A Multivariate Evaluation of _Ex-ante_ Risks Associated with Fed Cattle Production

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**Abstract:** The purpose of this study is to evaluate the risks faced by fed cattle producers. With the development of livestock insurance programs as part of the Agricultural Risk Protection Act of 2000, a thorough investigation into the probabilistic measures of individual risk factors is needed. This research jointly models cattle production yield risk factors, using a multivariate dynamic regression model. A multivariate framework is necessary to characterize yield risk in terms of four yield factors (dry matter feed conversion, average daily gain, mortality, and veterinary costs), which are highly correlated. Additionally, a conditional Tobit mode is used to handle censored yield variables (e.g., mortality). The proposed econometric model estimates parameters that influence the mean and variance of each production yield factor, as well as the covariance between variables. Following the model fitting using a maximum likelihood approach, simulation methods allow for profits, revenue, and gross margins to be evaluated given different assumptions concerning volatility among other shocks. The profit function is composed of random draws, based on conditioning variables, as well as parameter estimates. Shocks to variability, yield factors, or prices allow for a visual representation of the vulnerability of cattle feeder profits to these shocks.

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**1 Introduction**

Cattle feeding can be a risky venture. From the time of cattle placement to finishing, which usually lasts 3-5 months, the value and profitability of cattle can change immensely. Most of this risk comes in the form of fed and feeder cattle price risk, but can also come from large swings in feed prices. Both of these factors, which pose more than half of the variability in cattle feeder profits, are out of the cattle owners’ control. In addition, the overall productivity of the pen can present risks that are akin to yield risks with crops.

Research from the crop insurance literature has indicated that agricultural yields can be modeled in a number of different ways. These differ in the restrictions that are imposed on the data. For example, parametric methods assume a particular distributional assumption which is efficient when the form is correct, but biased when the assumption is incorrect. Different distributions have been argued to be the most accurate characterization of crop yields, which
include the normal, log-normal, beta, gamma, and weibull distributions to name a few.

In modeling the \textit{ex-ante} risks, variables known at the time of placement that may affect the expected mean or variance must be taken into account. Past cattle feeder research has shown that cattle feed conversion, average daily gains, mortality rates, and health are significantly affected by variables such as gender, location of the feedlot, average weight of the pen, and time of year the pen is placed. By conditioning on these variables, each pen of cattle can be modeled as a function using a multivariate regression model.

Cattle yields present some additional complexities when compared to crop yields. The first difference is that production risk can be represented by four separate measures. These four yield components are highly correlated and have a dynamic relationship. A recent study by Belasco et al. (2006) modeled all four yield measures separately then in a second step constructed the covariance matrix. Significant efficiency can potentially be gained in a multivariate framework, with the additional information that can be learned on the dynamic nature of the yields relationship.

The second complexity associated with cattle production yields is the introduction of a censored variable into the set of yield factors. More specifically, mortality rates can be modeled as a latent variable where the variable is observable for positive values and unobservable for small positive values of the distributional realizations. To account for this relationship, a dynamic multivariate Tobit model will serve to model the latent mortality variable.

Once the dynamic multivariate relationship is characterized with the previously mentioned, yield variables can be randomly drawn to simulate profits. The profit function will consist of the 6 previously mentioned random variables. In order to model profits, the random variables will need to be jointly modeled by allowing for covariance between the variables, particularly concerning the yield components. Once the profit model is identified and characterized, profit simulations will provide insights into the effects from shocks on profits and revenue.
Much of the past research in the area of cattle feeding risk has focused its attention to price risk management. Given that the majority of profit risk stems from price risk, this is not completely off the mark. However, we also find that production risk factors can play an important role in understanding overall profit risk. Studies investigating the risk factors associated with cattle feeding have keyed on the fact that risk comes from many different sources. To add to the complex nature of this risk, the variability can change as many key variables change.

One of the earlier studies focusing on cattle feeding profitability came from Swanson and West (1963). This study found that variation to returns are partially explained by price margin variation (38%) and feeding margin gain (44%), while 18% of the variation was unexplained. Schroeder et al. (1993) evaluated over 6,000 pens of steers from two major Kansas feedlots and concluded that 70 to 80 percent of the variation in cattle feeder profits came from variation in fed and feeder cattle prices, while the price of corn explained 6 to 16 percent of the variation, and cattle performance (which included average daily gain and feed efficiency) accounted for less than 10 percent. Including both steer and heifer pens in their sample, Langemeier, Schroeder and Mintert (1992) found that fed and feeder prices accounted for 50 and 25 percent of variability in cattle feeding profits, respectively. Meanwhile, corn price variability explained up to 22 percent of variability and animal performance explained less than 1 to 3.5 percent variability. This research also identified variables that affect the expected value and variability of profits. For example, 22 percent of the difference in steer and heifer profits are directly attributed to differences in feed conversion. It is important to point out that not only does profit variability come from a few different sources, but these impacts change with different pen characteristics.

With the two previously mentioned studies of cattle feeder variability using data from large Kansas feedlots, a study by Lawrence, Wang and Loy (1999) utilized data from smaller Midwest feedlots that span 5 states. With their data set consisting of 223 different feedlots and over 1,600 pens, fed and feeder cattle prices together still explained around 70 percent of profit variability. As expected, animal performance (average daily gain) played a larger role in profit
risk in explaining between 6 to 15 percent of the overall profit variation. The effects from corn variation fell below that of animal performance.

Mark, Schroeder and Jones (2000), analyzed over 14,000 pens from two major Kansas feedlots. This research indicates that variability can change for different values of placement weight, season, and gender. Additionally, Mark and Schroeder (2002) explicitly show that the season of placement has significant effects on profitability and animal performance.

With the previously mentioned studies in mind, we can point to a few facts that run throughout the literature. First, most cattle feeder profit risk stems from swings in fed and feeder cattle prices. With this being said, any risk management strategy must begin with managing the possibility of cattle prices dramatically dropping during the feeding period. Second, animal performance factors significantly contribute to risk and are the only identified sources of risk that the feeder can affect through operational and placement decisions. Third, variables such as gender, placement weight, and time of placement can have significant effects on expected profits and variability.

Therefore in order to fully understand profit risk, a clear understanding of production risk is needed. This research intends to model production risk to gain insights into the multidimensional relationships between cattle production yield factors. The next section develops a dynamic, multivariate Tobit model to explore the relationships that conditioning variables have on the mean, variance, and covariance of the yield factors. The following two sections focus on the data used for estimation as well as the estimation results. Using this information, the final section jointly models price and yield risk factors to create an overall profit risk model for fed cattle production.

2 Yield Modeling Framework

The variables that introduce production yield risk into the cattle feeders’ profit function, are dry matter feed conversion (DMFC), average daily gain (ADG), the mortality rate (MORT),
and veterinary cost per head (VCPH). As seen in past studies, these variables are influenced by pen characteristics such as gender, location, average in-weight, and season of placement. These factors affect both the expected value and variance for each yield measure. Additionally, there are significant correlations between each yield measure and the conditioning variables. The modeling strategy to follow will account for each of these complexities and characterize the probabilistic models of the cattle yield factors involved with fed cattle production. Additionally, this model is extended to account for correlation between yields as a multivariate normal is used, rather than the uncorrelated univariate normals that was originally proposed by Belasco et al. (2006).

