AJAE Appendix for “Pricing-to-Market: Price Discrimination or Product Differentiation?”

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October 20, 2006

Note: The material contained herein is supplementary to the article named in the title and published in the American Journal of Agricultural Economics (AJAE).

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Derivations of Equilibrium Prices and Quantities

Scenario 1

In country 1, the consumer indifferent between buying the low-quality product or buying nothing is defined by the value of $\theta$ solving $y + \theta q_l - p_l = y$, i.e., $\theta_u = \frac{p_l}{q_l}$. Similarly, the consumer indifferent between the low- and high-quality products is defined by the value of $\theta$ solving equation $y + \theta q_h - p_h = y + \theta q_l - p_l$, i.e. $\theta_{1h} = \frac{q_h - p_h}{q_h - q_l}$.

Thus the low-quality product is purchased by consumers with $\theta \in [\theta_u, \theta_{1h}]$ and the demand for the low-quality product is

$$d_u = \frac{\theta_{1h} - \theta_u}{\theta_1} = \frac{q_l p_h - p_l q_h}{(q_h - q_l) q_l \theta_1}.$$  

The high-quality product is purchased by consumers with $\theta \in (\theta_{1h}, \theta_1]$ and the demand for the high-quality product is

$$d_{1h} = \frac{\theta_1 - \theta_{1h}}{\theta_1} = \frac{1 - \frac{p_h - p_l}{(q_h - q_l) \theta_1}}.$$  

The demands for the low- and high-quality products in country 2 can be obtained in a similar manner. Note however that the demands of consumers in country 2 depend on the price of the product expressed in local currency, i.e., $p_l \cdot e$ and $p_h \cdot e$, where $e$ is the exchange rate expressed in units of country 2’s currency per unit of country 1’s currency.

The demands in country 2 can be represented as

$$d_{2l} = \frac{\theta_{2h} - \theta_{2l}}{\theta_2} = \frac{e q_l p_h - p_l q_h}{(q_h - q_l) q_l \theta_2}, \text{ and}$$

$$d_{2h} = \frac{\theta_2 - \theta_{2h}}{\theta_2} = \frac{1 - \frac{p_h - p_l}{(q_h - q_l) \theta_2}}.$$  

The firm’s profit is

$$\pi = (p_l - \frac{1}{2} q_l^2) \frac{q_l p_h - p_l q_h}{(q_h - q_l) q_l} \left( \frac{1}{\theta_1} + \frac{e}{\theta_2} \right) + (p_h - \frac{1}{2} q_h^2) \left[ 2 - \frac{p_h - p_l}{(q_h - q_l)} \left( \frac{1}{\theta_1} + \frac{e}{\theta_2} \right) \right].$$
with first-order conditions:

\[
\frac{\partial \pi}{\partial p_l} = \frac{1}{2} \left( \theta_2 + e \theta_1 \right) \frac{4(p_l q_l - p_h q_h) + q_h q_h (q_l - q_h)}{(q_h - q_l)q_l \theta_1 \theta_2} = 0, \quad \text{and}
\]

\[
\frac{\partial \pi}{\partial p_h} = \frac{1}{2} \left( \theta_2 + e \theta_1 \right) \frac{(4p_h - 4p_l - q_l^2 - q_h^2) - 4\theta_1 \theta_2 (q_h - q_l)}{(-q_h + q_l)q_l \theta_1 \theta_2} = 0.
\]

Solving these two equations simultaneously for \(p_l, p_h\), we obtain the equilibrium prices

\[
p_h^* = \frac{1}{4} \frac{4\theta_1 \theta_2 + q_h (\theta_2 + e \theta_1)}{\theta_2 + e \theta_1}, \quad \text{and}
\]

\[
p_l^* = \frac{1}{4} \frac{q_l [4\theta_1 \theta_2 + q_l (\theta_2 + e \theta_1)]]}{\theta_2 + e \theta_1}.
\]

