The Trade-Wage Debate in a Model with Nontraded Goods: Making Room for Labor Economists in Trade Theory

Sherman Robinson
International Food Policy Research Institute

Karen Thierfelder
U.S. Naval Academy

Trade and Macroeconomics Division
International Food Policy Research Institute
1200 Seventeenth Street, N.W.
Washington, D.C. 20036-3006 U.S.A.

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Table of Contents

Introduction .................................................. 1
The 1-2-2-3 Model ............................................. 4
  Shares and Elasticities .................................... 6
  Prices, Endowments, and the Balance of Trade ........... 8
  Stolper-Samuelson Theorem ................................ 9
  Rybczynski Theorem ........................................ 11
  Trade-Balance Effects ..................................... 11
  Multisectoral Extensions ................................... 12
Empirical Decomposition of Trade-Wage Links .......... 13
  Sensitivity Analysis ....................................... 13
  The U.S. Economy in the 1980s .......................... 15
Conclusion .................................................... 17
References ..................................................... 19
Annex: Derivation of the wage decomposition equation in the 1-2-2-3 model .................. 22

Abstract

The Heckscher-Ohlin-Samuelson (HOS) model in international trade theory provides a powerful general-equilibrium paradigm for analyzing the impact of changes in trade on factor returns. In the HOS model, factor returns are determined solely by commodity prices, which are determined on large world markets. Changes in factor supplies affect the structure of production and trade, but not relative factor returns. In this framework, there is little room for labor economists who focus on partial-equilibrium analysis of supply and demand in factor markets. We extend the HOS model to include “nontraded” goods, distinguishing them theoretically from “nontradable” goods. The resulting 1-2-2-3 model applies to one country with two production activities using two factors of production but consuming a third imported good. We show that the HOS model is a special case of the 1-2-2-3 model when imports and domestic goods are perfect substitutes. In the 1-2-2-3 model, the magnification effects in the Stolper-Samuelson and Rybczynski Theorems are greatly qualified. Furthermore, changes in relative wages depend on changes not only in world prices, but also in factor endowments and in the balance of trade—the model accommodates the approach of labor economists in a general-equilibrium trade model. We derive the analytic relationship and, using stylized data, do sensitivity analysis which indicates that wages are more sensitive to changes in factor supplies than to changes in prices or the trade balance. We also apply the model to aggregate data for the U.S. in the 1980s.

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Introduction

The link between changes in trade and changes in factor returns (wages and profits) has been a major theme in both trade theory and policy debate for over a century.1 Empirical work in the past 15-20 years indicates that there has been a widening of the gap between skilled and unskilled wages in the United States, rekindling the trade-wage debate.2 One school of thought argues that the widening wage gap is due to increased commodity trade between "North" and "South," with the developed North increasing exports of skilled-labor-intensive and capital-intensive goods to the less-developed South, and importing unskilled-labor-intensive goods from the South. [Wood (1994, 1995) and Leamer (1994, 1996).] Neoclassical trade theory —often summarized as the Heckscher-Ohlin-Samuelson (HOS) model— supports the view that such increased trade should widen the wage gap in the North.

The HOS model provides a powerful general-equilibrium paradigm that has framed the discussion of these issues by trade economists, and the policy debate has been strongly influenced by them as well. Three theorems in the HOS model have structured how trade theorists view the trade-wage debate. The Stolper-Samuelson Theorem states that, in a two-good, two-factor model, when a commodity price increases, the return to the factor used intensively in the production of that commodity will increase and the return to the other factor will decrease. The effect is magnified, with the change in relative wages exceeding the change in relative prices; a result that argues strongly for the importance of changes in trade in explaining changes in wages. [Deardorff and Stern (1994).] The Factor-Price Equalization Theorem makes an even stronger statement: assuming identical tastes and technology in all countries, free trade in commodities will lead to identical factor prices internationally, regardless of differing factor endowments across countries —a result often cited by protectionist pressure groups in the North. Finally, the Rybczynski Theorem shows that, with unchanged prices, changes in factor endowments will lead to a magnified change in the structure of production and trade, but no change in relative wages.

In contrast to trade theorists, labor economists tend to look internally at domestic factor markets in a partial-equilibrium framework. To explain shifts in relative wages, it seems natural to look at changes in supply and demand on labor markets. There is an extensive literature on changes in the skill composition of the labor force [Katz and Murphy (1992) and Freeman (1995)]. For example, Borjas, Freeman, and Katz (1992, 1996) argue that migration of unskilled labor from South to North has significantly affected labor markets in the North, increasing the relative supply of

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1The recent literature is surveyed by Burtless (1995). For a discussion of the debate in the late 19th and early 20th centuries, see Williamson (1995).

2The same argument is applied to Europe, where the effect is to increase unemployment in an environment of sticky wages. See Kosters (1994).
unskilled labor and so widening the skilled-unskilled wage gap. From the demand side, labor economists look at the factor content of domestic production and trade to see how trade affects factor markets. Changes in exports and imports are assumed to "displace" domestic production. Changes in the structure of production, in turn, are translated into implicit changes in aggregate factor demand, which are then assumed to affect equilibrium returns in factor markets. This factor-content approach has been widely used, perhaps most carefully by Wood (1994). In what can be seen as a corollary to this approach, Borjas (1994) and Borjas, Freeman, and Katz (1992) argue that changes in the U.S. trade balance in the 1980s effectively changed the skill-structure of the labor force and so changed relative wages.

There are variants within the trade-theory and labor-market approaches, and they are not mutually exclusive. For example, Williamson (1995) uses the term "globalization" to include increases in both commodity trade and international migration. Most empirical work attempts to evaluate their relative importance. Indeed, sorting out the different roles played by changes in technology, factor supplies, and increased international trade is a major theme of the policy debate. Richardson (1995, p. 35) identifies the challenge: "In short, the work that needs to be done in this area will bridge the gap between international and labor economics."

There are, however, serious theoretical problems in reconciling these different perspectives. While the HOS model embodies a consistent general-equilibrium theory of the operation of an open economy, which makes it an appealing candidate as an organizing framework for empirical work, many of the predictions from this theory are incompatible with the labor-market approach. In the HOS model, only changes in international prices will change factor returns. The chain of causation runs from prices, which are determined in large world markets, to production, to wages. Changes in commodity trade, rather than prices, clear domestic product markets. Changes in relative factor supplies (and changes in technology that are equivalent to changes in endowments) affect the structure of production and trade, probably greatly, given the magnification effect, but will not affect relative factor prices. In this framework, there is no room for factor content analysis—a point made by a number of trade economists.

There have been a number of attempts by trade theorists to loosen the strict HOS framework and provide models where changes in factor supplies can affect relative wages. Richardson (1995), Jones and Engerman (1995), and Leamer (1996) discuss models with biased technical change, ranges of specialization, and specific factors. Leamer probably stays closest to the HOS model, arguing for

3Robinson et al. (1993) and Burfisher, Robinson, and Thierfelder (1994) explore the impact of migration in the context of the North American Free Trade Agreement (NAFTA), and argue that empirically migration affects wages more than changes in commodity trade. See also Abowd and Freeman (1991).


