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New Zealand Agricultural and Resource Economics Society (Inc.)

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A Binomial Tree Approach to Valuing Fixed Rotation Forests and Flexible Rotation Forests Under a Mean Reverting Timber Price Process

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Abstract

NPV and LEV are established and common approaches to valuing single rotation and infinite rotation forests respectively, when the rotation age is fixed in advanced. More recently, Real Options approaches have been employed to value single and infinite rotation forests with a flexible harvest age.

Under a stochastic timber price process, it has been shown that the valuation of a flexible rotation forest is equal or higher than that of a fixed rotation forest, because a flexible harvest regime delays the harvest if the timber price is not favourable, whereas a fixed harvest regime would proceed to harvest regardless of the price.

Often, valuation of fixed and flexible rotation ages are compared using 2 different methods – NPV (or LEV) and Real Options. The latter tends to have higher data requirements, employ different assumptions and is much more complex to estimate. Because of these differences, it may be difficult to isolate the cause of the increased valuation.

In this work, we apply a relatively simple Binomial Tree method from Guthrie (2009) to value both fixed rotation and flexible rotation forests. This method uses the same data, with the same assumptions for both valuations. By holding everything equal, the difference in valuation is solely attributable to the fixed versus flexible harvesting decisions.

Original results for both single and infinite rotations are presented using New Zealand Radiata Pine data. Under a mean reverting timber price process, the Binomial Tree approach offers useful insights on the increased valuation due to flexible harvest decisions.

Keywords: NPV, LEV, Real Options, Optimal Harvest Decision

1. Introduction

1.1 Fixed Rotation

The NPV formula for valuing single stand with infinite rotations is also known as the Faustmann Formula, named after the German civil servant who correctly formulated the optimal rotation problem, Martin Faustmann (Faustmann, 1849). Faustmann approached the problem by posing a seemingly simple question "How much is a piece of land worth if it is devoted to the growing of trees?". He was first to identify the problem as "choosing the harvest period to maximize the NPV value of a series of future harvests". He showed that the NPV of a forest can be expressed as a sum of discounted net cash flow over an infinite time horizon. [Esa-Jussi (2006)]

The NPV approach is one where the value of the investment is determined at the present day by forecasting expected future cash flows and discounting them at a specific discount rate. This approach is one of the most commonly used methods to determine the optimal rotation and value of a forest. There are several advantages to using this approach:

- relatively simple to work with numerically, making implementation relatively easy.
- forward-looking, as it explicitly models the ability of the project to generate future cash flows.
- accounts for the riskiness of the investment and the time value of money.

Until recently, the NPV approach has been widely accepted as the key method in investment decision-making (including non-forestry investments). However, there are a few major weaknesses of this approach.

All adjustments for risks are captured by the discount rate, which is assumed to be constant throughout the forest's lifetime. The NPV approach also does not account for flexibility due to the assumption of a fixed investment path where decision is made in advanced, and remain unchanged, even when unexpected favourable or unfavourable events arise. It ignores the value that alternative (unexpected) opportunities and choices bring to the investment.

1.2 Flexible Rotation

Flexibility in decision-making is valuable when investors face risks and uncertainties about the future, especially when there is a degree of irreversibility attached to the decisions being made [Dixit and Pindyck (1995)].

Consider the situation faced in forestry where foresters must decide when to harvest the forest. Under the Faustmann NPV approach, the harvesting decision (based on the optimal rotation age calculated from the NPV) will need to be made regardless of the timber price at the time of expected harvest (i.e. it is already pre-decided upfront when the trees were first planted). The decision to replant will also have to be made immediately, as per the optimal rotation plan.

In addition, once harvested, the trees cannot be put back into the ground. The harvesting decision is irreversible, and should the timber price be low during the harvest, the "loss" in profits is also permanently irreversible.

Given that foresters face uncertainties and irreversibilities, it may be optimal for them to remain flexible about the harvesting decisions. If timber prices are low at the "expected" time of harvest, foresters may want to delay harvest, wait-and-see before making a harvesting decision. Likewise, if timber prices are unusually high before the "expected" time of harvest, foresters may want to

harvest early to take advantage of the high prices. Uncertainties and irreversibilities of an investment decision cannot be easily introduced into the NPV approach. In order to better manage the true potential of the returns, foresters should use a framework or tool that can accommodate a flexible investment decision. The Real Options approach offers such flexibility.

1.3 The Real Options Approach

Evaluation of an investment has traditionally been performed using cost-benefit analysis, a method that includes benefit-cost ratio, NPV of benefits and costs, internal rate of return, return on invested capital, and opportunity costs. Traditionally, cost-benefit analysis is oriented toward making a simple decision: should an investment project be undertaken today? This decision is basically a "now-or-never" decision. Missing from this traditional analysis is the possibility (or option) of delay.

A more recent method in financial economics, called contingent claims analysis, deals with the optimal timing of the investment, and/or when to exercise an investment option. This method places a value on the option of investing in the future. In addition, it is based on the notion that many possible states will be encountered throughout the investment, and it enables decision makers to change strategy in response to these various states.

The capacity to be flexible when uncertainties and irreversibilities exist increases the value of the investment. The greater the degree of flexibility, the more (potentially) valuable the investment can return. [Copeland and Antikarov (2001)]

A financial option is a derivative security whose value is derived from the worth and characteristics of another financial security, or the so-called underlying asset. By definition, a call option gives the holder the right, but not the obligation to buy the underlying asset at a specified price (i.e. the exercise price) on or before a given date (i.e. the expiration date). [Reuer and Tong (2007)]. In contrast, a put option gives the holder the right to sell the underlying asset.