The equilibrium quantities are

\[
d_{1h}^* = \frac{q_h}{\theta_2 + e \theta_1}, \quad d_{2h}^* = \frac{q_l e}{4\theta_2}, \quad \text{and}
\]

\[
d_{1h}^* = \frac{4e \theta_1^2 - (q_l + q_h)(\theta_2 + e \theta_1)}{4\theta_1 (\theta_2 + e \theta_1)}, \quad \text{and}
\]

\[
d_{2h}^* = \frac{4\theta_2^2 - e(q_l + q_h)(\theta_2 + e \theta_1)}{4\theta_2 (\theta_2 + e \theta_1)}.
\]

For \(d_{1h}^* > 0\) and \(d_{2h}^* > 0\), \(q_h + q_l < \min \left[ \frac{4e \theta_1^2}{\theta_2 + e \theta_1}, \frac{4\theta_2^2}{\theta_2 + e \theta_1} \right] \) must hold. We assume that this is the case throughout the article.

**Scenario 2**

The monopolist treats each market independently due to market segmentation and constant marginal cost. The firm’s problem in country 1 is

\[
\max_{p_{1l}, p_{1h}} (p_{1l} - \frac{1}{2}q_{l}^2) d_{1l} + (p_{1h} - \frac{1}{2}q_{h}^2) d_{1h}.
\]

Similarly, the firm’s problem in country 2 is

\[
\max_{p_{2l}, p_{2h}} (p_{2l} - \frac{1}{2}q_{l}^2) d_{2l} + (p_{2h} - \frac{1}{2}q_{h}^2) d_{2h}.
\]
We solve the firm’s problem in the market 1 first. The marginal consumers are, 
\( \theta_{1l} = \frac{p_{1l}}{q_l} \), \( \theta_{1h} = \frac{p_{1h} - p_{1l}}{q_h - q_l} \). Thus the demands can be represented by

\[
d_{1l} = \frac{\theta_{1h} - \theta_{1l}}{\theta_1} = \frac{q_h p_{1h} - p_{1l} q_h}{(q_h - q_l) q_h \theta_1}, \quad \text{and}
\]

\[
d_{1h} = \frac{\theta_1 - \theta_{1h}}{\theta_1} = 1 - \frac{p_{1h} - p_{1l}}{(q_h - q_l) \theta_1}.
\]

Firm’s profit is,

\[
\pi_1 = (p_u - \frac{1}{2} q_h^2) \frac{q_h p_{1h} - p_{1l} q_h}{(q_h - q_l) q_l \theta_1} + (p_{1h} - \frac{1}{2} q_h^2) (1 - \frac{p_{1h} - p_{1l}}{(q_h - q_l) \theta_1}).
\]

The first order conditions are

\[
\frac{\partial \pi_1}{\partial p_u} = 4 \frac{(p_{1h} q_h - p_{1l} q_h) + q_h q_h (q_l - q_h)}{2 (q_h - q_l) q_h \theta_1}, \quad \text{and}
\]

\[
\frac{\partial \pi_1}{\partial p_{1h}} = 4 \frac{(p_{1h} - p_{1l}) - (q_h - q_l) (q_h + q_h + 2 \theta_1)}{2 (-q_h + q_l) \theta_1}.
\]

Solving these two equations simultaneously for \( p_u \) and \( p_{1h} \), we have, \( p_{1h}^* = \frac{1}{4} q_h (2 \theta_1 + q_h) \) and \( p_u^* = \frac{1}{4} q_l (2 \theta_1 + q_l) \). Thus the equilibrium quantities are \( d_{1l}^* = \frac{q_h}{4 \theta_1} \) and \( d_{1h}^* = \frac{2 q_h - q_h - q_l}{4 \theta_1} \).

Similarly, by solving the maximization problem of the monopolist in country 2, we can obtain the following equilibrium prices and quantities,

\[
p_{2l}^* = \frac{1}{4} q_l (2 \theta_2 + \epsilon q_l) / e,
\]

\[
p_{2h}^* = \frac{1}{4} q_h (2 \theta_2 + \epsilon q_h) / e,
\]

\[
d_{2l}^* = \frac{\epsilon q_h}{4 \theta_2}, \quad \text{and}
\]
(22) \[ d_{2h}^{*} = \frac{2\theta_2 - e(q_l + q_h)}{4\theta_2}. \]

For \( d_{1h}^{*} > 0 \) and \( d_{2h}^{*} > 0 \), \( q_h + q_l < \min [2\theta_1, 2\theta_2/e] \) must hold. Note that this condition is less restrictive than \( q_h + q_l < \min \left[ \frac{4e\theta_1}{\theta_2 + e\theta_1}, \frac{4e\theta_2}{\theta_2 + e\theta_1} \right] \) established in scenario 1 for the quantities in market 2 to be positive. Thus, \( d_{1h}^{*} > 0 \) and \( d_{2h}^{*} > 0 \) in scenario 2 holds.

