Investigating affordability problems of utility services –
A theoretical study on the ratio measure

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Abstract

Unlike in developing countries, there tends to be no problem of access to water, electricity, and heating for private households in transition countries. However, transition countries have a considerable amount of low-income households, and the problem of affordability of these environmental-related utility services remains urgent. Welfare economics literature suggests to neglect affordability aspects by separating allocative from distributive impacts of pricing. In practice, this separation runs the risk of rendering impossible any sustainability-oriented price reform. An Institutional Economics approach takes competing objectives into account. From this viewpoint it appears to be worth investigating the affordability-concept.

Although the affordability-related research has escalated remarkably in recent years, the theoretical contributions are still limited. Hence, we focus on the simple ratio measure often used in practice. We analyze the arguments speaking for the ‘potential affordability approach’. But we find that - within that approach – adhering formally to the ratio measure is possible only under conditions that make no sense regarding the concern of the measure. Thus for most cases the ratio measure is misleading. Some considerations on practical use for governance conclude the paper.

Keywords: Affordability, Transition Countries, Utility Services.

JEL Classification: D11, H41, L97, O15, Q56
1 TRANSITION ECONOMIES, INSTITUTIONAL DESIGN, AND AFFORDABILITY

Considerations which connect disadvantaged households and environmental-related utility services, such as water, electricity, and heating, often distinguish the problem of access from the problem of affordability. The problem of access refers to the availability of the necessary infrastructure, whereas the affordability refers to household’s financial burden for the consumption. Access is a much more basic problem and affordability is a problem arising effectively, when the previous one is solved.

While in developing countries one struggles with the very basic problem of infrastructure, i.e. access, this aspect is ‘less relevant for transition countries, where connection rates are generally high.’ (FANKHAUSER/TEPIC 2005: 1038). However, the problem of affordability remains a major challenge for questions of governance and institutional design in transition economies like Eastern Europe and the former Soviet Union. This is due to remarkable escalation of poverty during the last two decades. FANKHAUSER and TEPIE found that on average (sic) a household has no affordability problem in these countries (see ibid.: 1046). But they stress a considerable number of low-income households for whom affordability of utility goods is an existential issue. MACOURS and SWINNEN (2008) investigated the fact, that and how poverty is outspread especially in rural areas of transition countries. In the future the affordability problems might be aggravated by the fact that price increases for environmental-related utility services are very likely.

The recent efforts in affordability research so far was mainly of empirical nature. However, economic theory appears to be stupendously ill-prepared for such an urgent problem.¹ Neoclassical welfare economics literature suggests to neglect affordability aspects by separating allocative from distributive impacts of pricing (second fundamental theorem of welfare economics). Yet in political practice, this approach runs the risk of rendering impossible any sustainability-oriented price reform. Therefore, we apply an Institutional Economics approach instead: In the following we want to investigate theoretically the simplest of the affordability measures, the so-called affordability ratio, with the aid of the microeconomic household model. Having the concern of the affordability-concept at the back of our mind, we intend to see, to what extent the ratio-measure may reasonably serve as underlying measure for institutional design of utility service provision.

This paper is organized as follows: In section 2 we show how the target ratio can be defined, with the aid of a standard minimum level of consumption. Section 3 discusses conceivable factual situations of household consumption. In section 4 we present a transformation of the factual into a fictitious consumption situation according to the mentioned standard level (‘potential affordability approach’). Section 5 describes reasonable variation of this standard level. Therein finally our main argument against the ratio measure is presented. Section 6 concludes and points to the possible relevance for institutional design in fighting unaffordability of utility services.

