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# A SURVEY OF AGRICULTURAL ECONOMICS LITERATURE

VOLUME 2

# Quantitative Methods in Agricultural Economics, 1940s to 1970s

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# Optimization Models in Agricultural and Resource Economics

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The application of optimization concepts to the economics of agriculture and resource use has a history as long as mathematical economics itself. It was in the context of agriculture in an "isolated state" that von Thünen [1966] in 1826 developed his own concept of gain and loss at the "margin" and used it to develop a theory of relative economic value and spatial diversity in the use of land, labor, and capital. Indeed, we have it on the good authority of Marshall [1890] that von Thünen, the first agricultural economist among economists, along with Cournot, provided the initial inspiration for marginalist economics.

On the other hand, von Thünen and his classical predecessors Smith, Malthus, and Ricardo were also employing concepts that are most effectively represented by the use of linear programming theory. It was not until the modern era that the full unity underlying these different classical and neoclassical optimization approaches could be brought out by means of a single mathematical structure which incorporated both points of view as special cases namely, the Kuhn-Tucker theorem (Kuhn and Tucker [1951]).

The balanced blend of analytical reasoning and careful empirical observation that characterized von Thünen's work was evident in the work of later economists who specialized in agriculture. An important example is provid-

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ed by "budgeting." Its development and widespread application in agricultural economics occurred in the first quarter of this century. Not only was it an extension of von Thünen's early studies, but it played a central role in the education of a generation of agricultural economists, thereby helping to prepare the discipline for the rapid adoption of modern optimization methods. Indeed, in the hands of its best practitioners, budgeting was more than a trivial special case of mathematical programming. It was an explicit arithmetic procedure for obtaining approximate optima of simple constrained optimization problems and for exploring the broader implications at the regional and national levels of economic behavior in response to changing economic conditions and policy controls.

The budgeting era may be said to have reached its culmination in 1951, for in that year Mighell and Black's masterly exercise in budgeting, *Interregional Competition in Agriculture*, was published. It is something of an irony that the modern optimization methods introduced in the same year effectively rendered obsolete that splendid monument to good economic thinking and patient arithmetic. While no one would think of doing it that way any more, it is clear that the modern approach has formalized economic optimization and has eased the computational burdens of using it but has added few if any insights into the nature of the problem not already fully appreciated in the economic literature.

The first specific application of modern optimization to agricultural economics was by Hildreth and Reiter in 1951, and the application to the spatial problems that had dominated much of von Thünen's original work came with Fox's study [1953] of the feed-livestock economy. But the rapid adoption and widespread application of modern optimization methods to the economic analysis of agriculture and resource use may have been largely the result of the extensive and varied examples produced by Earl Heady and his associates in the 1950s. From that period on the application of optimization concepts to the formulation and solution of substantive problems in agriculture and resource economics has led to a literature so vast that a comprehensive survey is impossible. Consequently, this survey is restricted to selected contributions (primarily from the American literature) that are of seminal importance from a historical point of view, that are representative of an important research area, or that are of contemporary interest. The references cited in the text are supplemented in the bibliography by a few key survey papers on research that could not be covered in this paper.

Modern optimization methods and their application in agriculture and resource management are of interest to specialists in many fields, and as a result articles published in the professional journals for general economics, engineering, operations research, and so on are relevant. Moreover, economists must

be aware of the bulletins emanating from various federal agencies, state experiment stations, world organizations such as the United Nations, and the International Bank for Reconstruction and Development, as well as journals published in other countries. With these observations in mind perhaps the reader may find it possible to forgive the authors for any oversights that occur in this paper and for the somewhat arbitrary nature of the selected references.

The literature reviewed is divided into categories of food and diet, farm and agribusiness management, farm firm development, production response, interregional and spatial economics, natural resources, and agricultural development problems. For convenience in researching the literature table 1, which follows the text of this chapter, classifies the references according to these categories.

Throughout the discussion "neoclassical optimizing" refers to maximizing smooth, unconstrained, or equation-constrained functions using the basic tools of marginal analysis, i.e., traditional calculus. "Classical optimizing" is used here to include the linear programming problem that underlies the classical rent and trade theories and the budgeting arithmetic of von Thünen and later economists. "Modern optimizing" refers to the maximization of objective functions constrained by inequalities or equalities, requiring generalized Lagrange techniques and including the classical and neoclassical approaches as special cases.

# Food and Diet

The diet problem seems an appropriate subject with which to begin a review of the literature on applications of modern optimization theory in agriculture and resource economics. Obviously, the efficient use of food resources is a goal of growing importance in our finite world with its rapidly growing, often ill-fed population. The diet problem is that of determining the least cost combination of foods that will meet dietary standards. According to Dorfman, Samuelson, and Solow [1958], Jerome Cornfield was the first to formulate this problem in an unpublished memorandum in 1941. Stigler presents a careful statement and discussion of the problem in "The Cost of Subsistence" [1945]. It is interesting to note that his solution of the problem was not cast explicitly in the modern mathematical programming framework, but instead involved a careful application of the arithmetic budgeting procedures known in agricultural economics for decades.

A thorough and illuminating explicit linear programming treatment of the problem (which cites an unpublished 1947 paper by Dantzig and Laderman) is given by Dorfman, Samuelson, and Solow [1958]. An elaborate empirical

study developed for the interesting and important problem of protein supply in a developing economy is found in the work of V. E. Smith [1974].

The economic dietary (food-mix) problem for animal production is logically the same as the one for humans, and Waugh [1951] was the first to exploit linear programming in its explication. The budgeting framework was effectively and independently applied to the same problem at about the same time by Christenson and Mighell [1951]. By way of contrast to the linear programming work, a neoclassical optimizing approach was the basis of Heady's analysis of least cost dairy cow rations [1951] and hog rations (Heady et al. [1953]). An ingenious by-product was the "pork costulator" that allowed farmers to take advantage, without great computational effort, of the marginalism that economic theorists had long supposed to be descriptive of farmers' behavior. Briefly, Heady and his collaborators statistically estimated production functions for animals using experimental data, solved the least cost feedmix problem for various input-output price combinations, and in this way located points on the derived economic demand function for various feed inputs. The input demand schedule was then represented by means of an inexpensive plastic circular slide rule.

