Indifference Pricing of Weather Insurance

Wei Xu, Martin Odening[*], and Oliver Mußhoff
[*] Faculty of Agriculture and Horticulture, Humboldt-Universität zu Berlin, Germany
m.odening@agrar.hu-berlin.de

Paper prepared for presentation at the 101[st] EAAE Seminar 'Management of Climate Risks in Agriculture', Berlin, Germany, July 5-6, 2007

INDIFFERENCE PRICING OF WEATHER INSURANCE

Wei Xu, Martin Odening* and Oliver Mußhoff***

Abstract

This article develops an Indifference Pricing model for a weather derivative that is traded over the counter. The model is used to calculate ask and bid prices for a put option on a weather index in Germany. We find that under moderate risk aversion the maximal bid prices of grain producers exceed the minimal sell prices of insurers only for a few regions and crops, due to the presence of basis risk. Another finding is that the actuarially fair price may lead to wrong conclusions about the market potential of weather insurance.

Keywords

Weather insurance, indifference pricing model, basis risk.

1 Introduction

Weather is undeniably one of the most important sources of risk in agriculture, and it seems that fluctuations of temperature and precipitation have even increased in the last decade due to global climate changes. Perhaps the most obvious impact of weather risk is on crop yields, but its relevance is not limited to crop production. The performance of livestock farms, the turnover of processors, the use of chemicals and fertilizers and the demand for many food products also depend on the weather. Hence, large parts of the agribusiness are affected by weather risks. In the past, the answer of producers to such bad-weather risks was to buy insurance. Another solution came in the mid-1990's with weather derivatives, a class of financial instruments that permit the trade of weather-related risks[1]. Weather derivatives allow payoffs to be determined with higher transparency and with lower transaction costs. They also relieve insurance companies offering crop insurance of the sometimes serious problems of moral hazard or adverse selection. Until now most transactions have taken place in the energy sector, but some promising applications also exist in agriculture (SKEES, 2001). Despite this recent progress in weather risk markets however, their future remains uncertain. A growing literature aims at assessing the hedging effectiveness of agricultural weather derivatives, as well as the farmer's willingness to pay for index-based weather insurance (EDWARDS and SIMMONS, 2004, VEDENOV and BARNETT, 2004, FLEEGE et al., 2004). Three interrelated issues are involved in the economic evaluation of these instruments: Firstly, the statistical modeling

* Wei Xu and Martin Odening, Faculty of Agriculture and Horticulture, Department of Agricultural Economics and Social Sciences, Humboldt-Universität zu Berlin, Germany.

** Oliver Mußhoff, Faculty of Natural Sciences III, Institute of agricultural and Nutritional Sciences, Martin-Luther-Universität Halle-Wittenberg, Germany.

1 Here we use the terms 'weather derivatives' and 'index based weather insurance' synonymously, though there is a controversial discussion about this issue (cf. TURVEY, 2005).

of relevant weather variables; secondly, the estimation of a weather-yield relationship and thirdly, the development of an adequate pricing model. While the first two problem areas can be handled fairly well with existing statistical tools, the pricing of weather derivatives is challenging theoretically. This is because weather cannot be traded, i.e. the market for weather risk is incomplete. Hence a straightforward application of standard pricing models for financial derivatives is impossible. Actually, the poor transparency of pricing algorithms employed by sellers is considered a major cause of the slow development of weather markets (VARANGIS, SKEES and BARNETT, 2002). This problem has been previously recognized and several proposals have been made in the literature for pricing weather derivatives. These include actuarial approaches (JEWSON and BRIX, 2005), extended risk neutral valuation (HULL 2006, TURVEY, 2005) and a consumption based asset-pricing model (CAO and WEI, 2004, RICHARDS, MANFREDO and SANDERS, 2004). Comparisons between these approaches indicate that remarkable differences in the resulting derivatives' prices may occur (cf. MYERS, LIU and HANSON, 2005). Nonetheless there is still no consensus about the "best" way to price weather derivatives, since all existing methods have their particular pitfalls. Some methods lack a sound theoretical basis, like the burn rate method, while others (in particular equilibrium models) have to resort to simplifying assumptions in order to become tractable. In this paper we want to contribute to this ongoing discussion by introducing a new approach, namely indifference pricing. Indifference pricing starts with the appealing idea that the amount of money at which a potential buyer (or seller) of weather insurance is indifferent, in terms of expected utility between buying (or selling) and not buying (selling), constitutes an upper (lower) limit for the contract price. Such an approach can take into account the particular economic situation of individual buyers (sellers). The purpose of this paper is to take up the general idea of indifference pricing and to develop a model that can be used for pricing weather insurance in an agricultural context. Furthermore, based on this model we want to assess the potential demand and supply for weather derivatives under different conditions. This assessment is clearly case specific, but it could also easily be generalized. Such an analysis may answer the aforementioned question, of whether weather derivatives will permeate agriculture or not.

The paper is organized as follows. The next section provides an overview about pricing methods for incomplete markets and introduces the indifference pricing approach in general. After this a specific indifference pricing model is developed that is applicable for an over-the-counter trade of weather derivatives in agriculture. The subsequent section applies this model to crop farms in Germany and derives farmer willingness to pay for weather insurance under varying conditions. The paper ends with conclusions about the potential trading volume of weather derivatives in agriculture.

2 Pricing in Incomplete Markets

Financial derivatives are usually priced in a risk-neutral valuation framework. Financial theory asserts that the price of a contingent claim F, which depends on a stochastic variable I, can be calculated according to (NEFTCI, 1996: 297):

$$F = E_Q\left(D \cdot W_T(I)\right) \tag{1}$$

I can be a traded asset like a stock or a non-traded asset like a weather index. $W_T(I)$ denotes the payoff of the derivative at expiration time T and D is a discount factor e^{-rT} with the risk-free interest rate r. E represents an expectation, conditional on the information available at present, where the subscript Q indicates that the expectation of the derivative payoff is to be calculated by means of a risk-neutral probability measure, instead of by a real world probability measure P. (1) can also be written as

$$F = E_P\left(\frac{dQ}{dP} \cdot D \cdot W_T(I)\right) \tag{2}$$

Therein dQ/dP denotes the Radon-Nikodym derivative of Q with respect to P.[2] This change of measure turns the stochastic process of the variable I into a martingale.