¹ Theoretical investigation can be found though so far in HANCOCK (1993), CHAPLIN/FREEMAN (1999), GLIED (2008), MINIACI ET AL. (2008), and KESSIDES ET AL. (2009).
2 DEFINING THE TARGET RATIO
The idea of affordability refers to the burden a household has to bear in order to attain a certain good. The very common formalization of affordability is the burden ratio. It is the ratio of expenditures \(pq\) for the considered utility service (index good \(u\)) to the total income \(x\) of the household \(k\).\(^2\) Let the ratio be \(r\).

\[ r^u_k = p^u q^u_k / x_k \]

Following the intention and concern of the idea, its flaws mentioned in the discussion are to be picked up:

1. There is no correlation to a certain minimum consumption level of the index good. Poor households consuming a very/too small quantity of the good are considered having no affordability problem.
2. Similarly, there is no correlation to a maximum consumption level of the index good. Wealthy households wasting the good are possibly considered as having an affordability problem.
3. Households are characterized by the following circumstances: different amounts of members (household size), different climatic/regional conditions, and different technological endowments. These conditions lead to different individual necessity (see CHAPLIN/FREEMAN 1999: 1950). We are talking about some constraints that are not illustrated by the budget line.

Subsequently, we will try to seize these suggestions 1-3, whereas the following is worked on in BRETSCHNEIDER 2010:

4. Important to (especially to disadvantaged) households is the quantity of the good.\(^3\) Since the numerator are the expenses (quantity multiplied by prices), different prices will distort the measure (see CHAPLIN/FREEMAN 1999: 1951).

We intend to contour the affordability concept – following its inherent concern – as a concept focusing on the household’s ability to buy a certain good. KESSIDES ET AL. note accordingly: ‘[...] [I]t is often pointed out, in essence correctly, the issue is the ability to pay and not the willingness to pay [...]’ (KESSIDES ET AL. 2009: 3) This should be taken seriously particularly in front of the background of microeconomic theory. This theory tells us that a household’s decision for purchasing accords a syllogism. This means the action of purchasing is the conclusion of two premises: ability (restriction, Können) and willingness (preference, Wollen).\(^4\) This is the difference the choice consists of. In the well-known diagram for defining the household’s optimum the first is traditionally represented by the budget line and the latter by the (family of) indifference curve(s). Thus we suggest to take care of the interplay of these two categories also particularly regarding the affordability issue.

It is the concern of the approach to identify somewhat ‘poor’ or ‘disadvantaged’ households; individuals for whom the consumption of certain good is unaffordable. To

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\(^2\) For purposes of empirical research, it could be discussed whether it is more convenient to use total expenditures instead of total income. For our theoretical considerations we regard these two categories as identic. Thus we use the symbol \(x\) for either category.

\(^3\) Like widely in microeconomics we ignore quality aspects (that do of course play a role in reality).

\(^4\) We are aware that widely in literature the term ‘willingness’ contains also the facet of ability. The ‘willingness to pay’ consists of willingness (preferences) and ability. However, for our purposes we use ‘willingness’ decided only meaning ‘preferences’.
define a differentiation, there is normatively introduced a critical level, the target burden \( r^{u*} \). Each household, whose ratio \( r \) is higher than the critical level \( r^{u*} \) is classified as suffering an affordability problem regarding \( u \) and is seen as disadvantaged or – in this respect – ‘poor’. We can see the relation to the general poverty-problem, if we regard the way the \( r \) decreases or increases. Pursuing the concern of the approach the relevant independent variable that determines the value \( r \) is the income \( x_k \). Figure 1 illustrates this correlation, based on the well-known two-goods-model, wherein \( q^u \) stands for the quantity of the index good \( u \). Reducing complexity drastically the abscissa \( q^c \) refers to the quantity of all other goods a household might be interested in as a ‘second good’.

![Fig. 1: Various income levels and accorded affordability-ratios based on a certain level of the index good.](image)

In figure 1 we can see several income levels represented by various budget lines; those parallel straight lines are showing a negative correlation of the two goods. And accordingly there are different affordability-ratios, represented by rays starting from the point of origin. The greater the ray’s slope, the greater is \( r \); i.e. the higher is the burden the household has to bear. In the figure each budget line is attributed to a certain burden ray, as they intersect in the points A, B, C, and S. It becomes obvious: The higher the income, the more the burden slope declines; which means, the more the burden-ratio declines. Depending on the income the household might end up either in point A, B, C, or S. However, if one pursues this connectivity, it is – consciously or not – based on the assumption that all other relevant parameters remain constant. Important here is, that the consumed quantity of the index good \( u \) remains at the constant level \( q^{u*} \). And this is probably what someone who applies the simple ratio measure has in mind implicitly.

