GROWTH OF THE AGRICULTURAL FIRM: PROBLEMS AND THEORIES

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This review of growth theories of the firm is directed towards those theories of interest for managerial direction of the growth process of agricultural firms. It starts with a discussion of growth, growth directions, the growth process and associated problems. From this an analytical procedure is built up, which states a set of desirable features that theories suitable to guide managers of agricultural firms should contain. In the light of these, the following theories of firm growth are discussed: the traditional neo-classical approach to firm size, extensions of this approach, the growth theories of Edith Penrose and Robin Marris, and the combination of investment and finance theory possible with various types of mathematical programming and cybernetic and behavioural theory models.

1. INTRODUCTION

This article aims at a review of growth theories of the firm. Irrespective of the original reason for building the theories reviewed, they will be analyzed here in terms of their possibilities for directing and controlling the growth process of an agricultural firm over time. These possibilities will be measured against a set of ideas on problems that are involved in the growth process of agricultural firms, which stem from the author's intuition and from earlier exploratory studies. They are still only to be looked upon as hypotheses (Renborg [43, part II]). Here they serve to indicate the author's criterion against which the various theories are analyzed: *If* the growth problems of the agricultural firm are such and such, *then* these are the abilities of the theories to direct and control the growth process.

This approach of course limits the domain of the review. Theories will for example not be discussed as to their ability to explain or predict the behaviour of growing firms as parts of a macroeconomic theory or as parts of a study on structural change within an industry. Also theories may be analyzed in terms of aims for which they were not constructed. The review has been restrained in this way to keep it within reasonable limits and to fit the author's recent research interests.

The *agricultural firm* that we are concerned with is a collection of the ordinary means of production, land, labour and real capital. Unlike most other firms land represents a considerable part of the financial capital invested in this firm. Real capital is not only buildings and

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machinery but also animals. The definition of "firm" to be followed here is one of an administrative unit (i.e. not a technical unit, a farm) subject to independent planning for the benefit of the unit as a whole. This firm is not a stock company.

Most agricultural firms in non-socialist countries are few person firms, where the manager often owns the major part of the means of production and also often supplies the lion's share of the manual labour in the firm. The decision maker in these firms is of the classical "*entrepreneur*" type. Dominant in the socialist countries (but also existing in non-socialist countries) are the larger agricultural firms where management and ownership of resources are separated, where the managerial staff mainly supplies intellectual labour and where many persons supply the manual labour. The *manager* is here the first member of a decision making team in an organization. The legal form of this organization can vary (limited company, co-operative etc.). As we shall see, growth problems are to some extent different in these two groups of agricultural firms.

Growth is the increase in size of the firm. Size is some measure of the total sum of all the means of production which the firm commands. If these means of production are optimally utilized the size can be measured either on the input side as total costs or on the output side as the value of total production. When, as in the present article, the direction and control of the growth process is under study, it seems relevant to measure the size of the firm on the resource side and at a somewhat less aggregated level than mentioned above. The growth process includes a choice of growth directions, among other things a choice between increases of various means of production. This being so, it seems enough to think of the size of the firm as specified by a vector B, whose elements are the amounts of each means of production measured in technical units. These can be acres of land, man hours, square feet of building space, number of animals, etc. The size of the firm is thus specified in as many dimensions as there are elements in B. The firm grows if one or more elements increase in size and the rest are unchanged. If some elements—say b_2 and b_3—decrease in size at the same time as others—say b_1, b_5, and b_6—increase, for example if a substitution is taking place, it is not possible to tell if the firm as a whole grows or contracts without some common base of evaluation. However, it is possible to say that the firm has grown in some dimensions—b_1, b_5, and b_6—and contracted in others—b_2 and b_3. If, on some occasion, it is necessary to estimate the total size change of the firm the B vector can be multiplied by a cost or a value vector and the total costs or total value compared before and after the change in size.

The *growth directions* of the firm are not only elements of B but also elements of a vector X, indicating the activities, that is the production processes amongst which the firm may choose.

The *growth process* of the agricultural firm is a process in time where the decision maker selects growth directions according to some goals.

2. GROWTH PROBLEMS

The growth theories to be reviewed in this article will be analyzed as to their ability to serve as guides for directing firm growth when the following growth problems are assumed to exist.

The growth process is assumed to be influenced by the *goals of the decision maker.* The growth of the firm can of course in some cases be a goal in itself. If so, the growth process may look different and include different problems if the decision maker is interested for example in the rate of growth, in a growth specified in absolute terms to be reached during a specified or unspecified time span, or in growth by expansion from within the firm or by acquisition of and merger with other firms. More probable is that growth is a subgoal, a means to achieving a goal at a higher level. These ends can be related to profit maximization or to the "satisficing" of one or a set of goals. In growth situations of the latter type the following set of goals are likely to be of interest to decision makers: (i) making yearly withdrawals of specified amounts of cash, goods and services; (ii) making yearly withdrawals of leisure time; (iii) ensuring an amount of savings necessary to guarantee the future withdrawal of cash, goods, services and leisure time; and (iv) keeping risk and uncertainty within such limits that the decision maker thinks the firm's future existence is guaranteed.

The growth process may look different and contain different problems for maximizers of profit and for satisficers of aspiration levels in the goal dimensions mentioned above. Also satisficers of different aspiration levels may face growth processes with quite different problems. This, then, means that when analyzing growth theories we have to indicate which of the decision maker's goals are and are not taken into account by the theories. As the author is most inclined to hypothesize that decision makers are satisficers of aspiration levels in many goal dimensions, the theories' possibilities for building images of goal situations of this type will be especially stressed in this article.

Another set of growth problems involve the ways in which to *identify the factors that give growth opportunities.* These problems partly lie with the capacity of the decision maker. His cognitive ability, creativity and intellectual capacity determine how many growth opportunities can be detected. His enterprising spirit determines how many possibilities can be utilized within the firm. The problems also arise, both outside and inside the firm, in other ways than those associated with the decision maker. External opportunities for growth stem from such factors as increasing demand for particular products, changes in technology which call for production on a larger scale than before, and discoveries and inventions whose exploitation looks particularly promising—Penrose [42]: for the growing firm two problems seem to be important in this connection. The possibilities for anticipating the development and, if this cannot be done, the possibilities for building up a preparedness for meeting external change in such a way that the greatest advantage is taken of it.

Internal opportunities for growth stem from the existence of unused resources within the firm. As Penrose [42] has pointed out unused resources always exist within firms, originating from the indivisibility of many means of production, from the fact that given means of production can be used in different ways in different situations, and from the fact that new productive services are constantly being produced within the firm [42, pp. 67–80]. These unused resources—men, machines, knowledge—include growth opportunities in that there are cost advantages in using them.

In the introduction it was proposed to measure the size of a firm in as many dimensions as there are means of production used, or possible to use, within the firm. The reason for choosing such a low level of aggregation is that the various means of production very often have different characteristics between which it is important to distinguish in the growth process. These *differences in characteristics of the resources* thus create growth problems in that they do not make it possible to aggregate the various means of production into one resource. Examples of important differences are the following. Increases over time in the value of land, buildings and machinery are seldom the same and do not always show up in market prices. Acquisition of various resources are often treated differently in tax calculations. Various resources may well have different contributions to offer to the fulfilment of aspiration levels in various goal dimensions. Also various resources have different accessibility; labour, buildings, machinery and livestock can as a rule always be bought to some price, whereas this is not always so with land. Access to resources can often be achieved by either renting or buying them. Differences in these two ways of getting access to resources can exist between planning situations and create different characteristics for various resources from one situation to another.

Empirical observations show that there are a number of growth problems associated with the *acquisition of financial capital.* These are not only associated with the necessary choice between retained earnings, borrowing or external equity capital to finance the growth process. In many cases this choice is influenced by other factors affecting the growth processes. Since an increasing retention of earnings decreases the amount available for withdrawal and consumption, this method of acquiring financial means has a direct effect on the fulfilment of the goals of the entrepreneur. For small agricultural firms, which do not issue stocks for sale on the stock market, an increase in borrowed funds raises the price of capital at the margin and increases the uncertainty involved. In many cases the amount which can be borrowed depends on the type of resource acquired. Thus, there exists an important interdependence between the choice of funds to finance growth and other factors affecting the growth process.

For the relatively small agricultural firms, growth may mean that *new abilities are required* by the farmer to manage his larger farm. There are marked differences between the managerial ability required by a farmer on a one man farm compared with a farm with two or more employed people where the entrepreneur or manager is further away from the technical operations.

Important sets of problems are associated with the growth process as such, i.e. with the fact that growth is a process over time. One group of such problems is associated with the fact that this process apparently creates what can be called *growth costs*. Penrose [42] speaks about such costs stemming from the increasing demand for managerial services when firms grow: planning for growth involves new efforts outside the daily routine and often requires new knowledge which involves costs to acquire. It is also possible to show empirically the existence of growth costs associated with adjustment of resources and organization to larger firm sizes. Examples are adjustments of layouts of roads, buildings, fields, low yields from crops and livestock during an initial period, etc.

Finally there are also a number of problems associated with the fact that external events are taking place over time when the growth process is going on. The variations in prices, yields and other parameters associated with random or non-random variations and with technological, economic and institutional change create the *uncertainty of the environmental conditions* which must be taken into account when planning for firm growth.

3. ANALYTICAL PROCEDURE

These growth problems constitute the background to the following somewhat arbitrary list of desirable features of theories suitable to guide managers of growing agricultural firms. Good theories should be able to take into account:

(1) the goals as formulated by the entrepreneur;
(2) the growth opportunities available;
(3) the differences in characteristics of various resources;
(4) the differences in characteristics of various sources of financial capital;
(5) the fact that larger firms often require new abilities by the farmer;
(6) the fact that growth is a process over time and that growth costs are associated with this process; and
(7) the fact that risk and uncertainty influence growth, as it is a process over time.

It has to be remembered that this list is only the author's means for analyzing differences between various theories of firm growth. It is not a list of necessary and sufficient conditions to be fulfilled by an efficient theory.

4. THE TRADITIONAL APPROACH

Problems associated with size and change in size—i.e. questions closely related to growth—have traditionally been treated by economists using the neo-classical average and marginal cost curves as the theoretical tool. This neo-classical theory of the firm gives some clues to the

treatment of the size problem, but must be looked upon as an insufficient planning instrument for the manager who wants to direct the growth of his firm. There are many obvious—and well known—reasons for this insufficiency. The neo-classical theory of the firm is formulated at a high level of abstraction. In its role as the macroeconomist's tool for a rough description of the firm's behaviour in a macroeconomic market model, it hardly suits as a decision model for a manager at the firm level. Thus it cannot take into account items 2 to 5 in the analysis list, which are all features at a lower level of abstraction closer to the decision maker in the firm. Moreover, the neo-classical cost curves describe efficient cost situations for firms at specified sizes, not changes in costs when firms carry through a growth process. As Penrose says ([42] p. 2), ". . . there may be advantages in *moving* from one position to another quite apart from the advantages of being in a different position", and (p. 100), ". . . economies of growth may exist at all sizes and some of them may have no relation either to the size of the firm before it undertakes an expansion based on them, or to any increase in efficiency due to a larger scale of production".

This then means that feature 6 in the analysis list above is not included in this theory. Finally, the neo-classical theory covers only profit maximization as a goal and assumes that all outcomes are known with certainty. This means that only parts of feature 1 and none of feature 7 exist for the theory.

5. EXTENSIONS OF THE NEO-CLASSICAL APPROACH

5.1 GENERAL

There are some interesting examples of writers who have tried to extend the neo-classical approach into the area of planning for growth. Baumol [3] and Williamson [51] have developed models[1] to determine the optimal permanent growth rate for an infinite planning period under certainty. G. B. Richardson [44] has proposed a model where optimal growth is determined by the point where marginal profit from increases of investments in the firm equals marginal cost of financial capital. Although these theories are still at too high a level of abstraction to serve as really good foundations for steering instruments in firm growth, they will be reported here, because they represent a bridge between the traditional approach and some theories to be discussed later. The bridging function lies in the fact that these extensions of the neo-classical approach include features such as growth costs and retention rates.

Both Baumol and Williamson assume that the future is known with certainty, that the planning period has an infinite length, that the firm operates under pure competition with a linear homogeneous production

[1] In this review "model" and "theory" will be used as synonyms in spite of the wider meaning of "theory".

function. They also postulate the existence of specific growth costs, dependent on the rate of growth and accelerating when the growth rate increases. The problem is to determine the optimal growth rate during the planning period—"a fixed percentage rate of growth to be continued into the indefinite future"—Baumol [3, p. 1079]. Baumol develops one model where the objective against which to measure optimality is traditional profit maximization over time. By this is meant the discounted difference between the streams of payments to and from the firm during the planning period. In another model developed in the same article the growth of the volume of sales is maximized. In his model, Williamson demonstrates cases where the objective varies. He covers profit maximization over time as well as maximization of growth rate and of sales volume. Both Baumol's and Williamson's models will be given here in the form where the objective is traditional profit maximization over time, this being sufficient to demonstrate the character of the models.

Richardson's model will not be discussed further in this review.