In the case of an option on a stock that follows a geometric Brownian motion, the change of measure is achieved by reducing the drift rate of the stochastic process to the risk-free interest rate. This means that more weight is given to unfavourable events, which makes sense in a risk-averse world. According to the first fundamental lemma of asset pricing, the use of risk-neutral probabilities ensures that the derivative price is arbitrage-free (DUFFIE, 2001). The change of measure of an asset's stochastic process is closely related to the concept of the market price of risk. Actually the drift rate of the asset's stochastic process is corrected by a parameter that reflects the market price of risk λ.[3]

If the capital market is complete, i.e. any contingent claim can be attained through a self-financing trading strategy, and then one can show that the risk-neutral measure (equivalent martingale measure) is unique (FÖLLMER and SCHIED, 2002: 23). Weather derivatives, however, are a typical example of an incomplete market, since weather is not a traded variable. In principle, (extended) risk-neutral valuation can be still carried out. The problem with pricing in incomplete financial markets is that the no-arbitrage condition does not result in a unique price. Many equivalent martingales exist and as a result, only bounds for prices on contingent claims can be provided (BENTH, 2004: 88, JENSEN and NIELSEN, 1996: 221-222). Formally stated, arbitrage-free prices lie in the range:

$$\left[\inf_{Q\in\mathbf{Q}} E_Q\left(D \cdot W_T(I)\right),\ \sup_{Q\in\mathbf{Q}} E_Q\left(D \cdot W_T(I)\right)\right] \tag{3}$$

where Q is the set of all equivalent martingales. Unfortunately, the no-arbitrage price interval is in general large and hence not useful (EBERLEIN and JACOD, 1997). It is important to realize that it is impossible to calculate a unique price for a derivative written on a non-traded asset without further information about the market participants' risk preference. Several proposals have been made in the literature in order to attain a unique valuation of financial claims in incomplete markets. ALATON, DJEHICHE and STILLBERGER (2002) determine the market price for weather risk to be an implicit parameter, such that the theoretical pricing model matches the observable market prices for some contracts. Of course this approach is only practical if a market already exists for weather derivatives. TURVEY (2005) proposed to estimate the market price of risk by using the capital asset pricing model, (CAPM). Due to the CAPM, the following equation is straightforward:

$$\mu = r + (\mu_M - r) \cdot \rho \cdot \frac{\sigma}{\sigma_M} \tag{4}$$

where μ and σ are the expected value and the standard deviation of the returns of an asset, respectively. Variables μ_M and σ_M stand for the corresponding values of the market portfo-

---

[2] $D \cdot (dQ/dP)$ is also called pricing kernel or state-price density.

[3] Note that the market price of risk already comes into play in a complete market setting. However, an explicit estimation of this parameter is not required, since the risk adjusted drift rate of the underlying is simply the risk neutral drift rate (see for example HULL, 2006: 715).

lio M, and ρ denotes the correlation between the asset and the market portfolio. Combining (4) with the definition $\lambda = (\mu - r)/\sigma$, an estimable expression for λ can be obtained:

$$\lambda = \rho \cdot \frac{(\mu_M - r)}{\sigma_M} \tag{5}$$

TURVEY argues in accordance with HULL (2006: 552) that the correlation between weather indexes and return in a capital market is small or negligible. That means that the weather variability is not a systematic risk. Consequently, the market price of the weather risk should be zero and no correction of the distribution of the weather index is necessary. This means the expectation in (1) can be calculated with real world probabilities.

CAO and WEI (2004) and RICHARDS, MANFREDO and SANDERS (2004) apply an extended version of LUCAS' (1978) equilibrium pricing model, where direct estimation of the weather risk's market price is avoided. Instead, pricing is based on the stochastic processes of the weather index and an aggregated dividend δ_t. In this model, a representative agent chooses a trading strategy that maximizes the present value of expected lifetime utility. The latter depends on consumption C_t, $t = 0,\cdots,T$. The first order conditions of this maximization problem, together with the equilibrium condition, $C_t = \delta_t$, imply the following price equation:

$$F = D \cdot E\left(\frac{\partial u(\delta_T)}{\partial \delta_T} \bigg/ \frac{\partial u(\delta_0)}{\partial \delta_0} \cdot W_T \right) \tag{6}$$

where $u(\delta_t)$ is the period utility. (6) resembles the structure of the pricing equation (2). The only difference is that the pricing kernel dQ/dP is replaced by a marginal rate of substitution.

MYERS, LIU and HANSON (2005) criticize the Lucas approach, since it remains unclear whose consumption should be used in the model (6). It is likely that the aggregation level of the consumption data will impact the correlation between the dividend process and the weather process. Moreover, the Lucas model prices the weather insurance contract as if it were traded on a liquid secondary market. This assumption seems unrealistic for the trading of index based weather insurance in agriculture.

The indifference pricing approach that we pursue here is also based on utility maximization. In contrast to the Lucas model it is not an equilibrium model. Instead of assuming a representative agent, the indifference pricing approach considers an individual agent who faces specific risks that depend on the weather's impact on their particular business. The basic idea is that agents (farmers or insurance companies) have an incentive to buy/sell weather insurance if thereby their utility (of final wealth) is increased. In this setting it is natural to define the indifference buy price F_b^i as the price at which an investor i is indifferent between a) paying F_b^i now and receiving the claim from the derivative at expiration and b) not having the claim and paying the price. This comparison has to take into account the initial wealth x of the investor as well as all possible investment and trading strategies $\vartheta \in \theta$. Define $V(x,k)$ as the maximal utility that can be achieved at time T starting from a given present endowment x and without having the derivative, then

$$V(x) = \sup_{\vartheta \in \theta} E\left(u\left(x + \int_0^T \vartheta dS_t \right) \right) \tag{7}$$

S_t is the price of the assets invested in the capital market. The maximization problem is to find a trading strategy $\vartheta(S_t)$ to maximize the utility of terminal wealth. Accordingly, the maximal utility that is attainable when buying k units of the derivative at price F is

$$V\left(x - F_b^i, k\right) = \sup_{\vartheta \in \theta} E\left(u\left(x + \int_0^T \vartheta dS_t - k \cdot F_b^i + k \cdot W(I)\right)\right) \tag{8}$$

The indifference price of the derivative is then implicitly expressed as

$$V\left(x - F_b^i, k\right) = V(x) \tag{9}$$

An indifference selling price F_s^i can be defined quite analogously. Neither F_b^i nor F_s^i should be interpreted as market prices for the derivative. Rather, they could mark the starting point for price negotiations between potential sellers and buyers. Note that so far, no specific restrictive assumptions about the stochastic process of the underlying weather index have been made.