Let us call the scenario shown above (especially in figure 1) an ability-related shifting of the burden ratio. Now, the microeconomic model offers a second way to let the burden ray shift. The burden ratio might change also on a given budget line. That would be a willingness-related shifting. Thus, if the affordability-ratio should be given

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the original intention it is necessary to define explicitly a certain quantity ‘necessary to reach a decent standard of living’ (KESSIDES ET AL. 2009: 11). In figure 1 this is the level $q^{u*}$. In this respect we follow MINIACI and KESSIDES with their ‘standard minimum level of consumption’ (MINIACI ET AL. 2008: 203 and KESSIDES ET AL. 2009: 14) within the ‘potential affordability approach’.

Now, there are aftermaths of this explicitness. We cannot say anymore, that this is the level households actually use; that this is a factual standard. In the syllogism mentioned above the actual use is the conclusion. Now we rather have to acknowledge that the level $q^{u*}$ is a normative standard. Nota bene: The methodic decision to engage in this standard is not driven by theoretical imperatives once one works within the microeconomic model. But it is driven by the original concern of affordability seen through the microeconomic approach ‘glasses’. In the syllogism mentioned above the normative standard belongs formally to the side of willingness (Wollen). But whereas economists are used to the normative category of the individual preference, we are entering the difficult field of and discussion on merit wants. And this is the methodical difficulty one catches with that decision: One has to define somehow a level of consumption, which a fictional benevolent dictator wants the citizens to reach. But instead of going into the discussion about criteria which might help finding this level, it is important for our purposes to observe that this normative level is (almost always) different to the factual individually purchased quantities. So this is the base on which we can shift the budget line/ray-intersection on in the diagram.

To go on we want to pick up the aspect of the target ratio $r^{u*}$. We mentioned that this is to be given normatively. Having a norm of minimum consumption only for the index good $u$, we still have no target ratio; the ray is still shifting. What we need is a second normative consumption level as reference level, namely for all other goods $c$. The point where the two minimum consumption levels intersect is then called the subsistence bundle; point $S$ in figure 1. And the ray of reference $r^{u*}$ is fixed then in the subsistence bundle. The rays of all households defined by budget and standard consumption level are compared to this ray of reference. Those whose ray’s slope is higher than $r^{u*}$ are considered as facing affordability problems. Inversely for households whose ray’s slope is less than $r^{u*}$ the index good $u$ is considered affordable.

The formal expression for the target affordability-ray may be formally defined as follows:

$$q^u = q^c q^{u*}/q^{c*}.$$  

And defining

$$r^{u*} = q^{u*}/q^{c*}$$

the equation can be expressed as

$$q^u = q^c r^{u*}.$$  

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6 Simplifying we could say – like often – that this definition should be given exogenously economists by the democratic public/society.

7 In MINIACI’s and KESSIDES’ diagram the ray of the target ratio does not cross the subsistence bundle (see MINIACI et al. 2008: 202 and KESSIDES et al. 2009, 12). We think that the target ray is to cross point D, in order to get a consistent notion of affordability ratio.
Consequently, if we uncover the normative expression \( r_u^* \) we can discover the normative levels \( q_{u^*} \) and \( q_{c^*} \). We want to stress this point: defining the target affordability requires two normative definitions of quantity levels in advance. In this (normative) way the idea of opportunity costs is inherent to the target affordability-ratio. Thus we think it is an inappropriate statement by HANCOCK who says that it ‘[…] makes little sense to define affordability in terms of the ratio […] if it is believed that opportunity cost is important.’ (HANCOCK 1993: 133) The phenomenon of opportunity costs is rather perfectly implemented in the burden ratio. One could positively talk about a ‘normative opportunity cost clip’. Furthermore with this background the following definition can be understood deeper. ‘Affordability’ is concerned with securing some given standard [consumption of the index good \( u \)] at a price […] which does not impose, in the eyes of some third party (usually government) an unreasonable burden on households incomes.’ (MACLENNAN/WILLIAMS 1990: 9, as cited in HANCOCK 1993: 129) This ‘unreasonable burden’, which is often mentioned regarding the affordability problem, refers to falling below the second normatively standardized level, the quantity \( q_{c^*} \) – considering \( q_{u^*} \) being fixed.

