051-2100	62
Avoided damages 2051-2100:	
Mean	69
Standard Deviation	81
Skewness	5
Kurtosis	53

Figure 4:
Distribution of avoided damages after 2050. Also shown is the strike price of the option corresponding to the NPV of mitigation costs after 2050



higher moments into account. But these pricing formulas require non-negative prices of the underlying asset. Therefore, we take the avoided damages (gross benefits) of the target after 2050 as underlying asset, which are non-negative, and expected mitigation costs after 2050 as the strike price (see Anda et al, 2009, for details).

Application of the Black-Scholes option pricing formula yields an option value of $33.5 trillion. Taking into account the net cost of the interim policy (between 2010 and 2050), this leads to a positive expanded NPV for the interim target of $11.5 trillion. As we can see in Fig. 4, the distribution of avoided damages shows a significant tail. The application of more sophisticated option pricing formulas that take into account all moments of the distribution (such as the Edgeworth binomial) yields an option value of $75 trillion.

In general, if there is an upper tail in the avoided damage distribution, even a deep out-of-the-money option (PV of expected cost higher than PV of expected avoided damage) could have a sufficient value to justify a relatively aggressive interim policy.

Until now we demonstrated how the option value can be applied to test a “prescribed” policy. It is more important, though, to consider the option value as a part of optimization processes. The following extension of the method just presented is possible: If experimentation with the model shows that the coefficient of variability of costs and benefits is independent of the interim target, then the expanded NPV for all interim targets can be specified as a function of the expected benefits and expected costs alone. This is the case in the DICE 2007 model (Nordhaus, 2007). One can then simply run a deterministic optimization of the interim target, where all parameters are fixed at their expected value, and adjust for the option value over this function. In this sense, ROA becomes an extension of deterministic optimization with perfect foresight. Higher relative variance of net benefits will justify a more restrictive interim target and vice versa.

4 Stochasticity

At present, most discussions focus on parametric uncertainty and uncertainty about potential future catastrophes. Additional uncertainty stems from the persistent stochasticity of both the climate and the economy, i.e. continuous system shocks drawn from a finite probability distribution. Stochasticity of the climatic system, albeit not the centerpiece of the current debate, could still be relevant in the context of the actual climate change that is taking place: an intuition may be drawn from the resemblance between the trends of economic growth and the actuality of climatic (or long-term weather) patterns, which show persistent random fluctuations despite the relatively smooth progression of human carbon dioxide emissions or atmospheric carbon dioxide concentrations. While such random fluctuations do not produce irreversible system behavior since the randomness-subjected parameters can return to the original levels, they still influence long-run developments of the system, as randomness affects the investment choices made by the risk-averse economic agent.

The analytical aspects of how random productivity shocks with given probability distributions affect the optimal long-term economic decisions are discussed elsewhere (for example, Weitzman, 2009, and Gollier and Weitzman, 2010, discuss this issue in relation to the climate change problem). In the simplest case, the effect is expressed by the so-called extended Ramsey rule, which is discussed by Gollier (2007) in detail. The rule says that the effect of randomness in utility (a constant relative risk aversion utility) following the distribution N(μ, σ) could be represented with the risk-free interest rate r^f given below:

$$r^f = \rho + \eta\mu - \eta^2\sigma^2/2, \qquad (5)$$

where ρ is the pure time preference, and η is the risk aversion parameter (intertemporal elasticity of substitution). The equation means that randomness (the variance) influences the agent's intertemporal decisions by reducing the effective interest rate taken into account by the risk-averse agent. A greater randomness (a greater σ) results in a smaller risk-free interest rate (a smaller r^f). Note that the third term of the right-hand side in the above formula is zero when η is zero – randomness does not cause any intertemporal effect if the agent is risk-neutral (η=0).

It is easy to infer from the extended Ramsey rule that continuous system shocks can generally affect optimal policy decisions taken by the risk-averse agent, and that such effects can be complex in a more elaborate model setting. For example, in a more relevant case to climate change, the agent may have a choice to conduct either investment in production capital or climate change abatement besides the current consumption. The randomness of climate change should influence the balance between the two by affecting their effective interest rates, and it is not obvious which alternative the randomness favors. However, representation of such multiple interactions under uncertainty is normally beyond the scope of simple analytical models, and this is an area where numerical analysis of stochastic dynamic optimization can play a role.

