ECONOMIC INTERPRETATION OF FERTILIZER RESPONSE DATA

J. R. ANDERSON*

1. INTRODUCTION

At least since Liebig first presented his "law of the minimum" in 1842 agricultural scientists and economists have been interested in the efficiency of fertilizer use. Over the years many concepts have been developed and clarified, especially with regard to the algebraic form of fertilizer production functions. The last decade, however, has seen a substantial expansion and much more general acceptance of research and discussion in this area.[1] Much stress has been laid on the likely benefits of co-operative research between agronomists, statisticians, and economists.

* Department of Farm Management, University of New England. The helpful comments of J. L. Dillon and P. A. Rickards are gratefully acknowledged. Residual errors are the responsibility of the author.

[1] Illustrated, for example, by such publications as R. D. Munson and J. P. Doll, "Economics of Fertilizer Use in Crop Production", *Advances in Agronomy*, Vol. 11 (1959), pp. 133-169, E. O. Heady and J. L. Dillon, *Agricultural Production Functions* (Iowa State University Press: Ames, 1961) and OECD *Documentation in Food and Agriculture*: No. 50, "Interdisciplinary Co-operation in Technical and Economic Agricultural Research" (Paris, 1962); and No. 65, "Co-operation between Research in Agricultural Natural Sciences and Agricultural Economics" (Paris, 1964).

Physical scientists, especially fertilizer agronomists, have been exhorted to employ surface-oriented designs in experiments and thence economic principles in interpretation. While major emphasis has been placed on statistical design and analysis, economic interpretation *per se* has been relatively neglected. As it seems that agronomists will make increasing use of experiments amenable to economic analysis[2], the relevant principles should be clearly outlined. The present paper has this as its major aim.

2. Background

A production function is an algebraic relationship between inputs (including fertilizers) and output (a crop or livestock product). Development of the modern approach to fertilizer production functions[3] was centred on the American Midwest States and North Carolina. However, it must be stressed that this was merely an extension of principles already long understood. In fact, Australian agriculturalists, such as Prescott[4], had helped build the foundations of this work. A comprehensive discussion and bibliography of production function techniques is given in *Agricultural Production Functions*[5]. Throsby[6] has further demonstrated the mechanics of fitting production functions to response data. Features of various experimental designs for estimating fertilizer production functions have recently been reviewed by Dillon[7].

In this paper we follow Dillon's[8] assumption of a simple theory of response, namely that:

(i) there is a continuous smooth relation between inputs and output;

(ii) diminishing returns prevail to each input factor, i.e., additional output from succeeding units of each input becomes less and less; and

(iii) decreasing returns to scale prevail, i.e., a proportionate increase in all inputs results in a less than proportionate increase in output.

---

[2] The trend for future use of such experiments in Australia is indicated by the approximately 150 nitrogen-phosphorus rate trials on wheat in New South Wales, Queensland and Tasmania during 1966 compared with less than 20 in 1960.

[3] See E. L. Baum, E. O. Heady and J. Blackmore, eds. *Methodolgical Procedures in the Economic Analysis of Fertilizer Use Data* (Iowa State College Press: Ames, 1956) and E. L. Baum, E. O. Heady, J. T. Pesek and C. C. Hildreth, eds. *Economic and Technical Analysis of Fertilizer Innovations and Resource Use* (Iowa State College: Ames, 1957). A good introduction to the techniques is given in W. B. Sundquist and L. S. Robertson Jr., "An Economic Analysis of Some Controlled Fertilizer Input-Output Experiments in Michigan", *Michigan Agr. Exp. Sta. Tech. Bul.* 269, 1959.

[4] J. A. Prescott, "The Law of Diminishing Returns in Agricultural Experiment", *Economic Record*, Vol. 4 (1928), pp. 85-89.

[5] Heady and Dillon, *op. cit.*

[6] C. D. Throsby, "Fitting Production Functions to Experimental Data", this *Review*, Vol. 29, No. 3 (September, 1961), pp. 112-247.

[7] J. L. Dillon, "Economic Considerations in the Design and Analysis of Agricultural Experiments", this *Review*, Vol. 34, No. 2 (June, 1966), pp. 64-75.

[8] J. L. Dillon, *Introduction to Crop and Livestock Response Analysis* (Permagon Press: Oxford, in press), Section 1.2.

The algebraic form of the production function[9], provided it is consistent with this theory of response, it is not crucial to the following discussion.

3. The Simple Classical Model

The production function

$$Y = f(X_1, X_2, \ldots X_n \mid X_{n+1}, \ldots X_m); \tag{1}$$

relates the quantity of the i-th fertilizer input X_i ($i = 1, 2, \ldots n$) to output Y, given the fixed factors[10] X_{n+1} through X_m, where the Y and X_i are specified on a per unit area basis.[11]

The profit function is defined as

$$\Pi = p_y Y - \sum_{i=1}^{m} p_i X_i \tag{2}$$

where p_y, $p_i > 0$ are the prices of Y and the X_i respectively.