# Farm and Agribusiness Management

In 1951 a simplified version of the optimal crop rotation problem using modern optimization methods was published. This was Hildreth and Reiter's contribution [1951] in the famous Koopmans volume. There appears to have been a lag between this seminal application and the widespread adoption of linear programming as a standard working tool in the profession, but the lag was short. With characteristic pragmatism and innovation agricultural economists were quick to see the utility in the new approach. A flood of effective studies in farm management appeared in the mid-fifties. Very early studies were made by King and Freund [1953] and King [1953], Swanson and Fox [1954], and Bowlen and Heady [1955]. Quick to follow were studies by Bishop [1956], Heady, McAlexander, and Schrader [1956], Swanson [1956], and Coutu [1957]. In 1958 Heady and Candler published one of the first comprehensive texts on applied linear programming. Comparative static analyses using parametric linear programming algorithms also appeared at this time (for example, McPherson and Faris [1958] and the elegant piece by Hildreth [1957b]). Somewhat more recent applications are discussed by Krenz, Baumann, and Heady [1962] and Bolton [1964]. This early work is still of interest, and indeed it is worth serious reconsideration for much of it deals with the economics of soil conservation practices which have not been in vogue for some time but which are receiving renewed attention.

The relationship between classical economic (budgeting) thinking and modern optimization theory is reflected in pieces by Mighell [1955] and Kottke [1961] that point out the similarity or equivalence between the budgeting and linear programming approaches. Edwards [1966] gives a lucid exposition of this relationship, and Swanson [1961] notes that linear programming logic has had a profound impact on budgeting procedures. It is also worthwhile to point out that workers in agriculture were quick to find the intellectual intrigue in simplex and parametric programming algorithms and contributed expository pieces as well as methodological wrinkles of considerable ingenuity. Heady's economic interpretation of the simplex algorithm [1954] is a prime example, as are the studies by Hildreth [1957b], Puterbaugh, Kehrberg, and Dunbar [1957], and Candler [1956, 1957, 1960].

The potential usefulness of integer and mixed integer programming in farm management has been recognized for some time. Edwards [1963] suggested a number of possible applications of the techniques to farm problems using Gomory's integer programming algorithm [1958]. There are several integer and mixed integer programming algorithms available, some of which were surveyed in Maruyama and Fuller [1964]. The most current and complete survey of integer programming algorithms is by Geoffrion and Marsten [1972]. Maruyama and Fuller [1964] proposed an "RHS" ("right-hand-side") method, which was essentially a computerized complete enumeration method. Candler and Manning [1961] and Musgrave [1962] used parametric linear programming to deal with decreasing costs and increasing returns. Giaever and Seagraves [1960] and Yaron and Heady [1961] used integer, mixed integer, and nonlinear programming to investigate decisions involving economies of scale.

Marketing analysis of agricultural commodities naturally involves transportation costs and spatial efficiency, aspects of optimization to be considered later in this paper. Several studies, however, may appropriately be mentioned here. Stollsteimer [1963] developed a linear programming model which determines the number, size, and location of plants processing a fixed amount of a single raw material into a single output. Economies of scale were dealt with through the use of parametric programming. King and Logan [1964] attacked the same problem and added an iterative, partly heuristic method to handle economies of scale. Candler, Snyder, and Faught [1972] dealt with a more general problem involving several raw materials and multiple outputs using a concave programming algorithm. The algorithm, a mechanized version of the King-Logan algorithm, is equipped to solve multiple local optima problems. Bressler and Hammerberg [1942] and Hammerberg, Parker, and Bressler [1942] used budgeting to specify optimal route organization and truck

sizes. Bressler [1952] applied the same methods to develop an efficient system of city milk distribution in Connecticut.

By the late 1950s rapidly developing computer technology was expanding the scope for sophisticated programming techniques. Bellman's dynamic programming [1957] was one approach which accordingly found increased application to management decisions in various industries; agriculture was no exception. The earliest agricultural applications of Bellman's approach were to optimal replacement problems, as described by White [1959], Faris [1960], and Halter and White [1962]. Burt and Allison [1963] applied the method to a Markov process in choosing wheat rotations, Minden [1968] proposed the use of dynamic programming as a tool for farm investment decisions, and Hinrichs [1972] discussed a recent application in West German agriculture. One of the most attractive features of dynamic programming is the facility with which stochastic parameters may be incorporated (for example, Burt [1965]). After a first flash of excitement induced by the flexibility and potential of Bellman's approach as typified by Throsby [1964], applications have been limited to rather simple subsystems of total farm systems. The explanation (Throsby [1968]) lies partly in the formidable computational requirements of dynamic programming. This is particularly true of allocation problems such as multiperiod farm investment. When both inputs and outputs are multiple, dynamic programming is beset by the "curse of dimensionality," because computational burdens increase exponentially with the number of outputs or inputs considered.

The significance of risk and uncertainty in the farm environment is underscored by the numerous efforts of agricultural economists to embody these concepts in decision models. Freund [1956] made the first application of active stochastic programming to a farm management problem. This technique is essentially the same as that of Markowitz's portfolio selection technique [1952]; the resultant problem is a quadratic program. Examples of applications of active stochastic programming are provided by McFarquhar [1961], Merrill [1965], and S. R. Johnson, Tefertiller, and Moore [1967]. Compared with linear programming, quadratic programming algorithms make heavy computational demands. Hazell [1971a] develops a technique leading to a linear programming problem which incorporates the mean absolute deviation of the objective function parameters, and Thomson and Hazell [1972] report a Monte Carlo study which indicates that Hazell's method gives results which are quite close to quadratic programming results. Chen's remarks [1971] and Hazell's reply should be read with Hazell [1971a]. A separable programming approach which also approximates Markowitz's E,V method was employed by Thomas et al. [1972]. A particularly important method for incorporating uncertainty is the focus-loss approach introduced into agricultural economics

by Boussard and Petit [1967]. Boussard [1969] later showed that the descriptive power of the model was at least as good as that of alternative models. In 1955 Dantzig offered a model of sequential programming under uncertainty which combined the merits of linear programming and sequential analysis; Cocks [1968], Rae [1971a, 1971b], and Yaron and Horowitz [1972] have applied this model and its extensions to problems of farm management.