**Derivations of Equations Associated with Corollary 3 and 4**

**Corollary 3**

First, we determine the sign of \( X - 1 \). Using the equation for the domestic-export price ratio with unit values, i.e., \( X = \frac{q^*_l + q^*_h(1-\sigma_1)}{\sigma_2 q^*_l + q^*_h(1-\sigma_2)} \), the sign of \( X - 1 \) corresponds to the sign of \( (p^*_l - p^*_h)(\sigma_1 - \sigma_2) \). It can be easily shown that \( (p^*_l - p^*_h) < 0 \) because \( q_l < q_h \). Moreover,

\[
\sigma_1 - \sigma_2 = \frac{4q_h(\theta_2 + e\theta_1)^2(\theta_2 - e\theta_1)}{[4e\theta_1^2 - q_l(\theta_2 + e\theta_1)][4\theta_2^2 - eq_l(\theta_2 + e\theta_1)]}
\]

and the two elements of the denominator are positive given the assumption we made for all quantities to be positive in equilibrium (see scenario 1 above). Thus, the sign of \( \sigma_1 - \sigma_2 \) depends on the sign of \( \theta_2 - e\theta_1 \). When \( \theta_2 < e\theta_1 \), \( \sigma_1 - \sigma_2 < 0 \), and \( X - 1 > 0 \). When \( \theta_2 > e\theta_1 \), \( \sigma_1 - \sigma_2 > 0 \), and \( X - 1 < 0 \).

Second, we determine the sign of \( \frac{\partial X}{\partial q_h} \). Because \( X = \frac{P_1}{P_2}, \frac{\partial X}{\partial q_h} = \frac{\partial P_1}{\partial q_h} P_2 - P_1 \frac{\partial P_2}{\partial q_h} \). Note that \( P_1 = \sigma_1(p^*_l - p^*_h) + p^*_h \), \( P_2 = \sigma_2(p^*_l - p^*_h) + p^*_h \), and \( q_h \) does not enter \( p^*_l \). Thus,

\[
\frac{\partial X}{\partial q_h} = \left[ \frac{\sigma_1}{\partial q_h} (p^*_l - p^*_h) + \frac{\partial p^*_h}{\partial q_h} (1-\sigma_1) \right] P_2 - P_1 \left[ \frac{\sigma_2}{\partial q_h} (p^*_l - p^*_h) + \frac{\partial p^*_h}{\partial q_h} (1-\sigma_2) \right].
\]

Rearranging we obtain:

\[
\frac{\partial X}{\partial q_h} = \frac{(p^*_l - p^*_h) \left( \frac{\partial p^*_l}{\partial q_h} P_2 - \frac{\partial p^*_h}{\partial q_h} P_1 \right) - \frac{\partial p^*_h}{\partial q_h} (P_2\sigma_1 - P_1\sigma_2 - P_2 + P_1)}{P_2^2}.
\]
where $\sigma_1 = \frac{\theta_1 + \theta_2 + \theta_3}{\theta_1 - \theta_2 + \theta_3}$, and $\sigma_2 = \frac{\theta_1 e(\theta_2 + \theta_3)}{-\theta_1 + \theta_2 + \theta_3}$.