When unlimited funds are available for investment in fertilizer, the economic problem is simply to maximize Π. That is, best operating conditions are specified by equating the first derivatives ($\partial\Pi/\partial X_i$, $i = 1, 2, \ldots m$) to zero[12]. Thus

$$p_y \partial Y/\partial X_i - p_i = 0;$$

or

$$\partial Y/\partial X_i = p_i/p_y;$$

which is often stated as "equate the marginal product of each variable factor to its inverse price ratio".

Analyses have sometimes been based on additional yield from fertilizer rather than total yield. If for

$$X_i = 0, (i = 1, 2, \ldots n), \; Y = Y_0;$$

and we define

$$\Delta Y = Y - Y_0;$$

[9] For a variety of reasons, polynomial functions are generally superior and, *ceteris paribus*, the simple second order polynomials are to be preferred for convenience in subsequent analyses. In particular, while the linear marginal product and isocline equations make the quadratic polynomial production function somewhat restrictive, this feature facilitates simultaneous solution for multi-factor economic optima.

[10] So called "trace" elements have generally been treated as fixed factors. The effect of using micro-nutrients is conveniently visualised as placing the process on a new (higher) production surface. However, micro-nutrient response surfaces have been explored, for example by R. J. Hader, M. E. Harward, D. D. Mason and D. P. Moore, "An Investigation of Some of the Relationships between Copper, Iron and Molybdenum in the Growth and Nutrition of Lettuce: 1. Experimental Design and Statistical Methods for Characterising the Response Surface", *Soil Science Society of America Proceedings*, Vol. 21 (1957), pp. 59-64.

[11] Each X_i is best considered as a weight per acre of elemental nutrient. For farmer recommendations the nutrients can then be converted to commercial fertilizers taking into account the multi-nutrient composition of many commercial fertilizers and the difference between actual and guaranteed analysis of fertilizers.

[12] Note that for the fixed inputs,

$$\partial Y/\partial X = 0, \quad i = n + 1, \quad \ldots m.$$

The necessary second order conditions

$$\partial^2\Pi/\partial X_i^2 < 0$$

hold from the assumptions of diminishing and decreasing returns.

the profit on fertilizer use is given by

$$\Pi_f = p_y \Delta Y - \sum_{i=1}^{n} p_i X_i; \tag{3}$$

and the profit maximizing conditions are

$$p_y \partial(\Delta Y)/\partial X_i = p_i.$$

The formulation of the profit function and its derivatives in terms of yield response is necessary when production functions like the Cobb-Douglas[13] are used but is unnecessary for polynomial functions since for these

$$\partial Y/\partial X_i = \partial(\Delta Y)/\partial X_i$$

4. Pricing Under Certainty

Reported economic analyses have seldom used appropriate price estimates.[14] Output prices have typically been city market prices with no regard to product quality and fertilizer prices have often been "factory gate" prices. In this section we discuss the reckoning of prices for marketable and intermediate products and consider briefly prices influenced by quality measures.

4.1 *Product prices*

Let H represent marketing costs per unit of output. Costs involved will include selling costs, transport costs and elements of harvest cost. If the relevant output price is defined as

$$P_y = p_y - H:$$

we have

$$\partial Y/\partial X_i = p_i/P_y;$$

4.2 *Intermediate product prices*

Many fertilizer response studies involve products for which the market is poorly defined (such as some hay crops) or is nearly non-existent (as with most pasture crops). These experiments have been confined to varying only a few of the relevant factors in the fertilizer-pasture-animal production process. But even relatively simple grazing models present analytic difficulties as has been noted by Dillon and Burley[15].

Economic interpretation requires imputation of a price to intermediate products such as pasture. This has most often been done by

[13] For example, for a Cobb-Douglas function

$$Y = b_o \prod_{i=1}^{n} X_i^{b_i},$$

$$X_i = 0 \text{ implies } Y = 0.$$

[14] Notable exceptions are E. O. Heady, and H. R. Jensen, *Farm Management Economics* (Prentice-Hall, Inc. Englewood Cliffs, N.J., 1954), p. 187, W. Y. Yang, *Methods of Farm Management Investigations* (FAO Agricultural Development Paper No. 64, Food and Agriculture Organization of the United Nations, Rome, 1958), p. 106 and D. F. Capstick, *et al.*, "Economic Analysis of Irrigation and Fertilization of Cotton on Sharkey Clay and Grenada Silt Loam Soils in Eastern Arkansas", *Arkansas Agr. Exp. Sta. Special Report* 12, 1963.

[15] J. L. Dillon and H. T. Burley, "A Note on the Economics of Grazing and its Experimental Investigation", *Australian Journal of Agricultural Economics*, Vol. 5, No. 2 (December, 1961), pp. 123-132.

using some variant of either the residual claimant method[16] or the substitution value method[17]. A lucid discussion of the pricing concepts in pasture evaluation has been given by Johnson and Hardin[18]. While the development of principles was necessarily broad, their frame of reference can be used to derive prices for a pound of pasture at a given time rather than for an acre of pasture over an accounting period (as they discussed). Briefly, pasture has the extreme values of acquisition cost and salvage value. The relevant price is the marginal value product (MVP) when this lies between the extreme values. MVP depends on the use to which forage is put relative to its supply.