Dillon's expository article [1971] reviews thoroughly the application of subjective probability theory to agriculture. This includes as special cases many of the approaches, mentioned above, including E,V analysis.

Early application of game theory to farm management were restricted to "games against nature." Probably the first example was Schickele's [1950] application to climatic uncertainty. Later, Swanson [1957] suggested application of game theoretic frameworks to the same problem. In a series of analyses Dillon and Heady [1961] applied the Wald, Laplace, and Savage criteria to farmers' choices of enterprises and found a poor descriptive fit. In an extensive application to weather uncertainty, Walker et al. [1960] showed how the various criteria suited different financial situations and attitudes toward risk. In 1962 Dillon wrote his excellent survey article of game theory applied to agriculture, detailing both suggested and actual applications. His conclusion that the use of game theory had nearly run its course was premised on the continued use of ordinary games against nature. More recently, however, McInerney [1967] suggested the use of constrained games against nature. Several theoretical works have followed, notably McInerney [1969], Hazell [1970], Maruyama [1972], and Kawaguchi and Maruyama [1972], and it appears that practical applications of constrained games may be in sight. In private correspondence Professor Maruyama informed the authors of this paper that constrained games were applied in Japanese agricultural economics literature as early as 1966 (see, for example, Imamura [1966]).

Interesting work involving modern techniques of farm management is not always reported in the professional literature, or it may appear in relatively obscure outlets, working memoranda, and so on. An example is the computeraided real-time farm management advisory service under the direction of John Schmidt of Wisconsin. Similar systems are operating at Purdue and Michigan State. Candler, Boehlje, and Saathoff [1970] outlined the problems of development and implementation of the top farmer program at Purdue. Nonetheless operational developments attest to the practical relevance of what might otherwise be thought of as elegant toys for mathematicians and economic theorists.

The above references only scratch the surface of a vast body of literature, but we hope they provide a sample adequate for illustrating the variety of uses to which modern optimization methods have been put in the study of

optimal farm management. Before proceeding to other major areas of application, it would be in the spirit of the present undertaking to comment briefly on the role of the more traditional neoclassical marginal analysis in the farm management setting. The tradition goes back to von Thünen; its definitive modern statement is in Black's Introduction to Production Economics [1926] and Heady's Economics of Agricultural Production and Resource Use [1952]. Modern developments in statistics have made possible the quantitative exploitation of the neoclassical point of view. The early examples not surprisingly came from the Ames School with a focus on crop-nutrient response (for example, Tintner [1944], Heady [1946], and Heady, Pesek, and Brown [1955]). Some of Glenn Johnson's work at Kentucky and at Michigan State in the 1950s, described in Bradford and Johnson [1953], Haffnar and Johnson [1966], and Johnson and Quance [1972], was also based on this model. Of course, the optimal feed-mix and feed-ration problems solved in either the linear programming way or the neoclassical way are also an important aspect of farm management. (Early work of this kind was mentioned in the section on food and diet.)

# Farm Firm Development

Economic development in agriculture (in the absence of a geographical frontier) usually involves the growth of some farm firms and the decline or abandonment of others. From the managerial point of view, in which firm policies to enhance growth are sought, and also from the production response point of view, in which aggregate implications of development and agricultural policy are the focus, farm growth and decline are of interest. Studies of farm development have much in common with the farm management studies already reviewed, and it is not always possible to categorize a model into one class or another unambiguously. For example, the early multiperiodic linear programming studies of farm growth and investment such as Swanson [1955] or Loftsgard and Heady [1959] had managerial, production response, and farm growth aspects. However, this field of application is important enough to consider separately, as is indicated by Irwin's review of various methods for farm growth modeling [1968].

Irwin and Baker [1962] marked the beginning of a noteworthy series of farm growth models which have emphasized financial aspects of farm firm growth. Martin and Plaxico [1967] report on a polyperiod model of farm growth with investment, capital markets, and consumption all considered in some detail. Johnson, Tefertiller, and Moore [1967] apply Monte Carlo techniques to a firm growth model with stochastic crop yields. White [1959] expands the Martin-Plaxico and Johnson-Tefertiller-Moore models by incorpo-

rating investment, credit, production, and consumption matrices. In two articles [1968a, 1968b] Baker extended and generalized the Irwin-Baker model. The work of Baker and his protégés is of particular interest as a behavioral approach to modeling firm growth. These studies focus on financial constraints or rules of thumb. Barry and Baker [1971], for example, use reservation prices on credit to infer attitudes toward uncertainty.

One problem common to multiperiod linear programming models of farm growth has been matrix size. Given a single period submatrix of any detail, the multiperiod model presents formidable problems in construction and in computation of solutions. J. M. Boussard has made significant strides on both of these problems: his matrix generating program GEMAGRI (Boussard [1972]) automatically generates a multiperiod linear program on punched cards from standard farm records, and his clever application of a turnpike theorem to the multiperiod linear programming model of farm growth derives a practical method for finding the "optimal" horizon for such a model (Boussard [1971]).

Heidhues [1966] focuses the recursive programming approach on the study of farm growth and decline in an analysis of West German farms. His study incorporates considerable technological and financial detail. It was followed by Steiger's study [1968], summarized in de Haen and Heidhues [1973 and forthcoming], which developed individual recursive programming models for all farms in two villages of an area where examples of growth and decay were evident. A recently completed study by Ahn and Singh [1972] uses a similar approach to study the differential effect of development policies on farms of different sizes in a developing agriculture.