One such example of randomness introduced in a stochastic model is Lontzek and Narita (2010), who perform a numerical analysis of stochastic dynamic optimization incorporating fluctuations of the climate system (which are translated to fluctuating climate change damage). Their model is a simplified Ramsey-type numerical representation of climate change and the economy, which estimates the optimal decisions balancing the investment in production capital, the abatement, and the current consumption. The maximization problem is expressed as follows:

$$\begin{aligned} &\max_{c,m} \int_t U(c,T)e^{-\rho t} \\ &s.t. \quad dK = G(K,m,T)dt \qquad , \\ &\qquad\quad dT = H(K,c,m)dt + \sigma dB \end{aligned} \tag{6}$$

where U is the utility, c the consumption, T the temperature, K the capital, m the emission control rate, and σ the degree of randomness.

Randomness of climate change affects the effective (risk-free) return to investment and abatement through indirect relationships linking uncertainty and both actions. In general, the above problem is not solvable analytically. Lonzek and Narita thus compute numerical solutions by using a collocation method. The value function, the emission control m and the consumption c could be described as functions of two state variables, K and T, Those functions are approximated as linear combinations of Chebyshev polynomials at certain nodes set in the K-T space. Coefficients of polynomials are optimized so that they satisfy the Hamilton-Jacobi-Bellman equation of the maximization problem.

Their results show that the relative effect of stochasticity to the deterministic solutions varies even qualitatively (in sign) throughout the state space (Fig. 5), implying that effects of uncertainty can be significantly different depending on the state of the system and also the model configurations. Nonetheless, a robust finding is that uncertainty could both increase and decrease the optimal level of abatement depending on the combination of state variables and the level of risk aversion.

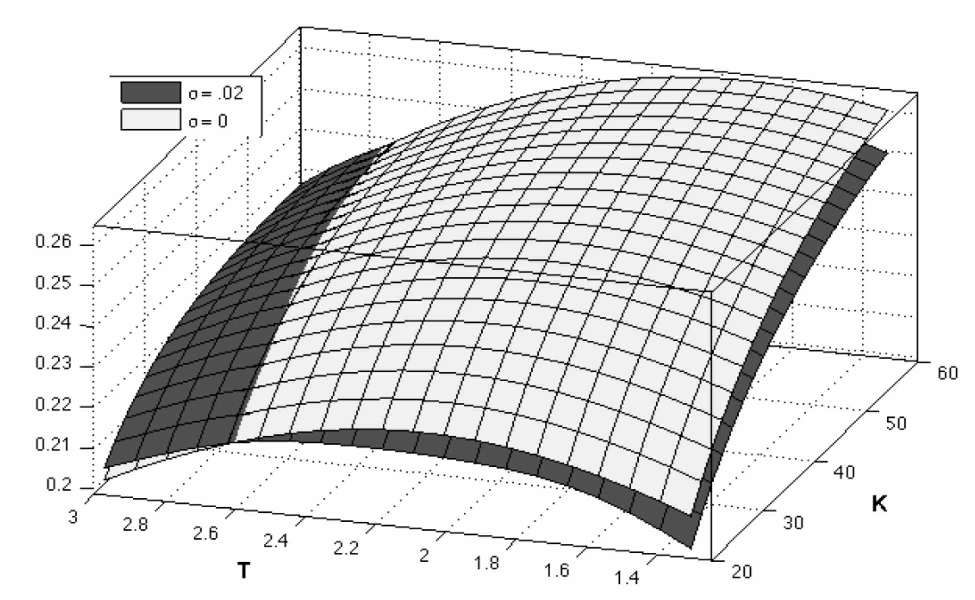


Figure 5 An example of effect of uncertainty on the optimal mitigation estimated by using stochastic dynamic optimization (taken from Lontzek and Narita, 2010). The dark-grey and light-grey surfaces signify the stochastic and deterministic cases, respectively. Each surface represents possible combinations of the global temperature increase (T), the capital (K) and the level of mitigation (the vertical axis: expressed as fraction of emissions). Note that the two surfaces intersect along a line in the graph – in this setting, uncertainty means larger optimal mitigation efforts at a high temperature and lower optimal mitigation efforts at a low temperature.

Applications of numerical stochastic dynamic optimization to the climate change problem are still in the beginning stage and could be extended in various ways in the coming years. For example, the approach has a potential to make a contribution to the literature of irreversible changes in the future. In the existing studies, irreversibility is usually introduced exogenously in forms such as catastrophic events and thresholds. In contrast, Figure 5 suggests that some threshold might endogenously emerge in some stochasticity models. In the model described above, uncertainty might "push" either abatement or emission levels higher depending on the current levels of state variables. An interesting question is whether this feature is preserved in presence of discrete uncertainty, and how the magnitude of uncertainty affect patterns in such a case – the investigation of this issue is left to future research.

5 Conclusions and Future Research Needs

We now give some conclusion and future research needs in the context of the three approaches discussed in Sections 3 and 4.