The model is specified as follows:

\[ Y_i = X_iB + \varepsilon_i \]  
\[ E[\varepsilon_i|X_i] = 0 \]  
\[ Var[\varepsilon_i|X_i] = \Sigma_i = \Sigma(X_i) \]

where \( Y_i = [DMFC_i, ADG_i, MORT_i, VCPH_i] \) and \( B \) is a px4 matrix containing the marginal effects of each conditioning variable on all four yield factors. \( X_i \) is a 1xp matrix containing the following conditioning variable values for each observation that include a constant term, the log of average in weight, and binary variables indicating gender, location, and season of placement.

\[ X'_i = \begin{bmatrix} 1 \\ Steers \\ Mixed \\ KS \\ Log(Inwt) \\ Winter \\ Fall \\ Spring \end{bmatrix} \]
\( \Sigma_i \) contains the variance and covariance elements and is a 4x4 positive definite (p.d) matrix. Notice that the covariance matrix is allowed to vary by observation. We propose to model \( \Sigma_i = \Sigma(X_i) \) using the following unique decomposition of a p.d. matrix (Lau, 1978):

\[
\Sigma_i = T_i' D_i T_i
\]  

(5)

where \( T_i \) is upper triangular with ones along the main diagonal and \( D_i \) is diagonal matrix with positive diagonal entries. More specifically,

\[
\begin{align*}
T_i &= \begin{pmatrix}
1 & t_{12i} & t_{13i} & t_{14i} \\
0 & 1 & t_{23i} & t_{24i} \\
0 & 0 & 1 & t_{34i} \\
0 & 0 & 0 & 1
\end{pmatrix} \\
D_i &= \begin{pmatrix}
d_{1i} & 0 & 0 & 0 \\
0 & d_{2i} & 0 & 0 \\
0 & 0 & d_{3i} & 0 \\
0 & 0 & 0 & d_{4i}
\end{pmatrix}
\end{align*}
\]  

(6) (7)

Upper off-diagonal elements of \( T_i \) are unrestricted while the diagonal elements of \( D_i \) are restricted to non-negative values. We use a linear regression to model \( T_i \) and \( D_i \) as follows:

\[
\log \begin{pmatrix}
d_{1i} \\
d_{2i} \\
d_{3i} \\
d_{4i}
\end{pmatrix} = GX_i'
\]  

(8)

where the 4xp matrix, G, is used to calculate the variance for each observation, conditional on the unique characteristics of each pen. The covariance terms are also a linear function of the
conditioning variables and have the following relationship

\[
\begin{pmatrix}
    t_{12i} \\
    t_{13i} \\
    t_{14i} \\
    t_{23i} \\
    t_{24i} \\
    t_{34i}
\end{pmatrix} = AX_i'
\]

(9)

where \( A \) is a 6xp matrix of regression coefficients. Based on equations (8) and (9), the covariance terms are first fully flexible within the regression framework. This model is an improvement from the two-step method used in Belasco et al. (2006) as it reduces to the earlier model when the elements of \( A = 0 \).

The implied maximum likelihood method to obtain parameter estimates uses the following likelihood function.

\[
L(B, A, G|Y, X) = \prod_{i=1}^{n} |\Sigma_i|^{-\frac{n}{2}} \exp \left( -\frac{1}{2} \sum_{i=1}^{n} (Y_i - X_iB) \Sigma_i^{-1}(Y_i - X_iB)' \right)
\]

(10)

Given \( n \) observations, this leads to the following negative log likelihood function (up to an additive constant) that is minimized with respect to elements within \( B, G, \) and \( A \).

\[
LL = \frac{n}{2} \ln |\Sigma_i| + \frac{1}{2} \sum_{i=1}^{n} (Y_i - X_iB)' \Sigma_i^{-1}(Y_i - X_iB)
\]

(11)

By modeling yields in the preceding manner, the model is flexible enough to allow for expected values, variances, and covariances between the yields to vary with the conditioning variables. These are key components to modeling the nature of risk in fed cattle production where the expected distributional properties can change, given the characteristics of the pen.

Thus far, the model assumes that all variables are completely observable across observa-
tions and free from any censoring or truncation bias. However, in the case of mortality rates, these values are censored at zero. This problem may cause us to underestimate mortality rates due to biased parameter estimates caused by the censoring mechanism (Greene, 2003). The most widely accepted solution to regressing censored dependant variables in the univariate case was first proposed by Tobin (1958) and is known as the Tobit Model. This method essentially assumes there is a latent variable, \( y_i^* \), which linearly depends on the associated independent variables, \( X_i \), where we only observe

\[
y_i = \begin{cases} 
0 & \text{if } y_i^* \leq c_i \\
y_i^* & \text{if } y_i^* > c_i 
\end{cases}
\]

where \( c_i \) is the unknown censored value. Notice that when \( y_i \) is censored at zero, the only information we have is that \( y_i > 0 \). So the likelihood function becomes

\[
L(\beta, \sigma^2|x_i) = \prod_{i:y_i>0} \phi \left( \frac{y_i - x_i \beta}{\sigma} \right) \frac{1}{\sigma} \prod_{i:y_i=0} \Phi \left( -\frac{x_i \beta}{\sigma} \right)
\]

where \( \Phi \) is the CDF of a standard normal. The use of maximum likelihood estimation has been shown to result in estimators that are consistent and asymptotically normal (Amemiya, 1973), provided the assumed parametric model is correct. This method has been useful in applications spanning consumption, production, and income.

Censored multivariate regressions have been extended and shown to possess the same attractive asymptotic properties as in the univariate case (Amemiya, 1974; Lee, 1993). The multivariate Tobit model has been considered in a number of recent studies. Cornick, Cox and Gould (1994) formulate a multivariate Tobit model in order to analyze fluid milk consumption expenditures and account for the correlations across milk types. Eiswerth and Shonkwiler (2006) investigate the success of plant seeding that follows wildfire on arid rangeland, where all types of grass do not typically grow together simultaneously due to geographical differences. Also, Chavas and Kim (2004) use a dynamic multivariate Tobit model to evaluate price dynamics when price floors exist in a given market. The dynamic component plays an important role in
this analysis as the data is evaluated over time, where the correlations between prices adjust over different time periods. While the covariance matrix changes over time, it is held constant for differing values of the other conditioning variables. Here, we expand on these studies by allowing for the interdependence between the dependent variables to be a function of the data. The idea is to model the latent variables through the use of a multivariate Tobit model, using a dynamic multivariate sampling distribution under conditional heteroskedasticity while allowing for interdependence between the residuals.