A crucial problem is empirical estimation of pasture MVP. Regression analysis of cross-sectional data has been used but is unsuited for estimating MVP's which characteristically vary through the year. These can be generated in solution of linear programming models.[19] Such models cast elements of both residual claimant and substitution value methods in a whole-farm planning context. The usefulness of this valuation method for response interpretation will depend on the applicability of bench-mark programming formulations.[20]

Intermediate pricing can sometimes be avoided by restricting the problem to one of minimizing the average cost of the product which becomes an input in the next production phase. This may be useful for a forage crop destined to be ensiled, since there is virtually no commercial market for silage.

Given the few relevant linear programming studies available and inherent difficulties with other methods of valuation, it is not surprising that some researchers have tested a wide range of forage prices in investigating pasture fertilizer economics.[21]

---

[16] An essentially residual claimant method was used by R. G. Moyle, "The Economic Interpretation of Pasture Responses to Superphosphate", *Quarterly Review of Agricultural Economics*, Vol. 14, No. 2 (April, 1961), pp. 79-87.

[17] One of the early workers to discuss the inadequacies of these methods, particularly of the crude assumptions of substitution values (i.e., substituting a nutritional "equivalent" of a livestock feedstuff for which a market exists), was E. J. Nesius, "Allocation of Farm Resources for Economic Production of Pasture Forage", *Kentucky Agr. Exp. Sta. Bul.* 568, 1951.

[18] G. L. Johnson and L. S. Hardin, "Economics of Forage Evaluation" *Purdue Agr. Exp. Sta. Bul.* 623, 1955.

[19] This use of linear programming for pasture pricing in fertilizer allocation is demonstrated by M. Nelson, E. N. Castle and W. G. Brown, "Use of the Production Function and Linear Programming in Valuation of Intermediate Products", *Land Economics*, Vol. 33, No. 4 (November, 1957), pp. 257-261.

[20] A note of caution should be sounded. Linear-programming-generated MVP's may be sensitive to changes in forage production induced by fertilizer use, in which case fertilized forage activities would have to be included in the programming formulation. However forage MVP's may be fairly stable as occurred in the study reported in P. A. Rickards and W. O. McCarthy, "Linear Programming and Practicable Farm Plans", *University of Queensland Papers* Vol. 1, No. 5 (August, 1966), pp. 175-197. (P. A. Rickards, private communication). A further likely difficulty is the difference between MVP's for the optimum bench-mark plan and MVP's for farm plans differing markedly from the optimum solution.

[21] R. C. Woodworth, R. E. Proctor, G. W. Burton and A. B. Mackie, "Profitable Use of Fertilizer in the Production of Coastal Bermuda in the Coastal Plain Area of Georgia", *Georgia Agr. Exp. Sta. Tech. Bul.* 13, 1957. Sensitivity of optima to varying prices should always be checked in empirical analyses. This has occasionally been done, for example by J. P. Doll, E. O. Heady and J. T. Pesek, "Fertilizer Production Functions for Corn and Oats; including an Analysis of Irrigated and Residual Response", *Iowa Agr. Exp. Sta. Res. Bul.* 463, 1958.

An infrequently used approach is to circumnavigate intermediate product pricing by relating livestock output directly to fertilizer inputs. This can be done through surveys or experiments[22] or it may be possible to combine fertilizer-pasture production functions with pasture-livestock functions[23]. However, relevant functions of the latter type are rarely available, especially in Australia.

4.3 *Input prices*

Many empirical analyses have employed list or factory prices of the X_i with no discussion of the implicit assumptions. We now define P_i as the total unit cost directly attributable to X_i.

Typically P_i will include:

(i) net price paid to fertilizer dealer;

(ii) transport cost to farm and paddock (small if slack farm labour and farm vehicles are used);

(iii) wastage cost (negligible for quality-bagged lots); and

(iv) variable application cost.

For aerial and other contract spreading where the charge is on a "per unit of X_i" basis, cost of application is most significant. Where farm labour and machinery are used, variable application costs are usually low since most of the application cost is incurred irrespective of the levels of X_i applied, *i.e.* most machinery and labour costs are fixed.[24] If fertilizer is applied during another operation, such as sowing, application costs are also low.

An element of variable application cost[25] which is sometimes significant, albeit ill-defined, is the opportunity cost of labour and machinery incurred during refilling. This occurs when timeliness of operation is important.