5Bhagwati and Dehejia (1994), Deardorff and Hakura (1994), and Deardorff and Staiger (1988) not only discuss the theoretical limitations of factor-content analysis in the HOS model, but also consider how stringent are the assumptions of the HOS model.
a large role for changes in relative prices. Richardson (1995, p. 42) focuses on the role of technical change, stating: "The debate increasingly and properly isolates trade prices and sectoral total factor productivity differences as the causes of long-run factor-price movements. Trade volumes are correspondingly treated endogenously. Neutral and factor-augmenting technological change is seen to be just like factor-supply change, with innocuous impacts on factor prices."6

A number of trade economists agree with Richardson that technical change is important [Lawrence and Slaughter (1993), Krugman and Lawrence (1993), and Baldwin (1995)]. Bhagwati and Dehejia (1994), for example, conclude (p. 71) "... that the empirical evidence to date fails to put the burden of the explanation for the observed decline in real wages of the unskilled on freer trade, leaving technology and technical change as the key culprits." Wood (1994, 1995) argues that there are causal links between technical change and trade, accepting the view that technical change is important, but linking it back to trade.

These attempts to stretch the HOS model provide a framework which retains the large role for price changes and adds a role for certain kinds of technical change. Changes in factor endowments are potentially important if there are specific factors, which seems a weak foundation in a world economy in which many argue that most factors are internationally mobile. It is hard not to conclude from this work that the HOS model is fundamentally not a hospitable framework for a labor economist, or anyone else, who wants to focus on the operation of factor markets and changes in factor endowments in explaining changes in relative wages.

In this paper, we offer an alternative approach, extending the HOS model to include nontraded goods. A fundamental difficulty with the HOS model is that it focuses on tradable commodities whose prices are determined in (large) world markets. It has long been recognized that the introduction of non-tradable goods qualifies the strong separation between production structure and demand structure in the HOS model. Some prices become endogenous and provide a potential link between changes in factor endowments and changes in factor returns. It seems sensible to address the wage-trade debate with a theoretical framework that includes non-tradables. The problem is that, empirically, non-tradables — defined as goods produced in sectors in which there are neither exports nor imports — are not very important, constituting a small share of total production in virtually all countries. It seems unreasonable to argue that a theoretical framework is more general and better suited for evaluating the effect of trade on wages because it distinguishes a few service sectors.

We avoid this dilemma by making a theoretical distinction between "nontraded" and "nontradable" goods. In our approach, instead of classifying production in sectors as nontradable only if there are no exports or imports, all domestically produced goods sold on the domestic market are viewed as nontraded goods which are imperfect substitutes for imports supplied to the same market. The resulting model distinguishes between domestically produced goods sold on the world market and the domestic market, with all domestic production not exported treated as nontraded. Even

6Jones and Engerman (1995) and Jones (1996) consider models in which various kinds of technical change affect relative factor prices.
within the same sector, goods supplied to the domestic and world markets are distinct, and different from imports, which are seen as imperfect substitutes on the domestic market. Nontraded goods are thus semi-tradable. In its simplest form, the model refers to one country with two production activities and three commodities, and has been termed the 1-2-3 model [Devarajan, Lewis, and Robinson (1993)].

We start from the 1-2-3 model and the standard 2x2 HOS model, as elegantly presented by Jones (1965), and show how the introduction of nontraded goods yields a new model that meets Richardson’s challenge to provide a framework that can bridge the gap between trade theory and labor economists. The model incorporates the standard HOS model as a special case. However, the magnification effects of the Stolper-Samuelson and Rybczynski Theorems are greatly qualified. Changes in factor endowments have a prominent role, without making any assumptions about specialization, specific factors, or biased technical change. Indeed, for simplicity, we assume there are no specific factors; there is no technical change; and there is an internal equilibrium, with all goods produced, using all factors in every sector. Even with these strong assumptions, the model provides an independent role for factor markets that labor economists will appreciate. As a bonus, the model also incorporates the trade balance explicitly, and so can be used to evaluate the effect of changes in the trade balance on relative wages. In the next two sections, we first present the model and then provide some empirical sensitivity analysis using stylized U.S. data.

The 1-2-2-3 Model

The 1-2-3 model was first described in de Melo and Robinson (1989), who specified explicit functional forms for the aggregate utility function (constant elasticity of substitution, CES) and the production possibility frontier (constant elasticity of transformation, CET). They did not include factor markets, focusing on product markets and the behavior of exports and imports. In order to consider links between trade and factor returns, we extend the model to include sectoral production functions and two factors of production, capital and labor — implicitly aggregating skilled labor with capital. Our specification closely follows Jones (1965), with two production sectors, but adds a different imported consumption good and the balance of trade. The resulting 1-2-2-3 model is described below.

The economy produces two goods, E and D. The good E is exported and is not consumed domestically. The good D is consumed domestically. Imports, M, represent a third commodity which is consumed, but not produced, domestically. The goods M and D are imperfect substitutes in demand. Aggregate absorption, Q, is given by:

---

7Devarajan, Lewis, and Robinson (1990, 1993) explore the properties of the model in detail, extending it to include tariffs and taxes, and also use the same basic framework to analyze issues concerning the appropriate definition of the equilibrium real exchange rate.

8Jones (1974) also presents a model with a nonproduced import and a nonconsumed export, as in the 1-2-2-3 model, but does not consider the role of factor markets.
\[ Q = F(M,D; \sigma_q) \]  

where \( \sigma_q \) is the elasticity of substitution in demand. Absorption represents aggregate utility in this model. In the 1-2-3 model, \( F(\cdot) \) was defined as a CES function. In the 1-2-2-3 model, we assume that the desired ratio of imports to domestic goods is given by:

\[ \frac{M}{D} = k \left( \frac{P_D}{P_M} \right)^{\sigma_q} \]  

where \( k \) is constant for a CES function and approximately constant otherwise, and \( P_M \) and \( P_D \) are the prices of M and D respectively.\(^9\)

Following Jones' notation, the technology for producing \( E \) and \( D \) is given by the coefficients matrix \( A \):

\[ A = \begin{bmatrix} A_{KE} & A_{KD} \\ A_{LE} & A_{LD} \end{bmatrix} \]  

where \( A_{ij} \) is the quantity of factor \( i \) required to produce a unit of good \( j \). We do not assume that these coefficients are constant. When the coefficients are variables, however, they are assumed to depend only on relative factor prices — there is no technical change.\(^{10}\) Given this technology, factor market clearing requires:

\[ A_{KE}E + A_{KD}D = K \]
\[ A_{LE}E + A_{LD}D = L \]

where \( K \) and \( L \) are aggregate supplies of capital and labor.

In competitive equilibrium, unit costs will equal market prices:

\[ A_{KE} W_K + A_{LE} W_L = P^E \]
\[ A_{KD} W_K + A_{LD} W_L = P^D \]

where \( W_K \) and \( W_L \) are the "wages" of capital and labor and \( P^E \) and \( P^D \) are output prices.

---

\(^9\)In the general case, we ignore income effects and assume the absorption function is well behaved (e.g. homothetic, convex, and twice-differentiable).

\(^{10}\)We also assume that both sectors use both factors in equilibrium, so there are no corner solutions.
To close this model, we require an equation linking exports and imports. We assume that the balance of trade is given by:

\[ P^M - P^E = \Phi P^E \]  \hspace{1cm} (6)

where \( \Phi \) is a parameter giving the ratio of import expenditures to export earnings. This specification extends the standard HOS model, allowing the balance of trade to affect consumption, production, and factor returns. When \( \Phi \) is one, trade is balanced, with export earnings exactly equaling import costs—as in the usual HOS model. The trade balance (the value of exports minus imports in world prices) equals \( (1 - \Phi) \) times export earnings. An increase in \( \Phi \) implies a worsening of the trade balance.

Assuming that the country is "small" so that we can assume world prices, \( P^M \) and \( P^E \), are fixed, the model is complete. There are seven equations for seven endogenous variables: \( Q, E, D, M, W_K, W_L, \) and \( \Pi \). Unlike the HOS model, one of the commodity prices is endogenous.