The literature on parametric uncertainty in IAMs is already more conclusive than the literature on stochasticity. Some fairly robust conclusions from the DUM literature have been discussed in the literature review in Section 2. At least in cost-benefit analysis, uncertainty and learning have only a small effect on the optimal near-term climate policy unless uncertain climate thresholds are included, which is an argument for stricter near-term policy. Hence, future learning is not an argument for delayed action.

Some remaining research needs in the context of parametric uncertainty and DUM are: What is the effect of endogenous resolution of uncertainty (or active learning)? This is particularly important for learning about technology uncertainty, whose resolution depends on the installed capacity, and for the risks associated with geoengineering, whose resolution depends on the application of geoengineering. The obstacle here is that an endogenous learning time cannot be modeled by DUM and demands recursive methods. The same is true for the question, how more regular updating of climate policy changes optimal near-term policy and the value of information. Another interesting question that has not yet received much attention is how alternative preferences, such as habit formation, direct utility from an environmental good, distinction between risk aversion and intertemporal elasticity of substitution, influence the optimal policy under uncertainty.

One potential extension of ROA of climate policy is the calculation of risk-adjusted cost and benefits of climate policy that allows us to solve a deterministic forward-looking model instead of another possibility – DUM with learning in IAMs. A critical issue is to better account for irreversibility. There are two ways to model irreversibility on the climate side. The first way is the modification of climatic module introducing positive feedback. The second alternative is a modification of the damage function. Even if the temperature eventually goes down, some damages may stay for a longer period of time, such as a disintegration of ice shield and losses of biodiversity. On the cost side, more attention should be dedicated to the lost opportunity to abate (and maybe bank abatement) earlier. Now the mainstream of the literature focuses on abatement sunk cost, but inertia of the economy should be taken into account too. It may be impossible to significantly increase abatement facilities at the time we learn that climatic changes are more severe than we expected.

Another important field for ROA is the estimation of option values for earlier adoption of an emission target for developing countries. More aggressive policy from the beginning will navigate developing countries into a low-carbon economic growth pathway. Then developing countries will avoid sunk cost by avoiding investments into carbon intensive production capital. These benefits could be estimated by applying the option methodology.

A general feature observed in stochasticity analysis is that the effects of uncertainty on abatement are not clear-cut (uncertainty may both enhance and decrease abatement) since stochasticity affects the agent's decisions through multiple and convoluted channels involving climate change damage, capital accumulation and utility obtained from consumption. Risk of random climatic patterns can induce both more abatement to reduce fluctuations of climate change damage, and less abatement to divert resource elsewhere (i.e., investment or consumption) from abatement. An interesting corollary to this thesis is that the response patterns of abatement to uncertainty could in fact show some thresholds reflecting the heterogeneity of uncertainty effects – thresholds are indeed seen in the example of Figure 4 where uncertainty leads to more aggressive climate policy in high temperatures but less aggressive policy in low temperatures relative to the deterministic benchmark. It is noteworthy that such thresholds could be produced endogenously, in other words, they could be generated without discrete future revelations of knowledge or catastrophes. In future research, this threshold feature should be examined also in different model formulations (including a variety of levels of σ) to verify robustness and clarify general characteristics.

Stochasticity analysis could be extended in two directions: *simplification* and *elaboration*. Stochasticity analysis, which tends to show complex, somehow unintuitive solutions as we have seen, could benefit from simplified representations that could isolate essential features. In a way, such analyses are to support the investigations carried out by analytical model studies in making use of numerical simulations as powerful methodological tool. On the other hand, stochasticity modeling should also be extended in such a way that it includes more complexity in order to represent the climatic and economic fluctuations more realistically. Through both simplification and elaboration, stochasticity analysis could bridge the gap between the general rules described by analytical models and the actual complexity of the climate-economy interactions as a random system – so the two directions of research are not mutually exclusive but complementary. Also, stochasticity could be modeled with an assumption of discrete policy updates (analogous to, for example, the assumption taken in the ROA approach discussed in Section 3.2); such an analysis might provide insights that could be integrated with those from the other analytical approaches of climate change uncertainty discussed in the previous sections.

Future research needs exist for the individual approaches of DUM, ROA, and stochasticity analysis with SDP, but in principle these could be analyzed in a unified fashion as well. Although a unified model seems still technically impractical at present, such a model, if developed in the coming years, may provide some original insights as well. Finally, we recognize some general unresolved questions in the context of uncertainty and the economics of climate change but outside the reach of standard (optimal growth) IAMs. They include: How does uncertainty influence the negotiations and optimal design of a global climate agreement (see e.g. Kolstad, 2007)? What are the implications of policy uncertainty on private sector investments (e.g. Blyth et al., 2007) and optimal policy instruments?

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