5.2 BAUMOL'S MODEL

Baumol [3] deals with a firm whose yearly profit is R before the growth process starts. This R contains all incomes minus all production costs, together with interest on and depreciation of capital employed in production.

The assumptions made as to production functions and length of planning period for this firm make it possible to speak of a relative rate of growth, g, which applies to any size measure of the firm; thus it also applies to R. To illustrate: For a growth rate of 5 per cent per annum, g is 0·05. It is required to determine the optimal magnitude of g, which is reached when profit is maximized over a planning period of infinite length.

As mentioned earlier the existence of growth costs, dependent on the growth rate, are assumed. The growth costs associated with a specific growth rate, g, discounted to the planning moment and added together, are called $C(g)$. For these discounted growth costs the following properties of the first and second derivatives are assumed:

$$C'(g) > 0 \qquad C''(g) > 0,$$

i.e. total growth cost increases at an increasing rate as the growth rate rises.

With a discount rate of k we may define P as the sum of profits accruing each year, (t), discounted to the planning moment, without taking account of the growth costs:

$$(1)\ P = \sum_{t=0}^{\infty} R\left(\frac{1+g}{1+k}\right)^t = R\frac{1}{1-\frac{1+g}{1+k}} = R\frac{1+k}{k-g}$$

for $g < k$, which is necessary for the series to converge[2].

Taking the growth costs into account the discounted profit, π, can be formulated:

$$(2)\ \pi = P - C(g) = R\frac{1+k}{k-g} - C(g)$$

In this function π is the dependent and g the independent variable. The conditions for profit maximization are:

$$\frac{\delta\pi}{\delta g} = R\frac{1+k}{(k-g)^2} - C'(g) = 0$$

and

$$\frac{\delta^2\pi}{\delta g^2} = 2R\frac{1+k}{(k-g)^3} - C''(g) < 0.$$

A graphical picture of this profit maximizing problem is given in figure 1, where g^* indicates the optimal growth rate which maximizes profit.

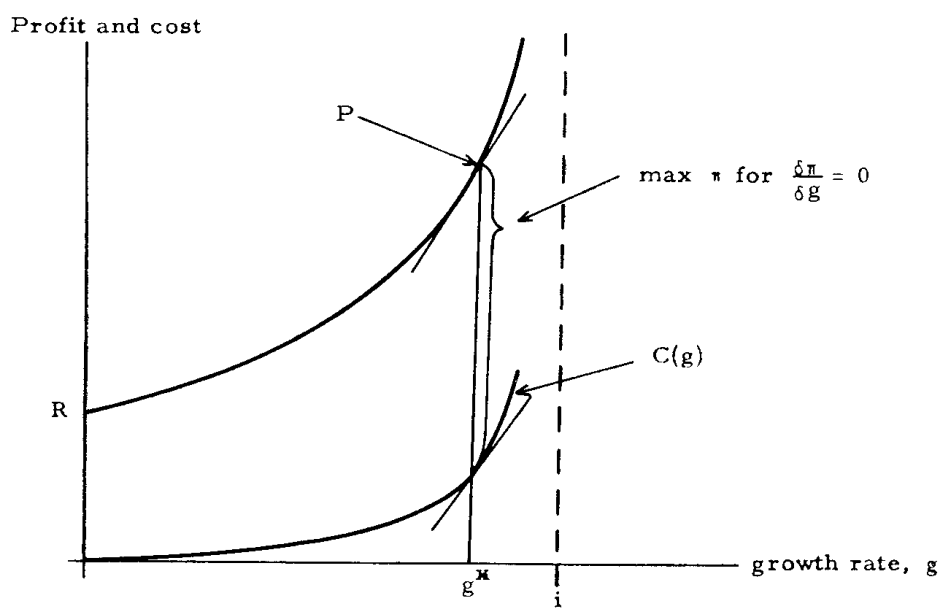

FIGURE 1: *Baumol's profit maximizing growth model*

[2] The probability that $g > k$ for agricultural firms, which only seldom are stock companies cannot be sidestepped by the arguments given by Modigliani and Miller, [37] which are reiterated by Baumol. This restricts the validity of the model. However, in the case where $g > k$, the planning problem at least seems soluble for finite planning periods.

5.3 WILLIAMSON'S MODEL

Williamson's model [51] is an improvement of Baumol's in that he introduces the link between profit in one year and capital investment in the following. He thereby fills in a gap in Baumol's theory and also stresses the important connection between the choice of consuming or saving one year's profit on one side and growth on the other. These features can be shown in the following, somewhat simplified, presentation of his model.

In this presentation g, k, and t are as defined above. R_t is the profit in year t and, thus, R_0 has the meaning which in Baumol's model was given to R. Due to the assumptions mentioned earlier S_t = the value of sales in year t, K_t = total capital invested in year t, and R_t will all grow at the same rate, g, over time. Williamson also assumes unchanging technology which gives a constant capital/output ratio over time.

Williamson postulates that the technical relations given in equations (3), (4), and (5) hold for his growing firm:

(3) $R = R(S) \qquad R' > 0 \qquad R'' < 0$

i.e. profit is an increasing function of the total value of sales but the rate of increase is falling.

The capital available for growth, X, comes from the proportion r, of the profit, R, which is not withdrawn for consumption from the firm; i.e.

(4) $X = rR$

This version of Williamson's assumptions means that no external equity capital is received and that the firm is at the maximum leverage ratio where no more borrowing is possible owing to the risk of being driven out of business[3].

The capital available for growth can be used for addition to capital, I, and to non-investment growth costs, C. The constant capital/output ratio means that $I = gK$. The growth costs are assumed to be a function of g similar to the one proposed by Baumol. The use of the available capital, thus, becomes:

$$(5)\quad X = I + C = gK + C(g) \qquad \begin{array}{l} C' > 0 \\ C'' > 0 \\ C(0) = 0 \\ C'(0) = 0 \end{array}$$

The inverse of function (5) gives:

(6) $g = g(X) \qquad g' > 0,\ g'' < 0$

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[3] Halter [21] has made an interesting attempt to introduce borrowing up to the maximum leverage ratio in a growth model of the Baumol-Williamson type.

In the profit maximization case Williamson maximizes the total value, M, of the firm. This value is the yearly withdrawals from the firm, $(1 - r)R$, added together after growth and discounting in the same way as in Baumol's model. Williamson's profit maximizing model is thus of the form:

Maximize:

$$(7)\quad M = \frac{1 + k}{k - g}(1 - r)R$$

subject to

$$g = g(rR) \qquad g' > 0 \qquad g'' < 0$$
$$R = R(S) \qquad R' > 0 \qquad R'' < 0$$

where M, r, and S are variables.

We observe here that R has the same meaning as in Baumol's model in equation (1), and that the growth costs are included in the side condition g. We can also observe that the value of the firm is dependent both on the yearly withdrawals of the profit $(1 - r)$, and via g on the proportion, r, of the profit reinvested in the firm.

The maximization problem involves choosing S and r so that the future stream of withdrawn profit is as large as possible. The optimization is achieved by partial derivation of M with respect to R and r.

We have

$$(8)\quad \frac{\delta M}{\delta R} = \frac{(k - g)(1 - r)(1 + k) + (1 - r)(1 + k)R\frac{\delta g}{\delta R}}{(k - g)^2} > 0$$

assuring that the profit maximizing firm selects that sales level, S^*, which maximizes, R^*, the profit; and

$$(9)\quad \frac{\delta M}{\delta r} = \frac{-(k - g)(1 + k)R + (1 - r)(1 + k)R\frac{\delta g}{\delta r}}{(k - g)^2} = 0$$

$$\text{or} \quad 1 - r = \frac{k - g}{\delta g/\delta r}$$

as the first order condition for a profit maximizing r.

These conditions are most easily demonstrated diagramatically, as in figure 2 parts a–c.

In this figure the retention ratio, r, is measured along the horizontal axis. Maximum r is $r = 1$, i.e. the whole yearly profit retained. Along the vertical axis the discount rate, k, and the growth rate, g, are measured. Maximum for g is $g = k$, where k is exogenously determined. Observe that g can come as close to k as desired, but that the value of the firm is undetermined for $g = k$.

In figure 2a the growth rate, g, has been introduced as a function of r and R with characteristics according to equation system (7). g

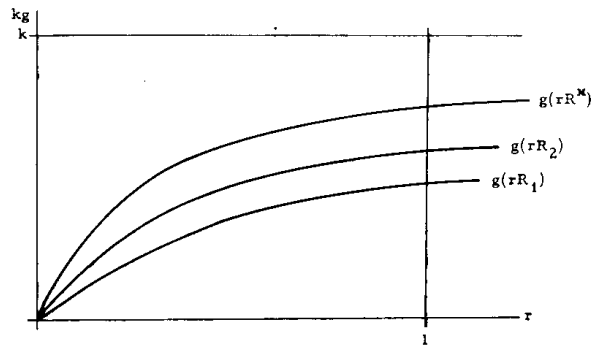

FIGURE 2a: *Williamson's profit maximizing growth model*

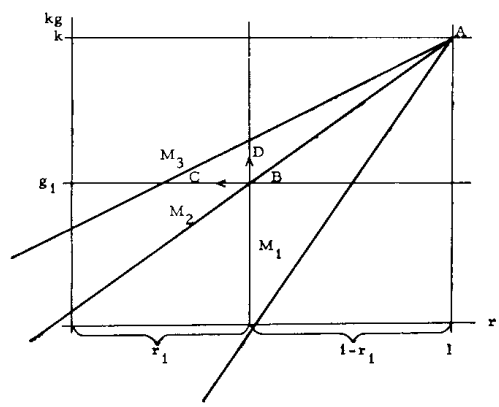

FIGURE 2b: *Williamson's profit maximizing growth model*

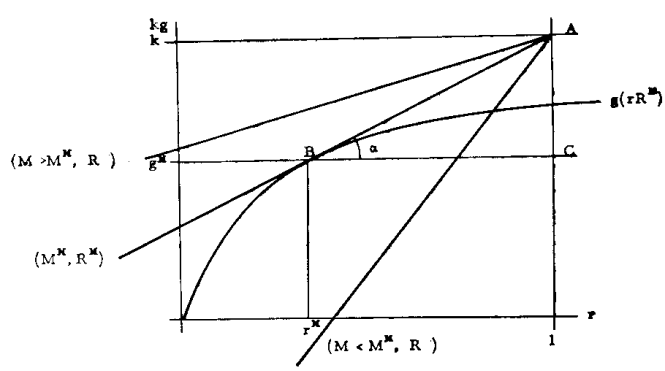

FIGURE 2c: *Williamson's profit maximizing growth model*

represents a whole family of lines through the origin, each increasing at a diminishing rate, due to the increasing growth costs. The highest of these lines, $g(rR^*)$, represents the technically possible combination of r and g which gives the maximum profit, R^*.

Figure 2b shows a series of "iso-value" lines. Each line represents combinations of g and r, which give the firm equal value. These lines are straight, which can be seen after rearranging the first equation in (7):

$$(10)\ g = k - \frac{(1 + k)R}{M} + \frac{(1 + k)R}{M} r$$

Here g is a function of r with k as a constant and R and M as parameters. The lines converge towards the point A, where $r = 1$ and $g = k$. $M_1 < M_2 < M_3$, showing that the value of the firm increases the smaller the slope of line. This can be seen from the equation (10), where increasing M gives decreasing slope, $\frac{(1 + k)R}{M}$, and increasing g-intercept, $k - \frac{(1 + k)R}{M}$.

The profit maximizing firm tries to reach the largest possible M. This is shown in figure 2c which combines figures 2a and 2b. Point B represents the maximum possible value M^* of the firm at the retention rate, r^*, and the growth rate, g^*, which are all thus optimal. Observe that at this point

$$BC = AC/tg\alpha$$

or

$$1 - r = \frac{(k - g)}{\delta g/\delta r},$$

which is the first order profit maximizing condition according to equation (9). The shape of $g(rR^*)$ ensures that the second order conditions for profit maximization are also fulfilled.

5.4 DISCUSSION OF BAUMOL'S AND WILLIAMSON'S MODELS

Let us now turn to the problem of providing help to the manager who wants to steer his—agricultural—firm through a growth process in which he sees growth problems mentioned earlier. These two models are still on too high a level of abstraction to really give any guidance. They only discuss growth under one goal (at a time), they are not directed towards exploration of the growth opportunities, and growth is still looked upon as occurring in one aggregated dimension. Moreover, the differences in abilities required by management in firms of various size is not explicitly treated by the models. Thus, items 1 to 3 and item 5 in the list of desirable features are not fulfilled by these models.

As compared to the original neo-classical model of the firm, Baumol's model and more especially Williamson's model, represent important developments in other features. Growth is looked upon as a process over time and growth costs play the important role of hampering the growth process. Feature 6 of our list of desirable items is thus included in the models. Williamson's model also includes important parts of item 4, the difference in characteristics of various sources of financial capital. In his model he tackles the important growth problems

associated with the financing of growth with savings from realized profits. However, by assuming that the firm is on the point of maximum leverage ratio, he assumes away the problems involved in financing growth with borrowed capital. A start on a treatment of this problem has been made by Halter [21].