**Shares and Elasticities**

We wish to analyze how these endogenous variables, particularly relative wages, change when we change the prices of the traded goods (\( P^E \) and \( P^M \)), factor supplies (\( K \) and \( L \)), and the balance of trade (\( \Phi \)). First, we define various share parameters and elasticities, following Jones (1965). Define \( \lambda_{ij} \) as the share of the total supply of factor \( i \) used in sector \( j \):

\[ \lambda_{KE} = \frac{A_{KE} E}{K}; \quad \lambda_{KD} = \frac{A_{KD} D}{K} \]

\[ \lambda_{LE} = \frac{A_{LE} E}{L}; \quad \lambda_{LD} = \frac{A_{LD} D}{L} \]  \hspace{1cm} (7)

Define \( \theta_i \) as the share of factor \( i \) in total income generated in sector \( j \).

\[ \theta_i = \frac{A_i W_i}{P_j} \]  \hspace{1cm} (8)

Given that factors are fully employed and that income is fully allocated to factors (zero profit condition), it follows that:

\[ \sum_j \lambda_{ij} = \sum_i \theta_i = 1 \]  \hspace{1cm} (9)
We will assume that $E$ is the capital-intensive sector and that $D$ is labor-intensive. Capital's value share in $E$ must be greater than its value share in $D$, ($\theta_{KE} > \theta_{KD}$), and also the share of the total capital stock used in $E$ must be greater than the share of the labor force used in $E$, ($\lambda_{KE} > \lambda_{LE}$). Define the following matrices:

$$
\lambda = \begin{bmatrix}
\lambda_{KE} & \lambda_{KD} \\
\lambda_{LE} & \lambda_{LD}
\end{bmatrix}
$$

(10)

$$
\theta = \begin{bmatrix}
\theta_{KE} & \theta_{LE} \\
\theta_{KD} & \theta_{LD}
\end{bmatrix}
$$

(11)

Given that the rows of each of these matrices sum to one, their determinants are given by:

$$
|\lambda| = \lambda_{KE} - \lambda_{LE}; \quad |\theta| = \theta_{KE} - \theta_{KD}
$$

(12)

Under the assumption that $E$ is capital intensive, both determinants are positive (and less than one).

Use a (') to denote the relative change in a variable or parameter (or its log derivative). The elasticities of substitution between capital and labor in production in the two sectors $E$ and $D$ can be defined by:

$$
\sigma_E = \frac{\hat{A}_{LE} - \hat{A}_{KE}}{\hat{W}_K - \hat{W}_L}
$$

$$
\sigma_D = \frac{\hat{A}_{LD} - \hat{A}_{KD}}{\hat{W}_K - \hat{W}_L}
$$

(13)

In addition, define two additional parameters:

$$
\delta_K = \lambda_{KE} \theta_{LE} \sigma_E + \lambda_{KD} \theta_{LD} \sigma_D
$$

$$
\delta_L = \lambda_{LE} \theta_{KE} \sigma_E + \lambda_{LD} \theta_{KD} \sigma_D
$$

(14)

Quoting Jones (1965, p. 35): "In general, $\delta_L$ is the aggregate percentage saving in labor inputs at unchanged outputs associated with a 1% rise in the relative wage rate, the saving resulting from the adjustment to less labor-intensive techniques in both industries as relative wages rise."

Finally, the elasticity of transformation between $E$ and $D$ (along the production possibility frontier) is given by:
\[ \Omega = \frac{\delta_k + \delta_L}{|\lambda| |\theta|} \]  

(15)

In the original 1-2-3 model, the production possibility frontier was defined as a CET function, with a constant \( \Omega \). In the 1-2-2-3 model, we only assume it to be approximately fixed when taking log derivatives.

**Prices, Endowments, and the Balance of Trade**

After some algebra, the model reduces to four relationships in changes in relative prices, production, and demand. The first is the link between changes in relative prices and relative wages along the contract curve underlying the production possibility frontier.

\[ (\dot{w}_k - \dot{w}_L) = \frac{1}{|\theta|} (\dot{p}^E - \dot{p}^D) \]  

(16)

In the standard HOS model, where both goods are tradeable and their prices are set in world markets, this equation demonstrates the Stolper-Samuelson Theorem. Relative wages depend only on relative prices and, since \(|\theta| < 1\), the change in relative wages is greater than the change in relative prices—the model incorporates the magnification effect.

Second, movements along the production possibility frontier are determined both by changes in relative prices and changes in relative factor endowments.

\[ (\dot{k} - \dot{d}) = \frac{1}{|\lambda|} (\dot{k} - \dot{d}) + \Omega (\dot{p}^E - \dot{p}^D) \]  

(17)

This equation demonstrates the Rybczynski Theorem. Since \(|\lambda| < 1\), with unchanged prices, changes in relative factor endowments will have a magnified effect on relative production.\(^{11}\)

The demand side of the model involves \( D \) and \( M \) rather than \( D \) and \( E \). Log differentiating equation 2 yields:

\[ (\dot{M} - \dot{d}) = \sigma_G (\dot{p}^M - \dot{p}^D) \]  

(18)

This equation shows how relative demand for \( M \) and \( D \) changes with changes in relative prices.

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\(^{11}\)Equations 16 and 17 present results for relative wages and production. When only one price changes, one can show that one wage goes up while the other falls (Stolper-Samuelson). Similarly, one can show that, with a change in one factor endowment, one output goes up while the other falls (Rybczynski).
Finally, the supply and demand sides are linked through the balance-of-trade equation. Log differentiating equation 6 yields:

\[
(\hat{E} - \hat{M}) = \beta^M - \beta^E - \Phi
\]

(19)

Eliminating \( \hat{N} \) in equations 17 and 18, and then substituting for \( \hat{E} \) and \( \hat{M} \) in equation 19, yields an expression for changes in relative prices of D and E as a function of changes in exogenous world prices, factor endowments, and the balance of trade.

\[
(\hat{p}^E - \hat{p}^D) = \frac{1}{(\sigma_{\Omega} + \Omega)} \left[ (\sigma_{\Omega} - 1) (\hat{p}^E - \hat{p}^M) - \Phi + \frac{1}{|\lambda|} (\ell - \hat{K}) \right]
\]

(20)

In this model, when world prices are fixed, \( \hat{p}^D \) is the relative price of nontraded (semi-tradable) goods to traded goods, and represents the real exchange rate. In the general case, there is effectively a different real exchange rate for imports and exports. Equation 20 refers to domestically produced goods supplied to domestic and world markets, and describes how the economy moves along the production possibility frontier as a function of changes in world prices, the balance of trade, and factor endowments.

Substituting equation 20 into equation 16 yields an equation relating changes in relative wages to changes in world prices, the balance of trade, and factor endowments.

\[
(\hat{w}_K - \hat{w}_L) = \frac{1}{|\Theta|} \left[ (\sigma_{\Omega} - 1) (\hat{p}^E - \hat{p}^M) - \Phi + \frac{1}{|\lambda|} (\ell - \hat{K}) \right]
\]

(21)

This is the fundamental result from the 1-2-2-3 model. In contrast to the HOS model, when nontraded goods are included, changes in relative wages depend not only on changes in world prices, but also on changes in factor endowments and the balance of trade.