In the univariate case, each observation can fall into one of two possible regimes where the dependant variable is either censored or not. However, within the framework of a generalized multivariate Tobit model, the possible censoring regimes increase to $2^m$ where $m$ is the number of censored dependent variables. For our purposes, four dependent variables lead to 16 possible regimes. Due to the fact that only one variable is censored, only two regimes are possible. For observations with multiple censored dependent variables, integration becomes more complex by adding a dimension for each censored variable. As long as this dimension is not greater than three, standard maximum likelihood methods can be used (Chavas and Kim, 2004).

To obtain the likelihood function, each observation must be ordered as censored or non-censored variables for each regime. To this end, $Y_i$ will be partitioned into its censored variables, $y_i^1$, and uncensored variables, $y_i^2$, under each regime:

$$
Y_i' = \begin{pmatrix} y_i^1 \\ y_i^2 \end{pmatrix}
$$

Further, the sample log-likelihood function corresponding to each individual, can be expressed as follows:

$$
LL = \sum_{i: Mort>0} \left\{ ln \left[ \phi(Y_i, \mu_i, \Sigma_i) \right] \right\} + \sum_{i: Mort=0} \left\{ ln \left[ \phi(y_i^2, \mu_i^2, \Sigma_{22i}) \right] + ln \left[ \Phi(\mu_i^1, \Sigma_i^1) \right] \right\}
$$

where $\phi_i(\cdot)$ refers to the multivariate normal probability density function over uncensored prices.
under the \( i \)th regimes, while \( \Phi_i(\cdot) \) is the multivariate cumulative distribution function over censored prices within the same regime. The censored variable is modeled based on a multivariate normal density and is a function of the observable variables within the same observation. For this reason, the conditional mean and variance for \( y_1^i \) given \( y_2^i \) are respectively:

\[
\mu_1^i \left( y_2^i \right) = E \left( y_1^i \right) + \Sigma_{12i} \Sigma_{22i}^{-1} \left( y_2^i - E(y_2^i) \right) \tag{16}
\]

\[
\Sigma_i^1 = \Sigma_{11i} - \Sigma_{12i} \Sigma_{22i}^{-1} \Sigma_{21i} \tag{17}
\]

where \( \Sigma \) can be decomposed into the following components where \( \Sigma_{22} \) contains the elements from variables with no censoring across observations, \( \Sigma_{12} \) relates the variables with and without censoring, and \( \Sigma_{11} \) contains elements from censored observations

\[
\Sigma = \begin{pmatrix}
\Sigma_{11} & \Sigma_{12} \\
\Sigma_{21} & \Sigma_{22}
\end{pmatrix}
\tag{18}
\]

This illustrates the major difference between the univariate and multivariate Tobit models, in that the expected mean and variance are a function of the other observed dependent variables. Following is the negative sample log likelihood function used for estimation.

\[
LL = \sum_{i: \text{Mort}_i > 0} \left\{ \ln \left( |\Sigma_i^{-1}| \right) + (Y_i - X_i B) \Sigma_i^{-1} (Y_i - X_i B)' \right\} + \\
\sum_{i: \text{Mort}_i = 0} \ln \left( |\Sigma_{22i}^{-1}| \right) + (y_2^i - X_2^i B_2) \Sigma_{22i}^{-1} \left( y_2^i - X_2^i B_2 \right)' + \\
2 \ln \left( \Phi \left[ -(\Sigma_i^1)^{-\frac{1}{2}} \mu_i^1 \right] \right) \tag{19}
\]

where \( B \) can be broken into two components containing the parameter estimates for the censored variable (e.g., \( \text{MORT} \)), \( B_1 \), and the parameter estimates for the uncensored variables (e.g., \( \text{DMFC}, \text{ADG}, \text{and VCPH} \)), \( B_2 \).


## 3 Data Description

This empirical analysis is applied to a comprehensive set of data collected from five commercial cattle feedlots located in Kansas and Nebraska. Proprietary production and cost data were obtained for 11,397 pens of cattle from 1995 to 2004. Table 1 contains summary statistics from the data sample.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Mean</th>
<th>Std</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMFC</td>
<td>Dry matter feed conversion (lbs feed / lbs gain)</td>
<td>6.19</td>
<td>0.72</td>
<td>4.00</td>
<td>24.00</td>
</tr>
<tr>
<td>ADG</td>
<td>Average Daily Gain (lbs gain / day)</td>
<td>3.36</td>
<td>0.48</td>
<td>0.74</td>
<td>5.78</td>
</tr>
<tr>
<td>VCPH</td>
<td>Veterinary cost per head ($)</td>
<td>11.83</td>
<td>6.25</td>
<td>0.00</td>
<td>60.00</td>
</tr>
<tr>
<td>MORT</td>
<td>Percentage of pen that die</td>
<td>0.93</td>
<td>1.53</td>
<td>0.00</td>
<td>25.83</td>
</tr>
<tr>
<td>InWeight</td>
<td>Average weight per head of cattle upon entrance (lbs)</td>
<td>737.50</td>
<td>87.22</td>
<td>500.00</td>
<td>900.00</td>
</tr>
<tr>
<td>OutWeight</td>
<td>Average weight per head of cattle upon exit (lbs)</td>
<td>1,177.91</td>
<td>88.10</td>
<td>910.00</td>
<td>1,472.00</td>
</tr>
<tr>
<td>Winter</td>
<td>Binary variable equal to 1 if entry between Dec-Feb</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spring</td>
<td>Binary variable equal to 1 if entry between Mar-May</td>
<td>0.23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Summer</td>
<td>Binary variable equal to 1 if entry between Jun-Aug</td>
<td>0.26</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fall</td>
<td>Binary variable equal to 1 if entry between Sep-Nov</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steers</td>
<td>Binary variable equal to 1 if entire pen were Steers</td>
<td>0.51</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heifers</td>
<td>Binary variable equal to 1 if entire pen were Heifers</td>
<td>0.37</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steers</td>
<td>Binary variable equal to 1 if pen was mixed gender</td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KS</td>
<td>Binary variable equal to 1 if Kansas feedlot location</td>
<td>0.80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NE</td>
<td>Binary variable equal to 1 if Nebraska feedlot location</td>
<td>0.20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total sample size n=11,397 pens of cattle

Dry Matter Feed Conversion (DMFC) measures the pounds of dry feed required per pound of live weight gain. To compute the average DMFC for a given pen, total dry feed
consumed is divided by the total weight gained during the feeding cycle. Average daily gain (ADG) captures the average pounds the cattle gain throughout the feeding period. Veterinary costs per head (VCPH) are calculated by dividing the total dollar amount spent on veterinary services by the pen size upon entry. The mortality rate (MORT) is a percentage calculated as the number of death losses during the feeding period divided by the number of head initially placed on feed. According to Smith (1998), cattle mortalities in feedlot settings come mostly from respiratory diseases and digestive disorders. Also a great majority of health problems occur in the first 90 days cattle are on feed.

The size of a pen of cattle averaged 134 head with an average placement weight of 737.5 pounds and an average finished weight of 1,178 pounds. InWeight is measured as the average weight per head in each pen upon placement on feed.\(^1\) The log of InWeight is used as a conditioning variable. To capture seasonal effects, binary variables are constructed to denote Winter, Spring, Summer, and Fall seasons. In this data, fall placements tend to be lighter than any other season, while spring placements tend to bring heavier weights.