What is the relationship between indifference pricing and the above discussion of determining an equivalent martingale in the case of incomplete markets? It can be shown that, under the assumption of an exponential utility function, the risk neutral measure Q implied by indifference pricing equals the minimal entropy measure (FRITTELLI, 2000). This means that among all equivalent martingale measures one chooses the measure Q that is closest to P. Apparently the indeterminacy of the equivalent martingale measure is resolved by the choice of a particular utility function. A nice property of indifference prices is that they recover familiar Black-Scholes prices in the case of complete markets (cf. HENDERSON and HOBSON, 2007). This is not true for incomplete markets. A distinguishing feature is that indifference prices are non-linear in the number of traded contracts k. This means that $2 \cdot F_b^i(k=1)$ is not the indifference price for $2 \cdot W$, and is simply due to the non-linearity of the utility function. DAVIS (1997) suggests a linear pricing rule, which is similar to the indifference price. He invokes the economic principle of "zero marginal rate of substitution", which in this context means that an investor cannot increase his expected utility by diverting a small amount of the derivative into his portfolio for price F^*. Formally stated, the marginal utility price F^* is defined by

$$\frac{\partial}{\partial k} V\left(x - F^*, k\right)\Big|_{k=0} = 0 \tag{10}$$

The marginal utility price (10) can be considered a special indifference price with an infinitesimal k.

Practical application of both indifference pricing and marginal-pricing are rare, since explicit solutions of (9) or (10) are difficult to obtain[4]. A major simplification of the calculation of indifference prices can be attained if the set of possible trading strategies is restricted. DAVIS (2001), for example, considers an energy supplier who produces and sells gas at the current (stochastic) market price, but makes no investment decisions. Assuming a logarithmic utility function and a geometric Brownian motion for the weather index, Davis proves that the marginal price F^* for an option on an HDD index can be calculated with a modified Black-Scholes formula. In an extreme scenario no trading of the derivative and the investment goods are allowed, i.e. $\theta = \{0\}$. BECHERER (2003) shows that in this case, the indifference pricing

[4] An exception is MUSIELA and ZARIPHOPOULOS (2004) who present a closed form solution for an indifference price, assuming an exponential utility function and a diffusion process for the non-traded asset.

approach coincides with the well known actuarial principle of equivalent utility. In the absence of dynamic trading strategies (8) simplifies to

$$E(u(x)) = E(u(x - k \cdot F + k \cdot W_T)) \tag{11}$$

The non-trading assumption is less restrictive than it appears at first glance, at least for agricultural applications. At the time being, the contracts that are available for agricultural producers are tailored products that are offered over the counter by insurance companies (SKEES, 2001). The objective for a typical farmer purchasing these contracts is to hedge weather related risks rather than to trade them. Once a contract has been purchased, the farmer keeps it until expiration. It is also unlikely that a liquid market for index-based insurance contracts will develop, because they are designed to meet a very specific demand. Moreover, agricultural production decisions are made at the beginning of a planning period and cannot be changed during the vegetation period. That means a farmer cannot adjust his "investment portfolio", even when new weather information becomes available. Based on this idea we develop an indifference pricing model for weather derivatives that are traded over the counter (OTC).

3 An Indifference Pricing Model for Weather Derivatives

In what follows we consider two market participants, a seller (bank or insurance company) and a buyer. In order to ease the calculation of the indifference price we follow BROCKET et al. (2006) and assume a two-date economy. At $t = 0$ both agents optimise their investment portfolios in order to maximize their terminal wealth at time T. No trading of the derivative or adjustment of the investment portfolio is allowed between these two dates. First we consider the decision-making process of the seller. At the beginning of the planning period the seller is endowed with initial wealth x_s. He has to determine the amount of capital, α_s, to be invested in a risky capital market portfolio. Additionally, he can sell k_s shares of the weather contract for a price $F_s(I)$. The residual capital is invested in a risk-free asset. The value of this portfolio at T is then

$$\begin{aligned} &X_s^{with} = (x_s - \alpha_s + k_s \cdot F_s) \cdot q_f + \alpha_s \cdot q_s - k_s \cdot W \\ &\text{with } q_f = 1 + r_f,\ q_s = 1 + r_s,\ W = W_T(I) \end{aligned} \tag{12}$$

r_s and r_f denote the return of the capital market investment and the return of the risk-free asset, respectively. Without the opportunity to sell weather contracts the terminal wealth of the insurer is

$$X_s^{without} = (x_s - \alpha_s) \cdot q_f + \alpha_s \cdot q_s \tag{13}$$

The decision-making process of the buyer is quite analogous, but instead of investing in a market portfolio he spends α_b shares of capital for a risky production activity, which to some extent depends on weather conditions. Additionally he can buy k_b units of the weather contract. The terminal value of this portfolio at T is

$$X_b^{with} = (x_b - \alpha_b - k_b \cdot F_b) \cdot q_f + \alpha_b \cdot q_b + k_b \cdot W, \text{ with } q_b = 1 + r_b \tag{14}$$

while r_b denotes the return on production. Without investing in weather contracts, the final value of his portfolio simplifies to:

$$X_b^{without} = (x_b - \alpha_b) \cdot q_f + \alpha_b \cdot q_b \tag{15}$$

Next we derive the seller's indifference price. According to (9) the "fair price" for the seller is given by

$$\sup_{\alpha_s} E\left[u\left(X_s^{with}\right)\right] = \sup_{\alpha_s} E\left[u\left(X_s^{without}\right)\right] \tag{16}$$

In the context of indifference pricing, risk preferences are usually modelled by an exponential utility function:

$$u(X) = -e^{-\gamma \cdot X}\text{, with absolute risk aversion parameter } \gamma > 0 \tag{17}$$

In order to get a closed form solution of the indifference price, we replace the expected utility in (17) by its certainty equivalent CE and approximate it using Pratt's Theorem:

$$CE = E(X) - \frac{\gamma}{2} \cdot \sigma^2(X) \tag{18}$$

$E(X)$ and $\sigma^2(X)$ are the expected value and the variance of the terminal wealth, respectively[5]. The definition (16) of the indifference price then becomes:

$$\sup_{\alpha_s}\left(E\left(X_s^{with}\right) - \frac{\gamma_s}{2} \cdot \sigma^2\left(X_s^{with}\right)\right) = \sup_{\alpha_s}\left(E\left(X_s^{without}\right) - \frac{\gamma_s}{2} \cdot \sigma^2\left(X_s^{without}\right)\right) \tag{19}$$