**Stolper-Samuelson Theorem**

As the elasticity of substitution in consumption, \( \sigma_{\Omega} \), goes to infinity, the last two terms in brackets in equation 21 go to zero. In the limit, the remaining term in world prices reduces to:

\[
(\hat{w}_K - \hat{w}_L) = \frac{1}{|\Theta|} (\hat{p}^E - \hat{p}^M)
\]

(22)

which corresponds to equation 16 with \( \hat{p}^D = \hat{p}^M \) since D and M are now perfect substitutes. This is exactly the HOS model, with changes in relative wages depending only on changes in world prices,  

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12This equation is equivalent to the equation for the real exchange rate in the 1-2-3 model derived in Devarajan, Lewis, and Robinson (1993), with the addition of a term for changes in factor endowments.
and the Stolper-Samuelson Theorem again applies. The HOS model can thus be seen as a special case of the 1-2-2-3 model when imports and domestic goods are perfect substitutes.

When there is no change in factor supplies and the balance of trade, equation 21 reduces to:

\[
(\hat{\hat{W}}_K - \hat{\hat{W}}_L) = \frac{1}{|\Theta|} \left[ \frac{(\sigma_0 - 1)}{(\sigma_0 + \Omega)} \right] (\hat{\beta}^K - \hat{\beta}^L).
\]  

(23)

Since \( \Omega \) is positive, the second term in this expression is always less than one. The result is that the magnification effect in the Stolper-Samuelson Theorem is reduced. The larger is the transformation elasticity \( \Omega \) and the closer is the elasticity of substitution in demand to one, the weaker is the link between changes in prices and changes in relative wages.

When the elasticity of substitution equals one, the right-hand side goes to zero and changes in world prices have no effect on relative wages. One way to see what is going on is to consider the country's offer curve, which shows the relationship between exports (on the horizontal axis) and imports (on the vertical axis) as world prices change. As de Melo and Robinson (1989) show, when \( \sigma_0 = 1 \) the country's offer curve becomes vertical. In that case, as the world price of imports changes, expenditure on imports remains fixed nominally and hence, with a fixed export price, real exports do not change.\(^{13}\) Hence, there is no movement along the production possibility frontier, and relative wages do not change. The link between changes in world prices and changes in relative wages is completely broken.

When \( \sigma_0 > 1 \), from equation 20, an increase in the price of imports leads to an increase in the price of \( D \), which corresponds to an appreciation of the real exchange rate. The offer curve slopes upwards. When imports become more expensive, it is worthwhile to produce more of the domestic substitute, moving resources away from the production of exports. The volume of trade declines and, from equation 23, the relative wage of capital falls. Such a situation might characterize a developed country when the price of its imports rise on world markets. The sign of the results is the same as in the HOS model, but the magnification effect is weakened or eliminated.

When \( \sigma_0 < 1 \), \( M \) and \( D \) are weak substitutes. In this case, an increase in the world price of \( M \) leads to a decrease in the price of \( D \) relative to \( E \), which is a depreciation of the real exchange rate. Production of \( D \) declines and exports increase. In effect, the country depreciates in order to shift resources into exports, increasing export earnings in order to pay for the more expensive, but essential, imports. The offer curve is backward bending. This situation is characteristic of developing countries which have to undergo a structural adjustment program in the face of an adverse terms-of-trade shock (for example, a large increase in the price of oil). In this case, changing a commodity

\(^{13}\)The model is characterized by Lerner symmetry. It does not matter whether world export or import prices change. In the cases discussed below, for expositional convenience, we will start from a change in the world price of imports, assuming export prices are fixed.

10
price has the opposite effect on wages than would be predicted by the HOS model. In a developing
country exporting labor-intensive goods, where \(|0| < 0\), an increase in the price of imports will lead
to an increase in the wage of labor relative to capital, while the HOS model would predict a decrease.
The 1-2-2-3 model seems much more realistic in this case.

Rybczynski Theorem

To consider the application of the Rybczynski Theorem in the 1-2-2-3 model, consider the
relationship between the change in domestic relative prices as factor endowments change when world
prices and the balance of trade do not change (equation 20). Substitute the resulting expression for
\(\Delta z - \Delta D\) into equation 17. The result is an expression relating the change in the structure of
production as a function of the change in factor endowments, all other exogenous variables held
constant:

\[
(\Delta z - \Delta D) = \frac{1}{|\lambda|} \left[ \frac{\sigma_q}{(\sigma_q + \Omega)} \right] (K - L) \tag{24}
\]

As with the Stolper-Samuelson Theorem, this equation reduces to the HOS version (equation 17, the
Rybczynski Theorem) as a special case in the limit when the elasticity of substitution \((\sigma_q)\) goes to
infinity. In general, however, the magnification effect in the Rybczynski Theorem is ameliorated.
Since \(\Omega\) is greater than zero, the term in brackets is less than one. The greater is \(\Omega\) and the lower is
\(\sigma_q\), the weaker is the link between changes in factor endowments and changes in the structure of
production. Unlike the Stolper-Samuelson Theorem, there is no sign reversal when \(\sigma_q < 1\).

Trade-Balance Effects

One view of the effect of changes in the balance of trade on relative wages is that a worsening
in the trade balance (increasing \(\Phi\)) leads to increased imports which displace domestic production of
low-skill, labor-intensive goods (D), and hence should widen the wage gap. Borjas, Freeman, and
Katz (1992), using the factor-content approach, compute the net implicit contribution of the trade
deficit to the supply of labor by different skill categories. They conclude (p. 214): "The annual
increase in implicit labor supply due to the mid- and late-1980s trade deficit in manufactures was on
the order of 1.5 percent for the economy as a whole and 6 percent for the manufacturing sector." They
then analyze the impact of these shifts on wages in a partial-equilibrium analysis of the labor
markets, concluding that "... from 15 to 25 percent of the 11 percentage point rise in the earnings
of college graduates relative to high school graduates from 1980 to 1985 can be attributed to the
massive increase in the trade deficit over the same period."

In equation 21, however, an increase in \(\Phi\) will lead to a narrowing of the wage gap, since the
sign of the coefficient on \(\Phi\) is negative. There is a serious conflict between the 1-2-2-3 model and
the labor-market approach. The problem is a conflict between partial- and general-equilibrium
models. The labor-market approach assumes that increased imports due to the worsening of the
balance of trade will “displace” domestic production of labor-intensive goods. However, increasing $\Phi$ implies that absorption will rise since the worsening trade balance shifts the consumption possibility frontier out, even though the production possibility frontier stays the same. Consumers have more to spend and, since $D$ is a normal good, they will demand more $D$ as well as more $M$. The effect will be to increase the relative price of $D$, as shown in equation 20, shifting resources away from $E$ to produce more $D$. The increase in $P^D$ represents an appreciation of the real exchange rate and the model demonstrates the Dutch disease.\(^\text{14}\) The production of $E$ declines, $D$ expands, and the relative wage gap narrows.\(^\text{15}\)

**Multisectoral Extensions**

The theoretical properties of the 1-2-2-3 model are appealing. It deals with a number of the unrealistic properties of the HOS model in a way that satisfies Occam’s Razor — minimal additional assumptions are imposed to yield a much better behaved model. The model can be extended to disaggregate commodities and factors of production. Adding more nontraded and imported goods causes no theoretical problems. The assumption of imperfect substitutability between imported and domestic goods is simply extended to many sectors. The variables $D$ and $M$ acquire sector subscripts, with sector-specific aggregation functions, which are often called Armington (1969) functions. This approach has been widely used in both single- and multi-country computable general equilibrium (CGE) models.\(^\text{16}\)

On the export side, the issue is a bit more complex. Adding additional export sectors whose prices are set in large world markets will make the model behave like the HOS model. With more export sectors than factors of production, relative factor prices are again determined by world prices, and are independent of endowments. One approach to generalizing the 1-2-2-3 model to include many export sectors which has become standard in the CGE literature is to specify a transformation function at the sectoral level between exports and goods supplied to the domestic market [Robinson (1989)]. Each sector engages in joint production, supplying a composite good that can be transformed into sectoral $E$ and $D$ according to a constant elasticity of transformation (CET) function. The sectoral composite good is produced with a standard neoclassical production function. The logic of the 1-2-2-3 model is applied sectorally — there is a nontraded good in each sector.