On average, pens spent 129 days on feed, meaning most pens are fed throughout more than one season. For example, a pen placed in the pleasant fall months will likely be on feed while temperatures drop towards colder winter averages. Binary variables are also used to differentiate pens by gender (Steers, Heifers, and Mixed) where pens comprised of all steers make up more than half of the data sample. Steers are more often used for feeding due to the faster pace with which they put on weight, whereas heifers put on weight slower, max out at a lower weight, and need to be used for reproduction.

Binary variables are also used to differentiate feedlot location by state, which include Kansas and Nebraska. Feedlot locations could additionally be split by feedlot, however within Nebraska and Kansas each feedlot does not appear to be significantly different. The two Kansas feedlots are relatively larger than the Nebraska feedlots. Additionally, the Nebraska feedlots keep their pens for more days on feed, resulting in lower DMFC and wider weight swings.

\(^1\)Pens with average placement weights below 500 pounds and above 900 pounds were excluded from our sample.
Histograms of the dependent variables and entry weight are shown in Figure 2. Here the positively skewed nature of DMFC, VCPH, and MORT and quite apparent. For this reason the log of DMFC and VCPH is taken in order to symmetrize the variables. Unfortunately, there is no mechanism to take the log of mortality rates since so many observations are zero. Alternatively, ADG is already distributed similar to a normal distribution centered at 3.4. These histograms also illustrate the importance in recognizing the yield factors are not fixed and should be thought of as components of risk in order to more accurately describe fed cattle production. Additionally, Figure 3 illustrates the need to model the production yield factors in a way that accurately captures the covariance structure.

4 Estimation Results

This particular system of equations must be estimated by taking into account the finding that each conditioning variable has an effect on the four cattle production yield measures and the dependent variables are also highly correlated. Due to this, it is necessary to discuss the results from each conditioning variable in the context of all yield measures. These results are displayed in Table 2. This section begins with an interpretation of the parameter estimates on both the mean and variance components of the system. The next section deals primarily with the covariance parameter estimates.

4.1 Performance Effects From Gender

Gender differences are known to play a large role in cattle feedlot performance. Steer cattle are known to gain weight at a much faster rate than heifer cattle and are commonly marketed at a higher weight. Additionally, some heifers need to be used to stock new generations of cattle. For these reasons, steer cattle are more prevalent in feedlots than their heifer counterparts. This is also true within the given data set where steer pens compose 51% of the pens placed on feed.
## Table 2: Maximum likelihood parameter estimates

<table>
<thead>
<tr>
<th>Variables</th>
<th>Dry Matter</th>
<th>Average Daily</th>
<th>Mortality</th>
<th>Vet Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Feed Conversion</td>
<td>Gain</td>
<td>Coefficient</td>
<td>Standard Error</td>
</tr>
<tr>
<td>Intercept:</td>
<td>0.675*</td>
<td>0.046</td>
<td>-3.445*</td>
<td>0.213</td>
</tr>
<tr>
<td>Steers:</td>
<td>-0.069*</td>
<td>0.002</td>
<td>0.300*</td>
<td>0.008</td>
</tr>
<tr>
<td>Mixed:</td>
<td>-0.027*</td>
<td>0.003</td>
<td>0.128*</td>
<td>0.013</td>
</tr>
<tr>
<td>Kansas:</td>
<td>-0.123*</td>
<td>0.002</td>
<td>0.169*</td>
<td>0.010</td>
</tr>
<tr>
<td>Log(inwt):</td>
<td>0.193*</td>
<td>0.007</td>
<td>1.002*</td>
<td>0.033</td>
</tr>
<tr>
<td>Winter:</td>
<td>-0.003</td>
<td>0.002</td>
<td>-0.171*</td>
<td>0.010</td>
</tr>
<tr>
<td>Fall:</td>
<td>0.050*</td>
<td>0.002</td>
<td>-0.221*</td>
<td>0.011</td>
</tr>
<tr>
<td>Spring:</td>
<td>-0.018*</td>
<td>0.002</td>
<td>-0.041*</td>
<td>0.010</td>
</tr>
</tbody>
</table>

**Heteroskedasticity:**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept:</td>
<td>-9.067*</td>
<td>0.739</td>
<td>-8.804*</td>
<td>0.723</td>
<td>12.716*</td>
<td>0.886</td>
</tr>
<tr>
<td>Steers:</td>
<td>-0.060</td>
<td>0.030</td>
<td>0.058</td>
<td>0.030</td>
<td>-0.042</td>
<td>0.038</td>
</tr>
<tr>
<td>Mixed:</td>
<td>0.481*</td>
<td>0.044</td>
<td>0.143*</td>
<td>0.044</td>
<td>0.583*</td>
<td>0.055</td>
</tr>
<tr>
<td>Kansas:</td>
<td>-0.127*</td>
<td>0.034</td>
<td>-0.038</td>
<td>0.034</td>
<td>0.133*</td>
<td>0.044</td>
</tr>
<tr>
<td>Log(inwt):</td>
<td>0.646*</td>
<td>0.113</td>
<td>0.890*</td>
<td>0.110</td>
<td>-1.760*</td>
<td>0.136</td>
</tr>
<tr>
<td>Winter:</td>
<td>0.013</td>
<td>0.037</td>
<td>0.085</td>
<td>0.037</td>
<td>-0.109</td>
<td>0.048</td>
</tr>
<tr>
<td>Fall:</td>
<td>0.356*</td>
<td>0.037</td>
<td>0.186*</td>
<td>0.037</td>
<td>0.312*</td>
<td>0.047</td>
</tr>
<tr>
<td>Spring:</td>
<td>-0.351*</td>
<td>0.038</td>
<td>0.127*</td>
<td>0.038</td>
<td>-0.117</td>
<td>0.052</td>
</tr>
</tbody>
</table>

*Denotes the estimate is statistically significant at the 0.05 level
To assist in capturing the effect that gender has on production, pens were identified as entirely steer, entirely heifer, or some mixture of the two. For estimation purposes, binary variables were developed for each type of pen. Results shown in Table 2 are relative to heifer pens. Not surprisingly, both steer and mixed pens have lower feed conversion rates and higher rates of average daily gain. More specifically, pounds of feed are converted into pounds of weight gain more efficiently by 6.9% and 2.7% for steer and mixed pens, respectively. This superior ability to convert feed into weight gain directly assists in the higher rates of ADG for steer and mixed pens, relative to heifer pens. The data suggests that steer pens gain weight faster than heifer pens by 0.30 pounds per day.\(^2\) Results from Mark, Schroeder and Jones (2000) indicate that steer pens had similar performance advantages with a feed conversion that was 4% lower than heifer pens, while gaining an average of 0.34 pounds more per day.