Taking these factors into account the profit function can then be restated in terms of appropriate input and output prices as

$$\Pi = P_y Y - \Sigma P_i X_i. \tag{4}$$

[22] For a survey example, see R. B. Jones and K. E. Hocknell, "Economics of Fertilizer Application to Permanent Pastures for Beef Production", *University of Nottingham, F.R. No.* 147, 1962, and for an experimental relation see L. A. Reuss, H. K. Roberts and R. E. L. Greene, "Pangola Grass Pastures for Beef Production in Florida—a Method of Determining the Economics of Establishing and Fertilizing Them", *Florida Agr. Exp. Sta. Bul.* 651, 1957.

[23] As for example in R. J. McConnen, C. O. McCorkle Jr. and D. D. Caton, "Feed-Livestock Relationships: A model for Analysing Management Decisions", *Agricultural Economics Research*, Vol. 15, No. 2 (April, 1963), pp. 41-48.

[24] Details of cost budgeting for decisions on liquid versus solid fertilizers are given by E. O. Heady and E. L. Baum, "Economic Comparison of Farm Application of Dry and Liquid Types of Nitrogen in Iowa", in Baum *et al.*, eds, 1957, *op. cit.*, pp. 89-114.

[25] When variable application cost of X_i is some function, $g_i(X_i)$, of the amount used,

$$P_i = p_i + g_i(X_i);$$

where p_i represents other costs (purchase, transport, etc.) per unit and the profit maximizing specification is

$$\partial Y/\partial X_i = [p_i + g_i(X_i) + g_i'(X_i)X_i]/P_y.$$

4.4 *Multiple products and quality constraints*

The above models have assumed a homogeneous product. However, output from a fertilizer activity invariably consists of a number of products, for example various grades in vegetable production. Depending on the technical and economic factors operating, such products are variously handled by

(i) aggregating into one composite product, if fixed proportions are produced,

(ii) treating as multiple technically related products[26], or

(iii) simply neglecting joint products of no immediate economic interest.

A situation not so readily dealt with is when a joint product provides a measure of quality important in economic interpretation of the response process. Milk production with quality (and price) determined by fat or solids-not-fat is such a process.

Solution to this class of problem is given by Umland and Smith[27] who developed the method for response surfaces in chemical manufacture. If for a response function

$$Y_1 = f_1(X_1, X_2 \ldots X_n);$$

a profit function

$$\Pi = P_y Y_1 - \Sigma P_i X_i;$$

is subject to the constraining relationship

$$Y_2 - Y_2^* = f_2(X_1, X_2, \ldots X_n) - Y_2^* = 0;$$

where Y_2^* is some critical level of the constraining quality measure Y_2, the optimum solution is found by differentiating

$$\Pi_c = \Pi + \lambda(Y_2 - Y_2^*);$$

with respect to the X_i and λ, an unknown Lagrange multiplier, setting these equal to zero and solving the resulting equations

$$\partial\Pi/\partial X_i + (\partial Y_2/\partial X_i) = 0, \qquad i = 1, 2, \ldots n;$$

$$Y_2 - Y_2^* = 0.$$

Many variants of the constraining relationship are possible. For a simple example, if Y_1 is wheat yield and Y_2 the crude protein analysis of the wheat, a price scheme may operate such that for

$$Y_2 \geqq Y_2^*, P_y = P_{yh}, \text{ a premium price}$$

and

$$Y_2 < Y_2^*, P_y = P_{ys}, \text{ a lower price.}$$

In this case two sets of equations with two Lagrange multipliers and two wheat prices must be solved. The profit maximizing solution is selected from these constrained optima.

Typically the equations to be solved are non-linear, even when the response functions are quadratic polynomials, since quality is often a function of the variable factors. This necessitates the use of numerical methods for solution.

[26] "Multi-ware" production has been discussed fully by R. Frisch, *Theory of Production* (Reidel, Dordrecht, Holland, 1965), Chapters 14 and 15. Agricultural multiple response processes are outlined in Dillon, *op. cit.*, Sections 2.7 and 2.8.

[27] A. W. Umland and W. N. Smith, "Use of Lagrange Multipliers with Response Surfaces", *Technometrics*, Vol. 1, No. 3 (August, 1959), pp. 289-292.

5. Pricing under Uncertainty

In contrast to input prices, output price may be uncertain. One procedure is to assume that any uncertain price situation can be resolved as a subjective risk problem. This implies that the decision maker holds a personal probability distribution for the possible price outcomes. The relationship between this distribution and any historical or objective probability distribution of price outcomes will depend on the experience and knowledge of the decision maker. The approach allows ready incorporation of any pertinent information (for example, institutional arrangements and market reports) into the decision.

Analysis is simplified by assuming a discrete probability distribution; *i.e.* by considering only a finite number of price outcomes. The expected price[28] for T prices

$$\sum_{j=1}^{T} (P_{yj} \text{ probability of } P_{yj}).$$

is then the correct decision price so long as the decision maker has a linear utility function for the payoffs involved in fertilizer investments.[29] Thus P_y is implicitly taken as the expected P_y where risk and uncertainty are involved.

6. Limited Funds

So far it has been assumed that farmers have unlimited funds for fertilizer investment and should therefore apply rates of fertilizers which maximize expected profits per acre—an obviously unrealistic situation. Alternative criteria are required for practical interpretations.