Recalling the definitions of X_s^{with} and $X_s^{without}$ given in equations (12) and (13) yields an explicit expression for the certainty equivalent of the terminal wealth, with and without the weather derivative, in terms of the expected returns, variances and covariances of the involved random variables

$$\begin{aligned} CE^{with} = \Big(& x_s \cdot q_f + k_s \cdot F_s \cdot q_f + \alpha_s \cdot \left(E(q_s) - q_f\right) - k_s \cdot E(W) \\ & - \frac{1}{2} \cdot \gamma_s \cdot \alpha_s^2 \cdot \sigma_{q_s}^2 - \frac{1}{2} \cdot \gamma_s \cdot k_s^2 \cdot \sigma_W^2 + \gamma_s \cdot \alpha_s \cdot k_s \cdot COV(q_s, W) \Big) \end{aligned} \tag{20}$$

and

$$CE^{without} = \left(x_s \cdot q_f + \alpha_s \cdot \left(E(q_s) - q_f\right) - \frac{\gamma_s}{2} \cdot \alpha_s^2 \cdot \sigma_{q_s}^2 \right) \tag{21}$$

where $E(q_s)$ and $E(W)$ stand respectively for the expected q_s and for the expected payoff of the weather derivative. $\sigma_{q_s}^2$ and σ_W^2 denote the corresponding variances and $COV(q_s, W)$ is the covariance between q_s and the derivative's payoff. Via the first order conditions we obtain the following solutions for the optimal shares of the seller's investment on the financial market α_s^{*with} and $\alpha_s^{*without}$:

5 With this specification our model is similar to that of EDWARDS and SIMMONS (2004). The main difference is that EDWARDS and SIMMONS assume an exogenous price when calculating the demand for weather derivatives of Australian wheat producers.

$$\alpha_s^{*without} = \frac{E(q_s) - q_f}{\gamma_s \cdot \sigma_{q_s}^2} \tag{22}$$

$$\alpha_s^{*with} = \frac{E(q_s) - q_f + \gamma_s \cdot k_s \cdot COV(q_s, W)}{\gamma_s \cdot \sigma_{q_s}^2} \tag{23}$$

(23) states that the optimal capital share, which is invested into the market portfolio, decreases in case of a negative covariance between q_s and W. This is not surprising since W means a negative payoff for the seller.

Inserting $\alpha_s^{*without}$ and α_s^{*with} into (20) and (21), equating these expressions and solving for F_s results in the desired equation for the price threshold of the seller:

$$F_s = \frac{1}{q_f} \cdot \left(E(W) + \pi_s\right) \tag{24}$$

with

$$\pi_s = -\frac{1}{2} \cdot \gamma_s \cdot k_s \cdot \sigma_W^2 \cdot \left(\rho_{q_s,W}^2 - 1\right) - \frac{\sigma_W}{\sigma_{q_s}} \cdot \left(E(q_s) - q_f\right) \cdot \rho_{q_s,W} \tag{25}$$

where $\rho_{q_s,W}$ measures the correlation between the derivative's payoff and the return of the market portfolio. (24) has a simple actuarial interpretation. The indifference price is the present value of the expected derivative payoff plus a risk premium, which can be positive or negative. The sign of π_s depends on the sign and the magnitude of the correlation $\rho_{q_s,W}$. If $\rho_{q_s,W}$ is zero or negative, the seller will require a positive premium in addition to the discounted expected value of the payoffs. This is under the assumption that the investment in the capital market will only be worthwhile if the expected return on the capital market, $E(q_s)$, is larger than the risk-free interest rate q_f. For a large positive value of $\rho_{q_s,W}$ it may happen that π_s becomes negative. If $\gamma_s = 0$ (risk neutrality) and $\rho_{q_s,W} = 0$, the risk premium is also zero. Note that the indifference price is independent of the initial wealth.

Carrying out quite similar steps for the buyer of the derivative results in optimal investment shares for his or her production activity, with and without the weather insurance:

$$\alpha_b^{*with} = \frac{E(q_b) - q_f - \gamma_b \cdot k_b \cdot COV(q_b, W)}{\gamma_b \cdot \sigma_{q_b}^2} \tag{26}$$

and

$$\alpha_b^{*without} = \frac{E(q_b) - q_f}{\gamma_b \cdot \sigma_{q_b}^2} \tag{27}$$

Herein are $E(q_b)$ and $\sigma_{q_b}^2$ the expected value and the variance of the return on production q_b, while $COV(q_b, W)$ is the covariance between q_b and the derivative's payoff. In contrast to the seller, the buyer invests more in the risky production activity if the covariance between the production revenues and the indemnity payments is negative. Using these results we can derive an indifference price for the buyer analogous to (24):

$$F_b = \frac{1}{q_f}\left(E(W) + \pi_b\right) \tag{28}$$

with

$$\pi_b = \frac{1}{2}\cdot\gamma_b\cdot k_b\cdot\sigma_W^2\cdot\left(\rho_{q_b,W}^2 - 1\right) - \frac{\sigma_W}{\sigma_{q_b}}\cdot\left(E(q_b) - q_f\right)\cdot\rho_{q_b,W} \tag{29}$$

The indifference price equation for the buyer has the same structure as for the seller. Again, the risk premium π_b can be negative or positive. While the first summand on the right side of (29) is always negative, the second is positive, if one reasonably assumes that $E(q_b) - q_f > 0$ and the correlation $\rho_{q_b,W} < 0$[6]. Hence the sign of the risk premium depends on the specific parameter values. For example, in the case of a highly risk averse decision maker, a rather small (negative) correlation between yields and payoffs and small expected returns on production, the risk premium will be negative. In other words, the producer will offer less than the expected value of the payoff to buy the weather insurance.

When do we observe trading of weather insurance between the seller and the buyer in the outlined model framework? The following proposition states a necessary condition.

PROPOSITION 1 *Trading of weather insurance will* only *arise if*

$$-\frac{\left(E(q_b) - q_f\right)\cdot\rho_{q_b,W}}{\sigma_{q_b}} > -\frac{\left(E(q_s) - q_f\right)\cdot\rho_{q_s,W}}{\sigma_{q_s}} \tag{30}$$

Proof: Trading requires that $F_b(k) > F_s(k)$, which implies, in view of (28) and (24), that $\pi_b > \pi_s$, i.e.

$$\begin{aligned}\pi_b = \frac{1}{2}\cdot\gamma_b\cdot k_b\cdot\sigma_W^2\cdot\left(\rho_{q_b,W}^2 - 1\right) - \frac{\sigma_W}{\sigma_{q_b}}\cdot\left(E(q_b) - q_f\right)\cdot\rho_{q_b,W} > \\ -\frac{1}{2}\cdot\gamma_s\cdot k_s\cdot\sigma_W^2\cdot\left(\rho_{q_s,W}^2 - 1\right) - \frac{\sigma_W}{\sigma_{q_s}}\cdot\left(E(q_s) - q_f\right)\cdot\rho_{q_s,W} = \pi_s\end{aligned} \tag{31}$$

Obviously π_b and π_s are both linear in k, but with opposite slope ($\partial\pi_b/\partial k < 0, \partial\pi_s/\partial k > 0$). Hence $F_b(k)$ and $F_s(k)$ can only intersect if the second term on the left side of the inequality (31) exceeds the second term on the right side. q.e.d.