While ADG and DMFC results make steer pens more desirable than heifer pens, the results from the regression equations for MORT and VCPH indicate that steer pens are inferior to heifer pens in general health measures. The percentage of mortality losses while on feed are higher for steer and mixed pens by 0.10 and 0.32, respectively.\(^3\) Given the higher mortality rates for steer and mixed pens, it is not surprising that veterinary costs are higher for these types of pens.\(^4\) Steer pens incur 6.5% higher veterinary costs than heifer pens, while mixed pens are quite expensive with 21% higher veterinary costs.

The heteroskedasticity parameter estimates offer insight into the influence conditioning variables have on the variance. A parameter estimate that is positive indicates an increasing effect on the variance. To evaluate the effect that a binary variable, say \(x_k\), has on the conditional

\(^2\)While 0.30 pounds per day may appear to be a small gain, it amounts to 39 extra pounds over 130 days on feed. This amounts to a 3% gain in out weight over the average heifer pen.

\(^3\)To interpret the marginal effects within the Tobit model, MLEs must be multiplied by the proportion of non-censored observations in the sample (Greene, 2003, pg. 766), which is 53.404% within the data.

\(^4\)Veterinary costs at cattle feedlots can be incurred due to precautionary checks, which are typically performed at the beginning of the feeding cycle and consist of vaccinations and health checks, and visits due to deteriorating health. While this data does not distinguish between the two, it is assumed that all feedlots incur similar expenses for precautionary visits, so that any variation in veterinary costs can be linked to the health of the pen.
variance for a given observation can be illustrated as follows:

\[
\sigma^2_{i|x_k=1} - \sigma^2_{i|x_k=0} = \exp(x_1\gamma_1 + \ldots + x_{k-1}\gamma_{k-1}) \times [\exp(\gamma_k) - 1]
\]  

(20)

It is evident from this equation that variance increases when \(\gamma_k > 0\) and decreases when \(\gamma_k < 0\). While steer pens do not differ significantly with variance, mixed pens bring on higher variance parameters for most variables, with the exception of veterinary costs.

4.2 Performance Effects From Location

As previously mentioned, the data contain results from five major cattle feedlots where two reside in Kansas and three in Nebraska. Differences in location are identified with binary variables indicating the state of residences. The main reason for this distinction is due to the geographic closeness of the feedlots within the same state and the similar management practices discussed earlier.\(^5\) The binary location variable is then intended to control for any differences due to different weather systems as well as different management practices.\(^6\)

One of the most distinguishing characteristics of the data is the higher entry weights and lower days on feed associated with feedlots residing in Kansas. This may be due to the practice of backgrounding on Kansas feedlots. According to Neville and McCormick (1981), calves that are weaned at an early age and well-fed need less time at the feedlot. The other more obvious reason is that cattle with higher placement weights need less time to reach the desire marketing weight. This finding appears consistent within our data where the Kansas lots have their cattle backgrounded to prepare them for the diet at feedlots. The data indicate that DMFC is 12\% lower and ADG is higher by 0.17 in Kansas feedlots. Cattle feeding in Kansas feedlots do not

\(^5\)Initially binary variables were used to distinguish between each feedlot until it was found that significant differences between the feedlots can be found by differentiating by state, since feedlots were typically not significantly different relative to feedlots within similar states.

\(^6\)Management practices within this data do not represent state-wide practices. In 1996, there was an estimated 670 feedlots with a 1,000+ capacity within the state of Nebraska (NASS, 1997, pg. 109). While fewer feedlots reside within Kansas, a higher proportion of the feedlots are operations with capacities over 32,000 head. (NASS, n.d.).
have a significantly different rate of mortality, however veterinary costs are lower due to the less
days on feed. Vet costs per head per day, which can be computed by dividing VCPH by days 
on feed, are roughly similar for each state at $0.09. Kansas feedlots within this data sample 
have mixed influences on variance for each dependent variable.

4.3 Performance Effects From Entry Weight

Entry weight is the only quantitative conditioning variable. This allows parameter estimates 
to be interpreted as elasticities for logged dependent variable regressions. The coefficient from 
the regression on DMFC implies that a 10% increase in entry weight corresponds to an increase 
in feed conversion by 1.9%. Similar results have been concluded by past studies (Mark et al., 
2000; Schroeder et al., 1993). Increases in entry weight by 10% lead to an increase in ADG by 
0.10. The increase in daily gain for heavier pens of cattle is partly due to the less time these 
types of pens spend on the feedlot. Higher feed conversion and daily gains imply an increase in 
intake for heavier placed cattle.\(^7\)

Pens of cattle that are more mature in age and weight tend to have less health problems. 
This is shown by the results contained within regression results for MORT and VCPH. Mortality 
rates fall significantly for heavier weights as a one percentage point increase in placement weight 
is associated with a decrease in the rate of mortality by 0.04 percentage points. Intake per head 
per day increases steadily with entry weight from 18.03 pounds for the smallest weight class 
(500 - 600 lbs) to 22.00 for the highest weight class (800 - 900 pounds). These results are shown 
in Table 3. Additionally, feed conversion does not appear to increase linearly as it is maximized 
for extremely low and high entry weight classes.

Different weight classes also appear to strongly effect other characteristics such as time 
of placement and gender. Heavier placements appear to be dominated by steer pens, while the 
lighter placements comprise mostly of heifer pens. More than 50% of the pen placements with 
average weight below 700 pounds are heifers, while 74% of the heaviest placements (800-900

\(^7\) Intake (lbs of feed / day) = DMFC (lbs feed / lbs gain) \times ADG (lbs gain / day).
Table 3: Comparison of Different Weight Classes

<table>
<thead>
<tr>
<th>Variable</th>
<th>500-600</th>
<th>600-700</th>
<th>700-800</th>
<th>800-900</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>815</td>
<td>2,876</td>
<td>4,545</td>
<td>3,061</td>
</tr>
<tr>
<td>DMFC</td>
<td>6.22</td>
<td>6.13</td>
<td>6.19</td>
<td>6.25</td>
</tr>
<tr>
<td>ADG</td>
<td>2.93</td>
<td>3.19</td>
<td>3.40</td>
<td>3.56</td>
</tr>
<tr>
<td>Intake</td>
<td>18.03</td>
<td>19.30</td>
<td>20.81</td>
<td>22.00</td>
</tr>
<tr>
<td>VCPH</td>
<td>17.79</td>
<td>13.86</td>
<td>10.91</td>
<td>9.73</td>
</tr>
<tr>
<td>MORT</td>
<td>1.97</td>
<td>1.22</td>
<td>0.79</td>
<td>0.61</td>
</tr>
<tr>
<td>InWt</td>
<td>561.09</td>
<td>656.93</td>
<td>749.90</td>
<td>841.34</td>
</tr>
<tr>
<td>OutWt</td>
<td>1,091.14</td>
<td>1,124.00</td>
<td>1,181.86</td>
<td>1,245.66</td>
</tr>
<tr>
<td>Days on Feed</td>
<td>176.23</td>
<td>143.01</td>
<td>123.78</td>
<td>110.55</td>
</tr>
</tbody>
</table>

Proportion of sample:

- Winter: 0.26, 0.28, 0.26, 0.22
- Spring: 0.10, 0.17, 0.25, 0.29
- Summer: 0.25, 0.25, 0.27, 0.28
- Fall: 0.39, 0.31, 0.22, 0.21
- Steers: 0.28, 0.35, 0.50, 0.74
- Heifers: 0.52, 0.50, 0.38, 0.19
- Mixed: 0.21, 0.15, 0.12, 0.07
- KS: 0.62, 0.80, 0.84, 0.81

pounds) consists of steer pens. Also, lighter pens are introduced more typically in the fall months and rarely in spring months. Pens on the heavier side (>700 pounds) are mostly placed in spring or summer months.