6.1 *A formal approach—Lagrange methods*

In a formal sense, outlay limitations are conveniently handled by using Lagrange multipliers. If expenditure per acre on fertilizer is limited to some level C, equation (4) is modified as follows:

$$\Pi = P_y Y - \Sigma P_i X_i + \lambda(\Sigma P_i X_i - C); \tag{5}$$

where λ is a Lagrange multiplier. Then, given that $\Sigma P_i X_i$ cannot exceed the restriction C, the constrained best operating conditions are defined by differentiating equation (5) with respect to the X_i and λ, equating the $(n + 1)$ derivatives to zero and solving simultaneously.[30]

This analysis can be extended to consider multiple activities and products, but becomes cumbersome. For practical interpretation of fertilizer response data the major limitation is the assumption of a well defined fertilizer expenditure constraint for each acre fertilized. If such constraints exist, they are unique to individuals and knowledge of them is not generally available.

---

[28] See E. N. Castle and M. H. Becker, *Farm Business Management* (New York: Macmillan, 1962), pp. 150-151 for examples of expected price computations.

[29] This assumption of linearity is not unreasonable when sums of money involved in fertilizer decisions are not large. Where fertilizer is a major portion of farm investment a more complicated economic analysis with explicit consideration of risk may be required (as outlined in Section 9 below).

[30] See Heady and Dillon, *op. cit.* pp. 54-55 and J. P. Doll, "The Allocation of Limited Quantities of Variable Resources among Competing Farm Enterprises", *Journal of Farm Economics*, Vol. 41, No. 4 (November, 1959), pp. 781-789.

6.2 *A practical approach—the opportunity cost principle*

Solutions are necessarily unique whenever fund limitations exist, but the following alternative method lends itself to ready use in practical decision making without specification of fixed outlay constraints. Whole farm profits are maximized when marginal returns from investment in all variable factors are equated. Thus determination of optimum fertilizer rates depends on knowledge of returns in alternative investment opportunities.

With generally diminishing marginal returns, the equalizing marginal net return[31], R, will depend on the intensity of investment in each alternative and consequently on the supply of funds. Normally R will thus be determined in part by the level of investment in fertilizer. However, for interpretation of an isolated response function we may know little of alternative investment possibilities and the values of R used may be merely realistic standards[32] derived from budgets of a few alternatives.

Marginal net profit per dollar invested in X_i is defined as $\partial\Pi/\partial(P_iX_i)$ and the optimum input levels are found by setting

$$\partial\Pi/\partial(P_iX_i) = R, \quad i = 1,2, \ldots n;$$

and solving for the X_i. From equation 4 (P_i constant)

$$(P_i^{-1})(\partial\Pi/\partial X_i) = (P_i^{-1})(P_y\partial Y/\partial X_i - P_i) = R;$$

so that

$$\partial Y/\partial X_i = P_i(1 + R)/P_y.$$

This means that the usual criterion, marginal product equals inverse price ratio, is unchanged for the limited capital situation provided the input prices are inflated by the factor $(1 + R)$.

6.3 *Alternative criteria*

Various criteria alternative to profit maximization (constrained and unconstrained) have been employed in response analysis. Of these "rules of thumb", one has received most attention and is worth discussing briefly. Pesek and Heady[33] showed that the ratio of net additional[34] return to investment in fertilizer is a useful criterion where there are fixed costs (K per acre) associated with fertilizer use. The rate of fertilizer which gives the highest return per dollar invested is taken as the minimum rate which should be used.

Use of this criterion has been restricted to single nutrient studies—perhaps because solution for minimum rates of more than one factor becomes very tedious, even for simple polynomial functions and two

[31] In J. D. Colwell and R. J. Esdaile, "The Application of Production Function Analysis for the Estimation of Fertilizer Requirements of Wheat in Northern New South Wales", *Australian Journal of Experimental Agriculture and Animal Husbandry*, Vol. 6, No. 23 (November, 1966), pp. 418-424, this opportunity cost is referred to as "minimal rate of return" and "limiting marginal cost ratio". These authors have recognized that "profit maximizing" rates are irrelevant for many farmers and they present soil-test based recommendations for a range of values of R.

[32] Frequently the lowest appropriate R will be that associated with off-farm investments including repayment of borrowed funds.

[33] J. T. Pesek and E. O. Heady, "Derivation and Application of a Method for Determining Minimum Recommended Rates of Fertilization", *Soil Science Society of American Proceedings*, Vol. 22 (1958), pp. 419-423.