Financial economists Black and Scholes (1973) and Merton (1973) pioneered a formula for valuation of a financial option, and their methodology has opened up subsequent research on the pricing of financial assets and paved the way for the development of Real Options theory.

The notion of Real Options was developed by Myers' (1977) seminal idea that one can view firms' discretionary investment opportunities as a call option on real assets, in much the same way as financial call option provides decision rights on financial assets. By way of analogy, a Real Option can have its underlying asset as the gross project value of expected operating cash flows, its exercise price as the investment required to obtain this underlying asset, and the time to maturity as the period of time during which the decision maker can defer the investment before the investment opportunity expires.

In short, Real Options are investments in real assets (as opposed to financial assets), which confer the investor the right, but not the obligation, to undertake certain actions in the future. [Schwartz amd Trigeorgis (2004)]

There are 3 approaches to implementing Real Options valuations:

- Partial Differential Equation (PDE): The PDE approach treats time as a continuous variable and expresses the present value of a cash flow stream as the solution to a PDE. The most famous such PDE appears in Black and Scholes (1973). This is the standard and most widely

used Real Options valuation method in academic research due to its mathematical elegance and insights. For example, in Pindyck (1993), the author studied the uncertain cost of investment in nuclear power plants, where he derived a decision rule for irreversible investments subject to technical and input uncertainties.

- Binomial Trees: Developed by Cox, Ross and Rubinstein (1979), the binomial tree approach treats time as a discrete variable and expresses the present value of a cash flow stream as the solution to a system of simple linear algebraic equations. This method's precision can be improved to a very high level by dividing the life span of an option into more stages. This discrete-time approach (to the timber price process) is mathematically simpler than the PDE method, yet it provides an efficient procedure for valuing options. Copeland and Antikarov (2001) applied binomial trees to value real projects and proved that this method is equivalent to the PDE solution. It is easy to use without losing the insights of the PDE model.

- Simulation: A simulation typically computes thousands of possible paths describing the evolution of the underlying asset's value from the start period to the end period. It can handle problems of higher dimensions [Gamba (2003)]. With the advancement of computing power, large simulation programs are being used to construct value options that are very difficult to solve using PDEs or binomial trees. Though powerful, this method is not very insightful (compared to the closed form PDE solutions) because it only provides the answer (valuation) without insights into the relationships between variables and the key drivers for the valuation.

1.4 The Real Options Approach Applied to Forestry

Traditional, Faustmann harvesting ignores the annual timber price fluctuations and prescribes harvest on the basis of expected prices. Brazee and Mendelsohn (1988) recognized the volatility of timber prices from year to year, and incorporated a stochastic timber price into their work. Their work concluded that the flexible price harvest policy significantly increases the present value of expected returns over the rigid Faustmann model. Clarke and Reed (1989) and Reed and Clarke (1990) further distinguished the stochastic uncertainties of the timber price and the timber growth.

Miller and Voltaire (1983) were two of the first authors to introduce Real Options into forestry. Morck, Schwartz and Stangeland (1989) used a PDE approach (contingent claims) to determine the optimal harvesting rate. Thomson (1992) employed a binomial tree to determine land rent endogenously assuming stumpage prices follow the Geometric Brownian Motion (GBM) process.

Plantinga (1998) highlighted the role of option values in influencing the optimal timing of harvests. The author treated an option value as a premium over the expected value of a timber stand reflecting the opportunity cost of harvesting now and foregoing the option to delay harvest until information on future stand values is revealed.

Gjolberg & Guttormsen (2002) applied the Real Options approach to the tree-cutting problem under the assumption of mean-reverting (rather than random-walk) stumpage prices.

Insley (2002) investigated the role of the timber price process on the rotation length in a single-rotation model. A dynamic programming approach and a general numerical solution technique were used to determine the value of the option to harvest a stand of trees and the optimal cutting time when timber prices follow a known stochastic process. In Insley and Rollins (2005), the authors extended the single-rotation work of Insley (2002) to multiple rotations, and analyzed forest stand value with stochastic timber prices and deterministic wood volume.

Duku-Kaakyire and Nanang (2004) compared a forestry investment using the Faustmann NPV model and the Real Options approach. They investigated four options: an option to delay deforestation, an option to expand the size of the wood processing plant, an option to abandon the processing plant if timber prices fall below a certain level, and an option that included all three of these individual options. This analysis was conducted using the binomial tree method. The results show that while the Faustmann analysis rejected investments as unprofitable, the real option analysis showed that all 4 options were highly valuable. It demonstrated the weakness of the Faustmann approach, namely, the lack of managerial flexibility to adjust for shocks, risks and uncertainty.

Commonly, valuations of fixed and flexible rotation ages are compared using different and separate methods: an NPV/LEV model and a Real Options model. In Meade et al. (2008), results from a simulation method called Bootstrapping Real Options Analysis (BROA) was compared to results from a NPV (discounted cash flow) calculation. In Manley and Niquidet (2010), the authors compared 3 real option value methods with the Faustmann method. In such comparisons, the Real Options models tend to have higher data requirements, employ different assumptions and is much more complex to estimate compared to NPV/LEV. Because of these differences, it may be difficult to isolate the cause of the increased valuation.

In Guthrie (2009), the author applied the binomial tree method to study the optimal harvest decision of forests in Oregon (USA) using a mean-reverting timber price process. The same binomial tree method was able to generate results for Real Options (flexible harvest decision) and NPV/LEV (fixed rotation), for both single and infinite rotations.