Like the 1-2-2-3 model, these multisectoral generalizations have the standard HOS model as a special case when sectoral elasticities of substitution and transformation go to infinity. As in the 1-2-

\(^{14}\)The Dutch disease occurs when an increase in availability of foreign exchange (arising, say, from the discovery of oil in the North Sea) worsens the trade balance and generates a real appreciation of the exchange rate, increasing imports, and reducing production of tradables.

\(^{15}\)Bhagwati and Dehejia (1994) also criticize Borjas, Freeman, and Katz, but from the perspective of the HOS model, arguing that changes in endowments should have no effect on relative wages. Learner (1996, pp. 11-12) considers a model with nontradables and notes that: “An external deficit raises the demand for nontradables and may or may not affect wages.” In a footnote, he worries about different causes of a change in the deficit, and how they might affect relative wages.

\(^{16}\)Robinson (1989) and de Melo (1988) survey such models.
2-3 model, however, trade shares matter a lot when elasticities are less than infinity. These models also easily accommodate special cases such as sectors with no exports and/or imports or sectors in which the country is "large" on world markets.

**Empirical Decomposition of Trade-Wage Links**

Equation 21 can be used to decompose the relative contributions of changes in world prices, the balance of trade, and factor endowments to changes in relative factor returns. This decomposition is at the heart of the policy debate about the causes of the widening of the skilled-unskilled wage gap in the U.S. The 1-2-2-3 model can be used as the organizing framework for detailed empirical work, using both econometric analysis and simulation models. In this paper, however, we will use aggregate data from the U.S. only to give a rough idea of: (1) the empirical sensitivity of the decomposition to variations in parameter values and (2) the potential changes in relative factor returns arising from changes in world prices, the balance of trade, and factor endowments to which the U.S. had to adjust in the 1980s.

**Sensitivity Analysis**

Tables 1 and 2 present data for an archetype economy patterned after the U.S. around 1980. All wages and prices are assumed to be one and GDP is set at 100. Exports (E) represent 10 percent of GDP and capital (K) is assumed to include both capital and skilled labor. The export sector (E) is assumed to be highly capital intensive relative to the domestic sector (D), and we assume a relatively low elasticity of substitution in production between capital and labor. The result is a transformation elasticity ($\Omega$) of 3.46 (Table 2). We assume an import-substitution of elasticity of 3. The resulting coefficients for the price, trade balance, and endowment terms in equation 21 are given in Table 2.

Note first that the Stolper-Samuelson magnification effect is nearly eliminated. The coefficient on the price term is 1.11, which implies that relative wages change only slightly more than relative prices. Second, the size of the coefficient for the endowment term, 5.51, is much larger. Not only do changes in endowments have a direct effect on relative wages, the effect is magnified. There is a lot of scope in this model for factor-market and endowment effects, and labor economists get top billing.

Finally, the trade balance (or Dutch disease) effect is smaller and of opposite sign than the price effect. The relative size of the trade balance effect is set by the choice of the import substitution elasticity. For a value of three, the effect is exactly half the value of the price effect. A given percent worsening of the trade balance will offset half of a comparable change in relative world prices. In the short run, one might well assume a lower import substitution elasticity, and so expect a larger offset.

---

17With wages and prices set to one, we start with the elements of $\theta$ for the E sector and total L and K. We then derive the other values of $\theta$ and $\lambda$, given the known aggregate supplies of L and K.
Table 1: Archetype Data and Parameters for U.S.

<table>
<thead>
<tr>
<th></th>
<th>Exports (E)</th>
<th>Domestic (D)</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>10</td>
<td>90</td>
<td>100</td>
</tr>
<tr>
<td>Capital (K)</td>
<td>8</td>
<td>47</td>
<td>55</td>
</tr>
<tr>
<td>Labor (L)</td>
<td>2</td>
<td>43</td>
<td>45</td>
</tr>
<tr>
<td>Value shares (θ)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>0.80</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>0.20</td>
<td>0.48</td>
<td></td>
</tr>
<tr>
<td>Factor shares (λ)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>0.15</td>
<td>0.85</td>
<td>1.00</td>
</tr>
<tr>
<td>L</td>
<td>0.04</td>
<td>0.96</td>
<td>1.00</td>
</tr>
<tr>
<td>K/L substitution elasticity (σₖ, σₗ)</td>
<td>0.10</td>
<td>0.10</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Relative Wage Change Equation

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Import substitution elasticity (σ_Q)</td>
<td>3.00</td>
</tr>
<tr>
<td>Transformation elasticity (Ω)</td>
<td>3.46</td>
</tr>
<tr>
<td>Determinant</td>
<td>θ</td>
</tr>
<tr>
<td>Determinant</td>
<td>λ</td>
</tr>
</tbody>
</table>

Coefficients for relative wage change equation:

\[
\left( \hat{w}_X - \hat{w}_L \right) = \frac{1}{|θ| (σ_Q + Ω)} \left[ + (σ_Q - 1) (P_E - P_M) \right.
\]

Prices \hspace{1cm} 1.11

\[- \hat{φ} \]

Trade balance \hspace{1cm} -0.56

\[+ \frac{1}{|λ|} (L - K) \]

Endowments \hspace{1cm} 5.51
Figures 1 to 4 explore the sensitivity of these coefficients to variations in initial conditions. In Figure 1, we create a series of archetype economies with increasing values of the import substitution parameter and plot the values of the three coefficients. As discussed earlier, when the elasticity is less than one, the sign of the price effect is negative and it equals zero when the elasticity is one. As the substitution elasticity increases, the size of both the endowment and trade-balance effects decreases and the price effect increases. As expected, the model moves toward the standard HOS model as the import substitution elasticity increases, but it is interesting to note how significant the endowment terms remains. Even when the substitution elasticity reaches a value of 32, the coefficient on the endowment term is around one.

Figure 2 shows how the coefficients change for archetype economies with larger trade shares. In the standard HOS model, the trade share is irrelevant—it matters only whether a good is tradable, not how much is traded. In the 1-2-2-3 model, the response of relative wages to changes in endowments is very sensitive to the trade share. The coefficients on the price and trade-balance terms level off as the trade share reaches 20 percent, while the coefficient on endowments falls steadily. At a trade share of 40 percent, the size of the endowment effect is close to the price effect. Labor economists and trade theorists share equal billing in countries with high trade shares.

Figure 3 explores the impact of assuming less difference in technologies between the two sectors. At a capital value share in E of 0.60, the corresponding share in D is 0.54, which is not much of a difference. When the technologies are identical, the contract curve is a straight line, as is the production possibility frontier. In this case, only endowment changes can affect wages (given that both goods are consumed, which is assumed). As the capital value share in E increases, the size of the coefficient on endowments drops rapidly, although remaining much larger than the price coefficient.18

Figure 4 shows how economies with different transformation elasticities respond to the three effects. The changes in Ω were achieved by varying the K/L substitution elasticities in production in both the E and D sectors together (equations 14 and 15). Substitution elasticities of zero in both sectors yields a transformation elasticity (Ω) of zero. As the transformation elasticity increases, the magnitudes of all three effects decrease. The endowment effect is relatively more important the lower is the transformation elasticity, but its coefficient remains significantly larger than the others for all values of Ω. From Figures 3 and 4, over a wide range of technology assumptions, changes in endowments have far larger effects on wages than do changes in world prices. The focus of labor economists on supply and demand in factor markets is well grounded, even in a general equilibrium model.