The optimization problem that underlies the indifference price implies that the buyer can adjust the optimal production level. One may argue that this is unrealistic in agriculture. The level of agricultural production is usually determined among other things by the factor endowment of the farm and crop rotation requirements. That means α_b is chosen prior to the hedging decision and does not depend on the availability of weather insurance. If α_b is treated as an exogenous parameter and not as a decision variable, the risk premium of the buyer simplifies to

[6] Otherwise hedging of production risk would not be possible at all.

$$\tilde{\pi}_b = -\frac{1}{2} \cdot \gamma_b \cdot k_b \cdot \sigma_W^2 - \gamma_b \cdot \alpha_b \cdot \sigma_{q_b} \cdot \sigma_W \cdot \rho_{q_b,W} \tag{32}$$

In the subsequent application we use the definition of the risk premium given in (32) instead of the one in (29). The returns on investing in agricultural production, r_b, are calculated on a per hectare basis. In this case, α_b reflects the farm size in hectare.

4 Empirical Application: The demand of German grain producers for weather insurance

Grain production in northeast Germany, Brandenburg in particular, is highly affected by weather risk. During the relevant growth period of April to June, the sum of average rainfall in Brandenburg varied over the last 60 years between 64 and 258 mm. Actually, drought is the major cause for bad grain harvests. The correlation between rainfall and yields results from the sandy soil, possessing little water-storing capacity, as well as the lack of irrigation. Currently there exists no opportunity for insuring against yield losses caused by rainfall or temperature. In view of a series of extreme crop failures in the drought years 2000, 2003 and 2006, where only governmental disaster relief prevented farmers from becoming insolvent, there is a pronounced interest in introducing some kind of weather insurance. In the subsequent application we calculate the willingness to pay for such insurance by means of indifference pricing. This information is valuable for a potential seller who contemplates entering this market segment. Since designing an insurance contract is costly, only a few different contracts will be offered. To keep things as easy as possible we will consider a single weather derivative. The specification of this derivative is chosen in such a way that it addresses the demand of wheat producers in Brandenburg in the best possible manner, i.e. maximizes the hedging effectiveness. The demand for this insurance from other farms and from other regions in Germany (Saxony, Thuringia and Baden-Wuerttemberg) is also assessed, though we expect less willingness to pay for such a contract than compared with wheat producers in Brandenburg.

For the specification of the relationship between weather and yield Y_t we follow VEDENOV and BARNETT (2004), who suggest the model:

$$Y_t = I_t + \varepsilon_t \tag{33}$$

with

$$\begin{aligned} I_t = \beta_0 &+ \beta_1 \cdot \Delta R_t + \beta_2 \cdot (\Delta R_t)^2 + \beta_3 \cdot \Delta T_t^{April} + \beta_4 \cdot \Delta T_t^{May} + \beta_5 \cdot \Delta T_t^{June} \\ &+ \beta_6 \cdot \left(\Delta T_t^{April}\right)^2 + \beta_7 \cdot \left(\Delta T_t^{May}\right)^2 + \beta_8 \cdot \left(\Delta T_t^{June}\right)^2 + \beta_9 \cdot \Delta R_t \cdot \Delta T_t^{June} \end{aligned} \tag{34}$$

and $t = 1,...,N$. Here I_t denotes a weather index, $\varepsilon_t \sim N[0, \sigma_\varepsilon]$ is a normally distributed error term and Δ measures the deviation of a weather variable from its long-time average. T_t^{April}, T_t^{May} and T_t^{June} are the average monthly temperatures of April, May and June. R_t represents the rainfall deficit defined as

$$R_t = \sum_{\tau=1}^{z} \min\left(0, \sum_{i=(\tau-1)\cdot s+1}^{\tau \cdot s} d_i - d^{min}\right) \tag{35}$$

(35) measures the shortfall of the sum of daily rainfall amounts d_i during a period of s-days, relative to a minimum rainfall requirement d^{min}. An accumulation period from April to June ($z = 13$), together with the parameter choice $s = 7$ days and $d^{min} = 7.4$ mm, maximizes the correlation between the rainfall deficit and the wheat yield.

Estimation of the parameters β_0 to β_9 in (34) is based on wheat yield data from a representative cash crop farm in Brandenburg for a time period between 1993 and 2006.[7] The weather variables are derived from daily temperature and daily precipitation data, recorded at the weather station in Berlin-Tempelhof, and are used for the estimation of the production function. The parameter estimates are summarized in Table 1.

Table 1: Estimated Production Function for Wheat in Brandenburg, Germany

Parameter	Estimate	p-value
β_0	72.51	0.0000
β_1	1.67	0.0109
β_2	-0.65	0.0140
β_3	10.10	0.0017
β_4	-4.06	0.0156
β_5	-2.01	0.1845
β_6	1.57	0.0632
β_7	-4.02	0.0053
β_8	-1.06	0.5335
β_9	-2.33	0.0241
σ_ε	3.16	

Apparently, higher-than-average rain deficits, higher-than-average temperatures in May and June as well as below-average temperatures in April are responsible for shortfalls in wheat yield. These results make sense from an agronomic viewpoint. An R^2 of 0.98 indicates that the selected weather index can powerfully explain the wheat yield in Brandenburg.

However, this model, which was calibrated for Brandenburg and fed with the weather data from Berlin, fits poorly with the wheat yield in other regions or with the yield of other crops (see the first column in Table 2). This is not surprising, since different crops have particular growth patterns. Moreover, the results are in accordance with the spatial de-correlation of weather variables that have been reported in other studies (ODENING, MUßHOFF and XU, 2007).

7 Reliable yield data for the New Federal States are not available before the German reunification.

Table 2: Correlation between Revenue and Indemnity Payments for Different Regions and Products

				1	2	3	4
	Region	Distance* (km)	Product	R^2	p (€/dt)	σ_ψ	$\rho_{\psi,W}$
1	Brandenburg	39	wheat	0.98	11.22	133.33	-0.89
2	Saxony	109		0.71		117.96	-0.82
3	Thuringia	237		0.29		64.08	-0.40
4	Baden-Wuerttemberg	537		0.03		65.22	0.07
5	Brandenburg	39	rye	0.24	11.11	103.15	-0.57
6	Brandenburg		crop rotation**	0.48	12.75	99.03	-0.65

*) Distance between reference weather station and farm location.