A line of work closely related to that of farm growth has long been pursued at the United States Department of Agriculture-namely, the analysis of resource requirements for achieving various income levels in various farm situations. The work of John Brewster [1957] and others must be mentioned in this context.

# Production Response

One can interpret the solution of an optimizing model as being a long run equilibrium toward which the economy is tending and/or toward which it might be encouraged by various incentives and controls. If reality can be described in this way, then optimizing becomes a powerful tool for policy analysis. This idea lies behind many important applications of optimization methods in agricultural economics. Many of the regional budgeting studies that originated in the 1920s were oriented to such production response purposes. With the advent of linear programming many joint USDA and state experi-

ment station studies were converted to the new approach. The effects of price supports, income controls, and varying technological, marketing, and pricing situations were investigated using linear programming and parametric programming techniques.

The production response work was generally conducted under the title of "adjustment," and many important examples were sponsored by the USDA. Thus we had one program of research involving the cotton states of the South, one focusing on wheat production in the West, one involving livestock and feedgrains in the Corn Belt, and two more concentrating on dairying in the Lake States and in New England. Only a small proportion of this work has ever been published, but no doubt a significant number of active agricultural economists gained their early training in part through participation in these undertakings. The work was described in general terms in Sundquist et al. [1963], Colyer and Irwin [1967], and the Northeast Dairy Adjustments Study Committee [1963].

A concern that emerged in the course of this work was the aggregation problem involving the question of how much estimates of regional responses were distorted by the use of linear programming models of whole regions or representative farms as opposed to "adding up" individual farm models. As the latter was uneconomic, the issue was one of great importance. The first analysis of the problem using the duality theory of linear programming was by Day [1963]. Further consideration was given by Miller [1966] and Lee [1966]. Buckwell and Hazell [1972] applied a clustering technique to identify groups of farms which could be legitimately aggregated according to an extension of Day's criteria. Empirical work addressing the same issues was reported by Sheehy and McAlexander [1965], Barker and Stanton [1965], and Frick and Andrews [1965].

Another problem encountered in the application of representative firm models was representation of investment and disinvestment. Glenn Johnson's fixed asset theory [1958] was an important step toward solution of this problem.

Parametric programming techniques were applied to the problem of inferring supply functions and resource allocation responses from both aggregative and representative firm models. Kottke [1967] summarizes work in this field.

A quite different point of view was taken by the developers of recursive programming. Their view, as initially applied in agriculture by Henderson [1959], was to use programming models augmented by behavioral constraints of the kind already used by Wood [1951], to estimate short-run behavior of farmers at the regional level in a disequilibrium situation. Henderson's original model was used to make a one-year forecast of the allocation of land to various crops for a hundred United States farming regions. The dynamic implica-

tions of Henderson's model were brought out by Day [1963], who then stated the general class of recursive linear programming models to which Henderson's model belonged as a special case. Day's study also gave the first example of how recursive linear programming could be used to trace out the evolution of an industry over time. Applications by Schaller and Dean [1965], Muto [1965], and Cigno [1971] followed. Nontechnical discussions of the general methodology were also contributed by Day [1961, 1962]. An ambitious application of the recursive programming approach was the national model which originated with Glen T. Barton's production response group at the USDA in 1958. Day's 1963 study was the prototype study for this undertaking, and after the follow-up test by Schaller and Dean [1965] a national model was planned and implemented. Sharples and Schaller [1968] described the project during its construction phase. The model is currently being used as an experimental working tool and is being replaced by a more complex general simulation model. An even more ambitious undertaking is Thoss's multisector, multiregional recursive programming model for short-run national planning in Germany [1970]. Henrichsmeyer and de Haen [1972] describe a "next-generation" effort that is currently in the planning stage.

# Interregional and Spatial Economics

Of extreme importance in agriculture and in resource economics generally is the study of interregional or spatial efficiency and development. Going all the way back to von Thünen for its conceptual foundation, the application of modern techniques came with the development of the Hitchcock-Koopmans transportation model, a special case of linear programming for which efficient computer algorithms were developed in the early 1950s. Early applications of this model to distribution and pricing are discussed by Judge [1956], Henry and Bishop [1957], Farris and King [1961], Snodgrass and French [1958], and Stemberger [1959].

Beckmann and Marschak [1961] used the more general activity analysis framework of Koopmans and Reiter [1951] to extend the spatial distribution model to include production. Building on this work, Lefeber [1958] specified a linear programming model to determine efficient allocation and shadow prices, given the regional prices of final products and the regional endowment of primary factors. Orden's transshipment problem [1956] is a special case of the Beckmann-Marschak model which King and Logan [1964] applied to determine the optimum location, number, and size of processing plants and factor and final product flows. Judge, Havlicek, and Rizek [1965] studied the optimum location of livestock slaughter and geographical flows of live animals and meat. In Snodgrass and French [1958] an aggregate model is used

to determine the optimum interregional flows of whole milk and the corresponding equilibrium prices for 1953. This general model is also applied individually to fluid milk, butter, cheese, evaporated milk, and nonfat dry milk solids. A second model minimizes transportation and processing costs in determining the location of processing plants, and a third model adds production costs to these and specifies optimal production location. In a series of well-known papers Egbert and Heady focus on the best location for producing a fixed final national bill of wheat and feed grains (Heady and Egbert [1959], Egbert and Heady [1961, 1963]). Buchholz and Judge [1966] focus on livestock using the same approach. The Egbert-Heady models were forerunners of a family of works relating to the national allocation of agricultural resources: Heady and Skold [1965], Heady and Whittlesey [1965], Eyvindson, Heady and Srivastava [1975], and Brokken and Heady [1968]. Birowo and Renborg [1965] supply an application to Swedish agriculture.