Williamson's model touches on risk and uncertainty problems without dealing with them. He realizes their existence and assumes them away by developing his model for a firm at the maximum leverage ratio. However, this is not enough for us to say that item 7 in our checking list is taken into account by this model.

6. UNUSED GROWTH OPPORTUNITIES AS INDUCEMENTS AND THE MANAGERIAL LIMIT AS LIMITS TO GROWTH

6.1 GENERAL

In her 1959 study Edith Penrose [42] has developed a theory of the process of growth of firms. This theory is not built on the neo-classical theory of the firm. Penrose visualizes a growth process over time. She does not see an economically motivated end of this process. Nor does she accept that it is sensible to speak of such a thing as an "optimal" firm size. Her theory does, on the other hand, assume that the amount of growth per unit of time, the growth rate, has some limit to it. The inducements to growth stem from unused growth opportunities outside and inside the firm. The growth rate is hampered by specific growth costs, stemming from the difficulties for management in planning and organizing the growth process. These growth costs are assumed to increase at a growing rate when the speed of the growth process is increasing.

Penrose develops her theory in two steps. In the first a theory of internal growth is developed, as distinct from a growth through merger and acquisition. A theory covering the latter process is developed in a second step. In this review only the first theory will be demonstrated and analyzed.

6.2 BASIC ASSUMPTIONS OF PENROSE'S GROWTH THEORY

We note the following five assumptions on which the theory is built.

(i) The firm is defined as an administrative unit, not as a physical "plant". This administrative unit is looked upon as a collection of productive resources under a unifying management autonomously planning and leading the firm for the benefit of the enterprise as a whole [42, pp. 15–26].

(ii) The firm is a limited liability company operating in industry [42, p. 6]. This, then, means that the firm can acquire equity capital by issuing stocks and bonds to be sold on the market. As we will see in the analysis of the theory this has some interesting consequences as to its coverage of the growth problems for the non-stock agricultural firm.

(iii) Management's goal is to maximize that profit which in the long run can be achieved through investments in the firm. This means that investments are only made within the firm. Investments outside the firm are not made even though they may yield a higher profit. The opportunity cost of financial capital invested outside the firm is thus looked upon as being zero. The consequences of these formulations are that every new marginal investment is made which gives a positive surplus after all marginal increases of costs have been paid [42, pp. 27–30].

(iv) Financial capital and other productive resources are available at some price. This means that no absolute physical obstacle to growth exists [42, p. 43]. We note that physical obstacles to growth can exist under certain circumstances for agricultural firms (e.g. in the supply of land). However, other growth directions are open; hence the assumption holds also for agricultural firms.

(v) Opportunities for profitable investments are open somewhere in the economy at existing prices [42, p. 43].

6.3 THE INDUCEMENTS TO GROWTH

According to Penrose the inducements to growth are unused "productive opportunities" [42, pp. 31–32]. A growth theory has thus to include an examination of these opportunities to find the profitable ones. This can be thought of as a search for growth directions and goes on both outside and inside the firm.

External inducements to growth include such things as ". . . growing demand for particular products, changes in technology which call for production on a larger scale than before, discoveries and inventions the exploitation of which seems particularly promising or which open up promising fields in supplementary directions, special opportunities to obtain a better market position or achieve some monopolistic advantage and similar conditions and opportunities. They also include changes which might adversely affect a firm's existing operations and against which it could protect itself through expansion in particular directions, for example through backward integration to control sources of supply, diversification of final products to spread risk, or expansion of existing or allied products to preclude the entry of new competitors". [42, p. 65].

The existence of unused productive resources is the important *internal* inducement to growth to which Penrose devotes her greatest attention. Her point of departure is that an inducement to growth exists when the productive resources commanded by the firm could be used more profitably through growth than without it. Penrose states that "unused, productive services, resources and special knowledge" [42, p. 66] will always be found within any firm, and thus inducements to growth always exist. She supports her statement with the following arguments [42, pp. 68 and 68–89]:

(i) The existence of indivisible resources.

(ii) The same resources can be used differently under different circumstances.

(iii) In the ordinary process of operation and expansion new productive services are continually being created.

Let us consider each of these in turn.

(i) In every firm there exists a great number of *indivisible resources*, buildings, machines and men. No matter how well the firm is organized unused parts of these resources will always exist. It is possible to imagine a situation where unused parts of indivisible resources exist but where the use of these parts is unprofitable. The marginal value of these resources would thus be zero and the firm would be in neo-classical equilibrium. Penrose argues that the firm never reaches this point in its development process over time, since this process is taken up with production and growth and exchange of indivisible resources in a dynamic pattern which never allows the firm to reach or stay in an equilibrium position. This is especially obvious when we take into account technological change going on over time. This change, which is very difficult to anticipate, increases the problems of combining indivisible resources without leaving some of these resources unprofitably unused at any point in time.

(ii) Penrose explains in two ways the reasons why *given resources used differently under different circumstances* can give rise to unused resources. Firstly, the increased specialization of resources is part of technical and economic development. A highly specialized resource, e.g. a skilled worker or a complicated machine must be efficiently used to yield maximum result. Nevertheless it may be profitable for a firm, which is small compared to the degree of specialization of the resource, to acquire it. The skilled worker may, for example, be occupied only partly in the job for which he is specialized. The rest of his time is used for jobs where less skilled labour could be used. Secondly, resources like land, labour and machines can render a heterogeneous variety of services depending on the skills and abilities of the personnel of a firm. Changes in personnel and other resources can thus change the amount of services available to the firm. This inefficiently used resource creates unused service capacities in the firm. Irrespective of the original profitability in acquiring the specialized or heterogeneous resource, it may very well be profitable for the firm to grow and thereby to better utilize the specialized resource.

(iii) The third reason for the existence of unused resources in the firm is that *new productive services are continually being created* in it. These increases in productive services can stem from managers and other persons working in the firm. With increasing experience and knowledge acquired through work within the firm the amount of services delivered by them to the production process within the firm increases through more rapid decisions and greater production per unit of time. This increase can be substantial especially through the important interaction between material and human resources. In other words the joint amounts of resources within the firm deliver increasing amounts of services and outputs, the more experience the personnel has.

6.4 THE LIMITS TO GROWTH

Economists have usually pointed to three conditions to explain the rise of the long run average cost curve beyond some level of production (i) limitations of management, (ii) limitations of markets, and (iii) increasing uncertainty with increased size of the firm. In this review lack of markets can be ignored as a reason for size limitation. Most agricultural firms are fairly close to a pure competition situation. Moreover, in more restrictive market situations, limitation in markets can be overcome by producing new products.

Penrose analyzes the importance of the two other obstacles to growth, concluding that in the long run it is possible to overcome both of them. Her arguments run as follows [42, pp. 44–64].

In the long run management can always be reorganized and adjusted to the specific needs of every firm size without loss of efficiency. This is possible through delegation of decision making downwards within the firm and through adjustment of the knowledge level of the managerial personnel to the requirements of every position in the firm. Possibilities also exist to delegate tasks to consultants outside the firm.

If there is a tendency for uncertainty to increase when the firm grows larger the specialized managerial staff can in the long run neutralize this tendency by better forecasts and more careful planning of the firm's activities than is possible in smaller firms with a non-specialized managerial staff.

As Penrose points out these long run adjustment possibilities do not preclude the existence of limits to the amount of expansion possible in any given period of time. These limits are associated with the increasing demand on management for labour input and knowledge increase. There are also physical obstacles to the rate of growth that the existing managerial staff can plan per unit of time. There are also limits to the rate with which new personnel can be integrated into the firm without risk of organizational disturbances. Also the gathering of new knowledge necessary for managing larger firms takes time. Thus, there is a limit to the amount of new knowledge that can be assimilated per unit of time. Now, the more rapid is growth the heavier is the burden on management of gathering new knowledge, of planning and of organizing the growth process. The overtime work, errors etc. that this creates lead to growth costs which after some point grow more rapidly than the profit increase, thus creating *limits to growth, not absolutely and in the long run*, but *per unit of time*.

6.5 DISCUSSION OF PENROSE'S MODEL

It is fairly obvious that with Penrose's model we come closer than in the earlier models to the level of abstraction necessary to be of help to the manager who wants to guide his firm in a growth process. This can be seen from the central place in her theory occupied by studies of the growth opportunities as initiators of expansion and by growth

costs as limits to the rate of growth. It can also be seen in the very tangible way in which the particulars of these growth opportunities and limitations are developed in her theory. This, then, means that features 2 and 6 in our list of desirable items of growth theories are covered. However, it is worth noting that Penrose's model deals more with internal than with external growth opportunities. It is also interesting to note that the growth costs mentioned can all be brought back to problems of management, whereas other reasons for growth costs can also be found (see section 2 above).

This theory also gives attention to feature 5 in the list of desirable items. The fact that larger firms require new managerial abilities is an obstacle to growth only in cases where management is incapable of acquiring the necessary knowledge and/or making the necessary adjustments. Of interest in many actual cases is the following conclusion by Penrose [42, p. 98]: "Only for firms incapable of adapting their managerial structure to the requirements of larger operations can one postulate *an* optimum size".

It seems fair to say that the differences in characteristics of various resources, item 3 in our list, is also partly taken into account in Penrose's model. One aspect of these differences is stressed when the heterogeneity of resources is put forward as one of the reasons for the existence of unused resources [42, pp. 74–76]. However, the externally influenced changes over time in the availability and value of resources, and the differences in these changes between resources, are not explicity treated in Penrose's theory.

Risk and uncertainty as a limit to growth, occupy a rather concealed place in Penrose's model, hidden behind the viewpoint that it is management's lack of knowledge as to how to deal with risk and uncertainty rather than risk and uncertainty themselves that restricts the growth of the firm (see especially [42, p. 65]). For our purposes this stops at too high a level of abstraction. The manager who wants to steer his firm in a growth process needs to know *what kind of knowledge* that is necessary to overcome the effects of risk and uncertainty. If he does not have this knowledge, no matter whether this is due to inability to acquire it or to too short a time or too high a cost to acquire it, it is risk and uncertainty that restricts the growth of the firm. This, then, means that Penrose's theory does not include item 7 in our list of desirable features for a theory suitable to guide the growth process of a specific firm.

As an explanation of this feature of Penrose's theory, we can again point out that it is built for the limited liability company with stocks on the stock market [42, pp. 6–7]. Such a company can, as a rule, even in a process of rapid growth, keep a satisfactory relation between external equity capital and borrowed capital by issuing new stocks on the market, thus keeping the risk and uncertainty element within reasonable limits. In this way these elements can be restrained on the financial side of the business. Generally, this is impossible for the

agricultural firm of the family firm type, which interests us in this review. Thus the lack of item 7 in our list of desirable features is an important drawback of Penrose's theory. It may also be that it is a drawback for the type of firm treated by Penrose. As we will see later, there are other theories—e.g. Marris [33]—which stress the risk of being driven out of business as an obstacle to growth also for that type of firm.

In addition the acquisition of financial capital and the differences in characteristics of financial sources such as reinvested savings, loans and external equity capital are hidden in Penrose's theory behind the view that it is extrepreneurial ability more than lack of capital *per se*, that restricts the growth of the firms. Thus this restriction, item 4 in our list of desirable features, is also looked upon by Penrose as a managerial limitation. As mentioned earlier this is at least to stop at too high a level of abstraction for the theory to be of help for the manager directing its firm through a growth process. This manager needs to have knowledge about how to arrange the financial side of his growth process. This is especially so for the family firm. For this firm it can be suspected that capital restrictions in many cases can be relevant as obstacles to growth. There also exists in these cases an optimal combination of different financial sources.

It now only remains to state that Penrose's theory assumes that management's goal with the firm's growth is maximization of long run profit. This goal, stated in the way given earlier in this review, is the only goal discussed. Many goals and goals of the satisficing type are thus not included in the theory.

7. INFLUENCING THE DEMAND FOR THE FIRM'S OUTPUT AS AN INDUCEMENT TO GROWTH AND THE RISK OF BEING DRIVEN OUT OF BUSINESS AS A LIMIT TO GROWTH

7.1 GENERAL

Robin Marris in his book on "Managerial" Capitalism published in 1964 [33], draws our attention to other factors affecting growth than those stressed by Penrose. He has developed a theory of growth for limited liability companies with stocks on the stock market and *where ownership and management are separated.* He assumes that management's objective in these firms is to maximize the firm's growth rate subject to a security constraint.

Marris builds his theory on two ideas. One of them, giving the inducements to growth, centres around the profitability of new production possibilities. The other is the risk for management of losing control over the firm, which limits the firm's rate of growth.

Marris' firm is not working in pure competition. Thus, management's efforts to increase the demand for the firm's products sooner or later

lead to lower prices and/or higher unit costs. Sustainable growth of the demand for the firm's products in this situation is only possible by a continuous introduction of new products on the market, or as Marris puts it (after Penrose), by an increase in "the rate of diversification" [33, p. 120]. The growth rate is, according to Marris, a function of (1) the speed with which the firm introduces new products on the market and (2) the profitability of this new production.