Given the expressions for $\sigma_1$ and $\sigma_2$, \(\frac{\partial \sigma_1}{\partial q_h} = \frac{\sigma_1}{q_h}, \frac{\partial \sigma_2}{\partial q_h} = \frac{\sigma_2}{q_h}.\) Substituting for $\frac{\partial \sigma_1}{\partial q_h}, \frac{\partial \sigma_2}{\partial q_h}, P_1,$ and $P_2$, equation (25) can be re-written as

\[
(26) \quad \frac{\partial X}{\partial q_h} = \frac{(\sigma_1 - \sigma_2) \left[ (p^*_h - p^*_h) \frac{p^*_h}{q_h} - \frac{\partial p^*_h}{\partial q_h} p^*_h \right]}{P_2^2}
\]

where $(p^*_h - p^*_h) \frac{p^*_h}{q_h} - \frac{\partial p^*_h}{\partial q_h} p^*_h < 0$ because $(p^*_h - p^*_h) < 0$ and $\frac{\partial p^*_h}{\partial q_h} = \frac{\theta_1 \theta_2 + 2 \theta_3 (\theta_2 + \theta_3)}{4(\theta_2 + \theta_3)} > 0$.

The above shows that the sign of $\frac{\partial X}{\partial q_h}$ also depends on the sign of $\sigma_1 - \sigma_2$, which we have already determined depends on the sign of $\theta_2 - \theta_1$.

Thus, when $\theta_2 < \theta_1, \sigma_1 - \sigma_2 < 0, X - 1 > 0$, and $\frac{\partial X}{\partial q_h} > 0$. When $\theta_2 > \theta_1, \sigma_1 - \sigma_2 > 0, X - 1 < 0$, and $\frac{\partial X}{\partial q_h} < 0$.

**Corollary 4**

Because

\[
X = \frac{p^*_t}{p^*_2} = \frac{(2\theta_1 + q_0)e}{2\theta_2 + e q_0},
\]

\[
(27) \quad X_t = \frac{p^*_t}{p^*_2} = \frac{(2\theta_1 + q_0)e}{2\theta_2 + e q_0},
\]

\[
(28) \quad X_h = \frac{p^*_h}{p^*_2} = \frac{(2\theta_1 + q_0)e}{2\theta_2 + e q_0}, \quad \text{and}
\]

\[
(29) \quad X = \frac{p^*_t \sigma_1 + p^*_h (1 - \sigma_1)}{p^*_2 \sigma_2 + p^*_2 (1 - \sigma_2)} = \frac{e (q_0^2 + 4\theta_1^2 - q_0 q_2 - q_2^2) (2\theta_2 - q_0)}{(e^2 q_0^2 + 4\theta_2^2 - e^2 q_0 q_2 - e^2 q_2^2)(2\theta_1 - q_0)},
\]

then, when $q_t = q_h = q, X_t = X_h = X = \frac{(2\theta_1 + q_0)e}{2\theta_2 + e q_0}$.

In what follows, the equations allowing us to sign $\frac{\partial X}{\partial q_h}$ are derived. Rewrite $X$ as

\[
X = \frac{p^*_t \sigma_1}{p^*_2 \sigma_2} A \quad \text{where} \quad A = 1 + \frac{p^*_t}{p^*_t} \frac{1 - \sigma_1}{\sigma_1} \quad \text{and} \quad B = 1 + \frac{p^*_h}{p^*_2} \frac{1 - \sigma_2}{\sigma_2}. \quad \text{Therefore},
\]

\[
(30) \quad \frac{\partial X}{\partial q_h} = \frac{p^*_t \sigma_1}{p^*_2 \sigma_2} \frac{1}{B^2} \left[ B \left( \frac{\partial (p^*_h / p^*_2)}{\partial q_h} \frac{(1 - \sigma_1)}{\sigma_1} + p^*_t \frac{\partial (1 - \sigma_1)}{\partial q_h} \right) - A \left( \frac{\partial (p^*_h / p^*_2)}{\partial q_h} \frac{(1 - \sigma_2)}{\sigma_2} + p^*_t \frac{\partial (1 - \sigma_2)}{\partial q_h} \right) \right]
\]