Characteristics of all of the above work were the exclusion of explicit demand functions and the treatment of prices as exogenous. Building on the theoretical work of Enke [1951] and Samuelson [1952], who showed how trade theory could be formulated in mathematical programming terms, Fox [1953] shows how interregional supply-demand equilibrium could be modeled and solved computationally. The initial model focused on livestock feed. The United States was divided into ten regions and the demand for feed was estimated for each. Using the 1949-50 figures for regional production of feed, numbers and prices of livestock, and their demand equations, Fox derives equilibrium consumption, price, and shipments of feed for each region. In a later article Fox and Taeuber [1955] extend the 1953 model to include livestock. Regional demand and supply functions for livestock are added to the previous model, and a joint equilibrium solution is derived for both feed and livestock.

Dunn [1954] broadens and applies von Thünen's theory of location to the agricultural segment of the economy. An equilibrium system which includes space is formalized and is designed to solve problems on an aggregated or industrial level. Dunn's framework takes multiple products and technological interrelationships into account. Judge [1956] uses the Enke-Samuelson formulation as a basis for determining the spatial equilibrium prices for eggs when the regional supplies of eggs are taken as fixed and the demand functions are explicitly included. He then uses the linear programming transportation model to determine the optimum geographical flows of the commodity. Judge and Wallace [1958] propose an iterative parametric solution procedure to solve for prices, consumption, supplies, and flows when regional demands are represented by functional relations and supplies are predetermined. Judge and Wallace [1959, 1960] develop an equilibrium model for beef and pork,

a model which incorporates given regional supplies, transport costs, and demand equations for twenty-one regions of the United States.

Tramel and Seale [1959] develop a reactive programming procedure for determining the competitive prices and flows for the Enke-Samuelson problem. This procedure was applied by Maruyama and Yoshida [1960] in Japan and then was developed into two interrelated sets of interregional quadratic programming models by Takayama and Judge [1964a, 1964b] and Maruyama and Fuller [1964, 1965]. The framework for the quadratic version of the modified Beckmann-Marschak interregional activity analysis model is contained in two 1964 articles by Takayama and Judge. Subsequent articles by Plessner and Heady [1965], Yaron, Plessner, and Heady [1965], and Plessner [1972] contributed to the development of the quadratic programming model and investigated approaches to the problem when market demand functions fail to satisfy the integrability condition. Applications of the Takayama and Judge model include an interregional analysis by Buchholz and Judge [1966] of the United States feed-livestock economy and a spatial equilibrium analysis by Hall, Heady, and Plessner [1968] of the field crop sector of United States agriculture. Applications to other areas: Louwes, Boot, and Wage [1963] apply quadratic programming to the solution of the problem of optimal use of milk in the Netherlands when there are monopolistic tendencies in the market; Bawden [1966] shows how multicommodity international trade problems may be solved by exploiting the quadratic programming model of Takayama and Judge [1964a, 1964b]; Plessner [1967] carries out purely theoretical work designed to show how these operational spatial models fit into the general equilibrium theory. There have been a number of large-scale applications of the Maruyama-Fuller model in the Japanese literature. For example, the studies by Maruyama [1967] and Muto [1965] both had direct impact on Japanese government policy. Dynamic interregional equilibrium using concepts of intertemporal optimality and multihorizon programming has been treated formally by Judge and Takayama [1973], although empirical applications have yet to be achieved.

Interregional economics with a focus on disequilibrium and comparative dynamics instead of equilibrium and comparative statics is proposed by Day [1967] and is given theoretical treatment by Day and Kennedy [1970]. Bawden's spatial model [1966] constitutes an interesting example of this recursive programming approach to the interregional equilibrium problem. He represents regional production by econometric equations that depend on prices which are determined by a transportation model that optimizes short-run trade patterns. It may be regarded as a complex type of cobweb approach to supply-demand interactions as opposed to the equilibrium theory following the Samuelson-Enke formulation. A more recent study by Schmitz and Baw-

den [1973] applies this methodology to the world wheat market. Quite similar to Bawden's approach is Kottke's application [1970] of a recursive version of the Takayama-Judge model to an imperfectly competitive dairy industry. We have already mentioned the related work by Thoss [1970] and by Henrichsmeyer and de Haen [1972].

There are at least two good survey articles on spatial equilibrium models: Bawden [1964], and Weinschenck, Henrichsmeyer, and Aldinger [1969]. In addition, Takayama and Judge [1971] and Judge and Takayama [1973] furnish extensive bibliographies, exposition of theory and methodology, and examples of applications of spatial and temporal price allocation models.

# Natural Resources

Recent applications of quantitative optimization techniques to allocation of natural resources have been numerous, and in particular Bellman's dynamic programming principle has been extensively applied. Underlying this recent work is the general economics of extractive resources. Hotelling's [1931] pioneering application of the calculus of variations to the theory of nonreplenishable resources was perhaps the earliest contribution to this theory. Subsequently, numerous works by S. V. Ciriacy-Wantrup and others laid further theoretical groundwork for the application of sophisticated optimization techniques during the 1960s.

Economic models of commercial fishing have played an important role in developing an approach to replenishable resources. Two seminal works are provided by H. S. Gordon [1954] and Scott [1955]. These neoclassical models were applied by Crutchfield and Zellner [1962] and Quirk and Smith [1969]. Optimal control was used to good effect by Clark [1973]. In 1968 V. L. Smith proposed a general economic model of production from natural resources, and in 1970 Burt and Cummings utilized Bellman's dynamic programming framework to state an even more general theory of production and investment for natural resources. Most of the applied work has been focused on water resources. We shall first review contributions here and then briefly consider pollution studies.