**) 25 % wheat, 25 % barley, 25 % rye and 25 % canola.

Based on this information one can design a weather derivative that maximizes the hedging effectiveness for a specific target group, in this case a wheat producer in Brandenburg. The derivative type that we choose is a put-option with payoff W_T at the expiration date T:

$$W_T = \max(0, K - I_T) \cdot L \tag{36}$$

I_T denotes the weather index at expiration, K the strike-level and L the tick-size. The tick-size L is determined such that the negative correlation between the payoff of the option and the revenues of the wheat production is maximal. This is achieved if L equals the slope of the revenue function for wheat production multiplied by the acreage. Assuming a farm size of 100 hectares, L amounts to 1122 € per index point. For reasons of simplicity it is assumed that the wheat price p is constant (fixed by forward contracts) and amounts to 11.22 €/dt (German Federal Ministry of Food, Agricultural and Consumer Protection 2006: 19). As usual the strike-level K equals the long-term average of the weather index (62.64 index points). The option expires after $T = 1$ year. The last column in Table 2 displays the correlation between the production revenues ψ and the payoff of the put option. Using empirical data, we are able to determine the expected value $E(W)$ and the standard deviation σ_W of the empirical payoffs of the weather contract W. They are 5258 € and 7980 €, respectively.

Further model assumptions concern the risk aversion parameter, the return of investments on capital markets and the empirical payoff of the derivative. Following MONOYIOS (2004: 251) we set the absolute risk aversion parameters for sellers γ_s to $1 \cdot 10^{-6}$. The buyer of the option is assumed to have a relative risk aversion parameter of 3, which is considered moderate (PRATT 1964). Dividing this by the average equity capital endowment of cash crop farms in Brandenburg (3764 €/ha times 100 ha) yields an absolute risk aversion parameter γ_b of $8 \cdot 10^{-6}$. The German Stock Index (DAX) approximates risky capital market investments. Expected value and volatility of the capital markets returns r_s are calculated using index quotations on the last trading day in June between 1990 and 2006. $E(q_s)$ amounts to 1.08 and σ_{q_s} to 0.22. Further parameters needed as input for equation (25) are $\rho_{q_s,W}$, which amounts to -0.46, and the risk-free interest rate r_f, which is assumed to be 5 %.

5 Results

Based on these data and assumptions, the indifference price of the seller and of the potential buyers for the put option are determined according to equations (24) and (32). Figure 1 shows the seller's willingness to accept as well as the producer's willingness to pay, in Brandenburg and other regions, as a function of the contract volume k. Figure 1 also displays the actuarially fair price, as a benchmark. The fair price was calculated by a simple burn analysis, i.e. it equals the average hypothetical payoff of the derivative $E(W)$, discounted by the risk-free interest rate r_f (cf. JEWSON and BRIX, 2005). It amounts to 5008 € and is independent of the contract volume.

The wheat producer's willingness to pay in Brandenburg is, below a contract size of $k = 2.96$, greater than the actuarially fair price. This is due to the pronounced negative correlation between ψ and W. However, the indifference price decreases with increasing contract volume, because the payoff of the weather contract is uncertain in itself. This effect eventually overrides the first term of (32) so that the risk premium becomes negative for $k > 2.96$.

The ask price of the seller is higher than the actuarially fair price for any k and increases with increasing contract volume. The positive risk premium reflects the negative correlation between q_s und W and the excess return of risky capital market investments ($r_s > r_f$).

At first glance, the indifference price functions look like supply and demand functions, but there is a conceptual difference. By construction the utility of the buyer (or seller) is constant along the indifference price curve, i.e. the graphs in Figure 1 can be interpreted as iso-utility lines[8]. Hence the intersection of the buyer and seller price should not be misunderstood as a unique equilibrium price. In the case of Brandenburg's wheat producers, Figure 1 simply says that a bid-ask-spread exists below a contract volume of $k = 0.96$ and hence a trading potential for the weather derivative is present. At this point one should recall that the indifference price for the seller constitutes a lower bound for the true ask price. In a more realistic setting the transaction costs for developing and launching the weather derivative also have to be taken into account. Some of these transaction costs are typically fixed, so that economies of scale will occur. Thus trading of the weather derivative seems more likely if the two indifference curves intersect at a high contract volume.

Note that if the insurer used the burn rate method instead of indifference pricing, he would underestimate the willingness to pay, at least for small contract volumes, and would probably be discouraged from offering the weather contract. This conclusion, however, depends crucially on the assumed degree of risk aversion. According to (32) the bid price curve becomes flatter and shifts downward if γ_b decreases. That means that the market potential erodes with the farmer's diminishing risk aversion, and that the difference between the fair price and the indifference price vanishes. Furthermore, the willingness to pay for weather insurance depends in our model on α_b, the farm size. Keeping all other parameters constant, a variation of the farm size leads to a parallel shift of the buyer's indifference price curve, which means that larger farms have a considerably higher bid price than smaller farms. However, if we fall back on the more plausible assumption that the relative rather than the absolute risk aversion is constant, the impact of a change of farm size will be mitigated by an opposite change in γ_b.