1.5 Approach Taken in This Paper

In this paper, we apply this Binomial Tree with Mean Reverting price process approach from Guthrie (2009) to Radiata Pine forests in New Zealand. Original results for optimal valuation for both flexible and fixed rotations, under a single and infinite rotation, are generated using the same model. This method uses the same data, with the same assumptions for both valuations. By holding everything equal, the difference in valuation is solely attributable to the fixed versus flexible harvesting decisions, rather than partially attributing the differences to the methodologies (arising from the use of two separate models, data and assumptions).

By setting the volatility of the Mean Reverting price parameter in the Binomial Tree equal to zero, we produce further valuation results for a constant timber price. This is compared with the result obtained via a standard NPV/LEV calculation using an Excel spreadsheet. We show that both the Binomial Tree method and the NPV/LEV calculation produce the same result when the timber price is constant.

2. Overview of the Binomial Tree Method

2.1 Basic Parameters of a Price Binomial Tree: U, D and θ_U

A Binomial Tree starts with the present price, X0. Each "branch" of the tree represents a different possible future price, at a certain time in future. At each timestep, there are only 2 possible price movements from the price at a node: the price can either rise by a multiplicative factor U, or fall by a multiplicative factor D. (D = 1/U). So, at timestep n=1, the current price can either rise to X0*U, or fall to X0*D.

A Binomial Tree is simplified using the X(i,n) labelling as shown in Figure 1 below, where i is the number of Down steps, and *n* is the timestep. For example, X(1,1) is the price at timestep 1 where 1 Down step has occurred (after 1 time step), and X(0,2) is the price at timestep 2 where 0 Down steps have occurred (i.e. the price has been going Up at every node leading to this node).

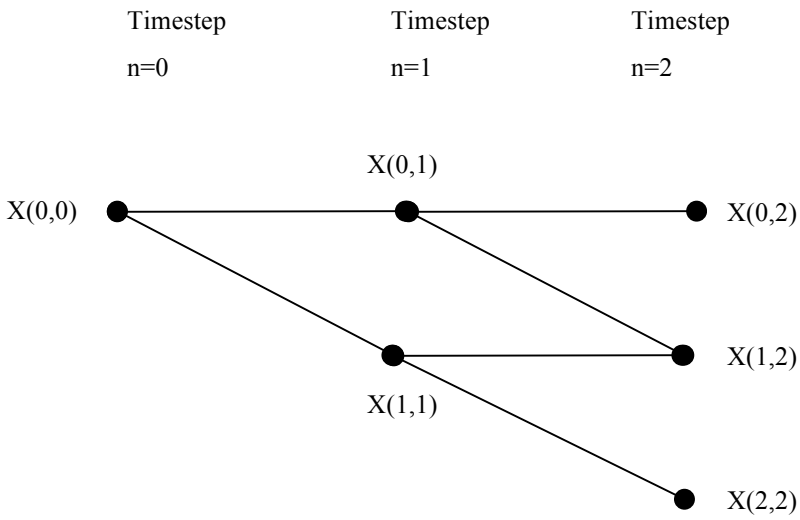

Figure 1: A Binomial Tree labeling convention.

Due to the multiplicative properties of the binomial tree, the order of the sequence of Ups and Downs are irrelevant, so that an Up step followed by a Down step has the same effect as a Down step followed by an Up state:

$$X(0,0)*U*D = X(0,0)*D*U = X(0,1)*D = X(1,1)*U = X(1,2)$$

Probabilities are assigned to the Up moves and also the Down moves. θ_U is the probability of an Up move, and θ_D is the probability of a Down move defined as $\theta_D = 1 - \theta_U$ such that θ_U and θ_D sum to unity.

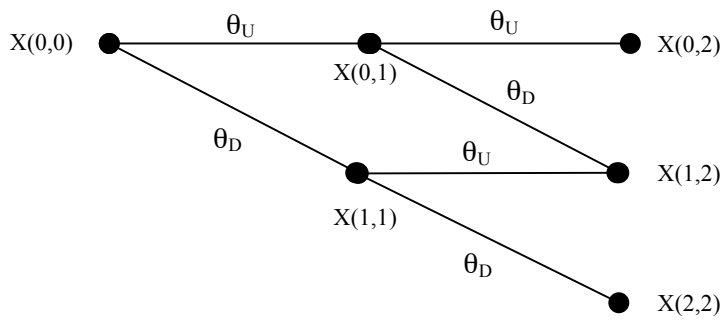

Figure 2: Probability of an Up move, θ_U, and probability of a Down move, θ_D.

2.2 Estimating U, D and θ_U of a Price Binomial Tree Using Historical Price Data

Assume the price series is mean reverting, such that the logarithm of the price follows a first-order autoregressive process (referred to as AR(1) process) [Guthrie (2009)]. That is, if p_j denotes the jth observation of the logarithm of the price, then:

$$\begin{aligned} p_{j+1} - p_j &= \alpha_0 + \alpha_1 p_j + u_{j+1} \\ u_{j+1} &\sim N(0,\phi^2) \end{aligned} \qquad \text{Equations (1)}$$

That is, changes in p are normally distributed with mean $\alpha_0+\alpha_1 p_j$ and variance ϕ^2, and u_{j+1} is a noise term. α_0, α_1 and ϕ are related to the Ornstein-Uhlenbeck parameters by the following equations:

$$\begin{aligned} \alpha_0 &= (1 - e^{-a\Delta t})b \\ \alpha_1 &= -(1 - e^{-a\Delta t}) \\ \phi^2 &= \frac{\sigma^2}{2a}(1 - e^{-2a\Delta t}) \end{aligned} \qquad \text{Equations (2)}$$

where a = rate of mean reversion, b = long-run level, σ = volatility of the Ornstein-Uhlenbeck process, and Δt = time step size.