18Note that this diagram tells nothing about what happens over time in an economy undergoing biased technological change. The archetype economies are all assumed to start from equilibrium. As the capital share value is increased along the horizontal axis, aggregate factor shares are also changed to achieve full employment of both K and L in the base. The diagrams indicate how different economies ranged along a particular dimension differ in their responsiveness to the three effects.
Figure 1: Import Substitution Elasticity

Figure 2: Export Share

Figure 3: Export Technology

Figure 4: Transformation Elasticity
The U.S. Economy in the 1980s

Figures 5 to 7 present aggregate data for the U.S. economy for the variables in the 1-2-2-3 model from national accounts data.\(^\text{19}\) The price data (Figure 5) indicate a major swing in the international terms of trade facing the U.S. in the period 1980-1986, with export prices rising much faster than import prices. After 1986, there was a slight relative fall in export prices, but then little change for the rest of the period. The ratios of \(P^E\) and \(P^M\) to \(P^D\) fell rapidly in the 1980-1986 period, rose slightly to 1988, and then continued to fall slowly thereafter. The dramatic fall in the 1980-1986 period reflects a major appreciation of the real exchange rate in that period, which shows up in Figure 6 as a decline in real exports and an increase in real imports (as ratios to domestic production sold on the domestic market, \(D\)). Figure 7 indicates a major Dutch-disease effect in 1981-1986, with rapid and large deterioration in the trade balance, which is also consistent with the observed appreciation of the real exchange rate and the movements in real exports and imports. The Dutch-disease effect is reversed in the 1987-1991 period.

Figure 8 shows the aggregate labor/capital ratio and also data on the ratio of production to non-production workers in manufacturing (where production workers are assumed to be unskilled).\(^\text{20}\) The recession in the early 1980s is evident, and unemployment, declines in real investment, and underutilization of capital dominate changes in relative endowments in that period. With recovery, investment increases and the capital stock grows faster than the labor force, which should narrow the wage gap. There is, however, a trend decline in the ratio of production to non-production workers throughout the period, with some volatility, which should also lead to a narrowing of the wage gap. Aggregating skilled or non-production labor with capital seems reasonable, given these trends.

Table 3 indicates the changes in relative prices, trade balance, and endowments (using both the \(L/K\) and production/non-production worker ratios) that occurred in the 1980-86 and 1986-91 periods. The data are drawn from Figures 5, 7, and 8. Using the coefficients for the archetype U.S. given in Table 2, Table 3 also presents the contributions of changes in relative prices, the trade balance, and relative endowments to changes in relative factor returns. The endowment change numbers are especially suspect given the difficulty in measuring changes in capital stocks and that the relative endowment of production/non-production workers refers only to manufacturing. Furthermore, in the 1980-86 period, part of the adjustment in the labor markets worked through changes in unemployment rather than changes in relative factor returns.

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\(^\text{19}\)The data are from national accounts tables reported in *Economic Report of the President, 1996*. The price indices for exports and imports are calculated from GDP deflators. The deflator for \(D\) is calculated by computing nominal and real GDP equal to GDP minus exports. Nominal exports and imports are for goods and non-factor services in the GDP accounts. All real variables are in 1992 prices.

\(^\text{20}\)The labor/capital data are based on aggregate employment data and a constructed capital stock series using real investment data from the national accounts. The employment data come from the Annual Survey of Manufacturing. See Lawrence and Slaughter (1993), who also report data on the ratio of non-production to production employment.

<table>
<thead>
<tr>
<th>Term</th>
<th>1980-86</th>
<th>1986-91</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{p}^E - \hat{p}^M )</td>
<td>0.15</td>
<td>-0.05</td>
</tr>
<tr>
<td>( \hat{\phi} )</td>
<td>0.30</td>
<td>-0.30</td>
</tr>
<tr>
<td>( \hat{L} - \hat{K} )</td>
<td>-0.05</td>
<td>-0.05</td>
</tr>
<tr>
<td>( \hat{L}<em>p - \hat{L}</em>{np} )</td>
<td>-0.10</td>
<td>-0.02</td>
</tr>
</tbody>
</table>

Effects on relative wages using coefficients from Table 2:

<table>
<thead>
<tr>
<th>Term</th>
<th>1980-86</th>
<th>1986-91</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prices</td>
<td>0.17</td>
<td>-0.06</td>
</tr>
<tr>
<td>Trade balance</td>
<td>-0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>( L/K ) endowments</td>
<td>-0.28</td>
<td>-0.28</td>
</tr>
<tr>
<td>( L_p/L_{np} ) endowments</td>
<td>-0.55</td>
<td>-0.11</td>
</tr>
<tr>
<td>Total: ( \hat{\gamma}_K - \hat{\gamma}_L )</td>
<td>-0.28</td>
<td>-0.17</td>
</tr>
<tr>
<td>Total: ( \hat{\gamma}_{np} - \hat{\gamma}_p )</td>
<td>-0.55</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Notes: "np" and "p" refer to non-production and production workers in manufacturing.
Note first the importance of the trade-balance or Dutch-disease effect. In the first period, it offsets the effect of changes in relative prices and dominates the price effect in the second period. If one assumed a lower import-substitution elasticity in the short run, the Dutch-disease effect would have been even more important compared to the relative-price term. When endowments change, however, the effect empirically dominates both the other two effects, even though the magnitudes of the changes in relative endowments are smaller than the changes in relative prices and the trade balance. Capital accumulation in the second period leads to an increase in the relative wage of labor, and the effect more than offsets the trade-balance effect, even though the swing in the trade balance was very large. Decreases in the ratio of production to non-production workers in both periods also leads to a decrease in the gap in relative wages, and the magnitude of the effect is far larger than the price and trade-balance effects.

As noted by Lawrence and Slaughter (1993) and others, the increase in the relative employment of non-production workers should lead to a narrowing of the wage gap, as in the 1-2-2-3 model, while the data indicate that the gap widened throughout the 1980s. They argue that the explanation lies in shifts in relative demand for labor due to technological change. In the HOS model, either sectorally biased technical change (e.g. total factor productivity growth in B versus D) or factor-biased sectoral technical change (changing the input coefficients and Θ parameters in a sector) will lead to changes in relative wages. Technical change that is equivalent to changing relative factor endowments (e.g., labor-saving technological change that makes labor more productive in all sectors) will have no effect on relative wages in the HOS model. In the 1-2-2-3 model, however, such economywide factor-augmenting technical change will affect relative wages — greatly, given the large coefficient on the endowment term. Compared to the HOS model, the 1-2-2-3 model allows an additional important channel by which technical change can affect relative wages.

While suggestive, these results must be seen as only illustrative given the simplicity of the 1-2-2-3 model. The trade-balance effect is important, since its sign is the opposite of that expected in partial-equilibrium, factor-content analysis of labor markets, which looks only at displacement effects [Borjas, Freeman, and Katz (1992)]. The enormous swings in the trade balance probably dominated any Stolper-Samuelson effects for this period, even though there were large changes in relative world prices. The existence and size of these trade-balance effects, unless properly taken into account, will confound any attempt to use econometric methods to sort out the relative sizes of the other effects.