Moore [1961] was one of the first to identify the problems of allocation of water over time. An important subset of the water conservation (temporal allocation) problem is the use of groundwater. In a series of works Burt [1964a, 1964b, 1966, 1967a, 1967b, 1970b] developed an approach to the groundwater problem using dynamic programming and employing stochastic state variables to represent stochastic elements in the supply of groundwater. Closely related to the groundwater problem is the allocation of irrigation water. Burt [1964b] and de Lucia [1969] both treat the case of conjunctive use

of groundwater and surface water, and Biere and Lee [1972] treat the case of reservoir water used to recharge groundwater in dynamic programming frameworks, but most of the studies of irrigation water are related to the management of reservoirs for water used directly in irrigation. The decision environment of the reservoir managers includes several elements of uncertainty including the weather and the demand for water. The authors of several articles (R. L. Anderson [1968], R. L. Anderson and Maass [1971], Butcher [1971], and W. A. Hall, Butcher, and Esogbue [1968] ) apply stochastic dynamic programming, assuming the supply of water to be stochastic and the demand determinant; Burt and Stauber [1971] assume a given inflow and a stochastic demand; de Lucia [1969], Dudley, Howell, and Musgrave [1971a, 1971b, 1972], Dudley [1970, 1972], and Dudley and Burt [1973] assume both stochastic supply and stochastic demand. The series of articles by Dudley alone and in collaboration with others culminates in the 1973 article by Dudley and Burt, which outlines a general stochastic dynamic programming model to determine optimal levels of intertemporal water application rates, intraseason irrigated acreage, and preseason acreage to be planted.

There have been other approaches to the problem of optimum reservoir management. One is the application of chance-constrained programming to single-purpose reservoirs by Eisel [1970, 1972], Loucks [1970], Joeres, Leibman, and Revelle [1971], and Nayak and Arora [1971]. Guise and Flinn [1970] employ a Takayama-Judge spatial equilibrium model to derive optimal prices for a water system. In an early application of stochastic linear programming Manne [1962] employed Markov process optimization to management of a multipurpose reservoir. Young [1967] was perhaps the first to apply linear decision rules to reservoir management.

Several studies have concentrated on selection, sequencing, and timing of investments in water resource projects. Jacoby and Loucks [1972] have described a technique combining simulation models of river basins and optimization routines to select and assess possible patterns of investment. Cummings and Winkelmann [1970] and Regev and Schwartz [1973] apply the dynamic programming framework of Burt and Cummings [1970] to the problem of interregional investment and allocation of water. Regev and Schwartz are also concerned with economies of scale and therefore apply mixed-integer programming. Young and Pisano [1970] apply nonlinear programming to minimize investment costs in water projects; Butcher, Haimes, and Hall [1969] and Morin and Esogbue [1971] propose special dynamic programming algorithms for sequencing and scheduling of water supply projects; and Erlenkotter [1973] formulates a dynamic programming model to minimize costs of developing a given hydroelectric capacity in a river basin. R. A. Young and Bredenhoft [1972] use a two-stage optimization model to simulate reactions

of economic decision makers in a river basin. A current study being conducted under the auspices of Heady at Iowa State is concerned with the allocation, of water resources between regions in the United States and the environmental effects of these allocations. A part of this study is reflected in the recent application of the Heady-Egbert regional adjustment model by Heady et al. [1973].

One "natural resource" that is currently in the public eye is the capacity of the environment to absorb society's pollution residuals. An imaginative approach to this problem is illustrated by d'Arge [1971] in his use of a parable of an astronaut irretrievably lost in space. To determine the astronaut's optimal pattern of consumption over time, d'Arge uses optimal control theory. On a more mundane level engineers and economists are developing models which will help to determine "optimal" levels of pollution. At the University of Illinois Earl Swanson and his colleagues have been conducting interdisciplinary work to determine the sedimentation effects of various cropping systems using linear programming models. Narayanan and Swanson [1972] report the results of a parametric linear programming study of the trade-offs between sedimentation and farm income. A similar work was undertaken by Seav [1970]. Graves, Hatfield, and Whinston [1972] outline an approach which employs nonlinear programming to determine optimal methods of water quality control for the Delaware River Estuary. Davidson and Bradshaw [1970] employ Pontrivagin's minimum principle to the treatment of polluted streams, and Hass [1970] proposed the Dantzig-Wolfe decomposition algorithm as a basis for a decentralized method of arriving at optimal water pollution taxes. It is certain that many more such applications will follow as the quality of data and the understanding of environmental systems improves.

# Agricultural Development Problems

National planning has been the predominant setting in which optimization techniques have been applied to problems in economic development. For computational reasons these studies have until quite recently been limited to linear programming methods. One of the earliest (and best-known) examples of such a model is that of Sandee [1960]. Some of the more frequently cited works in this field are those by Manne [1966], Manne and Weiskopf [1969], Eckaus and Parikh [1968], Chenery and MacEwan [1966], and Bruno [1967]. Many of these focus on the optimal resource allocation between agriculture and other sectors when such national goals as foreign exchange maximization are pursued. Often they build on and incorporate previous Leontief-style input-output models of the economy in question. This underlying input-output work is summarized in a series of conference proceedings beginning in 1951,

and continuing through Barna [1963], Carter and Brody [1972a, 1972b], and Brody and Carter [1973]. An illuminating example focusing on agriculture is the study by Fox, Sengupta, and Thorbecke [1966], who proposed imbedding an input-output model in a more general multisector analysis.

More recent planning models reflect advances in computer technology both in their increased attention to detail and in their use of more difficult optimizing techniques such as mixed-integer programming and decomposition. A number of important examples by such authors as Barraza, Bossoco, Duloy, Norton, Kutcher, Winkelmann, and others will be found in Goreux and Manne [1973]. Dynamic programming and mixed-integer programming techniques have begun to find application in sectoral or single-industry planning models; for example, Manne [1967] applies both techniques to several industries of the Indian economy, and Westphal [1971a] applies mixed-integer programming to the economy of South Korea.