From equation (8), an elasticity relating entry weight and conditional variance has the following form:

$$\frac{\partial \sigma^2_{ij}}{\partial \log(\text{weight})} \frac{1}{\sigma^2_{ij}} = \gamma_j$$

(21)

This relationship allows for a direct interpretation of the parameter estimate for entry weight. For example, a one percentage point increase in placement weight coincides with a 0.65% increase in feed conversion variance and a 0.89% increase in daily gain variance. Conversely, entry weight has a diminishing effect on the variability of mortality and veterinary costs. Many of these results may be tied to the fact that cattle placed at heavier weights spend less time on feed and have less time to deviate from expectations.
4.4 Performance Effects From Placement Season

Changes in temperature can have dramatic changes in cattle feedlot performance. Mark and Schroeder (2002) point out that optimal cattle performance typically occurs between 40 to 60 degrees. Deviations from this range, as well as variability in weather or precipitation, can lead to lower performance. Higher temperatures often result in less weight gain due to lower rates of consumption, while colder temperatures can lead to less efficient feeding as energy is used to maintain body heat. Feeding usually lasts anywhere from 3 to 5 months, meaning most pens will enter in one season and leave in another.

Parameter estimates for the DMFC regression infer that fall placements have the highest feed conversion rate and are significantly different from summer. This is not surprising given the fact that pens placed in the fall months are fed as the temperatures drop, so that the coldest months are likely near the end of their feeding cycle. A binary variable indicating summer placements is left out of the regression so that parameter estimates are relative to that season. The feed conversion rate for spring placements are significantly lower than summer placements, while winter placements are not significantly different. All seasons experienced significantly lower gains on a daily basis, relative to summer.

Pens placed in the spring months appear to have the fewest health problems as indicated by the significant negative parameter estimates in both mortality and vet cost regressions. Fall placements are not statistically different from summer concerning the mortality rate, while winter placements incur fewer veterinary costs.

4.5 Conditioning Variable Effects on Covariance Terms

One major benefit of the large set of data available for this research is the chance to allow covariance terms to be a function of the data. In a recent study by Belasco et al. (2006) covariance terms in this system of equations were assumed to be constant for all observations. However, OLS regressions indicated that the cross product residuals were correlated with the
conditioning variables. The individual-specific covariance matrix as defined in equation (5) allows for the added flexibility in the off-diagonal elements in equation (6) to be a function of the data.

Not all variables are expected to be highly correlated with one another, however one can make a strong case for a few relationships to be strongly correlated. For example, feed conversion rates and rates of average daily gain certainly complement one another, while veterinary costs and mortality rates can both arise with unhealthy or sick pens. Each of these examples are shown to have almost all conditioning variables significantly effecting the level of covariance as seen in Table 4.

Recall, covariance elements are contained in the matrix identified in equation (6). The relationship between the covariance estimate and the data is shown in equation (9). Based on this model, covariance estimates are linearly determined by the conditioning variables and parameter estimates from matrix A. For example, the covariance between DMFC and ADG for any individual is estimated through the following equation:

$$t_{12i} = \alpha_1 X_i^T$$  \hspace{1cm} (22)

where $\alpha_1$ is the first row of $A$ and is a $1 \times 8$ matrix containing the parameter estimates for the covariance level, based on the full sample. This form allows the unique characteristics of each pen to imply a different set of covariance parameters. The covariance terms account for effects that concurrently effect the cattle production yields. The high frequency of significant variables indicate the importance of including this flexibility.

To understand the correlation structure, a correlation matrix can be computed based on

---

8One way to test for heteroskedasticity is to regress the squared residual on the variables as defined by the White test (Greene, 2003). If variables are found to significantly effect an error term that is assumed to be independent, then heteroskedasticity must be controlled for. Alternatively, one may also take the cross product of residuals from a system of equations to determine if the covariance terms are in fact independent. This was the preliminary strategy which led to the finding that covariance terms were significantly effected by the conditioning variables.
Table 4: Maximum likelihood covariance parameter estimates

<table>
<thead>
<tr>
<th>Variables</th>
<th>Gain</th>
<th>Mortality</th>
<th>Vet Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>coeff.</td>
<td>se</td>
<td>coeff.</td>
</tr>
<tr>
<td>Covariance with DMFC:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept:</td>
<td>0.119</td>
<td>1.391</td>
<td>0.367</td>
</tr>
<tr>
<td>Steers:</td>
<td>-0.477*</td>
<td>0.058</td>
<td>-0.099</td>
</tr>
<tr>
<td>Mixed:</td>
<td>0.166*</td>
<td>0.076</td>
<td>0.170</td>
</tr>
<tr>
<td>Kansas:</td>
<td>0.166*</td>
<td>0.065</td>
<td>1.605*</td>
</tr>
<tr>
<td>Log(inwt):</td>
<td>-0.603*</td>
<td>0.213</td>
<td>0.909</td>
</tr>
<tr>
<td>Winter:</td>
<td>0.490*</td>
<td>0.070</td>
<td>0.413</td>
</tr>
<tr>
<td>Fall:</td>
<td>0.224*</td>
<td>0.067</td>
<td>-0.380</td>
</tr>
<tr>
<td>Spring:</td>
<td>0.136</td>
<td>0.083</td>
<td>1.281</td>
</tr>
<tr>
<td>Covariance with ADG:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept:</td>
<td>3.273</td>
<td>4.712</td>
<td>-0.930</td>
</tr>
<tr>
<td>Steers:</td>
<td>-0.452*</td>
<td>0.174</td>
<td>0.013</td>
</tr>
<tr>
<td>Mixed:</td>
<td>-0.509</td>
<td>0.320</td>
<td>-0.039</td>
</tr>
<tr>
<td>Kansas:</td>
<td>-0.020</td>
<td>0.187</td>
<td>-0.132*</td>
</tr>
<tr>
<td>Log(inwt):</td>
<td>-0.543</td>
<td>0.713</td>
<td>0.112</td>
</tr>
<tr>
<td>Winter:</td>
<td>-0.088</td>
<td>0.215</td>
<td>0.404*</td>
</tr>
<tr>
<td>Fall:</td>
<td>0.213</td>
<td>0.236</td>
<td>0.094*</td>
</tr>
<tr>
<td>Spring:</td>
<td>-0.156</td>
<td>0.216</td>
<td>0.184*</td>
</tr>
<tr>
<td>Covariance with MORT:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept:</td>
<td></td>
<td></td>
<td>0.701*</td>
</tr>
<tr>
<td>Steers:</td>
<td></td>
<td></td>
<td>-0.028*</td>
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<td></td>
<td>-0.010</td>
</tr>
<tr>
<td>Kansas:</td>
<td></td>
<td></td>
<td>0.023*</td>
</tr>
<tr>
<td>Inwtlog:</td>
<td></td>
<td></td>
<td>-0.095*</td>
</tr>
<tr>
<td>Winter:</td>
<td></td>
<td></td>
<td>-0.004</td>
</tr>
<tr>
<td>Fall:</td>
<td></td>
<td></td>
<td>0.005</td>
</tr>
<tr>
<td>Spring:</td>
<td></td>
<td></td>
<td>0.015*</td>
</tr>
</tbody>
</table>