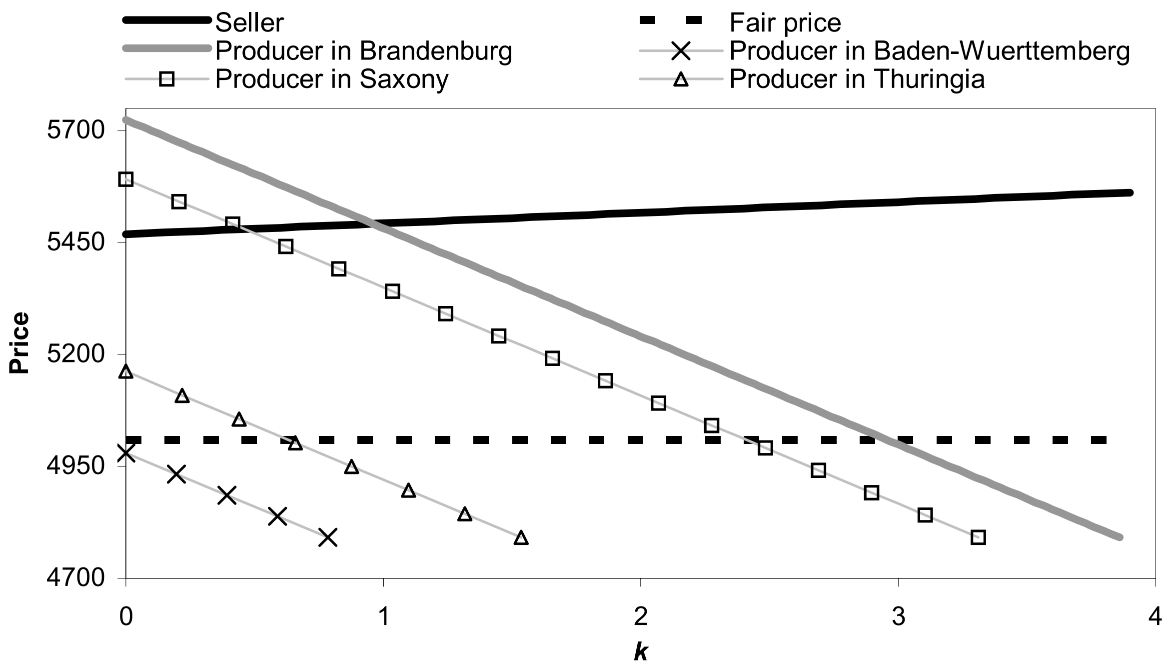
Figure 1: Indifference prices and fair price for a put option on a weather index in € (reference station: Berlin-Tempelhof). (a) Demand of wheat producers in

---

[8] This can be easily seen from the fact that (i) the certainty equivalent in eq. (21) is constant and (ii) the indifference price is derived from equating (20) and (21).

different regions in Germany. (b) Demand of different farm types in Brandenburg

(a)



(b)

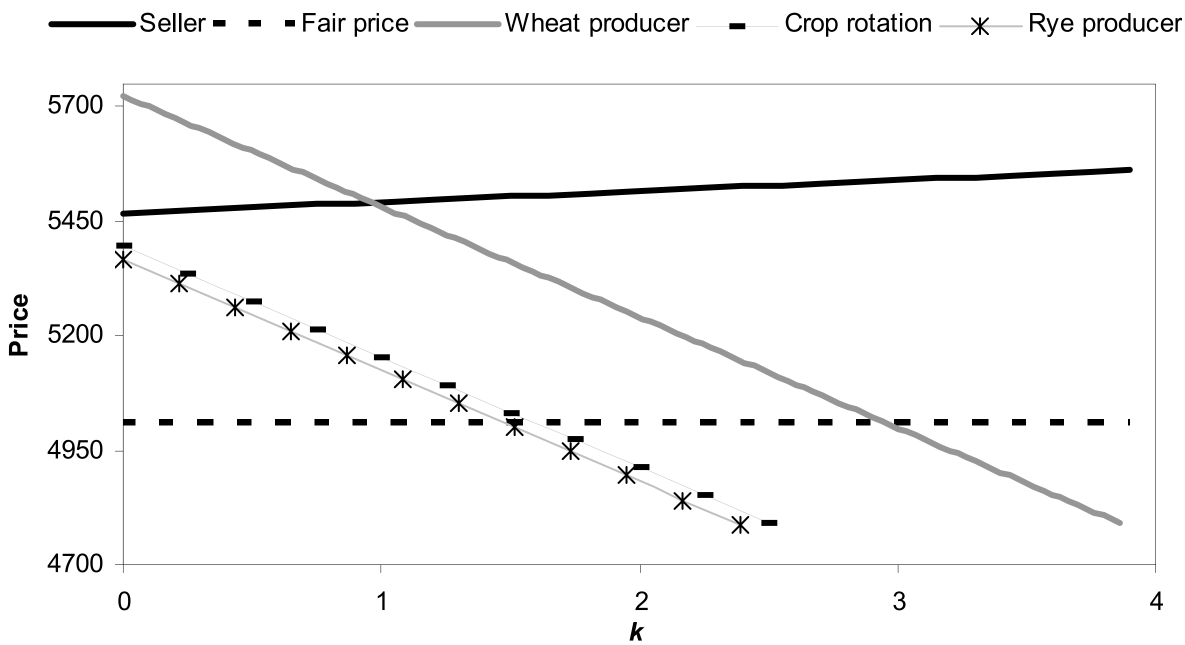


Figure 1(a) also depicts the indifference price of wheat producers in other regions in Germany, keeping the specification of the option constant. Compared to wheat producers in Brandenburg, the willingness to pay is lower in Saxony, Thuringia and Baden-Wuerttemberg. For the latter two states there is, in fact, no trading potential. This result can be explained by the higher geographical basis risk, i.e. lower hedging effectiveness, and the fact that production risk is lower than in Brandenburg. The slope of the buyer's price curve is unaffected, since it depends only on the risk aversion and on the volatility of the option payoff (cf. (32)).

Finally, Figure 1(b) shows the willingness to pay of different farm types in Brandenburg, keeping all other model assumptions constant. As before, we observe a parallel shift of the indifference price functions. The explanation is analogous to the regional comparison. The comparatively low indifference price functions for rye and for the crop rotation can be traced back to (i) the smaller correlation between production revenues and indemnity payments $\rho_{\psi,W}$ and (ii) a lower volatility of revenues σ_{ψ} (cf. Table 2). All in all, the results in Figure 1 highlight that the willingness to pay for index-based weather insurance is very specific to the production program and to the location of the farm.

1 Conclusions

DISCHEL (2002: 20) observes that potential market participants "are watching the weather market's progress from the sidelines". In this paper we take up the hypothesis that the lack of transparent pricing may explain why potential market participants hesitate to enter the weather market, and we suggest a rather new pricing approach, namely indifference pricing. On the one hand indifference pricing can be classified into the framework of financial pricing of derivatives, and hence has a strong theoretical basis. On the other hand, under some simplifying assumptions made here, it boils down to a rather plain model that has a straightforward actuarial interpretation. Insofar, indifference pricing bridges the financial and the actuarial approaches for the valuation of non-tradable assets. Our pricing approach has some advantages. Firstly, it circumvents the determination of the market price of risk. Clearly along with this comes the cost of specifying a utility function, but this is unavoidable whenever no-arbitrage arguments are insufficient to determine a unique price. Secondly, our model seems to be more adequate for an application to OTC markets than equilibrium pricing models. It takes into account individual basis risk and calculates its impact on the willingness to pay for a weather contract. We also find it more convenient to work with distributions of the relevant random variable, rather than to specify stochastic processes in a continuous time framework.