Application of optimization techniques to farm management in less developed countries to date has been limited. McFarguhar and Evans [1957] provide an early application of linear programming to combinations of enterprises in tropical agriculture. More recently Heyer [1971] has applied linear programming to the problem of allocating peasant resources in a small rural area of Kenya. In a second work Heyer [1972] extends her original model to account for uncertainty through the use of a game theoretic framework. Spencer [1973] has applied linear programming to a study of the allocation of labor resources to rice production in Sierra Leone. His study was based on farm management survey data, and its objective was to improve interregional allocation of labor resources. Baker [1973] employs linear programming in an analysis of the role of credit in smallholder farming. Probably nowhere will one find a greater output of useful optimization studies to problems of less developed agriculture than at the Punjab Agricultural University in Ludhiana. Most of these studies are by S. S. Johl and A. S. Kahlon (for example, Johl and Kahlon [1967]) and various of their students. This demonstrates the need for scholars to research local journals and experiment station reports for applied studies relevant to their special problems.

General systems simulation models such as those described by Halter, Hayenga, and Manetsch [1970] are of growing importance in less developed countries. The reason is that they make possible the systematic study of a model economy when data are inadequate, or when goal specification is difficult, or when the economy is simply too complex to optimize with existing algorithms and computers. They also are useful when, as a prelude to systematic planning, one wants to understand how the economy works and how it is likely to respond to policy controls.

General systems simulation includes, as a special category, models in which

given components are represented by optimizing submodels. This category also belongs to the class of recursive programming systems of Day and Kennedy [1970]. Examples of the recursive programming approach to the problem of tracking a developing agricultural economy include Singh [1969, 1971], and Ahn and Singh [1972]. Thoss [1970] focuses on multisector development using this technique.

> Table 1. Categorization of Survey References into Eight Branches of Agricultural Economics Research

Citations preceded by an	asterisk (*) indicate survey
articles or substantial	bibliographical sources.

- 1. Introduction. Day [1977]; Fox [1953]; Hildreth and Reiter [1951]; Kuhn and Tucker [1951]; Marshall [1890]; Mighell and Black [1951]; and von Thünen [1966].
- 2. Food and Diet. Christenson and Mighell [1951]; Heady [1951]; Heady, Woodworth, et al. [1953]; V. E. Smith [1974]; Stigler [1945]; and Waugh [1951].
- 3. Farm and Agribusiness Management. Agrawal and Heady [1968]; \*Agrawal and Heady [1972]; J. R. Anderson and Hardaker [1972]; Babbar [1955]; Bellman [1957]; Bishop [1956]; Black [1926]; Bowlen and Heady [1955]; Bradford and Johnson [1953]; Bressler [1952]; Bressler and Hammerberg [1942]; Brewster [1957]; Burt [1965]; Burt and Allison [1963]; Byerlee and Anderson [1969]; Candler [1956]; Candler [1957]; Candler [1960]; Candler [1972]; Candler, Boehlje, and Saathoff [1970]; Candler and Manning [1961]; Candler, Snyder, and Faught [1972]; Charnes [1953]; Charnes and Cooper [1959]; Chen [1971]; Cocks [1968]; Conner, Freund, and Godwin [1972]; Coutu [1957]; Dantzig [1951]; Dantzig [1955]; Dantzig and Wolfe [1960]; \*Dillon [1962]; \*Dillon [1971]; Dillon and Heady [1961]; Doll [1972]; Dorfman, Samuelson, and Solow [1958]; Edwards [1963]; Edwards [1966]; Faris [1960]; Freund [1956]; Geoffrion and Marsten [1972]; Giaever and Seagraves [1960]; Halter and Dean [1971]; Halter and White [1962]; Hammerberg, Parker, and Bressler [1942]; Hazell [1970]; Hazell [1971a]; Hazell [1971b]; Heady [1946]; Heady [1951]; Heady [1952]; Heady [1954]; Heady [1971]; Heady and Candler [1958], Heady and Egbert [1964]; Heady, McAlexander, and Schrader [1956]; Heady and Pesek [1954]; Heady, Pesek, and Brown [1955]; Heady, Woodworth, et al. [1953]; Hildreth [1957a]; Hildreth [1957b]; Hildreth and Reiter [1951]; Hinrichs [1972]; Hitchcock [1941]; Hutton [1963]; \*Hutton [1965]; Imamura [1966]; G. L. Johnson [1952a]; G. L. Johnson [1952b]; G. L. Johnson [1955]; G. L. Johnson and Haver [1953]; G. L. Johnson and Quance [1972]; Kawaguchi and Maruyama [1972]; G. A. King and Logan [1964]; R. A. King [1953]; R. A. King and Freund [1953]; Kottke [1961]; Langham [1963]; Loftsgard and Heady [1959]; Mc-Farquhar [1961]; McInerney [1967]; McInerney [1969]; McPherson and Faris [1958]; Markowitz [1952]; Markowitz [1959]; Maruyama [1972]; Maruyama and Fuller [1964]; Maruyama and Yoshida [1960]; Merrill [1965]; Mighell [1955]; Minden [1968]; Musgrave [1962]; Officer and Halter [1968]; Peterson [1955]; Puterbaugh, Kehrberg, and Dunbar [1957]; Rae [1971a]; Rae [1971b]; Roy [1952];

Table 1. Categorization of Survey References into Eight Branches of Agricultural Economics Research (Cont.)

Citations preceded by an asterisk (\*) indicate survey articles or substantial bibliographical sources.

Schickele [1950]; B. J. Smith [1973]; Stollsteimer [1963]; Swanson [1955]; Swanson [1956]; Swanson [1957]; Swanson [1961]; Swanson [1966]; Swanson and Fox [1954]; Thomas et al. [1972]; Thomson and Hazell [1972]; Throsby [1964]; \*Throsby [1968]; Tintner [1944]; Tintner [1955]; Walker et al. [1960]; Waugh [1951]; \*Weinschenck, Henrichsmeyer, and Aldinger [1969]; White [1959]; Yaron and Heady [1961]; Yaron and Horowitz [1972].