*Denotes the estimate is statistically significant at the 0.05 level
the following relationship and evaluated at the means:

$$\rho(e_i, e_j) = \frac{\text{Cov}(e_i, e_j)}{\sqrt{\text{var}(e_i)\text{var}(e_j)}}$$  \hspace{1cm} (23)$$

where the covariance terms are the off diagonal hessian elements and the variance terms are along the diagonal for the respective variables. The resulting correlation matrix is shown below in Table 5 to illustrate this correlation structure.

<table>
<thead>
<tr>
<th>Variable</th>
<th>DMFC</th>
<th>ADG</th>
<th>MORT</th>
<th>VCPH</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMFC</td>
<td>1.000</td>
<td>-0.801</td>
<td>0.341</td>
<td>0.026</td>
</tr>
<tr>
<td>ADG</td>
<td>1.000</td>
<td>-0.319</td>
<td>-0.064</td>
<td></td>
</tr>
<tr>
<td>MORT</td>
<td>1.000</td>
<td></td>
<td>0.363</td>
<td></td>
</tr>
<tr>
<td>VCPH</td>
<td></td>
<td></td>
<td></td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 5: Correlation matrix relationship evaluated at the means

It is no surprise to see high levels of correlation between vet costs / mortality rates and feed conversion / average daily gain for the reasons stated earlier. There also exists a high degree of positive correlation between feed conversion and mortality rates. This can be explained by the higher feed conversion rates that come from unhealthy cattle, while the healthy cattle are more efficient at gaining weight. Almost all correlation terms are above 20%, with the exception of VCPH with DMFC and ADG.

Due to the heterogeneous nature of the data, a unique covariance matrix corresponds to every unique set of variables, implying a unique correlation matrix. To illustrate this fact, two hypothetical pens are chosen at approximately one standard deviation from the mean entry weight. The results below in Table 6 demonstrate that these two observations have a distinct correlation structure.

Pen A corresponds to a pen that is placed into a Kansas feedlot in the fall, comprised fully of steers. Conversely, Pen B corresponds to a pen that is comprised of heifers and was placed into a Kansas feedlot during the summer months. Three major differences between these observations include the dramatic difference in weight, as well as different placement months.
Table 6: Comparison of correlation matrices for two separate pens

<table>
<thead>
<tr>
<th>Variable</th>
<th>Pen A&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Pen B&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DMFC</td>
<td>ADG</td>
</tr>
<tr>
<td>DMFC</td>
<td>1.000</td>
<td>-0.834</td>
</tr>
<tr>
<td>ADG</td>
<td>1.000</td>
<td>-0.263</td>
</tr>
<tr>
<td>MORT</td>
<td>1.000</td>
<td>0.459</td>
</tr>
<tr>
<td>VCPH</td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup>Pen A represents a pen entering with a low average weight of 650 pounds
<sup>b</sup>Pen B corresponds to a heavier pen with an average entry weight of 815 pounds

The proportion of statistically significant covariance parameter estimates provide evidence in favor of including these variables. A restricted case of this model is where covariance parameters are constant across individuals. This restriction is typical in studies of this nature due to the elimination of many parameter estimates (Belasco et al., 2006; Chavas and Kim, 2004). For our purposes, a restriction that assumes a constant covariance structure across observations increases degrees of freedom by 42, as the parameters estimates drops from 112 to 70. To test the effectiveness of this restriction on the given data set, a likelihood ratio test can be applied.

Within the likelihood ratio test framework, the flexible model described above will be the unrestricted case, while the restricted model will assume constant covariance terms. The restricted model reduces equation (9) to include only the covariance terms, which now are not a function of the data. Formally, the restriction can be stated as follows:

\[
H_0 \ : \ \alpha_{j,k} = 0 \\
H_A \ : \ \alpha_{j,k} \neq 0
\]

for all \( j = \{1, 2, ..., 6\} \) and \( k = \{2, 3, ..., 8\} \). \( \alpha_{j,k} \) elements are contained within the matrix A.

<sup>9</sup>Pen A contains an entry of 650 pounds, which is lower than 79% of the placement weights in the data sample. Also, Pen B weighs in at 815 pounds, which is lower than 18% of the observations.
\( \alpha_{j,k=1} \) is a 6 \times 1 vector containing ones and is the only remaining portion of matrix A in the restricted model. The results from the implied likelihood ratio test are shown in Table 7.

<table>
<thead>
<tr>
<th></th>
<th>Log Likelihood Values</th>
<th>( P )</th>
<th>Statistic</th>
<th>Critical Value (( \alpha = 0.05 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unrestricted</td>
<td>26,551.203</td>
<td>112</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant Covariance</td>
<td>26,306.716</td>
<td>70</td>
<td>488.974</td>
<td>55.76</td>
</tr>
</tbody>
</table>

These results strongly reject the notion of constant covariances within this data set. However, this restriction may be helpful when evaluating more homogenous production. The heterogeneity of cattle herds is an important aspect to this research and led to the usage of variance and covariance measures that were not constant across observations.

5 Modeling Profits

With accurate distributional modeling of cattle production yield characteristics, coupled with assumed distributional characteristics of price variation, and taking into account the joint correlation between these variables, conditional \textit{ex-ante} measures of profits can be computed. These profits will be a function of the unique characteristics of each pen, so that each set of characteristics lead to a unique profit distribution. This will be important in analyzing the extent of risk involved in overall profits. Simulation methods are used to incorporate the estimated distributional characteristics of yields, the assumed distributional characteristics of prices, and the marginal and joint effects from the conditioning variables. Here different shocks can occur that may affect the expected profits, variability, and covariance. An example of such a shock would be to the variability of fed cattle prices or corn prices. It is also worth mentioning that shocks to fed cattle and corn prices will be independent of yield shocks.\textsuperscript{10} Based on daily cash

\textsuperscript{10}This is mostly a simplifying assumption and may need further analysis. An argument has been formulated by Anderson and Trapp (2000) that changes in the price of corn can cause feedlots to substitute away from corn and towards other grains, like wheat. While this substitution will help the feedlots to keep their costs down, it may have an effect on feed conversion. Additionally, changes in corn prices may also effect the characteristics of cattle placed on feed. For example, placement weight may increase as a way to minimize days on feed.
prices from 1980 - 2005, a correlation of -0.16408 was used to characterize the relationship between fed cattle and corn prices.