The effect of changes in factor endowments is more difficult to sort out, given the inherent limitations of using a two-factor model and the noise introduced by the 1982 recession. In the 1980s, the fact that both the labor/capital and production/non-production labor ratios moved together lends some support to using the simple two-factor model where skilled labor and capital are assumed to go together. Disaggregation by skill, however, seems highly desirable. In the long run, the results from the 1-2-2-3 model indicate that the sorts of changes in the skill structure of the labor force reported by labor economists should have a major impact on relative wages. Extending the model to include technical change should only increase this sensitivity.
Conclusion

The debate about the impact of changes in world prices and trade on relative wages in developed countries has been strongly influenced by neoclassical trade theory, especially the Heckscher-Ohlin-Samuelson (HOS) model. In this general-equilibrium model, changes in factor supplies do not affect relative wages. Commodity prices are set on large world markets, trade clears domestic markets, and factor markets have no independent role to play. This model cannot provide a theoretical framework hospitable to labor economists, who focus on partial-equilibrium analysis of supply and demand on factor markets. In this paper, we extend the HOS model to provide a general-equilibrium framework that is more realistic and provides a major independent role for factor markets.

We specify a model with one country, two production activities, two factors, and three commodities (including imports)—the 1-2-2-3 model. We introduce nontraded goods and assume that imports and domestically produced goods sold on the domestic market are imperfect substitutes. We show that in the 1-2-2-3 model, the major trade theorems are seriously qualified. The magnification effects in the Stolper-Samuelson and Rybczynski Theorems are weakened. In the Stolper-Samuelson Theorem, even the sign of the effect of changes in prices on relative wages may be reversed from the HOS model. Even more important, changes in factor endowments and in the balance of trade cause relative wages to change. In this framework, there is room for labor economists.

We derive a decomposition equation showing the impact of changes in world prices, the balance of trade, and factor endowments on relative factor returns. Empirical analysis with an archetype model indicates that changes in relative wages are more sensitive to changes in factor endowments than to changes in world prices or the trade balance. Not only is there room for labor economists, but they have a prominent role. However, Stolper-Samuelson effects become more important as the trade share increases, which brings trade economists back into the picture. This result from the 1-2-2-3 model is intuitively appealing and contrary to the standard HOS model, where trade shares are irrelevant. Finally, changes in the trade balance do affect relative wages, but in the opposite direction from that derived from factor-content analysis, which assumes increased imports displace domestic production. The 1-2-2-3 model captures the general-equilibrium feedback from increasing aggregate absorption, which is ignored in the partial-equilibrium approach.

While the theoretical model is quite aggregated, and we chose not to consider the impact of changes in technology, empirical analysis of the experience of U.S. in the 1980s in this framework is suggestive. All three effects (changes in world prices, balance of trade, and factor endowments) were important during the period. Swings in the trade balance were especially large and, in the 1980-86 period, probably largely offset Stolper-Samuelson effects of price changes. The existence of such swings, not properly accounted for in partial-equilibrium models, undoubtedly introduced much noise and specification bias into econometric work based on such models.

The 1-2-2-3 model is simple, but can easily be expanded. It is certainly feasible and important to disaggregate the model framework to include many sectors and factors of production. Empirical
models based on this framework, such as single-country and multi-country computable general equilibrium (CGE) models, can easily incorporate biased technical change and disaggregation of the factor markets, incorporating features of specific-factor models. The resulting empirical models can provide a much better framework for evaluating the relative contributions of changes in trade, technology, and domestic factor market conditions than either the HOS model or any partial-equilibrium analysis.
References


Annex:
Derivation of the wage decomposition equation in the 1-2-2-3 model

Following Jones (1965), we consider two production goods, $E$, the export good, and $D$, the good sold on the domestic market. There are two inputs, labor ($L$) and capital ($K$). Assume that the export good is capital intensive and the domestic good is labor intensive. There is constant returns to scale technology described by the input coefficient $A_{ij}$ which indicates the quantity of factor $i$ needed to produce one unit of commodity $j$. The input coefficients depend only on factor prices. In equilibrium, full employment and zero profit conditions hold:

$$A_{KE}E + A_{KD}D = K$$

$$A_{LE}E + A_{LD}D = L$$

$$A_{KE}W_K + A_{LE}W_L = P^E$$

$$A_{KD}W_K + A_{LD}W_L = P^D$$

Totally differentiating the full employment conditions and dividing through by the appropriate forms of one and the endowment of either labor or capital, we find:

$$\lambda_{KE} \dot{E} + \lambda_{KD} \dot{D} = \dot{K} - [\lambda_{KE} \dot{A}_{KE} + \lambda_{KD} \dot{A}_{KD}]$$

$$\lambda_{LE} \dot{E} + \lambda_{LD} \dot{D} = \dot{L} - [\lambda_{LE} \dot{A}_{LE} + \lambda_{LD} \dot{A}_{LD}]$$

Totally differentiating the zero profit conditions and dividing by the appropriate forms of one and the price term, we find:

$$\theta_{LE} \dot{W}_L + \theta_{KE} \dot{W}_K = \dot{P}^E - [\theta_{LE} \dot{A}_{LE} + \theta_{KE} \dot{A}_{KE}]$$
\[ \theta_{LD} \hat{v}_L + \theta_{KD} \hat{v}_K = \beta \Delta - [\theta_{LD} \hat{A}_{LD} + \theta_{KD} \hat{A}_{KD}] \] (8)

For variable input coefficients, we need additional conditions. Take the derivative of the unit cost equations, holding factor and output prices constant. Rearranging terms, we find:

\[ \theta_{LE} \hat{A}_{LE} + \theta_{KE} \hat{A}_{KE} = 0 \] (9)

\[ \theta_{LD} \hat{A}_{LD} + \theta_{KD} \hat{A}_{KD} = 0 \] (10)

These equations indicate a movement along an isoquant. Note that the input coefficients are functions only of relative prices, there is no technical change.

The \( \lambda \)'s and the \( \theta \)'s embody the technology coefficients when relative changes are shown. The \( \lambda \) term is the share of the total supply of factor \( i \) used in sector \( j \). For example, for capital:

\[ \lambda_{KE} = \frac{A_{KE} E}{K}; \quad \lambda_{KD} = \frac{A_{KD} D}{K} \] (11)

We define \( \theta \) as the share of factor \( i \) in total income generated in sector \( j \).

\[ \theta_{ij} = \frac{A_{ij} W_i}{P_i} \] (12)

Given that factors are fully employed and income is fully allocated to factors, it follows that:

\[ \sum_j \lambda_{ij} = \sum_i \theta_{ij} = 1 \] (13)

We will assume that \( E \) is the capital-intensive sector and that \( D \) is labor-intensive. Capital’s value share in \( E \) must be greater than its value share in \( D \) (\( \theta_{KE} > \theta_{KD} \)). The share of the total capital stock used in \( E \) must be greater than the share of the labor force used in \( E \) (\( \lambda_{KE} > \lambda_{LE} \)).