An Ordinary Least Squares (OLS) regression of the price data produces estimates of α_0, α_1 and ϕ respectively as:

$$\begin{aligned} \hat{\alpha}_0 &= (1 - e^{-\hat{a}\Delta t_d})\hat{b} \\ \hat{\alpha}_1 &= -(1 - e^{-\hat{a}\Delta t_d}) \\ \hat{\phi}^2 &= \frac{\hat{\sigma}^2}{2\hat{a}}(1 - e^{-2\hat{a}\Delta t_d}) \end{aligned} \qquad \text{Equations (3)}$$

where Δt_d is the time step size of the price data. Solving for estimates of a, b and σ produces:

$$\begin{aligned} \hat{a} &= \frac{-\log(1+\hat{\alpha}_1)}{\Delta t_d} \\ \hat{b} &= \frac{-\hat{\alpha}_0}{\hat{\alpha}_1} \\ \hat{\sigma} &= \hat{\phi}\left(\frac{2\log(1+\hat{\alpha}_1)}{\hat{\alpha}_1(2+\hat{\alpha}_1)\Delta t_d}\right)^{1/2} \end{aligned} \qquad \text{Equations (4)}$$

From equations (4), the sizes of up and down moves (U and D) are derived as:

$$\begin{aligned} U &= e^{\hat{\sigma}\sqrt{\Delta t_m}} \\ D &= e^{-\hat{\sigma}\sqrt{\Delta t_m}} \end{aligned} \qquad \text{Equations (5)}$$

where Δt_m is the time step size of the binomial tree.

Each X(i,n) on the binomial tree is calculated by applying U and D to X(i,n) starting with X(0,0):

$$\begin{aligned} X(i,n+1) &= X(i,n)U \\ X(i+1,n+1) &= X(i,n)D \end{aligned} \qquad \text{Equations (6)}$$

$\theta_u(i,n)$ is then estimated as:

$$\theta_u(i,n) = \begin{cases} 0 & \text{if } \frac{1}{2} + \frac{(1-e^{-\hat{a}\Delta t_m})(\hat{b} - \log(X(i,n))}{2\hat{\sigma}\sqrt{\Delta t_m}} \le 0 \\ \frac{1}{2} + \frac{(1-e^{-\hat{a}\Delta t_m})(\hat{b} - \log(X(i,n))}{2\hat{\sigma}\sqrt{\Delta t_m}} & \text{if } 0 < \frac{1}{2} + \frac{(1-e^{-\hat{a}\Delta t_m})(\hat{b} - \log(X(i,n))}{2\hat{\sigma}\sqrt{\Delta t_m}} < 1 \\ 1 & \text{if } \frac{1}{2} + \frac{(1-e^{-\hat{a}\Delta t_m})(\hat{b} - \log(X(i,n))}{2\hat{\sigma}\sqrt{\Delta t_m}} \ge 1 \end{cases} \qquad \text{Equations (7)}$$

$\theta_u(i,n)$ is used to calculate the valuation, described in the next section.

2.3 Valuation Binomial Tree

Binomial trees are also used to implement valuation. The probability of an Up move (θ_U) and probability of a Down move (θ_D) in the *price binomial tree* are applied to the *valuation binomial tree*. These probabilities determine prices, and prices determine the cash flows, which in turn, determines the valuation.

The valuation binomial tree follows a similar labeling convention to the price binomial tree. Each node is labeled V(i,n), representing valuation at time step n, with i number of Down moves in the price. For example, the valuation binomial tree for N=2 is:

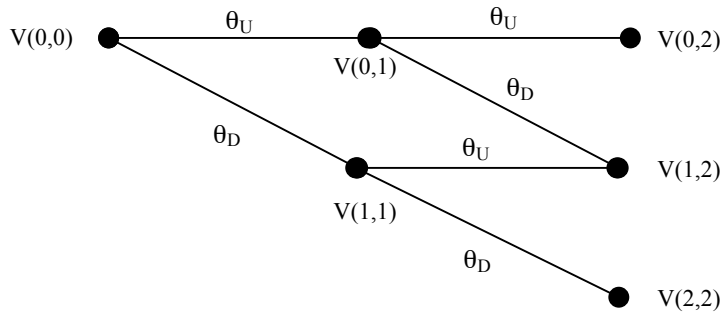

Figure 3: Valuation binomial tree.

In contrast to the price binomial tree which is calculated forward using X_0, U and D, the valuation binomial tree is calculated backwards (in reverse) starting from the terminal (last) nodes V(i,N). We define V(i,N), the harvest valuation function at terminal node N, as:

$$V(i,N) = (X(i,N) - H)Q(N\Delta t_m) + B \qquad \text{Equation (8)}$$

where X(i,N) is the price at time step N, H is the harvesting cost, Δt_m is the time step size of the binomial tree, $Q(N\Delta t_m)$ is the timber volume at time N, and B is the bareland value.

Discount rates are added to the valuation calculations to reflect the time value of money. For example:

$$V(0,1) = \frac{\theta_U V(0,2)}{R_f} + \frac{\theta_D V(1,2)}{R_f} \qquad \text{Equation (9)}$$

where R_f = (1+discount rate).

In addition to the discount rate, we can use the Capital Asset Pricing Model (CAPM) to further reflect a market risk premium into the valuation. A Market Risk Premium Adjustment (MRP_{Adj}) is subtracted from the θ_U to produce the so-called Risk Neutral probability Π_U [Guthrie (2009)]:

$$\begin{aligned} \Pi_U &= \theta_U - MRP_{Adj} \\ \Pi_D &= 1 - \Pi_U \end{aligned} \qquad \text{Equations (10)}$$

The MRP_{Adj} is obtained by regressing stock market data such as the Standard and Poors (S&P) 500 Total Returns Index [Guthrie (2009)].