[34] Net total return was mistakenly used in the other main reference to this criterion, namely Heady and Dillon, *op. cit.*, p. 46.

factors. However, since iterative solution on electronic computers is now often feasible, the multi-factor case is outlined. If K_i is the fixed cost associated with using the i-th factor, then we define ρ_i, the return on investment in this factor as

$$\rho_i = \Pi_f/(K_i + P_i X_i);$$

where equation 3 is modified to account for the fixed costs so that

$$\Pi_f = P_y \Delta Y - \Sigma P_i X_i - \Sigma K_i, \qquad i = 1,2, \ldots n.$$

The minimum rates (which maximize ρ_i) are found by setting

$$\partial \rho_i / \partial X_i = 0;$$

and solving these non-linear equations simultaneously for the X_i.[35]

While such "minimum" rates may often be more relevant practical recommendations than unconstrained profit maximizing rates—which can be regarded as "maximum" rates—the procedures involved are strictly heuristic and in no sense optimizing. Generally, the methods discussed in other sections, particularly using opportunity costs, are to be preferred for farmer recommendations.

7. Time in Fertilizer Response Analysis

Crop and livestock responses to fertilizer involve growth so that time may be important to the analysis. For example, milk production by cows grazing a nitrogen fertilized pasture is a time-dependent process where the continuously harvested product may also be subject to time-dependent price variations. Some of the difficulties associated with time which beset economic analyses of the grazing complex have been noted by Dillon and Burley.[36] A general review of time in response analysis is beyond the scope of this paper and the reader is referred to the comprehensive discussion by Dillon.[37] The present section looks briefly at the role of time preference in fertilizer decisions.

7.1 *Discounting for time preference*

Time has been ignored in most reported studies. Because these have usually involved a single-harvest crop response over a relatively short period, the effects of excluding time preference considerations have been small. Neglect is more serious where longer response periods are encountered.

The simplest procedure relevant to fertilizer response analysis is to discount future returns to present values, and then select rates which maximize "present" profits. The present value of a dollar N periods ahead is $1/(1 + r)^N$ where r is the interest rate per period. The relevant r is the internal rate of return[38] if this exceeds the cost of borrowed funds. Expressing equation (4) (*i.e.* a single harvest situation) in terms of present values gives

$$\Pi = [1/(1 + r)^N] P_y Y - \Sigma P_i X_i$$

[35] Often a fixed cost will be shared by two or more factors in a mixed fertilizer (e.g. phosphorus and potassium applied at maize planting). This is handled by specifying an optimum mixture (isocline) and determining the minimum rate of the mixed fertilizer.

[36] Dillon and Burley, *op. cit.*

[37] Dillon, *op. cit.*, Chapter 3.

[38] For a lucid discussion of rate of return computation see C. J. Grayson, *Decisions Under Uncertainty: Drilling Decisions by Oil and Gas Operators* (Harvard Business School, Division of Research, Boston, Mass., 1960), pp. 243-247.

which, after the usual maximization leads to the requirement

$$\partial Y/\partial X_i = [P_i(1 + r)^N]/P_y$$

where N time periods expire between outlay on fertilizer and receipt of payment for the product.

Again the usual criterion is unchanged except for inflation of the price ratio by a factor which accounts for time. This can be thought of as inflating the factor price to account for the cost of funds used in buying fertilizer. Where payment for a product is staggered over time (for example Australian wheat) the present value of P_y is computed by summing the discounted payments.

When fertilizers influence yield distribution over time, especially in multi-harvest crops such as tomatoes, time is best considered explicitly as a factor of production. Eidman, Lingle and Carter[39] in their demonstration of this used the criterion of equating the linear summation of the marginal value product for each period with the marginal factor cost, for each factor. That is

$$\sum_{t=1}^{M} [P_{yt}\partial Y_t/\partial X_i] = P_i, \qquad i = 1,2, \ldots n$$

where the net product price in the t-th harvest period is P_{yt}, and $t = 1, 2, \ldots M$. Each P_{yt} could be discounted to account for time preference.

7.2 *Residual Returns*

It is not uncommon that harvests succeeding a first crop following a fertilizer application benefit from this application. More rarely successive harvests will be adversely affected by previous fertilizer treatments. Generally residual effects diminish rapidly in successive crops. A further reason for the comparative neglect of residual returns in reported empirical studies is the difficulty experienced in deriving reliable production functions for residual response.[40]

The most common technique has been to consider successive responses jointly by summing the value products discounted in the manner outlined above. Clearly this is an extension of the multi-harvest case and the profit equation is

$$\Pi = \sum_{t=1}^{M} [(1/(1 + r)^{N_t})P_{yt}Y_t] - \sum_{i=1}^{n} P_iX_i$$

where the Y_t are successive harvests of the same crop[41] or of different crops[42] and the other symbols are as before.

[39] V. R. Eidman, J. C. Lingle and H. O. Carter, "Optimum Fertilization Rates for Crops with Multi-harvest Periods", *Journal of Farm Economics*, Vol. 45, No. 4 (November, 1963), pp. 823-830.

[40] Some problems encountered in residual fertility response measurement are discussed by B. R. Hoffnar and G. L. Johnson, "Summary and Evaluation of the Co-operative Agronomic—Economic Experimentation at Michigan State University", *Michigan Agr. Exp. Sta. Res. Bul.* 11, 1966.