- 4. Farm Firm Development. Baker [1968a]; Baker [1968b]; Baker [1973]; Barry and Baker [1971]; Boehlje [1967]; Boehlje and White [1969]; Bolton [1964]; Boussard [1969]; Boussard [1971]; Boussard [1972]; Boussard and Petit [1967]; Day and Cigno [forthcoming]; de Haen and Heidhues [1973]; de Haen and Heidhues [forthcoming]; \*Irwin [1968]; Irwin and Baker [1962]; S. R. Johnson, Tefertiller, and Moore [1967]; J. R. Martin and Plaxico [1967]; \*Renborg [1970]; Steiger [1968]; Yaron and Horowitz [1972].
- 5. Production Response. Barker and Stanton [1965]; Barry and Baker [1971]; Bolton [1964]; Boussard [1969]; Boussard [1971]; Boussard [1972]; Boussard and Petit [1967]; Brokken and Heady [1968]; Buchholz and Judge [1966]; Buckwell and Hazell [1972]; Cigno [1971]; Colyer and Irwin [1967]; Cowling and Baker [1963]; Day [1961]; Day [1962]; Day [1963]; Day [1967]; Day and Kennedy [1970]; de Haen [1973]; Egbert and Heady [1963]; Eyvindson, Heady and Srivastava [1975]; Frick and Andrews [1965]; Heady and Skold [1965]; Heady and Whittlesey [1965]; Heidhues [1966]; Henderson [1959]; G. L. Johnson and Hazel [1972]; G. L. Johnson [1958]; G. L. Johnson and Haver [1953]; Kottke [1967]; Kottke [1970]; Krenz, Baumann, and Heady [1962]; Lee [1966]; Maruyama and Fuller [1965]; Miller [1966]; Miller [1972]; Muto [1965]; Schaller and Dean [1965]; Schmitz and Bawden [1973]; Sharples and Schaller [1968]; Sheehy and McAlexander [1965]; Sundquist et al. [1963]; Thoss [1970]; Wood [1951].
- 6. Inter-regional and Spatial Economics. \*Bawden [1964]; Bawden [1966]; Bawden, Carter, and Dean [1966]; Beckmann and Marschak [1961]; Birowo and Renborg [1965]; Bressler [1952]; Bressler and Hammerberg [1942]; Brokken and Heady [1968]; Buchholz and Judge [1966]; Candler, Snyder, and Faught [1972]; Day [1962]; Dunn [1954]; Egbert and Heady [1961]; Egbert and Heady [1963]; Enke [1951]; Eyvindson, Heady, and Srivastava [1975]; Farris and King [1961]; Fox [1953]; Fox and Taeuber [1955]; Guise and Flinn [1970]; Haffnar and Johnson [1966]; Hall, Heady, and Plessner [1968]; Heady and Egbert [1959]; Heady and Skold [1965]; Heady and Whittlesey [1965]; Heidhues [1966]; Henrichsmeyer and de Haen [1972]; Henry and Bishop [1957]; Hitchcock [1941]; Judge [1956]; Judge, Havlicek, and Rizek [1965]; Judge and Takayama [1973]; Judge and Wallace [1958]; Judge and Wallace [1959]; Koopmans and Reiter [1951]; Kottke [1970]; Lefeber [1958]; \*Leuthold and Bawden [1966]; Louwes, Boot, and Wage [1963]; Maruyama [1967];

Table 1. Categorization of Survey References into Eight Branches of Agricultural Economics Research (Cont.)

Citations preceded by an asterisk (\*) indicate survey articles or substantial bibliographical sources.

Orden [1956]; Schmitz and Bawden [1973]; Takayama and Judge [1964a]; Takayama and Judge [1964b]; Takayama and Judge [1964c]; Takayama and Judge [1971]; \*Weinschenck, Henrichsmeyer, and Aldinger [1969]; Yaron, Plessner, and Heady [1965].

- 7. Natural Resources. R. L. Anderson [1968]; R. L. Anderson and Maass [1971]; Biere and Lee [1972]; Burt [1964a]; Burt [1964b]; Burt [1966]; Burt [1967a]; Burt [1967b]; Burt [1970a]; Burt [1970b]; Burt and Cummings [1970]; Burt and Stauber [1971]; Butcher [1971]; Butcher, Haimes, and Hall [1969]; Ciriacy-Wantrup [1952]; Clark [1973]; Crutchfield and Zellner [1962]; Cummings and Winkelmann [1970], d'Arge [1971]; Davidson and Bradshaw [1970]; de Lucia [1969]; Dudley [1970]; Dudley [1972]; Dudley and Burt [1973]; Dudley, Howell, and Musgrave [1971a]; Dudley, Howell, and Musgrave [1971b]; Dudley, Musgrave, and Howell [1972]; Eisel [1970]; Eisel [1972]; Erlenkotter [1973]; H. S. Gordon [1954]; R. L. Gordon [1967]; Graves, Hatfield, and Whinston [1969]; Graves, Hatfield, and Whinston [1972]; Guise and Flinn [1970]; Hall, Butcher, and Esogbue [1968]; Hass [1970]; Heady, Madsen, et al. [1973]; Hotelling [1931]; Jacoby and Loucks [1972]; Joeres, Leibman, and Revelle [1971]; Keckler and Larson [1968]; Loucks [1970]; Manne [1962]; Meier and Beightler [1967]; Moore [1961]; Morin and Esogbue [1971]; Narayanan and Swanson [1972]; Nayak and Arora [1971]; Quirk and Smith [1969], Regev and Schwartz [1973]; Revelle, Loucks, and Lyn [1968]; Riordan [1971]; Scott [1955]; Seay [1970]; V. L. Smith [1968]; Tolley and Hastings [1960]; G. K. Young, Jr. [1967]; G. K. Young, Jr., and Pisano [1970]; R. A. Young and Bredehoft [1972].
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