We note here that valuation using binomial tree could also be performed without the CAPM element (i.e. using θ_U and θ_D instead of Π_U and Π_D). In such a case, one would incorporate the appropriate level of risk premium by simply choosing a higher factor rate (R_f).

2.4 Incorporating Flexible Harvesting Decisions in the Valuation (Real Options Valuation)

When calculating the valuation (backwards), a decision on whether to harvest or not to harvest is re-evaluated at each and every node. If the cash flow from harvest (i.e. cash flow at the node) is more than the expected future cash flows (i.e. cash flows from not harvesting), then, the optimal decision is to harvest, and the valuation at the node equals the cash flow from harvest. If the present value of the expected future cash flows (i.e. those from not harvesting) is more than the present value of the cash flows from harvesting, then, the optimal decision is to not harvest, and the valuation at the node equals the present value of the corresponding expected future cash flows. That is:

$$V(i,n) = \max\left\{ (X(i,n) - H)Q(n\Delta t_m) + B,\ -C + \frac{\Pi_u(i,n)V(i,n+1) + \Pi_D(i,n)V(i+1,n+1)}{R_f} \right\} \qquad \text{Eqn (11)}$$

where C is the maintenance cost of the forest. The first argument of the max function represents the cash flow from harvesting, whereas the second argument represents the cash flow from not harvesting.

As mentioned previously, this process traverses backwards from n=N to n=0, ending with V(0,0). The Binomial Tree valuation is implemented backwards recursively over multiple iterations. Each iteration represents 1 harvest and replant rotation. During the calculation for the 1st iteration, the Bareland value, B, is assumed to be zero. At the end of the 1st iteration, a Bareland value is estimated by deducting the cost of (re-)planting the forest from V(0,0):

$$B = V(0,0) - G \qquad \text{Equation (12)}$$

where G is the cost of (re-)planting the forest. This 1st iteration Bareland value is the valuation for a single rotation forest with flexible harvesting age (real options valuation for single rotation).

To calculate the value for an infinite rotation forest, this 1st iteration Bareland value is then fed into the 2nd iteration (i.e. during the 2nd iteration of valuation calculations, B in the V(i,n) function of

Equation 11 is no longer zero). After this process is repeated for a certain amount of iterations (say, 15 iterations), the Bareland value converges to a steady state value (i.e. it no longer changes with subsequent iterations). This converged Bareland value is the valuation for an infinite rotation forest with flexible harvesting age (real option valuation for infinite rotation).

2.5 Applying the Valuation Method to a Fixed Harvest Age (Fixed Rotation NPV/LEV)

To apply this valuation method to a fixed harvest age, the same process is used with one modification. The harvest decision is fixed (pre-decided regardless of the price) at the node where t = fixed harvest age (i.e. use node t as the terminal node instead of N where t < N). All nodes on the valuation binomial tree to the right side of t (i.e. all nodes between t+1 and N) are ignored (truncated) and the backward traverse starts from node t (instead of node N for the case of flexible rotation forest).

During each node traverse, unlike the flexible harvest age case, there is no re-evaluation of a harvest decision (i.e. no harvest decision reconsidered at subsequent nodes) because there is already a fixed (pre-decided regardless of price) harvest decision at node t (= fixed harvest age). As such, the valuation for each node from n = (t-1) to n = 0 is:

$$V(i,n) = -C + \frac{\Pi_u(i,n)V(i,n+1) + \Pi_D(i,n)V(i+1,n+1)}{R_f} \qquad \text{Equation (13)}$$

This is the only modification required to compute the fixed harvest age results. The value of B after the 1st iteration is the single rotation NPV. After a certain number of iterations (say, 15 iterations), B converges to the infinite rotation LEV.

3. Data Used and Assumptions Made

For this work, the timber volume function (table) was sourced from the R300 Radiata Pine Calculator model from Kimberley et al (2005). Figure 4 shows the timber volume function.

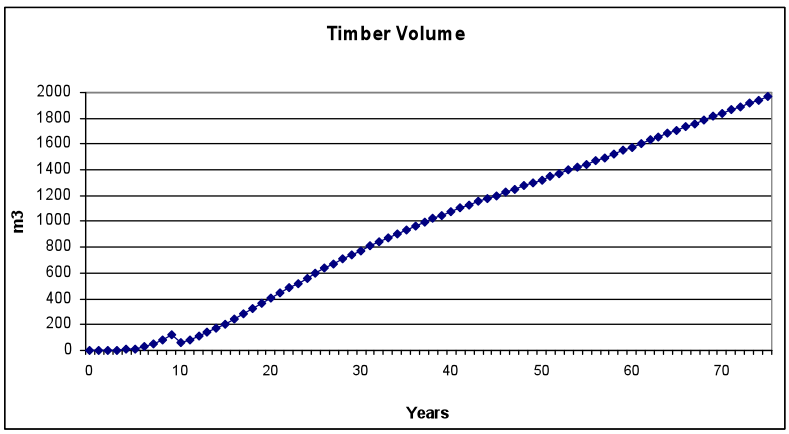

Figure 4: Timber volume function based on the R300 Radiata Pine calculator.

Figure 5 shows the MAF log price data [Horgan (2010)] aggregated into a single proxy timber price series, adjusted with the Consumer Price Index from Statistics New Zealand (2010).