[41] J. T. Pesek, E. O. Heady and L. C. Dumenil, "Influence of Residual Fertilizer Effects and Discounting upon Optimum Fertilizer Rates", *7th International Congress of Soil Science Transactions IV.* 28, (Madison, Wisconsin, 1960), pp. 220-227, and Heady and Dillon, *op. cit.*, p. 524.

[42] Doll, Heady and Pesek, *op. cit.*

Such analyses have not usually dealt with fertilizer applications to a crop benefiting from residual fertilizer. This situation is usually handled indirectly by attributing response to the complex of fixed factors, thus highlighting the importance of defining and measuring fixed factors.

The apportioning of a fertilizer investment among a number of successive crops according to the relative economic effects in each crop is intuitively appealing, consistent with accounting doctrine and indeed may be useful as an approximating tool. However, even for quite simple situations such apportioning is unsatisfactory as a formal optimizing procedure.

An alternative approach which may have practical value is to make the simplifying assumption that residual response can be described by constant carryover parameters. If X_i^* is applied to crop (J), the effect in crop $(J + 1)$ is as if an amount $C_iX_i^*$ is applied to crop $(J + 1)$, where C_i $(0 \leqq C_i < 1)$ is a carry-over constant. Then in the production function for crop $(J + 1)$ the variable X_i is replaced by $X_i + C_iX_i^*$. Optimum levels for the X_i in each production period can be determined through a profit function which discounts future nutrient injections as well as future receipts. The analysis obviously becomes unwieldy when a parameter C_i is some function of X_i.

8. Initial Fertility and Soil Testing

Invariably soils of agricultural interest contain a store of plant nutrients. The fertilizer allocation problem discussed above is thus: "—given the initial soil nutrients, what are the optimum fertilizer inputs?"

Consider a single variable factor response function

$$Y = f(X);$$

where the factor X consists of available soil nutrient (X') and applied nutrients $(X - X')$ and let the economic optimum nutrient level be X''. Then the higher the initial fertility X', the less will be the optimum application $X'' - X'$. If X' can be measured, the predictive power of the production function is increased and, if the measurement is available before the fertilizer decision is taken, the recommendation can be more precise.

A not insignificant problem is the measurement of initial fertility—the province of the science of soil (and tissue) testing. Nutrients are held in soils by complex chemical and physical systems which result in varying profile distribution, rate of release and availability to plants. These factors also vary with climatic conditions, the crop and its growth. By its nature soil testing is an imprecise tool—but the importance of further research on soil testing in conjunction with fertilizer response studies cannot be over-emphasised.[43] Herein lies the main hope for

[43] Even today, most soil testing services make fertilizer recommendations on the basis of "standards" of measured soil nutrients—in explicit ignorance of the economics of fertilizer response.

extending the usefulness of both soil testing and response studies. Australia is well advanced in this regard, particularly for phosphate[44] in wheat production, following the work of Colwell.[45]

Various techniques have been employed to incorporate initial fertility measurements in response functions. Most have involved expressing the response function in terms of total (initial plus applied) nutrients and defining a (usually linear) relation between initial nutrient level and soil test. For example, such a relationship is

$$X_i = x_i + a_i f_i$$

where, for the i-th nutrient, X is the total nutrient available, x is the applied nutrient, f is the soil test and a is a constant.

The a_i may be directly determined in independent experiments or evaluated experimentally along with the other constants in the regression equation.[46] Alternatively, the soil tests may be included as separate independent variables in the response function for which coefficients can be estimated.[47]

Research resources may be limited so that soil tests cannot be considered in this way. However, all relevant soil tests should be made in conjunction with any fertilizer trial—particularly if it is designed for economic interpretation. Knowledge of fertilizer response conditional on measured soil nutrients can thus be built up, with consequent impact on precision of recommendation.

9. Response Variability

Variability in response to fertilizers may arise from two sources. Firstly, controllable factors such as operational timing and spatial distribution[48] of fertilizer and plant propagation materials may contribute significantly to variability and if so should be incorporated in multi-factor experiments.

Secondly, treatment of variability arising from uncontrollable factors depends on their assessability prior to the time of making fertilizer decisions. In this respect, initial fertility factors are discussed above. Variability arising from within-paddock soil variations can be partly

[44] Phosphatic fertilizers have been the most important in Australia. Also soil scientists have been able to measure "available" soil phosphorus more successfully than other soil nutrients.

[45] J. D. Colwell, "The Estimation of the Phosphorus Fertilizer Requirements of Wheat in Southern New South Wales by Soil Analysis", *Australian Journal of Experimental Agriculture and Animal Husbandry*, Vol. 3, No. 10 (August, 1963), pp. 190-197, and J. D. Colwell and R. J. Esdaile, "The Calibration, Interpretation and Evaluation of a Soil Test for Phosphorus Fertilizer Requirements", in press.

[46] See R. Voss and J. Pesek, "Estimation of Effect Coefficients Relating Soil Test Values and Units of Added Fertilizer", *Agronomy Journal*, Vol. 54 (1962), pp. 339-341.