In the following section we will take a step back investigating factual situations households may end up in compared to these normative standards. The considered differences are underconsumption vs. non-underconsumption and ability-related underconsumption vs. willingness-related underconsumption. Beside the subsistence bundle (see point S in figure 1) the budget line which intersects the subsistence bundle plays a major role. We want to call it target budget (see straight line \( b^* \) in figure 1).

### 3 THE ATTRIBUTES OF FACTUAL CONSUMPTION BUNDLES

This income represented by the target budget line \( b^* \) is necessary to reach the subsistence bundle. We will now examine its implications in combination with the quantitative standard levels of consumption for both goods \( u \) and \( c \) (see figure 2). Remember that these are the levels some third party wishes all households to consume at least. However, the factual consumption bundle of the individual household may be realized in each of the raising areas, caused – following the syllogism – by the household’s budget line (restriction, ability) as well as his individual utility function (preference, willingness). We want to describe what it means a household ending up in a certain area by considering four cases (see figure 2): (a) non-underconsumption (grided area), (b) willingness deficiency-related underconsumption (striped area), (c) mixed-conditioned underconsumption (light grey area), and (d) ability deficiency-related underconsumption (dark grey area).

(a) Non-underconsumption (grided area): This is the area a benevolent dictator wants all individuals to end up in. Households therein are not facing any underconsumption problem, there is no problem for social policy. This is caused by two conditions: First, the household has a budget in his disposal which is greater than the target budget \( b^* \). Second, on his budget line the household chooses a consumption bundle which avoids an underconsumption for both \( u \) and \( c \). Considering critic no. 2 to the ratio measure on page 4, herein we have the cases of wasting-related unaffordability. In the diagram these are grided area-households ending up above the (in figure 2: imaginary) target ray. In all the other areas, the households underconsume either the good \( u \) or \( c \) or both. The different causes for the underconsumption are examined in the following paragraphs.
(b) **Willingness deficiency-related underconsumption** (striped area): Households in the striped areas possess an income which actually enables them to achieve an appropriate consumption level for both goods, like the households in the grided top right area. However, they just do not choose accordingly. These are the households HANCOCK takes into consideration in her article ‘Can pay? Won’t pay?’ (HANCOCK 1993) where she indicates a ‘perversity of preferences’ (ibid: 131).

What we are facing here is not a deficiency of ability (Könnensdefizit), rather a pure deficiency of willingness (Wollensdefizit). So we are dealing with the pure problem of meritoric wants and not with questions of distribution in a narrow sense. Considered graphically the paternalistic third party would like the household to move on his budget line until the grided area starts. This is to reduce the consumption of the ‘overconsumed’ good in favour of the underconsumed one. The liberal tradition starting with JOHN STEWART MILL has been warning against interfering with the individual’s autonomy. At least the barriers of legitimation for an intervention are highly demanding.

(c) **Mixed-conditioned underconsumption** (light grey area): In the monochrome filled areas the lack in ability plays a major role finally. Here the households have an income available smaller than the target budget $b^*$. So the households in this light grey area cannot reach the subsistence bundle, but they are already somehow making a wrong decision anyway. They are underconsuming one good but, at the same time already consuming more than is necessary from the other. Here we are facing a deficiency of both ability and willingness. Accordingly, a meritoric and a distributional problem arise at the same time. Considered graphically the paternalistic third party would like

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Fig. 2: Areas between the target budget line and the levels of minimum consumption.

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8 As an additional reason for ending up in the brindled areas HANCOCK introduces the category of ‘non-income constraints’ (see HANCOCK 1993: 130). We will pick up this problem below in section 5.

9 At the outset we stressed the difference between the two premises ability and willingness. All the more it is interesting now, when problems in this category coincide. In such a way RICHARD MUSGRAVE observes factual casual coincidence of meritoric and distributive problems. JOHN HEAD rather sees a more essential connectivity (see HEAD 1988: 6).
the household to shift on his budget line at least until the dark grey area starts. That is to say, to reduce the consumption of the ‘overconsumed’ good in favour of the underconsumed one; to underconsume the latter less intense. Systematically after this motion – that is the difference to the same scenario in the striped area – the problem of ability could be tackled.