### 5.1 The Profit Function

In order to model profitability risk, the following *ex-ante* profit function takes into account both the revenue and costs specific to cattle feeding. Following is the set of equations that explain the fed cattle production profit function. Per head cattle feeding profits are simply the net difference between revenue and costs accrued during the cattle feeding period.

\[
\pi = TR - FDRC - YC - FC - IC - VC
\]

(26)

where \( \pi \) are per head profits, \( TR \) is the total revenue per head from cattle feeding, \( FDRC \) is the per head cost of purchasing feeder cattle, \( YC \) is the per head fixed cost (yardage cost) of feeding cattle, \( FC \) is the per head feed cost, \( IC \) is an interest cost, and \( VC \) are the per head costs associated with veterinary care. \( TR \) is defined as

\[
TR = FP \times (0.96) \times CSW \times (1 - MORT)
\]

(27)

where \( FP \) is the price per hundred weight ($/cwt) of fed cattle and \( CSW \) is the average sell weight of the finished cattle, which is estimated based on the following equation

\[
CSW = CPW + ADG \times DOF.
\]

(28)

\( CPW \) is the average weight of the feeder cattle at placement and \( DOF \) is the number of days the pen of cattle is in the feedlot.

\( TR \) is adjusted for death loss using the MORT variable and a standard 4% live-weight shrinkage factor is applied to reflect the expected loss in weight during transport from the feedlot to the packing plant. Sell weight is a function of a random performance variable (\( ADG \))
and therefore is not fixed. This profit function allows days on feed to be specified, while allowing
sell weight to be determined by the average weight upon entry, \( ADG \), and the length of time
on feed. Cattle are assumed to be marketed on a cash basis as opposed to a price based on
dressed weight or grid pricing.\(^{11}\) To capture the expected \( FP \) at the time of placement, the
futures price from the CME can be used to proxy the price for the expected end date. \( FDRC \)
is defined as

\[
FDRC = FRP \times CPW
\]  

(29)

where \( FRP \) is the price per hundred weight of feeder cattle. This cost is a large portion of total
costs and reflects the value of the cattle upon entering the feedlot. On a per pound basis, \( FRP \)
is greater than \( FP \). \( YC \) is defined as

\[
YC = (0.40) \times DOF
\]  

(30)

which assumes that \$0.40 is a typical per head day cost for feedlots in Kansas and Nebraska.
\( FC \) is defined as

\[
FC = CP \times \left\{ \frac{DMFC}{0.88} \left[ CSW \times (1 - MORT) - CPW \right] \right\}
\]  

(31)

where \( FC \) is the price per bushel of corn and is divided by 56 to convert this price into a
per pound measure. The expected price of corn is based on the futures price for corn from
the CBOT halfway through the feeding period. The reason for this timing is the capture an
average price of corn over the entire feeding period. Further, dry matter is multiplied by the
corn-based feed ration, which is assumed to 12% moisture. \( DMFC \) is adjusted to reflect the
"as fed" feed conversion. \( IC \) is defined as

\[
IC = \left\{ \frac{1}{2} \left[ YC + FC + VC \right] + FRC \right\} \times IntRate \times \frac{DOF}{365}
\]  

(32)

\(^{11}\) For cattle sold on a grid, quality risk must enter the profit function. For the purposes of this research, quality
risk is not taken into account. Cash prices are based on the average weight of the pen, without regard for the
quality of the carcass. Evaluating quality risk remains an area of future research.
where \( IR \) is the interest rate. This expression assumes that an interest charge is applied to the full amount of the feeder cattle cost, \( FRC \), and half the total cost of yardage, feed, and veterinary fees. This assumption is based on the need to purchase feed throughout the feeding period, while the feeder cattle must be entirely purchased at the beginning of the feeding period.

### 5.2 Simulation of Profits

Random draws from a multivariate normal distribution simulate a collection of predictions for each of the yield factors. Given this information, the profit model described in equations (26)-(32) can be simulated with entry pen characteristics. In practice, this profit function can serve as a means for cattle owners or those in the cattle industry to understand expected profits that are a function of the unique characteristics of a pen of cattle placed on feed. To illustrate, a sample pen consists of its own unique characteristics, such as location, gender, entry weight, and placement season. This information influences the inferences made on production yield factors that define the multivariate normal distribution that describes production risk. The multivariate normal is four dimensional, where the mean values are a function of the mean parameter estimates and the variance is a function of both variance and covariance parameter estimates.

In addition to production risk, the model must also account for price risk. The expected prices and variance for fed cattle and corn can be obtained by using futures and options measures from the CME and CBOT. This is all the information necessary to characterize the profit function for simulation. Repeated random draws are taken in order to illustrate profits as a distribution.\(^{12}\) To further illustrate, the characteristics denoted in Table(8) are used to emulate a pen of cattle entering a Kansas feedlot on 2/14/2007.

Futures prices are used to approximate the price expectations for corn and fed cattle. For corn, a 3-month futures price is used to approximate the average cost of corn over the entire

\(^{12}\)For purposes of this study, 100,000 random draws were found to be enough to obtain a suitable profit distribution.
Table 8: Characteristics of Simulated Pen of Cattle

<table>
<thead>
<tr>
<th>Placement Characteristics</th>
<th>Price Characteristics$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date</td>
<td>2/14/2007</td>
</tr>
<tr>
<td>Weight(lbs)</td>
<td>750</td>
</tr>
<tr>
<td>Gender</td>
<td>Steer</td>
</tr>
<tr>
<td>Location</td>
<td>Kansas</td>
</tr>
<tr>
<td>Season</td>
<td>Winter</td>
</tr>
</tbody>
</table>

$^a$Expected prices based on 3 and 4 month futures price for corn and fed cattle, respectively.

feeding period. Fed cattle price expectations are estimated using a 4-month futures price, to denote the expected value of a fed steer when it is ready to be marketed. The Fed cattle volatility measure was assumed to be 20%, while the rise in corn volatility over the past year has led to the higher rate of 30%.$^{13}$

While this simulation has aspects unique to the current production situation, inputs can be changed for other purposes. Figure 1 shows the expected distribution of profits, given the previously mentioned inputs. It is not surprising that profits are near zero, with wide tails. Expected profits are centered at $17.14 per head, with a standard deviation of $251.71.

$^{13}$The rise in corn price and volatility over the last year is largely the result of a strong summer drought in the midwest during 2006, coupled with the increased demand for ethanol production.
References

Amemiya, T. (1973) ‘Regression analysis when the dependent variable is truncated normal.’ *Econometrica* 41(6), 997–1016

— (1974) ‘Multivariate regression and simultaneous equation models when the dependent variables are truncated normal.’ *Econometrica* 42(6), 999–1012


Figure 1: Distribution of ex-ante conditional profits

mean = 17.14
stdv = 251.71
Figure 2: Histograms of quantitative variables
Figure 3: Scatter plots of dependent variables