The model that we developed is rather stylised and might be refined in several directions. For example, the mean variance approximation of the certainty equivalent requires an exponential utility function and normally distributed random variables. However, there is much empirical evidence that many weather variables do not follow a normal distribution. (e.g. ODENING, MUßHOFF and XU, 2007). Using other distributions or other utility functions within the proposed pricing framework does not constitute a theoretical problem, but it would require numerical solution procedures and considerably increase the computational burden. Moreover, the consideration of portfolio effects in the decision-making process of farmers and insurers should be enhanced. Farms usually have more diversified production programs than here assumed, and they encounter additional risks, in particular price risks. Likewise, insurers may find risks in their portfolios showing a higher correlation with the weather variable than the market portfolio shows. Nevertheless, we believe that the main findings of the model application are rather robust with these simplifying assumptions. Our calculations confirm results of previous studies, showing a considerable magnitude of basis risk inherent to index-based weather insurance in agribusiness (ODENING, MUßHOFF and XU, 2007). Geographical basis risk, in conjunction with production related basis risk, erodes the potential advantages of weather derivatives over traditional crop insurance. To overcome this problem, insurers should offer tailored products that match the specific demand of individual producers. However, this is only a realistic scenario if the design of individual insurance contracts does not entail high transaction costs. The proposed indifference pricing approach may facilitate reducing such costs.

References

ALATON, P., B. DJEHICHE, and D. STILLBERGER (2002): On Modeling and Pricing Weather Derivatives. Applied Mathematical Finance 9:1–20.

BECHERER, D. (2003): Rational Hedging and Valuation of Integrated Risks under Constant Absolute Risk Aversion. Insurance: Mathematics & Economics 33:1-28.

BENTH, F.E. (2004): Option Theory with Stochastic Analysis: An Introduction to Mathematical Finance. Springer, Berlin.

BROCKETT, P.L., M. WANG, C. YANG, and H. ZOU (2006): Portfolio Effects and Valuation of Weather Derivatives. The Financial Review 41:55-76.

CAO, M., and J. WEI (2004): Weather Derivatives Valuation and Market Price of Weather Risk. Journal of Futures Markets 24:1065-1089.

DAVIS, M. (1997): Option Pricing in Incomplete Markets. In: M.A.H. Dempster, and S.R. Pliska, eds. Mathematics of Derivative Securities. Cambridge: Cambridge University Press, 227-254.

DAVIS, M. (2001): Pricing Weather Derivatives by Marginal Value. Quantitative Finance 1: 305-308.

DISCHEL, B. (2002): Dry Market in Need of Liquidity. Risk Magazine: Risk Management for Investor, September, 20-22.

DUFFIE, D. (2001): Dynamic Asset Pricing Theory, 3rd. ed. Princeton: Princeton University Press.

EBERLEIN, E., and J. JACOD (1997): On the Range of Options Prices. Finance and Stochastics 1: 131-40.

EDWARDS, M.J., and P. SIMMONS (2004): Preliminary Results for the Measurement of Willingness to Pay for Climate Derivatives. Paper presented at the 48th Annual Conference of the Australian Agricultural and Resource Economics Society, Melbourne, Australia,February,2004.

FLEEGE, T.A., T.J. RICHARDS, M.R. MANFREDO, and D.R. SANDERS (2004): The Performance of Weather Derivatives in Managing Risks of Specialty Crops. Paper presented at the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management, St. Louis, Missouri, 19-20 April.

FÖLLMER, H., and A. SCHIED (2002): Stochastic Finance: An Introduction in Discrete Time. Berlin: Walter de Gruyter.

FRITTELLI, M. (2000): The Minimal Entropy Martingale Measure and the Valuation Problem in Incomplete Markets. Mathematical Finance 10:39-52.

Germany, Federal Ministry of Food, Agricultural and Consumer Protection (2006): Ernte 2006: Mengen und Preise. Berlin: German Government. Available at http://www.bmelv.de/.

HENDERSON, V., and D. HOBSON (2007): Utility Indifference Pricing. In R. Carmona, ed. Volume on Indifference Pricing. Princeton: Princeton University Press: in press.

HULL, J.C. (2006): Options, Futures & Other Derivatives, 6th. ed. New Jersey: Prentice Hall International.

JENSEN, B.A., and J.A. NIELSEN (1996): Pricing by no arbitrage. In D.R. Cox, D.V. Hinkley, and O.E. Barndorff-Nielsen, eds. Time Series Models in Economics, Finance and Other Fields. London: Chapman&Hall, 177-225.

JEWSON, S., and A. BRIX (2005): Weather Derivative Valuation: The Meteorological, Statistical, Financial and Mathematical Foundations. Cambridge: Cambridge University Press.

LUCAS, R.E. (1978): Asset Prices in an Exchange Economy. Econometrica 46:1429-1445.

MONOYIOS, M. (2004): Performance of Utility-Based Strategies for Hedging Basis Risk. Quantitative Finance 4:245-255.

MUSIELA, M., and T. ZARIPHOPOULOU (2004): An Example of Indifference Prices under Exponential Preferences. Financial and Stochastics 8:229-239.

MYERS, R.J., Y. LIU, and S. HANSON (2005): How should We Value Agricultural Insurance Contracts. Paper prepared at the American Agricultural Economics Association Annual Meeting, Providence, Rhode Island, 24-27 July.

NEFTCI, S.N. (1996): An Introduction to the Mathematics of Financial Derivatives. San Diego: Academic Press.

ODENING, M., O. MUßHOFF, and W. XU (2007): Analysis of Rainfall Derivatives Using Daily Precipitation Models: Opportunities and Pitfalls. Agricultural Finance Review, Spring 2007: in press.

PRATT, J.W. (1964): Risk Aversion in the Small and in the Large. Econometrica 32:122-136.

RICHARDS, T.J., M.R. MANFREDO, and D.R. SANDERS (2004): Pricing Weather Derivatives. American Journal of Agricultural Economics 86:1005-1017.

SKEES, J. (2001): The Potential Role of Weather Markets for U.S. Agriculture. The Climate Report 2. Available at http://www.guaranteedweather.com/.

TURVEY, C.G. (2005): The Pricing of Degree-Day Weather Options. Agricultural Finance Review, Spring 2005:59-85.

VARANGIS, P., J. SKEES, and B. BARNETT (2002): Weather Indexes for Developing Countries. In R. Dischel, ed. Climate Risk and the Weather Market. London: Risk Books.

VEDENOV, D.V., and B.J. BARNETT (2004): Efficiency of Weather Derivatives as Primary Crop Insurance Instruments. Journal of Agricultural and Resource Economics 29:387-403.