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where \( \frac{p_{1h}}{p_{1l}} = \frac{q_{h}(2q_{l} + q_{h})}{q_{l}(2q_{h} + q_{l})} \), \( \frac{1-\sigma_{1}}{\sigma_{1}} = \frac{2q_{l}-q_{h}}{q_{h}} \), \( p_{2h}^{*} = \frac{q_{h}(2q_{l} + q_{h})}{q_{l}(2q_{h} + q_{l})} \), and \( \frac{1-\sigma_{2}}{\sigma_{2}} = \frac{2q_{h}-q_{l}}{q_{l}} \). The derivative of these expressions with respect to \( q_{h} \) can be written as: \( \frac{\partial (\frac{p_{1h}}{p_{1l}})}{\partial q_{h}} = \frac{p_{1h}}{p_{1l}q_{h}} + \frac{q_{h}}{4p_{1l}} \), and \( \frac{\partial (\frac{1-\sigma_{1}}{\sigma_{1}})}{\partial q_{h}} = \frac{1}{\sigma_{1}q_{h}} \).

Substituting these last four expressions into (30), we obtain

\[
(31) \quad \frac{\partial X}{\partial q_{h}} = \frac{p_{1l}}{p_{2l}} \frac{\sigma_{1}}{\sigma_{2}} \frac{1}{B^{2}} \left\{ B \left[ -\frac{p_{1h}}{p_{1l}q_{h}} + \frac{q_{h}}{4p_{1l}} \left( \frac{1-\sigma_{1}}{\sigma_{1}} \right) \right] - A \left[ -\frac{p_{2h}}{p_{2l}q_{h}} + \frac{q_{h}}{4p_{2l}} \left( \frac{1-\sigma_{2}}{\sigma_{2}} \right) \right] \right\}
\]

which can be rewritten as

\[
(32) \quad \frac{\partial X}{\partial q_{h}} = \frac{p_{1l}}{p_{2l}} \frac{\sigma_{1}}{\sigma_{2}} \frac{1}{B^{2}} \left\{ \frac{1}{q_{h}} \left( \frac{p_{2h}}{p_{2l}} A - \frac{p_{1h}}{p_{1l}} B \right) + \frac{q_{h}}{4p_{1l}} \left[ \frac{B}{p_{1l}} \left( \frac{1-\sigma_{1}}{\sigma_{1}} \right) - A \left( \frac{1-\sigma_{2}}{\sigma_{2}} \right) \right] \right\}.
\]

After substituting for \( A \) and \( B \) within the curly brackets and rearranging we obtain:

\[
(33) \quad \frac{\partial X}{\partial q_{h}} = \frac{p_{1l}}{p_{2l}} \frac{\sigma_{1}}{\sigma_{2}} \frac{1}{B^{2}} \left( \frac{C}{q_{h}} + \frac{q_{h}}{4} D \right), \quad \text{where}
\]

\[
(34) \quad C = \left( \frac{p_{2h}}{p_{2l}} - \frac{p_{1h}}{p_{1l}} \right) + \frac{p_{1h}p_{2h}}{p_{1l}p_{2l}} \left( \frac{1-\sigma_{1}}{\sigma_{1}} - \frac{1-\sigma_{2}}{\sigma_{2}} \right) \quad \text{and}
\]

\[
(35) \quad D = \left( \frac{1-\sigma_{1}}{\sigma_{1}p_{1l}^{2}} - \frac{1-\sigma_{2}}{\sigma_{2}p_{2l}^{2}} \right) + \frac{p_{2h}^{*} - p_{1h}^{*}}{p_{1l}^{*}p_{2l}^{*}} \left( \frac{1-\sigma_{1}}{\sigma_{1}} \right) \left( \frac{1-\sigma_{2}}{\sigma_{2}} \right).
\]

Then, we substitute for the equilibrium prices and market shares and simplify to obtain:

\[
(36) \quad \frac{\partial X}{\partial q_{h}} = \frac{1}{p_{2l}^{*} \sigma_{2}} \frac{1}{B^{2}} \frac{1}{\sigma_{2}} \left( \frac{e\theta_{1} - \theta_{2}}{e\theta_{1} + \theta_{2}} \right) \left( \frac{2q_{h} + q_{l}}{eq_{l}(2\theta_{2} + e\theta_{l})} \right).
\]

The sign of \( \frac{\partial X}{\partial q_{h}} \) is the sign of \( e\theta_{1} - \theta_{2} \).