(d) **Ability deficiency-related underconsumption** (dark grey area): These households consume insufficient quantities of both goods due to a pure distributional problem, with an actual deficiency of ability (Könnensdefizit). This area includes those who ‘do not even have the opportunity to make [an] inappropriate decision.’ (GLIED 2008: 15). The part below the imaginary target ray represents those households the objective no. 1 aims at, above on page 4.

In the following section we will see what happens if we ‘guide’ such a factual consumption bundle to a fictitious situation, according a certain standardized minimum level, as regarded in section 2.

4 **DIAGNOSIS OF A HOUSEHOLDS AFFORDABILITY SITUATION**

In this section we finally aim at getting the affordability ratio that expresses exclusively the deficiency of ability (Könnensdefizit) of a household. Doing this we pursue the ‘potential affordability approach’ and we will see how it actually works in the model. After our considerations in previous sections we now are able to combine the standard level-oriented definition of the r-ray (section 2) and the classifications of the factual household’s situation (section 3).

Above, in section 1 we mentioned that it is necessary to shift from the factual consumption bundle to a certain fictive consumption bundle. Namely to end up on the parallel line of the abscissa on the level \( q^{u*} \), in order to be able to scale the affordability-ratio right there. But this motion is not to be taken perpendicular, i.e. parallel to \( q^{u*} \)-straight line. It is rather to be taken along the budget line of the household (see figure 3). In other words: In order to end up in the fictional consumption bundle on the \( q^{u*} \)-straight line, we move from the factual bundle right on the household’s budget line until the point where this budget line intersects the \( q^{u*} \)-straight line.

If we look at the areas examined above, we see that – employing this method – none of the households with a factual consumption bundle in the grided top right area is now considered facing unaffordability. They all end up fictionally to the right of S. Thus the definition problem of unaffordability due to over-consumption is solved (objection no. 2 is solved). In the same manner all households with a pure deficiency of willingness are ending up to the right of S. In figure 3 there is a household with a factual consumption bundle in the formerly brindled top left area, represented by the point at the arrow’s origin. Shifting on his budget line until the \( q^{u*} \)-straight line he ends up fictionally with the affordability-ray \( r^{u} \), to the right of S; that is, with a affordability ratio smaller than the target affordability \( r^{u*} \). Hence, the problem of objection no. 4 is solved.

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10 In figure 1 these are households with a budget equal and smaller than the budget that intersects the ordinate at .
Maybe the most interesting cases are the households in the light grey area of figure 2. Therein we observed a deficiency of both ability and willingness. All these households are considered as facing affordability problems; they do all end up to the right of S. But since the deficiency of willingness is eliminated they end up not that far to the left. In figure 3 we can see this for the household, which ends up with the affordability-ratio $r^2$, having a higher slope than $ru^*$. With this motion we feign the unconditional priority for the standard consumption level $q^u*$. And by doing this we eliminate at the same time uno actu all deficiency of willingness. Thus we come to an affordability definition that refers to the deficiency of ability only.

Then, having all households on $q^u*$-line we can scale the affordability problem of the household, either by measuring the distance to point S (subsistence bundle) or by measuring the slope of the corresponding r-ray. All households that end up to the left of S and whose r-ray features a greater slope than $ru^*$, are facing an affordability problem. And accordingly: All households ending up to the right of S and whose r-ray features a smaller slope than $ru^*$ are not facing an affordability problem. Nota bene: Additionally we can scale the ‘depth’ of the affordability problem. The greater the distance to the point S on the left side, the bigger the affordability problem. Accordingly this consideration asks for an institutional design that helps all households on the left reaching point S.

5 VARIATION JUSTIFIED BY NON-INCOME RESTRICTIONS
The potential affordability approach helps to contour the affordability concept as a concept considering ability-deficiency. This approach is the answer to objections 1 and 2 mentioned on page 4. This is the point where we are so far. But somehow we ended up in a dead end street regarding the somehow shrewd feature of this proportional term to react flexible. And that is what a ratio and proportion intends to be: To represent a

\[11\] For eliminating the deficiency of willingness only it already would be sufficient on the motion to the $q^u*$-straight line, to stop at the $q^c*$-line, if this line is intersected first.
common measure for different fundamentals. The target point S makes sense as giving an orientation. But our former considerations suggest, that the ray in the dark grey area as well in the grided area (see figure 2) make are pretty much useless. Thus the question which arises is: Why and how would it make sense to adhere to a target ratio and target line \( r^{(u*)} \) instead of just a target point S?