To quantify management's risk of losing control over the company Marris builds up a "theory of take-over", based upon two different groups' evaluation of the company's stocks. One of these groups is the majority of buyers on the stock market. The other is an imagined group of take-over raiders who are assumed to plan the acquisition of sufficient stocks to be able to decide independently on the management and policy of the firm. The large majority of buyers on the stock market evaluate the stocks according to the expected returns under the existing management. This market value ratio is the combined effect of expected dividends and expected capital gains. The raiding group's valuation ratio is based on the expected returns from the management and the policy that the group would pursue if the raid were successful. If the latter valuation ratio is higher than the market valuation ratio there exists a risk of take-over; the greater the difference, the greater the risk. The existing management is thus interested in keeping the market valuation ratio of the company stocks equal to or just above the imagined raider valuation ratio. "Higher ratios yield no gain in satisfaction" for management in Marris' theory [33, p. 45]. This requirement of keeping the market valuation ratio just above a certain point is a restricting constraint in the management's objective of maximizing the growth rate of the firm, since it restricts management's possibilities of devoting resources to research and development of new products, the source for growth in Marris' theory.

The reader may ask why a theory of this type is reviewed here, when we are interested in directing the growth process of agricultural family firms. The individual agricultural firm is as a rule in a market situation where growth is possible without change of production. It is very seldom so large that it can continuously grow by diversification. Moreover, it is not often a "managerial" firm with stocks on the stock market. Thus the growth of the agricultural firm is neither induced by the introduction of new products, nor is it limited by Marris' type of take-over risk.

Nevertheless the author finds it important to include Marris' theory here, for two reasons. The theory quantifies some of the important *external* inducements to growth, a family of growth possibilities also open to the agricultural firm. It also emphasizes the risk of being driven out of business as a limit to growth, a risk which is of importance for the agricultural firm, although it takes on other forms than those facing a limited liability company with stocks on the stock market. Thus we shall proceed to review those parts of Marris' theory of greatest interest to us.

7.2 A "POLICY MODEL FOR GROWTH RATE MAXIMIZATION SUBJECT TO MINIMUM SECURITY CONSTRAINT"[4]

Marris' theory will be reviewed by describing his model for growth rate maximization together with those assumptions underlying it not yet mentioned. This model is as follows:[5]

(11) $D^{\cdot} = D(d, m)$

(12) $K^{\cdot} = r\,p' + N^{\cdot}v$

(13) $p' = \left(\frac{m}{c} - il\right)/(1 - l)$

(14) $c = c(d)$

(15) $v = (1 - r)/\left(\frac{k}{p'} - r\right)$

(16) $k = \alpha_1 + \alpha_2 N^{\cdot} + \alpha_3 r^2$

(17) $v \geqslant \bar{v}$

(18) $l \leqslant \bar{l}$

(19) $K^{\cdot} = D^{\cdot}$

where

c = capital-output ratio, i.e. the ratio of book value of gross assets to some measure of the quantum of total output [33, p. 228];

$D^{\cdot}$ = the compound growth rate of demand, $d(\log D)/dt$, where D is measured as the book value of gross assets required to meet the total volume of demand for all the firm's products at a standard rate of utilization. "Book value" is used as a physical measure [33, pp. xvi, xvii, 119, 131];

d = the diversification rate. By this is meant the rate at which diversification is attempted, i.e. the ratio of some volume measure of all new products attempted to a similar volume measure for all products already in production [33, p. 131];

i = average interest rate on debt [33, p. 9];

$K^{\cdot}$ = the compound growth rate of capital measured as gross assets [33, pp. xvii, 119, 206];

k = the rate of discount, which for a certain firm is the total yield in relative terms, i.e. the annual dividend plus the capital gain in relation to the market value of the firm [33, p. 209];

l = leverage ratio, i.e. the ratio of liabilities (debts) less liquid assets to gross assets [33, p. 132];

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[4] Marris, [33, pp. 226–249].

[5] This is Marris' equation system (6.1) to (6.9) [33, pp. 234–235]. To conform with notation in the rest of this review article the following changes in Marris' symbols have been made: his C has been changed to K, his g has been changed to l.

$\bar{l}$ = maximum leverage ratio. A constraint with functions similar to $\bar{v}$ [33, pp. 8–9];

m = profit margin, measured as the ratio of gross profits to the quantum of output measured by the same convention as used for defining the capital–output ratio [33, p. 228];

$N^{\cdot}$ = the compound growth rate of aggregate par value of all issued securities not included in debt [33, p. xviii];

p' = net rate of return = ratio of earnings to net assets. Net assets are gross assets less debt [33, pp. xviii, xvii];

r = retention ratio, i.e. ratio of retained earnings to earnings. Earnings = profits less interest on debt [33, p. xviii];

v = valuation ratio, i.e. the ratio of the market value of the firm to the book value of its net assets [33, pp. 22, 209];

$\bar{v}$ = minimum valuation ratio. A constraint to provide freedom from fear of take-over and/or from loss of confidence from shareholders [33, p. 206].

α_1, α_2, α_3 in (16) are coefficients in the relation between k, $N^{\cdot}$, and r. In this system equation (11) represents the inducements to growth and equation (14) the effects on capital requirements from the demand side D. Equations (12) through (16) excluding equation (14) represent the supply of funds, K, to finance the growth. Equations (17) and (18) are the financial restrictions to growth, minimum valuation ratio and maximum leverage ratio respectively, and (19) is the balance equation between the growth rates for financial capital, $K^{\cdot}$, and the growth rate for demand, $D^{\cdot}$.

Equation (11) states that the growth rate of the firm's total demand is some function of the relative number of new production possibilities introduced per unit of time and the profitability of the production. The variables d and m are policy variables. What, then, are the actions behind these variables that cause deliberate variation in demand growth? We can find interesting viewpoints on this in Marris' chapter four. First we have to remember that according to both Penrose and Marris the new products forming the numerator in the rate of diversification are products which *do not compete with the old products* in the firm's production. The creation of new products includes investments in intelligence (taken in its military meaning) and research and development on both the production and the market side. The necessary introduction of new products rests on a process of want creation within a small number of pioneering consumers and a chain reaction from these pioneers to larger groups of consumers. It includes stimulus of this reaction via price policy and advertising to reach a critical volume of each product. Criticality is here defined "as a condition where the probability of a continuing chain reaction tends to unity" [33, p. 146]. This criticality can involve different effort for different products. The diversification process may also include a choice of an optimal mix of new products. This is so when there are many possible products each with their expected capacity requirement, a specific expected contribution to profits and a specific size of the anticipated future market. The

most efficient diversification process can also include a choice between development of new products within the firm or early limitation of other firms' new products.

As Marris points out [33, pp. 124–126] firms growing by diversification in this way have to exist within an environment "where both consumer and producer techniques are capable of responding to continuous development and change" [33, p. 124], "an environment with continual change and new product creation, and in which new products both final and intermediate, are responsible for the lion's share of incremental output and demand" [33, p. 125]. This means that his theory only holds for modern, developed economies.

The growth process with which Marris is concerned is what he calls "sustainable", that is the same growth rate can be maintained "indefinitely or at least until there is some change in data" [33, p. 118]. Under these conditions he assumes that the long-run relative change in size—i.e. the growth rates over longer periods—of sales, gross assets and profits are equal [33, pp. 118–119].

As we see from equation (14) Marris also assumes a relationship between the diversification rate and the capital output ratio. Here he introduces what he calls the "Penrose effect" [33, p. 233 and discussions, pp. 114–118, 230–234], i.e. overstrain in high-level decision taking at an increasing diversification rate will cause the overall capital output ratio of the firm to rise. This is the relationship depicted in equation (14).

Let us now turn to the rest of the equations (12) to (16) and examine how Marris visualizes the acquisition of funds—the supply of finance, as he calls it—for the growth process. His starting point is the three well-known ways of acquiring funds, retained earnings, borrowing and acquisition of equity capital. As he is dealing with corporations the last mentioned source is available via new issues of the company's shares on the stock market. The growth rate of total capital is thus [33, p. 206]:

$$\frac{\text{Retained Earnings} + \text{Net Borrowing} + \text{Proceeds of New Share Issues}}{\text{Gross Capital at Beginning of Period}}$$

As we will see later from equation (18) Marris' model assumes that the leverage will be kept constant at its maximum, or constant at a lower value if equation (17) is the most restrictive constraint. Since at constant leverage ratio the growth rates of gross and net capital are the same, the growth rate of total capital is: The retained part of the net rate of return, rp[6] plus the growth rate of the value of all shares, $N \cdot v$[7], as expressed in equation (12).

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[6] The net rate of return is *calculated* on net assets. When, as here, the retained part of the net return is used as a rate of increase for gross assets it also includes net borrowing up to an amount which leaves the leverage ratio constant.

[7] This growth rate is composed of the relative increase in issued stocks at par value multiplied by the valuation ratio.

Equations (13), (15), and (16) explain the formation of the firm's net return (13) and value (15) and of the discount rate used in the firm (16). Thus they form the foundation of the basic equation (12). Remembering that m/c is the gross rate of return on total capital[8] it is fairly easy to check (for example by multiplying numerator and denominator by total capital) that the net rate of return is a function of the gross rate of return, the interest rate on debts and the leverage ratio, as is shown in equation (13).

In this review it is not necessary to go too far in explaining equations (15) and (16), which show "how stock-market behaviour is assumed to determine the valuation ratio" [33, p. 235]. There are two reasons for this. Firstly, the mechanism of explaining the total yield and valuation ratios of the non-stock agricultural firm cannot be put forward in the terms used by Marris. Secondly, Marris himself later on [33, p. 236] simplifies his model by deleting these two equations, accepting that the exogenously determined minimum valuation ratio is an effective constraint and leaving the $N^{\cdot}$ to be determined uniquely by r for each resulting valuation ratio independently of the rest of the system.

Equations (17) and (18) are the "risk" restrictions which hamper the growth rate of the firm. It is of interest to note that Marris introduces two restrictions of this type into his system. Equation (17) is the precaution against raiders on the stock market, whereas equation (18) is the maximum leverage ratio. The latter is assumed to be exogenously determined and is based on the probability distribution of earnings typical for the firm and the risk of being driven into insolvency. With the objective "maximum sustainable growth rate", given by Marris' management team, one or both of these two restrictions are effective.

It is of interest to note that in the first chapter of his book Marris [33, pp. 7–9] introduces the maximum leverage ratio as an important side condition in the growth process of the traditional, entrepreneurial non-stock firms. If we exclude gifts, inherited funds etc. these firms can only acquire financial capital via retained earnings and borrowing. For any given leverage ratio the maximum growth rate is in this case the same as the growth rate of the equity capital in the firm, i.e. the entrepreneur's own capital. If the maximum leverage ratio can be empirically determined then the maximum growth rate of the firm is determined by the growth rate of the equity capital at this maximum leverage ratio. This is so as at a specified leverage ratio the relative increase of borrowed capital has to be of exactly the same size as the relative increase of owned capital to keep the leverage ratio at the given value.

Let us now go back to the somewhat simplified version of Marris' model which consists of the system (11) to (19) with the exception of equations (15) and (16) and analyse its variables [33, pp. 235–236]. This is a system of five equations and two inequalities and with eleven variables.

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[8] $\frac{m}{c} = \frac{\text{gross profit}}{\text{total output}} \Big/ \frac{\text{total capital}}{\text{total output}} = \frac{\text{gross profit}}{\text{total capital}}$

Of these variables the interest rate, i, is exogenously determined, the valuation ratio, v, is kept at the exogenously determined minimum valuation ratio, $\bar{v}$, and the growth rate of new issues, $N^{\cdot}$, is uniquely determined by r. This leaves us with eight variables, of which four are policy variables—the diversification rate, d, the profit margin, m, the retention ratio, r, and the leverage ratio, l—and four endogenous variables—the growth rate of demand, $D^{\cdot}$, and of capacity, $K^{\cdot}$, the net rate of return, p', and the capital-output ratio, c. Leaving the inequalities aside we have a system of five equations and eight unknowns of which four are policy variables. A determination of three of the policy variables would then determine the system. In our situation it is obvious that the policy variables to be determined are the diversification rate, the profit margin and the retention ratio leaving the leverage ratio to be determined within its restriction. If, however, the leverage ratio is also restrictive there exist only two policy variables to be freely chosen.

In this last situation the problem is to choose that set of d, m, and r which maximizes the growth rate of demand or of capacity, which due to equation (19) amounts to the same thing.