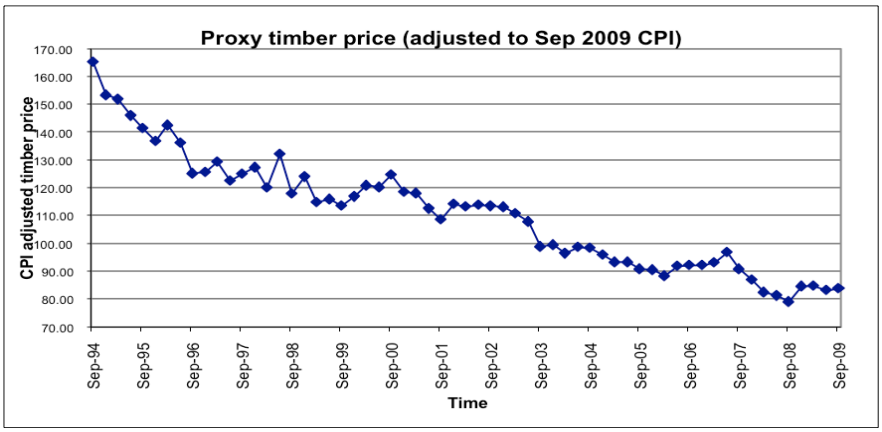

Figure 5: Aggregated proxy timber price, CPI adjusted.

The forest management costs are assumed to be:

- Planting costs, G = \$1,251/ha
- Pruning costs = \$473/ha (age 6), \$674/ha (age 7), \$684/ha (age 8)
- Thinning costs = \$370/ha (age 9)
- Maintenance cost of forest per year, C = \$50/ha
- Harvesting cost (clearfell logging), H = \$40/m^3

The Ordinary Least Squares (OLS) regression of the proxy timber price series produced:

$$\hat{\alpha}_0 = 0.235634$$

$$\hat{\alpha}_1 = -0.052569$$

$$\hat{\phi} = 0.039287$$

Substitution of these variables into Equations (4) produced:

$$\hat{a} = 0.216006$$

$$\hat{b} = 4.482340$$

$$\hat{\sigma} = 0.080705$$

From these values, U and D are estimated as 1.0236 and 0.9770 respectively, which as used to calculate X(i,n) and θ_U of the price binomial tree. The long run timber price is $e^{\hat{b}} = \$88.44$

The cash flow discount rate is assumed to be 4%, such that $R_f = 1.04$.

Based on Graham and Harvey (2009), a Market Risk Premium of 4.3% is assumed, and MRP_{Adj} is estimated to be -0.0038, which is used to calculate Π_U and Π_D of the valuation binomial tree.

Results for rotation ages of up to 75 years were generated. 21 harvest-and-replant cycles are used to represent infinite rotation. For example, if the rotation age is 30 years, then, an approximation for infinite rotation is 21 x 30 = 630 years. If the rotation age is 75 years, then, an approximation for infinite rotation is 21 x 75 = 1575 years.

4. Results for Binomial Tree Method (Mean Reverting Timber Price)

4.1 Binomial Tree Method with Fixed Harvest Age (Mean Reverting Price)

Figure 6(a) shows the results for fixed harvest age cases of single and infinite rotations, with Figure 6(b) zooming into the maxima points between ages 24 and 35. For a single rotation with fixed harvest age, the optimal rotation age is 31 years, with an NPV valuation of $6,385. For an infinite rotation with fixed harvest age, the optimal rotation age is 27 years (i.e. the Faustmann rotation age), with an LEV valuation of $9,419.

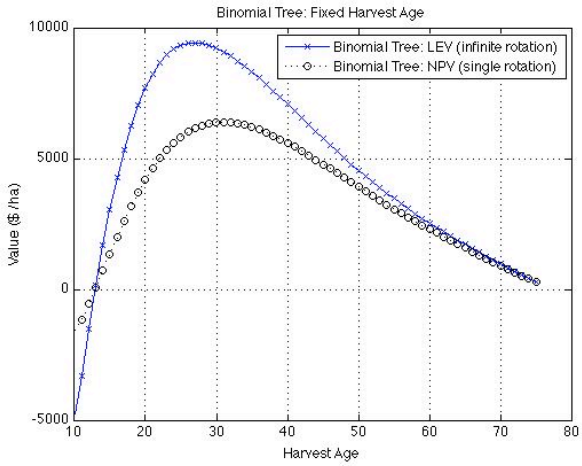

Figure 6(a): Results for fixed harvest age cases of single and infinite rotations.

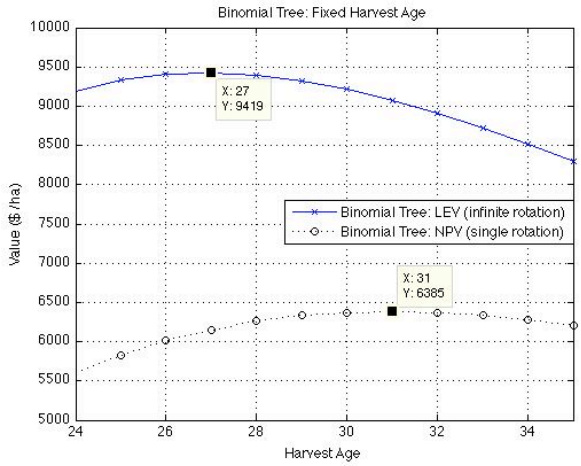

Figure 6(b): Enlargement on the maxima points of Figure 6(a).