[47] As in R. Voss and J. Pesek, "Generalization of Yield Equations in Two Variables: III. Application of Yield Data from 30 Initial Fertility Levels", *Agronomy Journal*, Vol. 54 (1962), pp. 267-271 and Colwell, *op. cit.*

[48] D. Jensen and J. T. Pesek, "Inefficiency of Fertilizer Use Resulting from Nonuniform Spatial Distribution: I. Theory. II. Yield Losses Under Selected Distribution Patterns. III. Fractional Segregation in Fertilizer Materials", *Soil Science Society of America Proceedings*, Vol. 26 (1962), pp. 170-182.

avoided by using large experimental plots as in controlled survey[49] trials. Many other edaphically important soil characteristics can (potentially at least) be assessed.[50] Production functions should be estimated for each combination of uncontrollable factors (*i.e.* each "domain").[51]

However, some uncontrollable factors are not suited to defining such domains and are not measurable prior to fertilizer decisions. Climatic factors[52], particularly rainfall, are most important and present challenging research problems. Experimental replication over a number of seasons is an obvious albeit slow and imperfect approach. Alternatively, climatic simulation in irrigated field plots or phytotron growth chambers may find limited use in fertilizer experiments.

By employing these various experimental methods, a log of conditional response functions could be developed for a wide cross-section of climatic conditions. Choice of input rates from these functions depends on the decision maker's attitude to risk—the shape of his utility function.

The simplest example would be for a farmer indifferent to risk; that is with a utility function linear over the range of payoffs from fertilizer investments. Expected profit is then the relevant decision criterion and optimum rates are computed from a composite response function of conditional functions weighted by the subjective probability distribution of the climatic variables.[53] This corresponds to determining optima by weighting the conditional optima by the probability distribution.

When risk preferences are important (implying a non-linear utility function) analytic treatment is more complex. Whether response functions are considered independently or embedded in a stochastic programming formulation, the analyst must know to what extent the decision maker is prepared to sacrifice expected profit in order to reduce risk (say profit variance). This can be determined through using utility theory[54] although some aspects of empirical estimation of utility functions remain unresolved.[55] An alternative but less satisfactory procedure recently demonstrated by Fuller[56] is to choose input levels which minimize profit variance.

---

[49] B. R. Hoffnar and G. L. Johnson, *op. cit.*, Chapter 5.

[50] B. E. Butler, "Assessing the Soil Factor in Agricultural Production", *Journal of the Australian Institute of Agricultural Science*, Vol. 30, No. 4 (December, 1964), pp. 232-240.

[51] H. O. Hartley, "Experimental Designs for Estimating the Characteristics of Response Functions", O.E.C.D., No. 65, *op. cit.*, pp. 163-176.

[52] Incorporation of several factors (including rainfall) has recently been demonstrated by J. S. Russell, "Nitrogen Fertilizer and Wheat in a Semi-arid Environment 3. Prediction of Profitable Rate of Application", *Australian Journal of Experimental Agriculture and Animal Husbandry*, in press.

[53] For an analogous use of a "drought-index" probability distribution in a nitrogen-maize process see J. L. Knetsch, "Moisture Uncertainties and Fertility Response Studies", *Journal of Farm Economics*, Vol. 41, No. 1 (February, 1959), pp. 70-76.

[54] H. M. Markowitz, *Portfolio Selection:* Cowles Foundation Monograph No. 16 (New York: John Wiley, 1959).

[55] As demonstrated, for example, by D. Davidson, P. Suppes and S. Siegel, *Decision Making: An Experimental Approach* (Stanford University Press, 1957) and C. J. Grayson, *op. cit.*

[56] W. A. Fuller, "Stochastic Fertilizer Production Functions for Continuous Corn", *Journal of Farm Economics*, Vol. 47, No. 1 (February, 1965), pp. 105-119.

Analysis of response variability leads to lower optimum fertilizer inputs for farmers who elect to reduce risks. But measurement of this influence poses several problems for which no simple panacea exists.

10. Conclusion

Interest in fertilizer experiments designed to provide response data amenable to economic analysis has been a feature of the increasing inter-disciplinary co-operation in agricultural research. Agronomists will be more and more involved with economic interpretation of the fertilizer trials they conduct.

It has been attempted to clarify some of the frequently overlooked details of such economic analyses. It was shown that in equating marginal products to their inverse price ratios, these ratios should (where relevant) be adjusted for product marketing costs, application costs, stochastic product price, opportunity costs of alternative use of funds and time preference.

It is not without significance that to account for most of the topics discussed would result in selection of optimum rates lower than those specified in most reported analyses. The effect may be slight, as in considering time preference, or may be considerable if, say, opportunity costs are high. This in part explains the common observation that farmers use fertilizers at lower than "profit maximizing" rates. The extent to which rates are lowered in discounting for variability in response is not easily measurable.

Specification of and experiments involving the many associated factors in fertilizer response must lead to increased precision of economic interpretation of fertilizer response data.