Marris illustrates the character of the solution of his system for a simplified model. Although the simplifications employed suit the stock company and not the simpler non-stock companies which interest us here, Marris' solution will be given in its diagrammatic form. The simplified system appears as follows:

(20) $D^{\cdot} = D(d, m)$

(21) $K^{\cdot *} = \alpha\frac{m}{c} - \beta\bar{v}$

(22) $c = c(d)$

(23) $K^{\cdot *} = D^{\cdot}$

In this system equations (20) and (22) are the same as (11) and (14) in the earlier system. (21) represents the maximum growth rate of the firm, expressed in growth rate of financial capital, for every specified combination of gross profit rate, $p = m/c$, and minimum valuation ratio, $\bar{v}$. This relationship has been shown by Marris [33, pp. 216–219] to have approximately the form given by equation (21). The approximations include among other things that the main financial source for growth stems from new issues of stocks and *not* from retention of earnings. This, then, is the point where the solution to be demonstrated is of interest for the stock company but not of interest for the traditional entrepreneurial firm without stocks on the stock market. Equation (21) which summarizes the information from equations (12), (13), (15) to (18), also holds only for a solution where the maximum leverage ratio is reached. Equation (23) is the new identity equation between maximum growth rates of financial capital for given gross rates of return at the minimum valuation ratio, and the growth rate of demand.

Marris' diagrammatic solution of the problem involves choosing that combination of diversification rate, d, and profit margin, m, which

gives the highest of the possible maximum growth rates as shown in figure 3[9]. The character of the solution is of course dependent on the shape of the equations, which are thus important in explaining the diagram.

Figure 3a illustrates equation (20). For any specified m, the growth rate of demand is a rising function of the diversification rate. The decreasing first derivative of this function can be explained by Marris' assumption that any firm tries to choose new products in optimal order [33, pp. 178–179]; thus, with the overall growth goal given, the firm first develops products with the largest expected contribution to growth. Equation (20) is represented in figure 3a by a whole family of curves, each representing a given profit margin, m. In this family $m_1 < m_2 < m_3$ etc; thus m_5 represents the highest and m_1 the lowest profit margin. This order depends on the assumption that for every given diversification rate and profit margin the resulting growth rate can be raised by increasing the costs for research and development of new products and for marketing costs (advertising, lower prices). This results in an increase in costs which decreases the profit margin.

In figure 3b equation (22) is given. Starting at some capital-output ratio at the diversification rate zero this ratio increases at an increasing rate as the diversification rate grows, a result of the "Penrose-effect" mentioned above.

Figure 3c shows the combined effect of equations (21) and (22). For any given profit margin, m, there exists a certain maximum possible growth rate of financial capital at zero diversification rate. As the diversification rate increases this maximum possible growth rate decreases due to the increasing capital output ratio needed according to equation (22). At some point this capital output ratio has reached such a size that zero growth rate is reached. Also here the combination of equations (21) and (22) is represented by the same family of profit margin curves, m_1, as in figure 3a. Here the highest profit, m_5, is at the top and the lowest, m_1, at the bottom of the family, indicating the fact that on the capital "supply" side of the model available capital is directly proportionate to the profit margin of the firm in this case via the effect of profit margin on the new issue rate of the firm.

Figure 3d represents the combination of the "demand" and the "supply" side of the model given in equation (23). It is a combination of figures 3a and 3c. In this diagram it is possible to connect the intersections of curves with the same profit margin into one line representing points where the growth rate of financial capital and of demand are the same, thus fulfilling equation (23). On this line it is possible to determine that combination of diversification rate, d^*, and profit margin, m^*, which gives the overall maximum growth rate, D^{**}, of the firm. In the diagram this point is somewhere between 10 and 15 per cent for d and lies somewhere between m_2 and m_3.

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[9] The four parts of Figure 3, *a* to *d*, are the same as Marris' diagrams 6.1 to 6.4, pp. 239–240.

FIGURE 3. MARRIS' MODEL FOR GROWTH RATE MAXIMIZATION WITH SECURITY CONSTRAINT

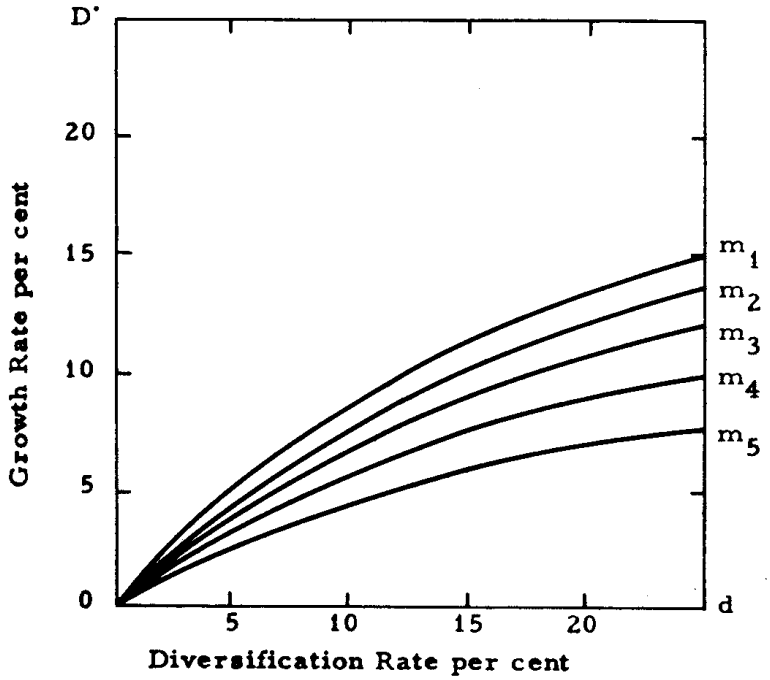

FIGURE 3a: *Family of Demand-Growth Curves*

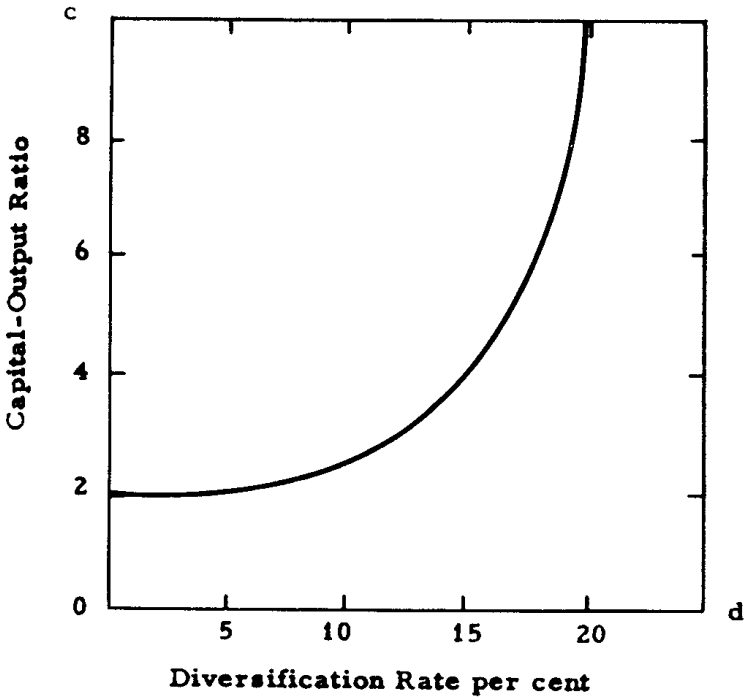

FIGURE 3b: *Internal Efficiency Relation*

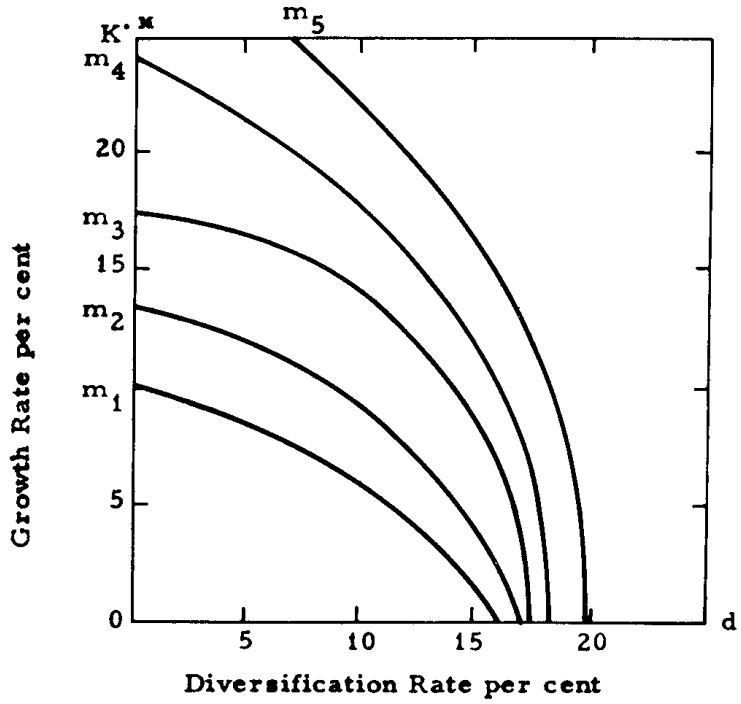

FIGURE 3c: *Family of Finance-Supply Curves*

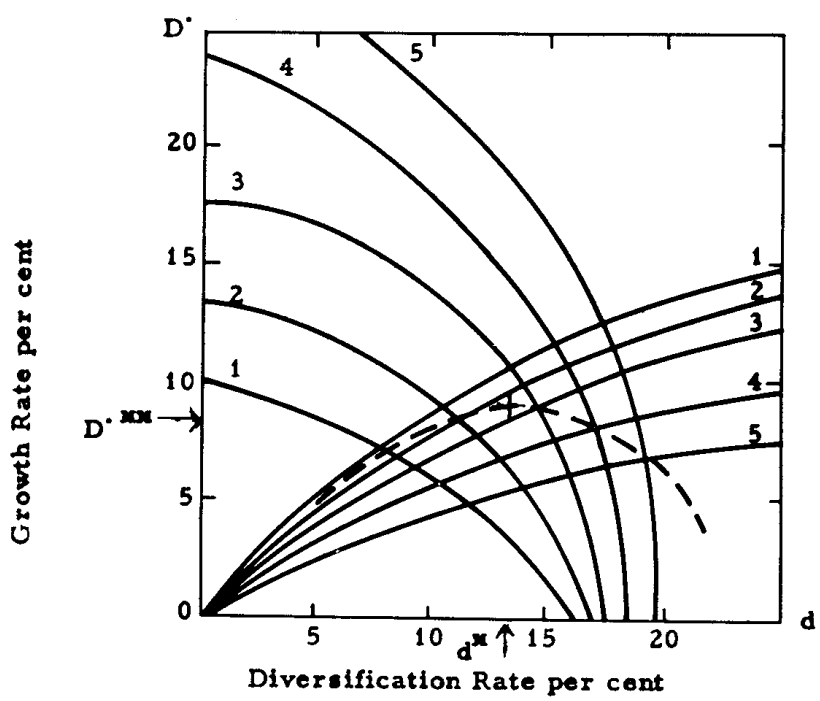

FIGURE 3d: *Balanced-Growth Curve*

7.3 DISCUSSION OF MARRIS' MODEL

Let us now turn to the analysis of this model as a means of steering the agricultural firm in a growth process. We can observe, as for Penrose's model, that we are closer to the reality of the manager than in the models where we started our review. Diversification rates, maximum leverage ratios and minimum valuation ratios bring the model to an abstraction level where managerial decision in the steering process are made. At the same time it is obvious that it is a choice only of *some* of the relevant factors which must be taken into account. Penrose, on the other hand, has devoted her interest to another subset of the relevant factors. It is still an open question whether these factors, Marris' or Penrose's choice, some combination or subset of them, or some other factors, are the most important ones to pay attention to in the growth process of the firm. In this review we are mainly interested in the family type agricultural firm; Marris' model includes specific features more suited in the steering of the growth process of limited liability companies than for the agricultural firm. We shall return to this point later in the review.

When analyzing Marris' model we can see that the *goal* structure in it is an overall goal of growth rate maximization, with the sub-goals minimum valuation ratio and maximum leverage ratio guaranteeing the survival of the firm. In an extension of his model not described here, Marris moves away from the point of maximum growth rate and indicates an alternative solution for maximum profit rate within his restrictions of a "sustainable and safe" growth rate. Drawing from our earlier description of the goal structure of farmers we can see that both these goals formulations are relevant in our case, except for one feature. Remembering the way in which the minimum valuation ratio was formulated as the result of various groups' valuation of the firm's shares on the stock market, it is obvious that a valuation ratio of this type as a restriction to growth does not belong to the necessary features of a growth model for the agricultural firm.

In this simplified version of Marris' model, an essential feature of the goal structure of the agricultural firm is assumed away. Marris assumes that issuing new shares is a more important way of acquiring financial funds for growth than retention of earlier earnings [33, p. 218]. In doing this he stresses a feature which is of importance for the firm he deals with, the limited liability company with stocks on the market. Unfortunately, he at the same time makes his solution of less interest to us in this review. Thus, the competition between retention and withdrawals of various goods and services, which is of interest in the goal structure of the agricultural family firm, needs a somewhat different set of restrictions and a different way of achieving a solution of the adjusted model from the one used by Marris.