We define the following matrices:

\[
\lambda = \begin{bmatrix}
\lambda_{KB} & \lambda_{KD} \\
\lambda_{LE} & \lambda_{LD}
\end{bmatrix}
\] (14)
\[ \theta = \begin{bmatrix} \theta_{KE} & \theta_{LE} \\ \theta_{KD} & \theta_{LD} \end{bmatrix} \]  

(15)

Note that the determinants of the matrices are:

\[ | \lambda | = \lambda_{KE} \lambda_{LD} - \lambda_{LE} \lambda_{KD} \]
\[ | \lambda | = \lambda_{KE} - \lambda_{LE} \]
\[ | \lambda | = \lambda_{LD} - \lambda_{KD} \]  

(16)

and

\[ | \theta | = \theta_{KE} \theta_{LD} - \theta_{KD} \theta_{LE} \]
\[ | \theta | = \theta_{LD} - \theta_{LE} \]
\[ | \theta | = \theta_{KE} - \theta_{KD} \]  

(17)

The elasticity of substitution for factors of production in each industry is:

\[ \sigma_E = \frac{(\hat{A}_{LE} - \hat{A}_{KE})}{(\hat{W}_K - \hat{W}_L)} \]  

(18)

\[ \sigma_D = \frac{(\hat{A}_{LD} - \hat{A}_{KD})}{(\hat{W}_K - \hat{W}_L)} \]  

(19)

Combining equations (9) and (18), we find:

\[ (\hat{W}_K - \hat{W}_L) \sigma_E = -\frac{\theta_{KE} \hat{A}_{KE} - \hat{A}_{KE}}{\theta_{LE}} \]  

(20)

or:

\[ \hat{A}_{KE} = -\theta_{LE}(\hat{W}_K - \hat{W}_L) \sigma_E \]  

(21)

and
\[ \hat{A}_{KL} = \theta_{KL}(\hat{W}_K - \hat{W}_L)\sigma_E \]  \hfill (22)

Likewise, combining equations (10) and (19) we find:

\[ \hat{A}_{KD} = -\theta_{LD}(\hat{W}_K - \hat{W}_L)\sigma_D \]  \hfill (23)

\[ \hat{A}_{LD} = \theta_{KD}(\hat{W}_K - \hat{W}_L)\sigma_D \]  \hfill (24)

Substituting equations (21) and (23) into equation (5):

\[ \lambda_{KE} \hat{\mathcal{E}} + \lambda_{KD} \hat{\mathcal{D}} = \hat{\mathcal{K}} - \left[-\lambda_{KE} \theta_{LE}(\hat{W}_K - \hat{W}_L)\sigma_E - \lambda_{KD} \theta_{LD}(\hat{W}_K - \hat{W}_L)\sigma_D\right] \]  \hfill (25)

or,

\[ \lambda_{KE} \hat{\mathcal{E}} + \lambda_{KD} \hat{\mathcal{D}} = \hat{\mathcal{K}} + \delta_{K} (\hat{W}_K - \hat{W}_L) \]  \hfill (26)

Likewise, substituting equations (22) and (24) into equation (6):

\[ \lambda_{KE} \hat{\mathcal{E}} + \lambda_{LD} \hat{\mathcal{D}} = \hat{\mathcal{L}} - \delta_{L} (\hat{W}_K - \hat{W}_L) \]  \hfill (27)

where:

\[ \delta_{K} = \lambda_{KE} \theta_{LE}\sigma_E + \lambda_{KD} \theta_{LD}\sigma_D \]  \hfill (28)

\[ \delta_{L} = \lambda_{KE} \theta_{KE}\sigma_E + \lambda_{LD} \theta_{KD}\sigma_D \]  \hfill (29)

Substituting equations (9) and (10) into equations (7) and (8), we find:

\[ \theta_{LE}\hat{W}_L + \theta_{KE}\hat{W}_K = \hat{\rho}^E \]  \hfill (30)

\[ \theta_{LD}\hat{W}_L + \theta_{KD}\hat{W}_K = \hat{\rho}^D \]  \hfill (31)

Subtracting the endowment changes, equation (27) from equation (26) we find:
\[(\lambda_{LE} - \lambda_{KE})\hat{E} + (\lambda_{LD} - \lambda_{KD})\hat{D} = (\hat{L} - \hat{K}) - (\delta_L + \delta_K)(\hat{W}_K - \hat{W}_L)\] (32)

and equation (16) implies:
\[-|\lambda|(\hat{E} - \hat{D}) = (\hat{L} - \hat{K}) - (\delta_L + \delta_K)(\hat{W}_K - \hat{W}_L)\] (33)

\[(\hat{E} - \hat{D}) = -\frac{1}{|\lambda|}(\hat{L} - \hat{K}) + \frac{(\delta_L + \delta_K)}{|\lambda|}(\hat{W}_K - \hat{W}_L)\] (34)

Subtracting the zero profit conditions, equation (31) from equation (30) we find:

\[(\theta_{LE} - \theta_{LD})\hat{W}_L + (\theta_{KE} - \theta_{KD})\hat{W}_K = (\bar{\theta}^E - \bar{\theta}^D)\] (35)

and equation (17) implies:
\[(\hat{W}_K - \hat{W}_L) = \frac{1}{|\theta|}(\bar{\theta}^E - \bar{\theta}^D)\] (36)

Substituting equation (36) into equation (34), we find an expression for the production possibilities frontier:

\[(\hat{E} - \hat{D}) = -\frac{1}{|\lambda|}(\hat{L} - \hat{K}) + \Omega(\bar{\theta}^E - \bar{\theta}^D)\] (37)

where,
\[\Omega = \frac{(\delta_K + \delta_L)}{|\lambda| |\theta|}\] (38)

Building on Jones' description of production, there are four key equation in the 1-2-2-3 model, expressed in log differentiated form:

\[\text{Contract curve:}\]

26
\[ (\tilde{W}_K - \tilde{W}_L) = \frac{1}{|\theta|} (\beta^E - \beta^D) \] (39)

Supply:
\[ (\tilde{E} - \tilde{D}) = -\frac{1}{|\lambda|} (\tilde{L} - \tilde{K}) + \Omega (\beta^E - \beta^D) \] (40)

Demand:
\[ (\tilde{M} - \tilde{D}) = -\sigma_q (\beta^M - \beta^D) \] (41)

Balance of Trade:
\[ (\tilde{E} - \tilde{M}) = \beta^M - \beta^E \] (42)

Rearranging equation (40), the supply equation, we find:
\[ \tilde{D} = \tilde{E} + \frac{1}{|\lambda|} (\tilde{L} - \tilde{K}) - \Omega (\beta^E - \beta^D) \] (43)

Likewise, rearranging equation (41), the demand equation, we find:
\[ \tilde{D} = \tilde{M} + \sigma_q (\beta^M - \beta^D) \] (44)

Combining equations (43) and (44):
\[ (\tilde{E} - \tilde{M}) = -\frac{1}{|\lambda|} (\tilde{L} - \tilde{K}) + \Omega (\beta^E - \beta^D) + \sigma_q (\beta^M - \beta^D) \] (45)

or,
\[ (\beta^E - \beta^D) = [(\tilde{E} - \tilde{M}) + \frac{1}{|\lambda|} (\tilde{L} - \tilde{K}) - \sigma_q (\beta^M - \beta^D)] \frac{1}{\Omega} \] (46)
Combining this with the balance of trade equation (42):

\[
(\hat{\beta}^E - \hat{\beta}^D) = \frac{1}{\Omega} \left( (\hat{\beta}^M - \hat{\beta}^E - \Phi) + \frac{1}{|\lambda|} (\hat{L} - \hat{K}) - \sigma_q (\hat{\beta}^M - \hat{\beta}^D) \right)
\] (47)

\[
(\beta^E - \beta^D) = \frac{1}{\Omega} \left[ (1 - \sigma_q)(\beta^M - \beta^E) - \Phi + \frac{1}{|\lambda|} (\hat{L} - \hat{K}) - \sigma_q (\beta^E - \beta^D) \right]
\] (48)

\[
(\beta^E - \beta^D) = \frac{1}{(\sigma_q + \Omega)} \left[ (\sigma_q - 1)(\beta^E - \beta^M) - \Phi + \frac{1}{|\lambda|} (\hat{L} - \hat{K}) \right]
\] (49)

Substituting equation (49) for the price term in the contract curve, equation (39), we find:

\[
(\hat{W}_x - \hat{W}_z) = \frac{1}{|\theta|} \frac{1}{(\sigma_q + \Omega)} \left[ (\sigma_q - 1)(\beta^E - \beta^M) - \Phi + \frac{1}{|\lambda|} (\hat{L} - \hat{K}) \right]
\] (50)
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