4.2 Binomial Tree Method with Flexible Harvest Age (Mean Reverting Price)

Figure 7 shows the results for flexible harvest age cases of single and infinite rotations. For a single rotation, the optimal valuation is the bareland value after 1 iteration (as described in Section 2.4), which is $8,188. For an infinite rotation, the bareland value converges to $11,590 after about 8 harvest-and-replant cycles.

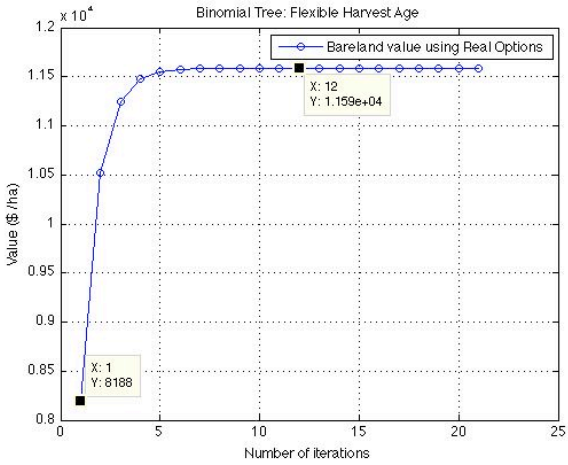

Figure 7: Results for flexible harvest age cases for single and infinite rotations.

Compared to the case with a fixed harvest age, allowing a flexible harvest age results in 28% and 23% higher valuations for single and infinite rotations respectively.

Figure 8(a) and 8(b) show the optimal harvest thresholds for single and infinite rotations respectively. The area on the graphs above the dotted line shows the timber prices that favour a

harvest decision for a given forest age. As a example in Figure 8(a), the threshold for a 25 year old single rotation forest is $83.90 such that if the timber price at that time (age 25) is above this threshold, it would be optimal to harvest, whereas if the timber price at that time (age 25) is below this threshold, it would not be optimal to harvest (and the optimal decision would be to defer harvest).

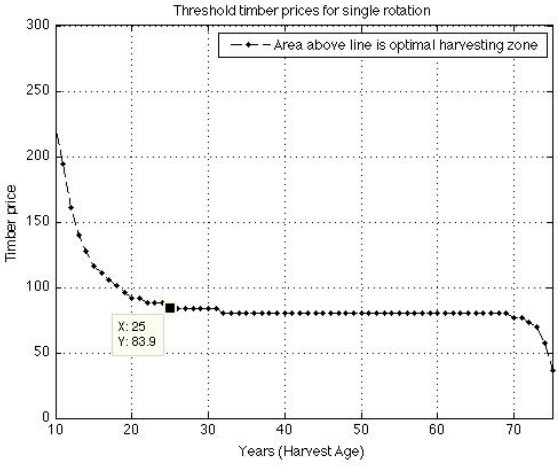

Figure 8(a): Optimal harvest threshold for single rotation.

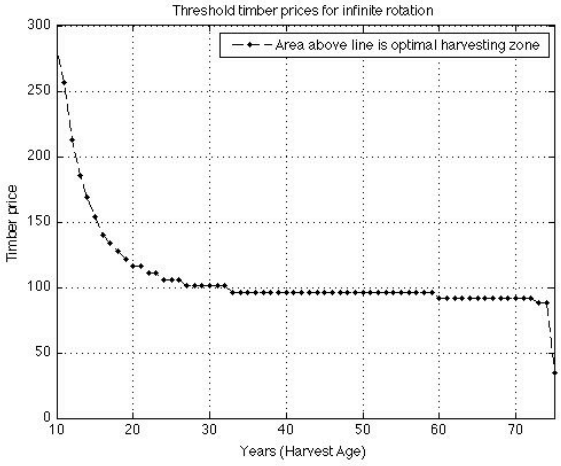

Figure 8(b): Optimal harvest threshold for infinite rotation.

5. Results for Binomial Tree Method (Constant Timber Price)

If the volatility of the mean reverting price process is set equal to zero, then, it results in a constant timber price. In this section, the volatility of the price process is set equal to zero, and results from the Binomial Tree valuation is presented for a constant timber price, assumed to be the current timber price value, X(0,0) = $83.90.

5.1 Binomial Tree Method with Fixed Harvest Age (Constant Price)

Figure 9(a) and 9b) show the NPV and LEV results obtained from the Binomial Tree valuation with fixed harvest age at a constant price. For single rotation, the optimal age is 31 with a valuation of $5,141. For infinite rotation, the optimal age is 27 with a valuation of $7,572.

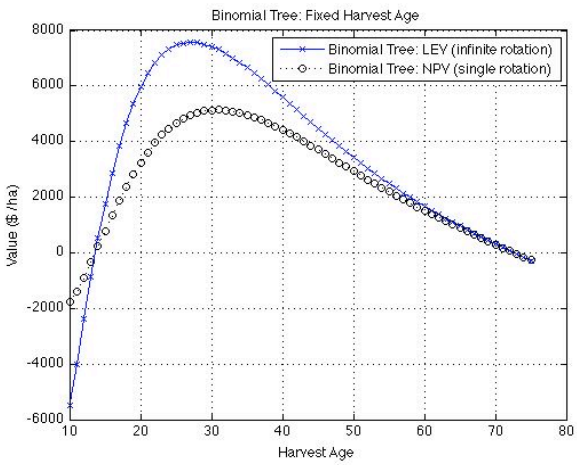

Figure 9(a): Results for fixed harvest age cases of single and infinite rotations at a constant price

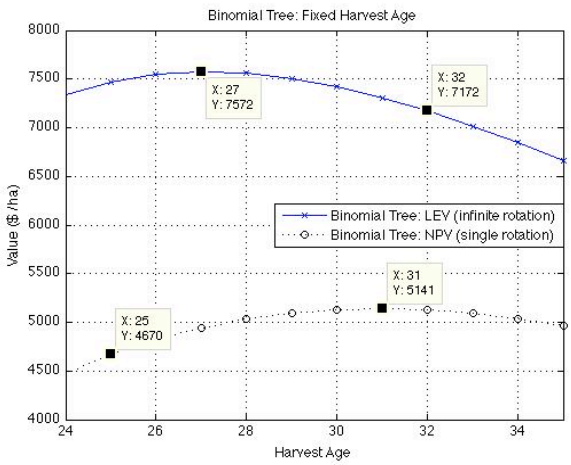

Figure 9(b): Enlargement on the maxima points of Figure 9(a).

