A REPLY

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In his comments on a section of the article "Some Notes on 'Dynamic' Linear Programming", E. J. Waring makes two major points: (*a*) any programming solution is only as valid as the assumptions on which it is based; and (*b*) it is difficult to make allowance for stocking rates in linear programming analyses. The remainder of his contribution, apart from some suggestions for extending the range of the technique, seems to be based on incomplete reading of the paper.

Even the most cursory scanning of the assumptions upon which the numerical example in the article is constructed indicates that the solution would be incapable of direct application in the real world. No reader, whether he be "practical man" or "more experienced analyst", could be so naive as to suppose that the complete complex of the farm's operations could be adequately described (for a one-year period) by what amounts to a 6 x 6 matrix! Rather, as stated in the article, the example is presented as an illustration of the use of a particular methodology. Nevertheless, in so far as the data employed relate to a real farm, the example demonstrates one very early link in the long chain which makes up a practical farm management analysis: the solution is suggestive of further refinements which would be necessary in the basic model, if real-world answers were actually being pursued. A number of such refinements, which might bear inclusion in subsequent programming runs, were mentioned in the article (page 140). But if Waring did in fact interpret the example as the "finished product"—a blueprint for immediate action—rather than as a methodological illustration, I apologize to him for not making my intentions even more clear.

Before dealing with the next major point, it should be mentioned that a sentence, noting that "sheep" throughout the calculations were measured in "dry sheep equivalent", was unintentionally omitted from the published version of the paper. I apologize if this omission led to any confusion. In addition, it is regretted that a printing error caused the second portion of Table 6 (shown on pages 134-5) to be placed before, instead of after, the first portion (shown on pages 136-7).

A number of Waring's remarks concern the treatment of stocking rates and it is agreed that in linear programming analyses these questions are of great importance. But his comments imply a rather too literal interpretation of the carrying capacity co-efficients in a formulation such as that in the article. For example, he asks ". . . what becomes of the seventy sheep to be grazed on newly sown wheat-with-improved pasture before the crop is harvested in year two". It should be understood that the "sheep row" coefficient for a given activity need not be an absolute measure (in terms of sheep using that activity for the whole period), but can be a *relative* assessment of the "sheep requirement" of that activity, *provided* that: (a) all relative "sheep row" coefficients are measured in such terms; (b) the correct contribution has been incorporated in the $z - c$ row; and (c) it is realized that any interpretation of the solution must

not strain the linearity assumptions nor violate technical feasibilities. This was the approach used in the article. Examination of the example in this light shows that in year one, for instance, we have in the optimum plan 518 "sheep" carried on a farm which has 100 acres of newly established pasture, is stripping 200 acres of wheat under which improved pasture is sown and has 600 acres of unimproved pasture. Likewise, similar pictures emerge for years two, three and four. These figures can hadly be construed as "technical absurdities". The alternative method for handling stocking rates which Waring suggests—the inclusion of a "carrying capacity" constraint and "sheep" activities—would certainly be a step towards more versatile description, although it must be remembered that such additions increase computational times.

Let us turn now to some more specific comments. Firstly, Waring's argument against the use of "gross" profit as a criterion is very weak. "Slack" resources can be more realistically handled in other ways (e.g., in our example by the inclusion of sheep and capital selling activities), and by using "gross" profit in the objective function one does not risk double-counting of the capital coefficients. Secondly, he proposes rearrangement of the optimum plan so that the farmer might finish the four years in a better position than that recommended by the original solution. But surely it would be more appropriate to extend the time horizon of the analysis and let linear programming calculate these rearrangements optimally. *Then* the "practicability" criteria could be applied. Thirdly it is difficult to imagine "more experienced analysts" wondering why research is undertaken into the application of a time-orientated decision-making technique in the context of pasture improvement[1].

The remainder of Waring's contribution consists of some patently misdirected criticisms, and some suggestions for extending the range of dynamic linear programming. In the former category, consider first his observation that the eight "Household, etc.," vectors could be replaced by a single column. This point was made in paragraph (*d*), page 138, together with the reason why this procedure was not used. Secondly, taken as it stands, his statement that "no provision is made for carry-forward of capital to later years" is refuted by paragraph (*b*), page 138, and the negative coefficients of the capital rows of the matrix in Table 5. If he means *unused* capital, this is covered by the stated assumption that capital has zero opportunity cost. To relax this assumption a column vector for capital substitution between years employing a suitable interest rate (cf. the "borrowing" vectors) could be introduced. Thirdly, the sheep he refers to as "producing a fat lamb and eleven or twelve pounds of wool" before reaching twelve months old, do not in fact begin producing until *after* they have reached this age. It is conceded, however, that the text may not have put this point clearly enough, although closer inspection of the figures reveals it. The natural increase is completing the first year of its life during the year before the unsold portion appears in the programme as productive sheep; they are accounted for in the stocking rates of the previous year

[1] cf. F. H. Gruen, "Pasture Improvement—the Farmer's Economic Choice," *Australian Journal of Agricultural Economics,* Vol. 3, No. 2, (December, 1959), p. 36, and K. O. Campbell and R. T. Shand, *An Economic Study of Pasture Improvement on Some Farms in New South Wales,* (Sydney: University of Sydney, Department of Agricultural Economics; Mimeographed Report No. 2) 1958, p. 68.

via the ewe-with-lamb to dry-sheep-equivalent conversion and do not appear as "adult sheep" until they are over one year old. Sheep purchased are assumed to be one year of age or greater (which is suggested by their price).

The extensions and modifications to the formulation which Waring suggests were either mentioned in the article (e.g., supplementary feeding), or have been discussed above. It is agreed, as stated in the paper itself, that such extensions would lead to greater realism.

Undoubtedly there are still many drawbacks associated with dynamic linear programming. Computational burden is one whose severity may be gradually lessened with improved algorithms, such as those cited in the article, and with advances in computer technology. Also it is agreed that "considerable finesse" will be required in the specification of coefficients. But clearly the purpose of my empirical exercise was not to show the practical use of dynamic linear programming in deriving directly applicable recommendations. Rather it was, at stated in the text, a numerical example, designed to illustrate the theoretical methodology and based on a highly simplified but explicitly stated set of assumptions. To the extent that Waring re-emphasizes the existing shortcomings of the technique, his comments are welcome. But it is hoped that his pessimism will not deter the interested reader from further exploration of this potentially fruitful field.