At this point another objection to the affordability ratio might help (sic). An aspect we did not integrate so far is objection 3 on page 4. It refers to different circumstances different households are facing, like
- different amounts of household members (size),
- different climatic/regional conditions, and
- different technological endowments (see KESSIDES et al. 2009: 15).

It is advisable here to go back to the difference of ability and willingness, and to see what we are dealing with now. The side of ability is represented by the budget (line). However, the side of willingness was a bit more complicated. On the one hand we have the individual preferences (i.e. utility functions), which determine the realized point on the household’s budget line. But on the other hand there are (more) ‘reasonable’ and standardized norms for certain quantities, given by a third party. It is typical for these two kinds of preferences that they are in conflict with each other.

What we are facing now are individual household’s circumstances that drive a household to some preference. Consequently, on the market these motives emerge as individual preferences. But ‘lebensweltlich’ these ‘individual preferences’ are restrictions though. In order to name this phenomenon we want to pick up HANCOCK’s term ‘non-income constraints’ (see HANCOCK 1993: 130) could be utilized. This means they are to be taken into account on the side of ability (deficiency). Thus we have the income as a constraint inherent in the model, and now additionally some constraints that have to be additionally introduced into the model. The implications are illustrated in figure 4.

![Figure 4: Variation of the standard level of the index good u and possible consequences for the standard level of all other goods](image-url)
There may be a household with exact the target budget $b^*$. Normally the standard minimum level of consumption is given with $q_u^*$. But due to certain particular necessities, the standard level for these cases may be at $q^{u*}_2$. But on this line, in point $T$, the household falls below the level of reference $q^c*$. But what has to be done now is a bit unclear. The institutional design could orientate either on the level $q^c*$ or on the affordability target ray $r^{u*}$. And this could mean quite a difference between the distance $TS_2$ and the distance $TU$ (see figure 4). Actually it appears to be reasonable to stick to the level $q^c*$. Why should there be any proportional increase of $q^c*$? Thus a ratio approach is a somehow misleading measure. And this is the point, as we argue, where it becomes finally recommendable to leave any ratio approach, even a potential affordability approach.\(^{12}\) A ‘residual income approach’ (see MENIACI et al. 2008: 206 and KESSIDES et al. 2009: 15) appears to be more appropriate. It works with a difference instead of a ratio. But we cannot go into this further in this paper.

6 ORIENTATION FOR DESIGNING INSTITUTIONS

The affordability ratio has become a commonly used measure in social-oriented empirical studies. These efforts target at being the basic for according governance strategies and instruments. With our considerations we arrived at the conclusion so far, that there are good reasons to employ it at least carefully. There are basically ways to account for major objections within the logic of the ratio. But there remain doubts and our investigation suggests further research, in order to find more accurate ways to diagnose the underlying problem of disadvantage considering the consumption of a certain good. If the ratio measure as investigated was the basic for subsidies, an oversubsidization is conceivable at least. Thus more exactness regarding concepts of affordability is preferable.

At the outset we referred to the problems of low-income households in transition countries, prevalent particularly in rural areas. Now ending up advising against oversubsidization of disadvantaged households is of course no cynic rebound. The warning mentioned above is not meant against the social concern, but in contrary for the purpose of this concern. Against the background of scarcity, institutional designs in general and institutional designs particularly in regard to poverty have to be pinpoint. And scarcity here refers especially also to poverty somewhere else. It is no antagonism that social concerns should be pursued by taking aspects of efficiency into account.

On top of that a better accuracy is preferable also because of the aspect of ecological sustainability. In this respect future prices of environmental-related utility services will probably increase. And this will happen with good cause. Hence institutions have to consider different important, but competing objectives. It remains the difficult task for social sciences to supply prudent orientation.

\(^{12}\) Nota bene: We are arguing like this as far as there is no sound justification for a proportional increase of $q^c*$. 

12
References