5.2 Binomial Tree Method with Flexible Harvest Age (Constant Price)

Figure 10 shows the results obtained from the Binomial Tree valuation with flexible harvest age at a constant price. The single and infinite rotation bareland values are $5,141 and $7,572 respectively. These are identical figures to the maximum NPV and maximum LEV values of Figures 9(b).

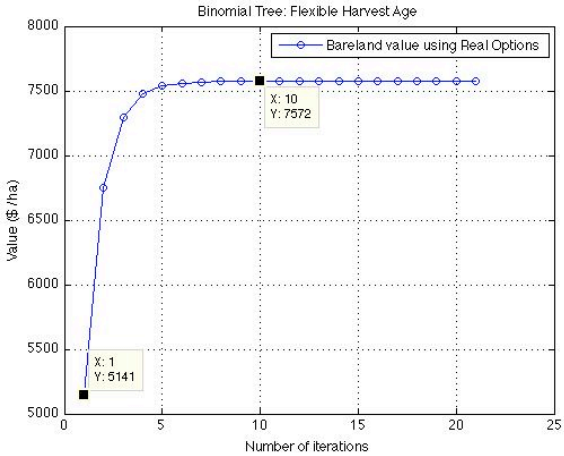

Figure 10: Results from Binomial Tree valuation with flexible harvest age at a constant price.

These results show and confirm that when prices are constant, fixed harvest age valuation (NPV and LEV) produces the same result as flexible harvest age valuation (Real Options). This is because when the price is constant, there is no flexibility because prices do not rise/fall, and therefore, there is no additional value from delaying harvest. The optimal flexible harvesting decision produces the same results as the optimal fixed harvesting decision, which are the NPV and LEV values.

6. Results from Standard NPV and LEV at a Constant Price

In this section, we generate results for fixed harvest age at a constant price using the standard Excel spreadsheet NPV and LEV calculation. Results are summarized in Table 1. For single rotation, the optimal age (i.e. highest NPV) is 31 years with an NPV of $5,141, whereas for infinite rotation, the optimal age (i.e. highest LEV) is 27 years with an LEV of $7,572.

Harvest Age	*NPV*	*LEV*
25 years	$4,670	$7,473
26 years	$4,825	$7,547
27 years	$4,946	**$7,572**
28 years	$5,037	$7,556
29 years	$5,098	$7,504
30 years	$5,132	$7,419
31 years	**$5,141**	$7,307
32 years	$5,127	$7,172
33 years	$5,093	$7,016
34 years	$5,040	$6,844
35 years	$4,970	$6,657

$$LEV_n = NPV_n \frac{R_f^{\,n}}{R_f^{\,n} - 1}$$

where n is the harvest age

Table 1: Results for fixed harvest age at a constant price using the standard Excel spreadsheet NPV and LEV calculation.

What is worth noting here is that the valuations in the NPV and LEV columns of Table 1 are identical to each of the values generated from the Binomial Tree valuation in Figure 9(b). To illustrate this, we highlight 4 data points in Figure 9(b) – $4,670 at 25 years, $5,141 at 31 years, $7,572 at 27 years and $7,172 at 32 years – which are equal to the same points in Table 1. These numbers show that both methods are equivalent when prices are constant.

7. Conclusions

Results in section 4.2 show that when compared to fixed harvesting, flexible harvesting results in 28% and 23% higher valuations for single and infinite rotations respectively because it takes advantage of price fluctuations by deferring harvest when prices are low and accelerating it when they are high, generating the higher valuation. For flexible harvest age, the binomial tree method also produces the price thresholds for optimal harvesting, which is a potentially useful tool for foresters in making an optimal harvest/no-harvest decision given a timber price at a certain age.

From section 5.2, we conclude that when the price is constant, the fixed (NPV/LEV) and flexible (Real Option) harvesting decisions produce the same valuations. When prices do not rise or fall, there is no additional value from delaying harvest. The optimal flexible harvesting decision produces the same results as the optimal fixed harvesting decision, which are equal to the NPV and LEV values.

From section 6, we concluded further that the Binomial Tree method is equivalent to the standard NPV/LEV method when the price is constant. Therefore, the differences in valuations that we obtain (in section 4.2) can be attributed to the more flexible harvest policies that we allow, rather than differences in factors such as price forecasts and discount rates.

The Binomial Tree method is a promising method for modeling and studying the effects of fixed versus flexible harvesting, for both single and infinite rotations, under a mean reverting timber price process. It is a relatively simple way to implement Real Options analysis. NPV and LEV results can also be produced using this same model, data, and assumptions, allowing for differences in valuations to be attributed solely to the fixed versus flexible decision rather than differences in model sophistication, data requirement or assumptions employed.

Going forward, as we advance our understanding and modeling of the timber price process in New Zealand, the Binomial Tree method can offer useful insights, allowing a forester to take into account contemporary pricing information to make a potentially better investment decision.

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