It is obvious that Marris' theory brings in something new in growth models when he stresses and so closely examines the external growth opportunities stemming from the introduction of new products. It is also obvious that this feature of his theory is mainly relevant to the

large firms with which he is dealing and which can carry through a continuous and presumably costly research and development activity to acquire new products to produce and sell. The agricultural firm is as a rule too small to be able to embark on continuous research and development activity of this type. Does this, then, mean that it may be difficult to find a continuous measure of the diversification ratio, d, in our types of firms? The present author does not think so. Marris shows that growth through imitation [33, pp. 186–200] and non-diversifying growth [33, p. 200], for example by cost-reducing innovations and by increasing consumer appeal for old products etc., can also be treated by the same type of analysis as the one he gives. For firms in the market situation typical of the individual agricultural firm it seems adequate to let the diversification rate, d, include not only strictly new products but also early imitated products, products produced with new or early-imitated techniques, products with new features which increase consumer appeal, etc. This then means that the growth rate of the agricultural firm is a function of the number of innovations and imitations on the product *and the cost* side that can be introduced per unit of time. This extension of the meaning of d in Marris' theory makes it more possible to speak of a continuous d also for the agricultural firms, since imitation costs less in research and development than internal development of new products and as imitation and new production techniques increase the number of directions in which to find new growth opportunities.

Marris devotes his whole chapter five to the problems involved in the acquisition of capital. This treatment is, of course, directed towards the problems of the type of firm he is discussing which unfortunately for us is not the family firm. He thus "asks" the stock market, as purveyors of financial funds, for the desired choice between retained earnings and new issues [33, p. 216] a problem which does not exist for the non-stock family firm. The discussions of the formation of the valuation ratio and its "safety" limit, the minimum valuation ratio, $\bar{v}$, which are important for Marris, are of interest for us only as a reminder of the existence of a similar type of minimum valuation ratio for any firm. For the agricultural firm it is not so much of interest in terms of fear of take-over raiders as of a basis for credit valuation of the firm from banks and other credit institutions.

As we have seen Marris carries through his analysis in a situation where the maximum leverage ratio is reached. This gives his analysis a uniqueness condition of much the same type as that of the neo-classical production function.[10] On the other hand, he leaves two related problems outside the analysis: firstly the difficult problem of estimating the maximum leverage ratio, secondly, the problem of the choice between growth on retained earnings and loans *before* the firm reaches its specific maximum leverage ratio. This is of interest as the owners may not be indifferent as to *when* in time the withdrawn and retained

[10] In which output has to be interpreted as the *maximum* amount that can be produced with any given set of input amounts available. Compare for example Danø [13].

carnings are falling. It is finally worth remembering that Marris formulates his simplified solution around the case where financial means are obtained via new issues of stocks and not from retained earnings (compare page 78 above). This decreases our specific interests in this part of his analysis.

To summarize this part of our analysis, we are forced to say that Marris devotes much attention in his book to the supply of funds for growth, and that large parts of this analysis are of limited interest to us here.

To go on to another area in our search for features in Marris' theory desirable in guiding the growth of agricultural firms it is obvious from our earlier discussion that *risk and uncertainty* as limits to growth play an important role in his theory. The two critical limits, the minimum valuation ratio, $\bar{v}$, and the maximum leverage ratio, $\bar{l}$, are both of interest to us here; however, the former is of interest only after a change in meaning. As we have indicated above, the minimum valuation ratio in the family firm case will preferably have as its basis the credit valuation of the firm by banks and other credit institutions.

With this discussion we have covered points 1, 2, 4, and 7 in our list of analytical items. We have seen that Marris' theory has valuable features in these areas. As to the rest of the points, it can be said that Marris does not pay any specific attention to the differences in characteristics of various resources which can form growth direction for the firm (item 3 in our list). Nor does he discuss the effects on the growth process of the fact that larger firms require new abilities by the manager (item 5). As regards item 6 in our list, the existence of growth costs and their effect on firm growth, Marris includes this feature in his theory as the "Penrose-effect" which increases the capital output ratio during rapid growth.

8. INTEGRATION OF INVESTMENT AND FINANCE THEORY

8.1 GENERAL

Firm growth often includes additions of durable means of production to the amounts originally available. Traditionally, calculations regarding the profitability of such additions follow the rules of *investment theory*. Classical contributions in this area are those by Fisher in 1930 [20], Lutz & Lutz in 1951 [32] and Dean also in 1951 [15]. This theory specifies rules for estimating when an investment is profitable and how a choice between investment alternatives may be made. Essential in such calculations are comparisons between the expected future streams of cash payments into and out of the firm during specified time periods, caused by each alternative investment. Traditionally the attention in these calculations is primarily directed towards the best avenues of investment and the points in time to make the investment. The methods for financing the investments have been of less interest in these calculations.

Parallel to this a *theory of finance* has been developed, directed towards studies of the qualities of various forms of financial capital. It has also given rules for the most profitable combination of these various forms when covering a given demand for funds. Contributions typical of this approach can be found for example in Solomon, ed. [45].

A combination of these two approaches is now common and has become operational in calculations through the development of mathematical programming methods. An early illustration of these possibilities was given by Charnes, Cooper and Miller [9]. A number of experiments have been made since then to solve investment, finance and growth problems with multiperiod linear programming, integer linear programming, various forms of stochastic programming, recursive programming etc. These experiments show that this group of programming methods offers efficient and flexible tools for operational formulation and solution of firm growth problems. It is thus appropriate to devote part of this review to an excursion into this experimental field. As there are many experiments, only a few may be chosen to illustrate the main features and problems of these types of models. The author has chosen to illustrate the multiperiod linear programming approach with Weingartner's thesis of 1963 [49] and the recursive linear programming approach with Heidhues' 1966 article [22]. The large group of experiments will be discussed as examples of how the problems of our manager of the growing agricultural firm have been taken into account in these types of models.

8.2 WEINGARTNER'S MODEL

Weingartner [49] gives the most rigorously formulated presentation to date of the class of multiperiod linear programming models simultaneously treating both investment and finance problems of interest for planning of firm growth. His models will be developed stepwise here to cover the three essential features in which we are interested: the investments which increase firm size; the financing of these investments; and the transfer of capital, real capital and funds, to future periods.

The basic model includes transfer of capital between subperiods. The well known model is given in equation (24) for a planning period where there are h[11] subperiods.

(24) Max. $f = EX$

under the side conditions

$$FX - S^{tr} + S = K$$

$$0 \leqslant X \leqslant 1$$

$$S^{tr}, S \geqslant 0$$

[11] In equations (24) through (29) small letters indicate scalar quantities and capital letters matrices of the given dimensions.

This is Weingartner's equation system 7.1 [49, p. 124]. Here

E is a $1 \times n$ matrix $\{e_j\}$, representing the net result of each project j discounted to the present time;

X is a $n \times 1$ matrix $\{x_j\}$, representing the scale of every investment project j;

F is a $h \times n$ matrix $\{f_t\}$, representing the capital outlay discounted to the present time per unit of every project j during the respective subperiod t;

S is a $h \times 1$ matrix $\{s_t\}$, representing unutilized capital during every subperiod;

S^{tr} is a $h \times 1$ matrix $\{s_{t-1}\}$, representing capital not utilized during subperiod $t - 1$ transferred to subperiod t, $(s_0 = 0)$;

K is a $h \times 1$ matrix $\{k_t\}$, of capital available during every subperiod t, discounted to the present time.

The essential capital transfer can be examined more closely by specifying the first two rows in the first side condition:

$$\text{Period 1: } F_1X + s_1 = k_1$$
$$\text{Period 2: } F_2X - s_1 + s_2 = k_2$$

This model allows capital transfers to the next subperiod. However, it does not allow accumulation of capital via production within the model. Thus it is not a model for endogenous determination of the growth possibilities. The growth character of the model is given through exogenously determined values of K for the various periods.

Weingartner also specifies a model where capital is generated from within the model, is transferred to future subperiods and can be used for investments in the future. The model is completely built on cash flows over time. It is given in equation system (25).

(25) Max. $f = \hat{A}X + v_h - w_h$

subject to

$$CX - (1 + r)V^{tr} + V + (1 + r)W^{tr} - W \leqslant D$$
$$0 \leqslant X \leqslant 1$$
$$V^{tr}, V, W^{tr}, W \geqslant 0$$

This is Weingartner's equation system 8.1 [49, p. 142]. Here,

$\hat{A}$ is a $1 \times n$ matrix, $\{a_j\}$, of net cash flows from every project j *after* subperiod h, discounted to h;

X is a $n \times 1$ matrix, $\{x_j\}$, representing the scale of each project j;

v_h is a unutilized capital during subperiod h;

w_h is borrowed capital during subperiod h;

C is a $h \times n$ matrix, $\{c_{tj}\}$, of net cash flows from every project j during the subperiods t. Negative c indicates payment into the firm, positive c indicates payment out from the firm;

V is a $h \times 1$ matrix, $\{v_t\}$, of capital unutilized during every subperiod t;

V^{tr} is a $h \times 1$ matrix, $\{v_{t-1}\}$, of capital not utilized during subperiod $t - 1$, transferred to period t,$(v_o = 0)$;

W is a $h \times 1$ matrix, $\{w_t\}$, of borrowed capital during every subperiod t;

W^{tr} is a $h \times 1$ matrix, $\{w_{t-1}\}$, of capital borrowed during subperiod $t - 1$ and transferred to subperiod t,$(w_o = 0)$;

D is a $h \times 1$ matrix, $\{d_t\}$, of capital generated from other parts of the firm during every subperiod t.

All c, v, w, and d are given in current values, i.e. without discounting. This is possible as the objective function only contains values pertaining to one subperiod, h, the last one in the planning period. In this model the net worth at this point in time of all payments in to and out of the firm, is maximized. The rate with which investment projects are introduced in this model is related to the size of D, to the endogenously generated capital stream CX and to the profitability of the capital borrowing W.

Here also the capital transfer can be examined more closely by a specification of rows one and two in the first side condition:

$$\text{Period 1: } C_1X + v_1 - w_1 \leqslant d_1$$

$$\text{Period 2: } C_2X - (1 + r)v_1 + v_2 + (1 + r)w_1 - w_2 \leqslant d_2$$

It is possible to extend this model further. Weingartner gives a number of proposals of interest for us.

To system (25) can be added an absolute limit to borrowing by the introduction of another restriction. This addition gives system (26).

$$(26) \quad \begin{array}{l} \text{System (25)} \\ W \leqslant B \end{array}$$

This is Weingartner's equation system 9.1 [49, p. 162]. Added here is:

B is a $h \times 1$ matrix, $\{b_t\}$, of the maximum amount of capital which can be borrowed during every subperiod t.

The resulting system (26) can also be extended to handle an increasing supply function for capital where increasing amounts of borrowed capital are associated with increasing prices. Such a supply function is exemplified in figure 4 with r_i as the given interest rate for every level and w_i the capital amount of capital borrowed at this rate.

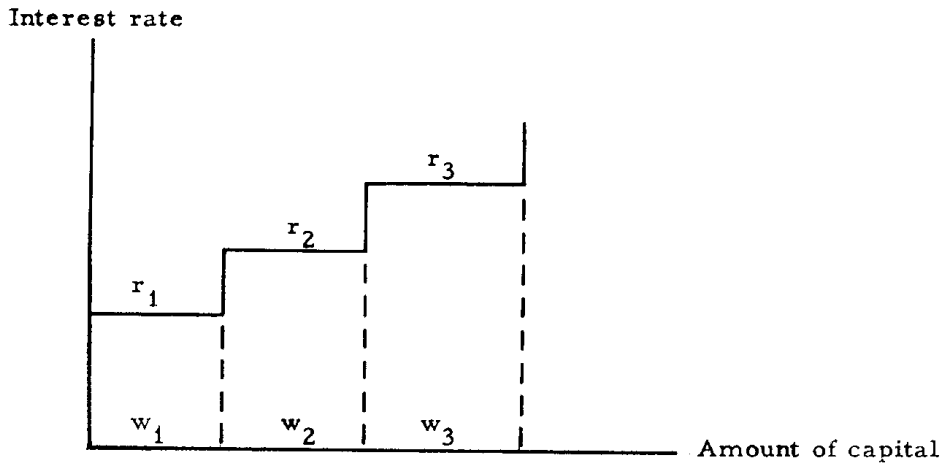

FIGURE 4: *A supply function for borrowed capital*

After this addition system (26) will appear as follows:

(27) Max. $f = AX + v_h - W_h^s\theta$

subject to

$CX - (1 + r)V^{tr} + V + (1 + r)W^{s,tr}\theta - W^s\theta \leqslant D$

$W_h^s \leqslant B^s$

$0 \leqslant X \leqslant 1$

$V, V^{tr}, W^s, W^{s,tr} \geqslant 0$

This is Weingartner's equation system 9.14 [49, p. 169].

In system (27) the new symbols are:

W_h^s is a $h \times m$ matrix, $\{w_{ti}\}$, of amounts of capital borrowed during every subperiod t at the interest rate r_i where $r_{i-1} < r_i < r_{i+1}$.

W^s is row h in W^s

θ is a $m \times 1$ matrix, where every element has the value 1

$W^{s,tr}$ is a $h \times m$ matrix, $\{w_{t-1,i}\}$, of amounts of capital borrowed during subperiod t_{-1} and transferred to period t

B^s is a $h \times m$ matrix, $\{b_{ti}\}$, of maximum amounts of capital which may be borrowed at a given interest rate r_i during every subperiod t.

As the systems (24) to (27) are built up it is only possible to introduce the investment projects up to unit level. In the majority of agricultural applications the side conditions $0 \leqslant X \leqslant 1$ have been changed to $X \geqslant 0$. In this way the projects are allowed to grow beyond the unit level. It is also possible without changing model to partition the matrix X into two parts, where one part represents the units in the product mix problem and one part the units in the investment problem of the firm. In this way the combined product mix, investment and finance problem can be solved.

We stop our review of Weingartner's models at this point, having shown the main features of the class of models of interest to us here. We have seen the possibilities for treating simultaneously the product and investment mix during series of periods, capital transfer between periods, lending and borrowing activities and capital accumulation. Readers interested in Weingartner's other expansions are referred to his stimulating 1963 publication [49].

8.3 HEIDHUES' MODEL

This model is of the recursive linear programming type, the well known programming procedure developed by Day [14]. It consists of a sequence of one-period planning problems solved with ordinary linear programming and connected over time with a starting situation and with one another through a series of relations so that each sub-problem is dependent on the solution to the foregoing. This method was used by Heidhues in 1966 [22] to solve growth planning problems for agricultural firms. The traditional linear programme for the individual subperiod may, for example, look like equation system (28).

$$(28)\quad \pi_t^* = \text{Max.}\, f = Z_t X_t \qquad t = 1, \ldots, h$$

subject to

$$G_t X_t \leqslant H_t$$

$$X_t \geqslant 0$$

where

π_t^* is the value of the objective function during period t at the optimal solution X_t^*;

Z_t is a $1 \times n$ matrix, $\{z_j\}$, of incomes minus variable costs during period t from activities j;

X_t is a $n \times 1$ matrix, $\{x_j\}$, of the amount of each activity during period t;

G_t is a $m \times n$ matrix, $\{g_{ij}\}$, of technical coefficients giving the required amount of fixed resources i for each unit of activity j during period t;

H_t is a $m \times 1$ matrix, $\{h_j\}$, of available amounts of fixed resources i and numerical values of other constraints at the start of period t.

The vector H_1 of available resources and other constraints at the start of period 1 specifies the starting solution.

In Heidhues' model the vector H_t of every period is dependent on the foregoing solution via the following relationship:

$$(29)\quad H_t = G_{(t-1)}\, \Lambda X^*_{(t-1)} + \Gamma H_{(t-1)} + T_t$$

In this equation the new notation is:

Λ is a $n \times n$ diagonal transformation matrix, $\{\lambda_{jj}\}$, with three aims: (1) to transfer the investments in means of production as available

amounts of fixed resources between periods, (2) to transfer accumulated capital between periods, (3) to remove resources no longer available from the model.

Γ is a $m \times m$ diagonal transformation matrix, $\{\gamma_{ii}\}$, which transfers whole or parts of the fixed resources available during one period to the next.

T_t is a $m \times 1$ matrix, $\{\tau_i\}$, with the help of which exogenous changes in every h_i can be made.

Heidhues' model works in the following way: assuming that the starting values of H_t for $t = 0$ are available for a firm to be planned, h_t for $t = 1$ are calculated from equation (29). The linear programming problem of system (28) is solved. From the optimal solution X_1^*, the fixed resources H_1 and exogenous influences T_2, the new vector H_2 of available resources and constraints is calculated from equation (29). In this way the planning is solved sequentially for subperiod after subperiod until the whole planning period is worked through.

8.4 SOLUTION OF GROWTH PLANNING PROBLEMS

A whole series of experiments with growth planning of agricultural firms using mathematical programming methods have been made during the 1960s. Most of them have been reported in easily accessible journal articles and research reports and are thus well known. A review[12] of mainly U.S. experiments was published by Irwin [26] in 1968. Thus, the integration of investment and finance theory made possible by mathematical programming is well known. It is therefore not thought necessary to go into a detailed discussion of these experiments here. The discussion will only focus on the degree to which the growth problems according to the list of items in the above analytical procedure have been treated in these experiments.[13]

The experiments indicate that a wide and flexible range of possibilities exist for formulating the *goals of the entrepreneur* using mathematical programming. In general, goals are formulated as a combination of the objective function and a set of restrictions in three different ways:

(i) In the traditional investment theory way, influenced by Hicks [23], where the difference between payments into and out of the firm during each subperiod is summarized after discounting at some specified point in time and maximized. This is the procedure used in system (24). It has also been used in a number of

[12] It covers multiperiod linear programming methods in general and not only those directed towards a study of firm growth.

[13] The author is indebted to Roif Olsson, Dept of Economics and Statistics at the Swedish Agricultural College, Uppsala, for his help in this part of the review. In a forthcoming study, Olsson develops a multiperiod linear programming model designed for further systematic studies of growth problems. His knowledge of current literature and the treatment in it of growth problems have been used by the present author in compiling this part of the review.

agricultural examples, e.g. by Loftsgard and Heady in 1959 [31], Throsby in 1962 [48], Dean and de Benedictis in 1964 [16], Köhne in 1966 [29], Martin in 1966 [35], and Martin and Plaxico in 1967 [36].

(ii) As maximization of the accumulated net worth at the end of the planning period. This is used in equation systems (25), (26), (27), (28), and also in agricultural examples, e.g. by Köhne in 1966 [29] and in 1968 [30], Johnson in 1966 [27], Martin in 1966 [35], Martin and Plaxico in 1967 [36], Johnson, Tefertiller and Moore in 1967 [28], Boehlje in 1967 [5], Eidman *et al.* in 1968 [18], and Boehlje and White in 1969 [6].

(iii) As maximization of a stream of money withdrawals for consumption for each subperiod during the planning period. This procedure was proposed by Baumol and Quandt in 1965 [4]. A similar model was also formulated at the same time, and most probably independent of Baumol and Quandt, by Cocks [10].

To the author's knowledge the last mentioned model has not been used in agricultural examples other than by Cocks, and thus deserves some explaining. Following Baumol and Quandt [4][14] the model is given in equation system (30). Here utility of a stream of withdrawals for consumption is maximized subject to relevant restrictions:

(30) Maximize $f = UK$

subject to

$$CX + K - (1 + r)V^{tr} + V + (1 + r)W^{tr} - W \leq D$$

$$X, K, V^{tr}, C \geq 0$$

In this model

U is a $1 \times h$ matrix, $\{u_t\}$, representing the utility of one money unit for consumption during every subperiod t;

K is a $h \times 1$ matrix, $\{k_t\}$, of variables representing the volume of consumption in money units during each subperiod t;

and all other notation is the same as that of equation system (25).

In this system the elements of C in the last row of the first set of restrictions, C_h, contain not only the cash payments into and out of the firm during subperiod h for each project x_j but also the future cash stream after period h discounted to this subperiod, i.e. the value of h of all the projects. This, then, means that k_h, the withdrawals for "consumption", at the end of the planning period consist of the net worth of the firm at that point in time. We see from the model that for $U = 1$ the value of the consumption in money units without discounting is maximized. When the elements of U are given values

[14] This model is somewhat more general than Cocks' in that it uses utilities assigned to each subperiod's withdrawal whereas Cocks uses a common subjective discount rate over the whole planning period.

1 or other values according to the utility preferences of the entrepreneur, the utility of the consumption possibilities is maximized. The choice between various values of the elements in U depends on the planning situation and will not be discussed here. Interesting and difficult preference problems arise when estimating the utility of withdrawals of consumption during each subperiod as compared to the utility of the "consumption" of owned capital at the end of the planning period.

In the models maximizing discounted cash flow, i.e. system (24) the utility of the whole surplus at each subperiod, not only that withdrawn for consumption, is measured with the discount rate. In the model maximizing accumulated net worth at the end of the planning period, i.e. system (25), the utility of the surpluses at each subperiod is measured in the model as to their contribution to the maximization of the accumulated net worth. These two models, then, only represent *specific* ways of setting up intra-time period preference systems. The model proposed by Baumol and Quandt in 1965 [4] gives more flexible possibilities of introducing varying withdrawal preference systems over time.

For example via a proper determination of the values of the elements in U, it can include the two other models discussed above. As was mentioned earlier, goals in mathematical programming are formulated as a rule as a combination of the objective function and a set of restrictions. In some cases this is necessary to avoid absurdities. Thus, for example, in the two systems (25) and (30) it would generally be necessary to include restrictions which guarantee that the desired withdrawal for consumption during each subperiod can be made. Otherwise all accumulated capital could well be invested during earlier subperiods and withdrawn only during the last period. This would mean that all consumption would occur only during the last subperiod, which is an obvious absurdity.

In other cases the possibilities for formulating goal dimensions in the side conditions of the mathematical programming models can contribute much to a refined and flexible formulation of even complicated goal structures. The agricultural applications published to date show many good examples of these possibilities. Consumption goals including for every subperiod a minimum consumption and additional consumption of a part of the possible surplus incorporating a predetermined marginal propensity to consume have been used in growth models by for example Cowling and Baker 1963 [11], Johnson 1966 [27], Martin 1966 [35], Martin and Plaxico 1967 [36], Johnson, Tefertiller and Moore 1967 [28], Boehlje 1967 [5], and Boehlje and White 1969 [6]. Solidity goals restricting the level of indebtedness have been used, for example, by Johnson 1966 [27], Heidhues 1966 [22], and Eddleman and Golden 1967 [17]. Liquidity restrictions guaranteeing that the stream of payments out of the firm does not exceed the payments into it have been used for example by Heidhues 1966 [22].

All these examples indicate that mathematical programming models of growing firms are flexible tools for handling of multi goal situations.

The mathematical programming experiments on firm growth planning go far enough in *identification of factors giving growth opportunities* that a simultanous choice of production processes and investments in new durable production resources is made. (See for example Köhne [29] and [30], Boehlje [5], Boehlje and White [6], whereas Martin [35], and Johnson [27] only treat the investment side.) Further extensions to include the ideas in Penrose [42] and Marris' [33] theories are hitherto untested. However, it seems possible to include at least some of these ideas rather easily in these types of models. Penrose's viewpoint that unused resources are the bearers of growth possibilities is included by the very way a linear programming model is constructed. The growth process might appear more obviously in the solution if the problem is set up so that the initial vector of available resources also includes the existing production plan. In this way both the employment of unused resources and the reallocation of resources in existing uses can be illustrated. To include Penrose's idea that new production resources are continuously produced within the firm and that new technology carries growth opportunities may require successive changes in the technical coefficients in the model according to some forecasting procedure. Testing the growth potential of anticipated new products may also be included in the model. The high preparedness necessary to take advantage of new growth opportunities when they appear stresses the interest in including flexibility constraints in the mathematical programming models. This is an area where still more needs to be done.[15]

It seems clear that the *differences in the growth in value over time* of various durable resources—land, buildings, livestock—are of great importance in firm growth planning in agriculture. Few if any studies are published in this area up to now.[15] It can be included in the programming models via forecasting of the differences in growth in value of the various resources. The problems associated with *acquisition of financial capital* were introduced at an early stage into the mathematical programming models for firm growth in agriculture, at first in capital market models (Irwin and Baker [25], Stewart and Thornton in 1962 [47]). Later this feature of growth was treated together with product mix and investment problems in both multiperiod linear programming (Stewart and Thornton in 1962 [47], Boehlje [5], Boehlje and White [6], Köhne [29] and [30]) and in recursive linear programming models (Heidhues [22], Steiger [46]). This problem area lends itself easily to treatment in these types of models.

Another desirable feature of growth models is that attention should be paid to the fact that *larger farms often require new abilities* by the farmer. The author has found no mathematical programming study formalizing and investigating this problem area.

What, then, are the possibilities for studying *growth costs* with these

[15] A start is made in these two areas by R. Olsson in the forthcoming study mentioned in footnote 13 above.

types of models, and have such studies been undertaken? The second question is the easier one to answer. Up to time of writing (March 1970), no such published studies have been found by the author.[16] However, it seems possible that these type of problems can be formalized and thus treated with mathematical programming models.

The existence of *risk and uncertainty* in the planning environment of the agricultural firm has great significance for the firm growth process. This stems from the fact that the net stream of cash payments into and out of the firm over time, which, together with borrowed funds, creates the financial capital from which growth originates, varies due to variations over time in inputs and outputs. In the multiperiod and recursive linear programming models under discussion these random and non-random variations appear in the technical coefficients of the cash flow matrix C in equation (25) and the matrix G in equation (28). To date no general technique for finding optimal solutions of these kinds of problems exist. Information on the effect of risk and uncertainty on the firm growth process can thus only be analyzed in these models with the help of approximation methods applicable to problems of specific types or by simulation techniques. Thus Eidman *et al.* [18] have used the so called chance constrained programming technique of Charnes and Cooper [8] to take account of risk variations in a specific problem. An example of a simulation study is the one made by Johnson 1966 [27] and [28].

9. POTENTIAL OF THE BEHAVIOURAL THEORY OF THE FIRM

9.1 GENERAL

When reviewing theories as to their ability to direct and control the growth process of a firm the biological analogies, cybernetics and the behavioural theory of the firm must be mentioned.

Three *biological analogies* existing in economics are of interest when firm growth problems are discussed. One of them is the *life-cycle-analogy*, where the firm is compared to a tree which is born, grows, ripens and dies as proposed by Marshall, 1920 [34] and Boulding 1950 [7] and as criticized by Penrose, 1952 [40] and 1953 [41]. Another is the *evolution analogy*, where the surviving firm is the one which can best adjust to a changing environment. A modern formulation of this has been given by Alchian, 1950 [1] and 1953 [2] and criticized by Penrose, [40] and [41]. This is analogous to the Darwinian principle of "the survival of the fittest". The third is the *homeostasis analogy* according to which firms, like living organisms, retain an inner balance under varied external conditions via a series of adjustment mechanisms

[16] Some experiments have been made by R. Olsson in his forthcoming study, *op. cit.*

aimed at smoothing and adjusting to the effects of these variations in the environment. The best known version is probably the one developed by Boulding [7].

Economists tend to have different viewpoints as to the usefulness of analogies of this type. A closer review of these analogies can be left aside here as our aim is not a discussion of theories useful in positive studies. However, with our interest in theories for directing and controlling firm growth, these analogies represent a point of departure to a group of related theories, namely cybernetic theories (Wiener, 1948 [50]) and the behavioural theory of the firm (Cyert and March, 1963 [12]). These two theories are briefly discussed below, not because they have yet formed any basis for extensive work in the firm growth field, but because of the interest of several examples, (Eisgruber [19], Hutton [24], Patrick [38]) and their potentialities for providing a basis for more work in the future.

9.2 CYBERNETIC MODELS

In cybernetics the firm is looked upon as a *controlled system of production.* The model of the firm assumes the existence of a controlled unit, for example for production, and another unit controlling the flow of inputs to and outputs from the controlled unit. The producing unit is influenced by an environment which creates disturbances in production. The controlling unit receives information on these disturbances both *ex ante* and *ex post.* The *ex ante* information is received as forecasts of expected disturbances. The controlling unit compensates the controlled unit for these disturbances. *Ex post* the controlling unit receives information through a feedback channel on the number of units produced, compares it with the required number of units (i.e. with the goal), and if necessary orders supplementary production. The controlling unit also regulates production according to received goals. A simple controlled system is given in figure 5.

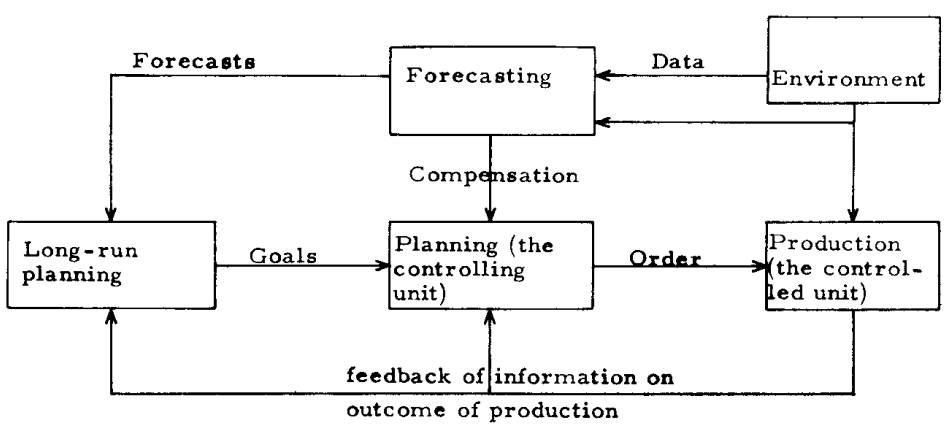

FIGURE 5: *The firm as a controlled system*

Generally, the flow of inputs and outputs within a firm is divided up in subsystems; for example acquisition of inputs; the technical flow of inputs, outputs and resources; the flow of labour services; the sales organization; and the economic control. Subsystems can also be plans covering different length of time. All these subsystems are parts of the total control system and connected via a flow of information between the systems.

There can be many goals in this system, both economic and non-economic and usually of the satisficing type. To function, the goals have to be broken down into subgoals for the various parts of the system.

9.3 THE BEHAVIOURAL THEORY OF THE FIRM

The decision making process in the firm forms the backbone to the *behavioural theory of the firm* developed by Cyert and March [12]. This decision making process is assumed to be represented by three sets of variables and four sets of relations.

The three sets of variables affect the goals, expectations and choice of the organization [Cyert and March 12, pp. 115–116]:

The organizational goals are assumed to have both *dimensions* and *aspiration levels* on any particular dimension. The dimension of the goals is influenced by the composition of the members of the organization and by the subunits of the organization that make the decisions. They are evoked by the planning or decision problems that arise. The aspiration levels are some weighted functions of past goals, past performances of the organization and past performances of other "comparable" organizations.

The *organizational expectations* are affected by the process of drawing inferences from available information and by factors influencing the search for problem solutions. In the process of drawing inferences variables such as simple trend studies are recognized. Variables influencing the intensity and success of the search process are the extent to which goals are achieved and the amount of organizational slack in the firm. The smaller the extent to which goals are achieved and the smaller the organizational slack, the more intensive the search, and thus also the formation of new expectations. Variables influencing the direction of search are the nature and location of the problem, stimulating the search.

The *organizational choice* takes place in response to a problem, using standard operating rules and involving identification of an alternative that is acceptable from the point of view of evoked goals. Variables affecting these three factors influence the choice of action.

The four sets of relations are [12, pp. 116–125]:

(i) *Quasi resolution of conflict* between goals held by various members of the organization. The conflict is solved by dividing its decision problems into subproblems and assigning them to subunits for solution (local rationality); by following acceptable level goals

instead of maximization type goals, and by attending to different goals at different times (sequential attention to goals).

(ii) *Avoidance of uncertainty.* Organizations are assumed not to maximize expected values or to use decision criteria under uncertainty when planning their activities. Instead they are assumed to avoid uncertainty by avoiding planning where plans depend on predictions of uncertain future events and by emphasizing planning where the plans can be self-confirming through some control device (feedback-react decision procedures).

(iii) *Search for good alternatives is stimulated by problems* and directed towards finding solutions to the problems. This search process is stimulated by the problem and depressed by the solution of the problem. The search process proceeds on the basis of a simple model unless it is driven to more complex models by the seriousness of the problem. Finally the search is biased, i.e. influenced by the training, experience and goals of the members of the organization.

(iv) *A learning process is continually going on.* This process includes adaptation of the directions and aspiration levels of goals. It also includes changes in decision criteria and in attention to different parts of the environment. Also search rules change.

The basic structure of the organizational decision making process is given in figure 6. This figure is an outline of the firm model in the above behavioural theory of the firm. The language used to picture this model is the language of a computer programme.

9.4 DISCUSSION OF THESE TYPES OF MODELS

Both the cybernetic and the behavioural theory approaches stress the *process* of production as it develops over time and the actions necessary for management to take to direct this process. This is the new feature of these approaches as compared with the theories reviewed earlier in this article. Let us end our review with a tentative discussion on how these steering theories can be used in the firm growth case. In this discussion we follow the structure of the decision making process in the behavioural theory given in figure 6 and in our original analytical procedure.

Action from management is evoked after observation of feedbacks from the environment. In the firm growth case this means on the positive side observation of the appearance of growth possibilities. These may stem from exgenous sources such as those discussed by Marris [33], or from the endogenous growth opportunities mentioned by Penrose [42]. On the negative side feedbacks can bring information on unfortunate price variations, poor yields in production due to weather variations, errors in the organization of production etc. Management's action is directed towards taking advantage of the new growth opportunities and neutralizing the unfavourable changes in the environment and production which threaten to hamper the growth. Applied to the firm growth

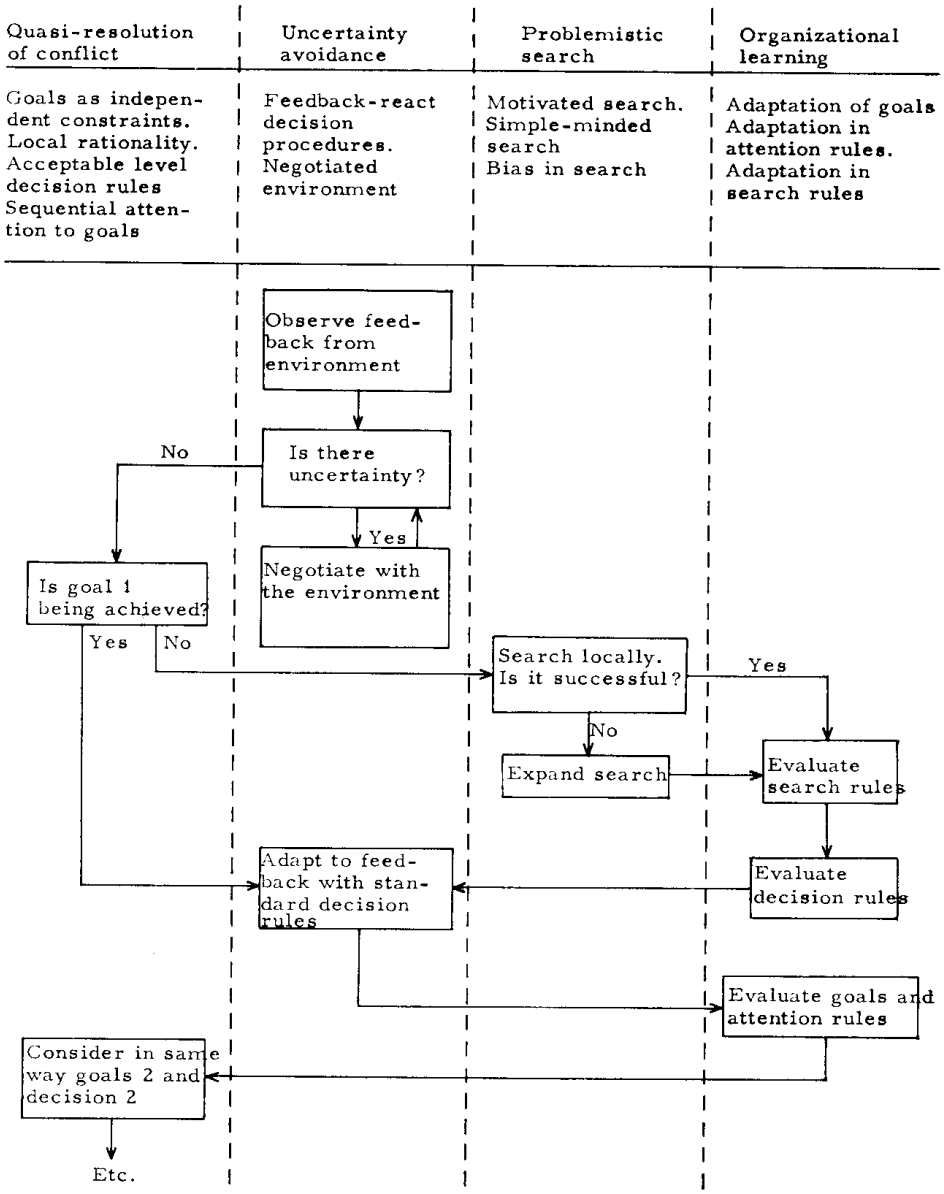

FIGURE 6: *Basic structure of the decision making process in the behavioural firm model*

Source: Cyert & March, 1963, figure 6.1, p. 126

case the behavioural theory thus includes a continuous scanning procedure to find good growth opportunities. It also assumes that the risk and uncertainty elements are partly taken care of by avoiding and neutralizing them.

Both the cybernetic and the behavioural theories assume that management sets a number of goals for its action within the firm. These

goals are of the satisficing type. In the growth case some of them can be related to the rate of growth and other dimensions related to the growth process. In the decision making process management controls the effects of the positive and unavoidable negative feedbacks from the environment for each one of these goals, one at a time.

If the goals can be satisfied also after the environmental change, management reacts according to standard rules established within the firm. Such rules can include for example acquisition of land up to specified total amounts as long as the price per acre is below a given value. Another example is reorganization of the feeding plans for livestock production following crop failures. If the goals cannot be satisfied search for new growth possibilities starts. This search can include attention to differences in characteristics of resources and various types of financial capital, the entrepreneur's ability to manage larger firms, the growth costs associated with various growth directions, etc.

A learning process is tied to the search for new growth opportunities. In this, management successively increases its knowledge of how to avoid risk and uncertainty, how to find good growth possibilities, how to improve standard decision rules and the possibilities for satisfying and adjusting the aspiration levels of various goals.

A return to the list of desirable items in the procedure for analyzing growth theories shows that important parts of them are included in the behavioural theory of the firm. Thus, this theory seems to have potentials for handling firm growth planning problems. The following new viewpoints not shown by the earlier theories are especially worth noticing: The use of a feedback mechanism to neutralize disturbances due to risk and uncertainty; the importance of standard decision rules and prepared alternative plans as precautions for sudden changes in the environment; the acceptation of "good" rather than "best" growth plans; and the existence of a learning process.

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